Introduction to Data Mining

Chapter 4
Association Analysis:
Basic Concepts and Algorithms

Source: revised from slides provided by Tan, Steinbach, Karpatne, and Kumar

Association Rule Mining

- Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction

Market-Basket transactions

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Beer, Eggs**
3	**Milk, Diaper, Beer, Coke**
4	**Bread, Milk, Diaper, Beer**
5	**Bread, Milk, Diaper, Coke**

Example of Association Rules

{Diaper} $\rightarrow$ {Beer},
{Milk, Bread} $\rightarrow$ {Eggs,Coke},
{Beer, Bread} $\rightarrow$ {Milk},

Implication means co-occurrence, not causality!

Definition: Frequent Itemset

- **Itemset**
  - A collection of one or more items
    - Example: {Milk, Bread, Diaper}
  - k-itemset
    - An itemset that contains k items
- **Support count (σ)**
  - Frequency of occurrence of an itemset
  - E.g. σ({Milk, Bread,Diaper}) = 2
- **Support**
  - Fraction of transactions that contain an itemset
  - E.g. s({Milk, Bread, Diaper}) = 2/5
- **Frequent Itemset**
  - An itemset whose support is greater than or equal to a *minsup* threshold

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Definition: Association Rule

- **Association Rule**
  - An implication expression of the form $X \rightarrow Y$, where X and Y are itemsets
  - Example:
    $\{\text{Milk, Diaper}\} \rightarrow \{\text{Beer}\}$

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

- **Rule Evaluation Metrics**
  - Support (s)
    - Fraction of transactions that contain both X and Y
  - Confidence (c)
    - Measures how often items in Y appear in transactions that contain X

Example:

$$\{\text{Milk}, \text{Diaper}\} \Rightarrow \{\text{Beer}\}$$

$$s = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{|\text{T}|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

Association Rule Mining Task

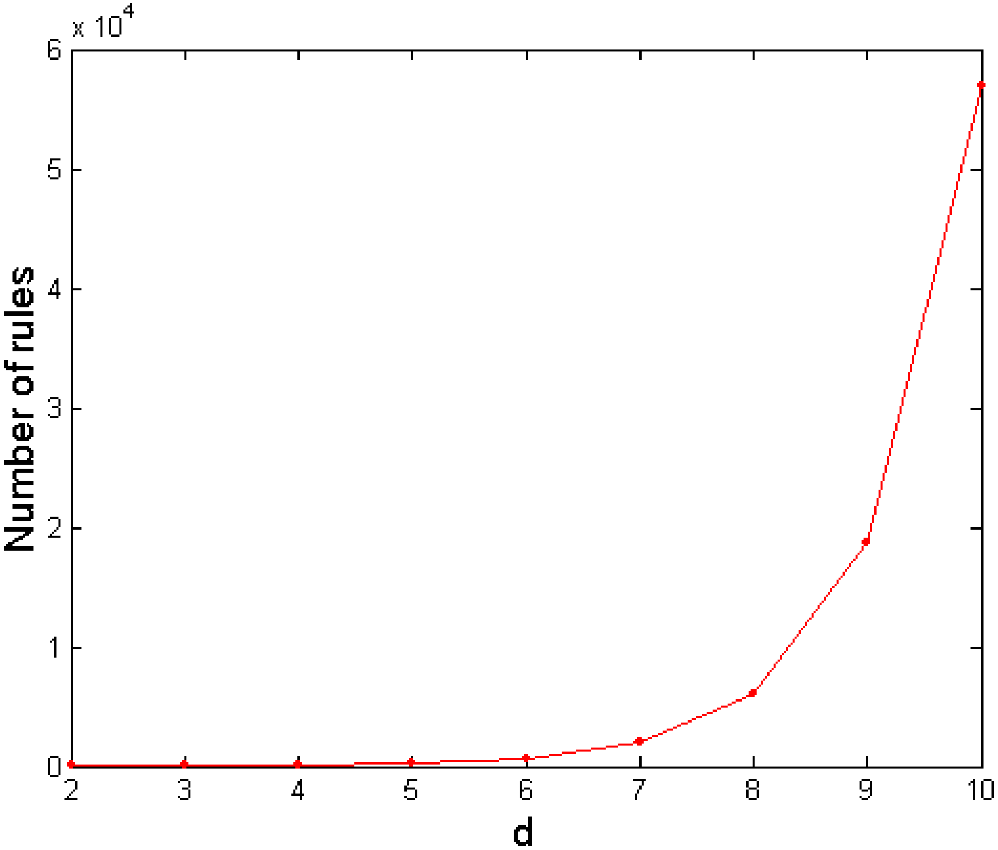
- Given a set of transactions T, the goal of association rule mining is to find all rules having
  - support ≥ *minsup* threshold
  - confidence ≥ *minconf* threshold

- Brute-force approach:
  - List all possible association rules
  - Compute the support and confidence for each rule
  - Prune rules that fail the *minsup* and *minconf* thresholds

  ⇒ Computationally prohibitive!

Computational Complexity

- Given d unique items:
  - Total number of itemsets = $2^d - 1$
  - Total number of possible association rules:



$$R = \sum_{k=1}^{d-1}\left[\binom{d}{k} \times \sum_{j=1}^{d-k}\binom{d-k}{j}\right] = 3^d - 2^{d+1} + 1$$

If d=6, R = 602 rules

Mining Association Rules

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Rules:

{Milk,Diaper} $\rightarrow$ {Beer} (s=0.4, c=0.67)
{Milk,Beer} $\rightarrow$ {Diaper} (s=0.4, c=1.0)
{Diaper,Beer} $\rightarrow$ {Milk} (s=0.4, c=0.67)
{Beer} $\rightarrow$ {Milk,Diaper} (s=0.4, c=0.67)
{Diaper} $\rightarrow$ {Milk,Beer} (s=0.4, c=0.5)
{Milk} $\rightarrow$ {Diaper,Beer} (s=0.4, c=0.5)

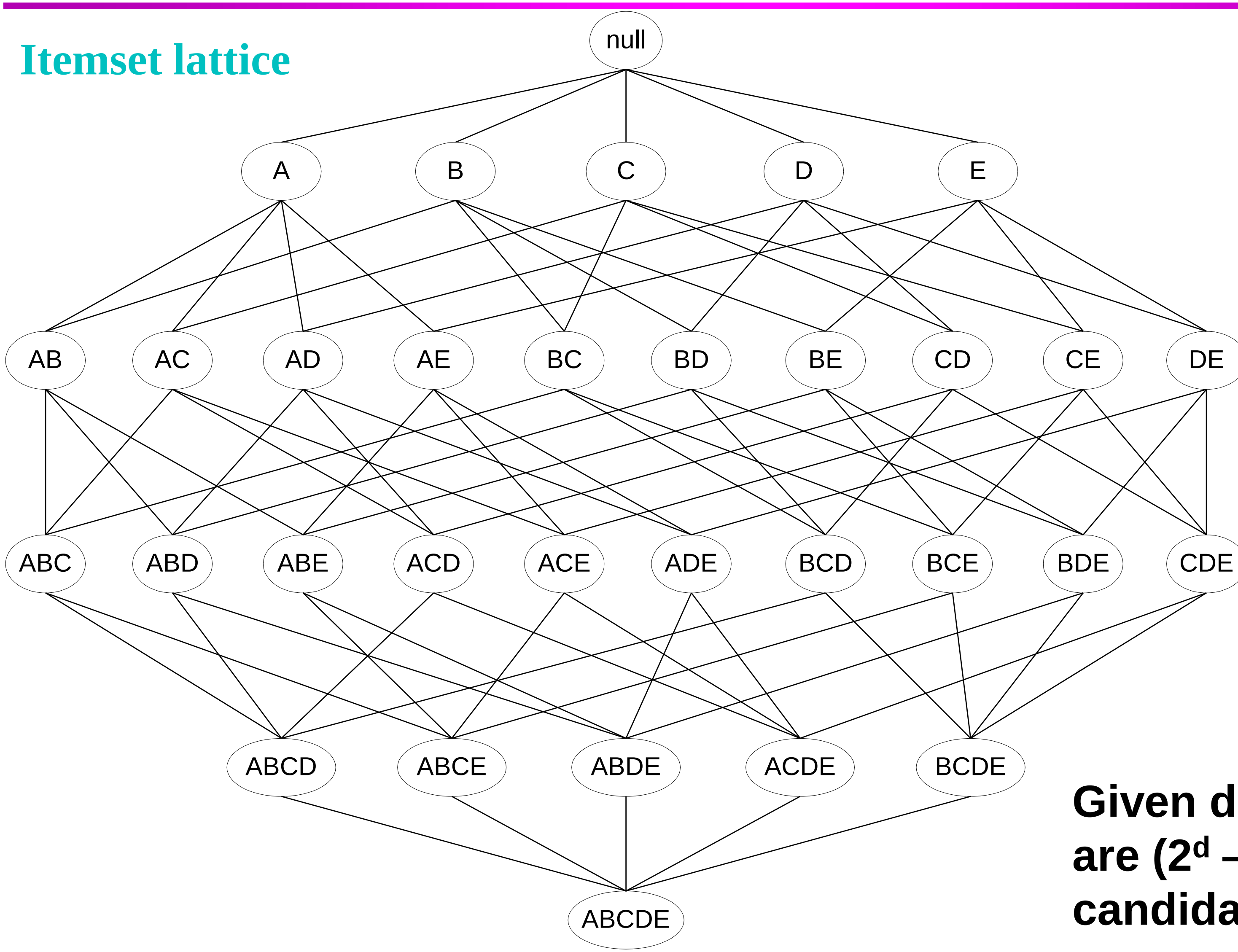
Observations:

- All the above rules are binary partitions of the same itemset:
  {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

Mining Association Rules

- Two-step approach:
  1. Frequent Itemset Generation
     - Generate all itemsets whose support ≥ minsup
  2. Rule Generation
     - Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset
- Frequent itemset generation is still computationally expensive

Frequent Itemset Generation



Given d items, there are (2^d – 1) possible candidate itemsets

Frequent Itemset Generation

- Brute-force approach:
  - Each itemset in the lattice is a candidate frequent itemset
  - Count the support of each candidate by scanning the database

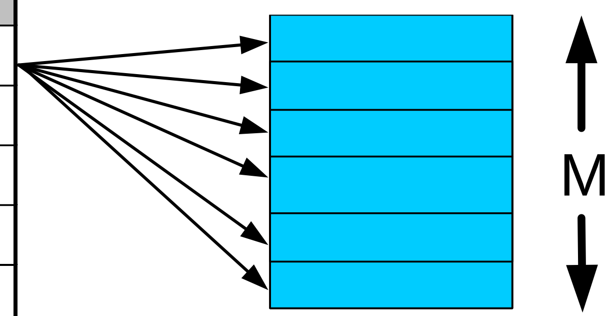
Transactions

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

N

w

List of Candidates



  - Match each transaction against every candidate
  - Complexity ~ O(NMw) => Expensive since $M = 2^d$!!!

Frequent Itemset Generation Strategies

- Reduce the number of candidates (M)
  - Complete search: $M=2^d$
  - Use pruning techniques to reduce M

- Reduce the number of transactions (N)
  - Reduce size of N as the size of itemset increases
  - If the width of a transaction is small (e.g., 2 items), this transaction can be removed before searching for longer itemsets (e.g., itemsets of size 3 and larger)
- Reduce the number of comparisons (NM)
  - Use efficient data structures to store the candidates or transactions
  - No need to match every candidate against every transaction

Reducing Number of Candidates

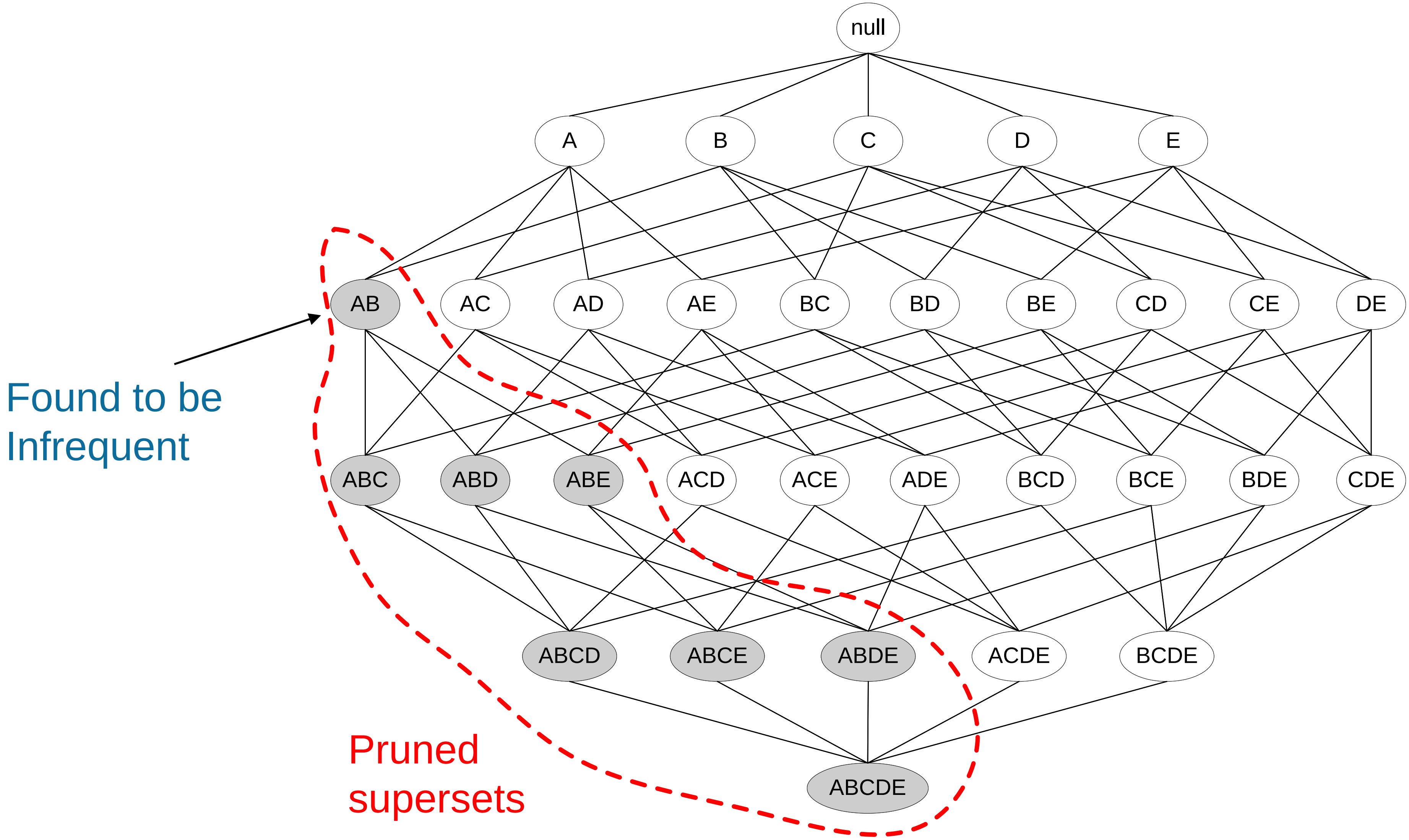
- Apriori principle:
  - If an itemset is frequent, then all of its subsets must also be frequent

- Apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

  - Support of an itemset never exceeds the support of its subsets
  - This is known as the anti-monotone property of support

Illustrating Apriori Principle



Illustrating Apriori Principle

TID	*Items*
1	Bread, Milk
2	Beer, Bread, Diaper, Eggs
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Bread, Coke, Diaper, Milk

Candidate: 6

Items (1-itemsets)

Item	Count
Bread	**4**
Coke	**2**
Milk	**4**
Beer	**3**
Diaper	**4**
Eggs	**1**

6 distinct items

Minimum Support = 3

If every subset is considered,

$$\binom{6}{1} + \binom{6}{2} + \binom{6}{3}$$

$$6 + 15 + 20 = 41$$

Illustrating Apriori Principle

TID	*Items*
1	Bread, Milk
2	Beer, Bread, Diaper, Eggs
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Bread, Coke, Diaper, Milk

Frequent: 4

Items (1-itemsets)

Item	Count
Bread	**4**
Coke	**2**
Milk	**4**
Beer	**3**
Diaper	**4**
Eggs	**1**

Minimum Support = 3

If every subset is considered,

$$\binom{6}{1} + \binom{6}{2} + \binom{6}{3}$$

$$6 + 15 + 20 = 41$$

With support-based pruning,

$$\binom{6}{1} + \binom{4}{2} + \binom{4}{3}$$

$$6 + 6 + 4 = 16$$

Illustrating Apriori Principle

Frequent: 4

Items (1-itemsets)

Item	Count
Bread	**4**
Coke	**2**
Milk	**4**
Beer	**3**
Diaper	**4**
Eggs	**1**

Candidate: 6

Pairs (2-itemsets)

Itemset
{Bread,Milk}
{Bread, Beer }
{Bread,Diaper}
{Beer, Milk}
{Diaper, Milk}
{Beer,Diaper}

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

If every subset is considered,

$$\binom{6}{1} + \binom{6}{2} + \binom{6}{3}$$

$$6 + 15 + 20 = 41$$

With support-based pruning,

$$\binom{6}{1} + \binom{4}{2} + \binom{4}{3}$$

$$6 + 6 + 4 = 16$$

Illustrating Apriori Principle

Item	Count
Bread	**4**
Coke	**2**
Milk	**4**
Beer	**3**
Diaper	**4**
Eggs	**1**

Items (1-itemsets)

Frequent: 4

Itemset	Count
{Bread,Milk}	**3**
{Beer, Bread}	**2**
{Bread,Diaper}	**3**
{Beer,Milk}	**2**
{Diaper,Milk}	**3**
{Beer,Diaper}	**3**

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

If every subset is considered,

$$\binom{6}{1} + \binom{6}{2} + \binom{6}{3}$$

$$6 + 15 + 20 = 41$$

With support-based pruning,

$$\binom{6}{1} + \binom{4}{2} + \binom{4}{3}$$

$$6 + 6 + 4 = 16$$

Illustrating Apriori Principle

Item	Count
Bread	**4**
Coke	**2**
Milk	**4**
Beer	**3**
Diaper	**4**
Eggs	**1**

Items (1-itemsets)

Itemset	Count
{Bread,Milk}	**3**
{Bread,Beer}	**2**
{Bread,Diaper}	**3**
{Milk,Beer}	**2**
{Milk,Diaper}	**3**
{Beer,Diaper}	**3**

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Candidate: 4

Triplets (3-itemsets)

Itemset
{ Beer, Diaper, Milk}
{ Beer,Bread,Diaper}
{Bread, Diaper, Milk}
{ Beer, Bread, Milk}

Candidate: 1

Minimum Support = 3

If every subset is considered,

$$\binom{6}{1} + \binom{6}{2} + \binom{6}{3}$$

$$6 + 15 + 20 = 41$$

With support-based pruning,

$$6 + 6 + 4 = 16$$

$$6 + 6 + 1 = 13$$

Illustrating Apriori Principle

Candidate: 6, frequent: 4

Items (1-itemsets)

Item	Count
Bread	**4**
Coke	**2**
Milk	**4**
Beer	**3**
Diaper	**4**
Eggs	**1**

Candidate: 6, frequent: 4

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Itemset	Count
{Bread,Milk}	**3**
{Bread,Beer}	**2**
{Bread,Diaper}	**3**
{Milk,Beer}	**2**
{Milk,Diaper}	**3**
{Beer,Diaper}	**3**

Candidate: 1, frequent: 0

Triplets (3-itemsets)

Itemset	Count
{Bread, Diaper, Milk}	**2**

Minimum Support = 3

If every subset is considered,

$$\binom{6}{1} + \binom{6}{2} + \binom{6}{3}$$

$$6 + 15 + 20 = 41$$

With support-based pruning,

$$6 + 6 + 4 = 16$$

$$6 + 6 + 1 = 13$$

Apriori Algorithm

- F_k: frequent k-itemsets
- C_k: candidate k-itemsets

- Algorithm
  - Let k=1
  - Generate F_1 = {frequent 1-itemsets}
  - Repeat until F_k is empty
    - **Candidate Generation**: Generate C_{k+1} from F_k
    - **Candidate Pruning**: Prune candidate itemsets in C_{k+1} containing subsets of length k that are infrequent
    - **Support Counting**: Count the support of each candidate in C_{k+1} by scanning the DB
    - **Candidate Elimination**: Eliminate candidates in C_{k+1} that are infrequent, leaving only those that are frequent => F_{k+1}

Candidate Generation: Brute-force method

Items

Item
Beer
Bread
Cola
Diapers
Milk
Eggs

Candidate Generation

Itemset
{Beer, Bread, Cola}
{Beer, Bread, Diapers}
{Beer, Bread, Milk}
{Beer, Bread, Eggs}
{Beer, Cola, Diapers}
{Beer, Cola, Milk}
{Beer, Cola, Eggs}
{Beer, Diapers, Milk}
{Beer, Diapers, Eggs}
{Beer, Milk, Eggs}
{Bread, Cola, Diapers}
{Bread, Cola, Milk}
{Bread, Cola, Eggs}
{Bread, Diapers, Milk}
{Bread, Diapers, Eggs}
{Bread, Milk, Eggs}
{Cola, Diapers, Milk}
{Cola, Diapers, Eggs}
{Cola, Milk, Eggs}
{Diapers, Milk, Eggs}

Frequent
2-itemset

Itemset
{Beer, Diapers}
{Bread, Diapers}
{Bread, Milk}
{Diapers, Milk}

Candidate
Pruning

Itemset
{Bread, Diapers, Milk}

Figure 6.6. A brute-force method for generating candidate 3-itemsets.

Candidate Generation: Merge F_{k-1} and F_1 itemsets

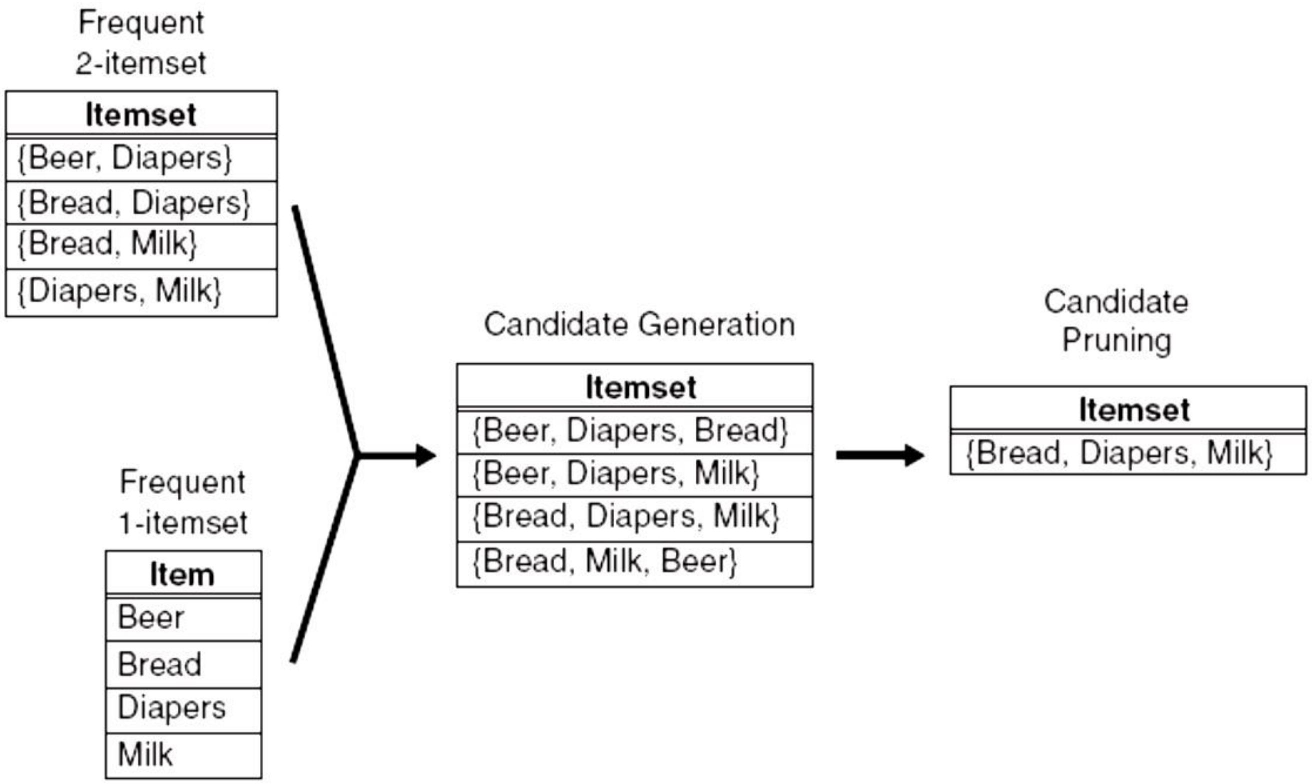


Figure 6.7. Generating and pruning candidate k-itemsets by merging a frequent $(k-1)$-itemset with a frequent item. Note that some of the candidates are unnecessary because their subsets are infrequent.

Candidate Generation: F_{k-1} x F_{k-1} Method

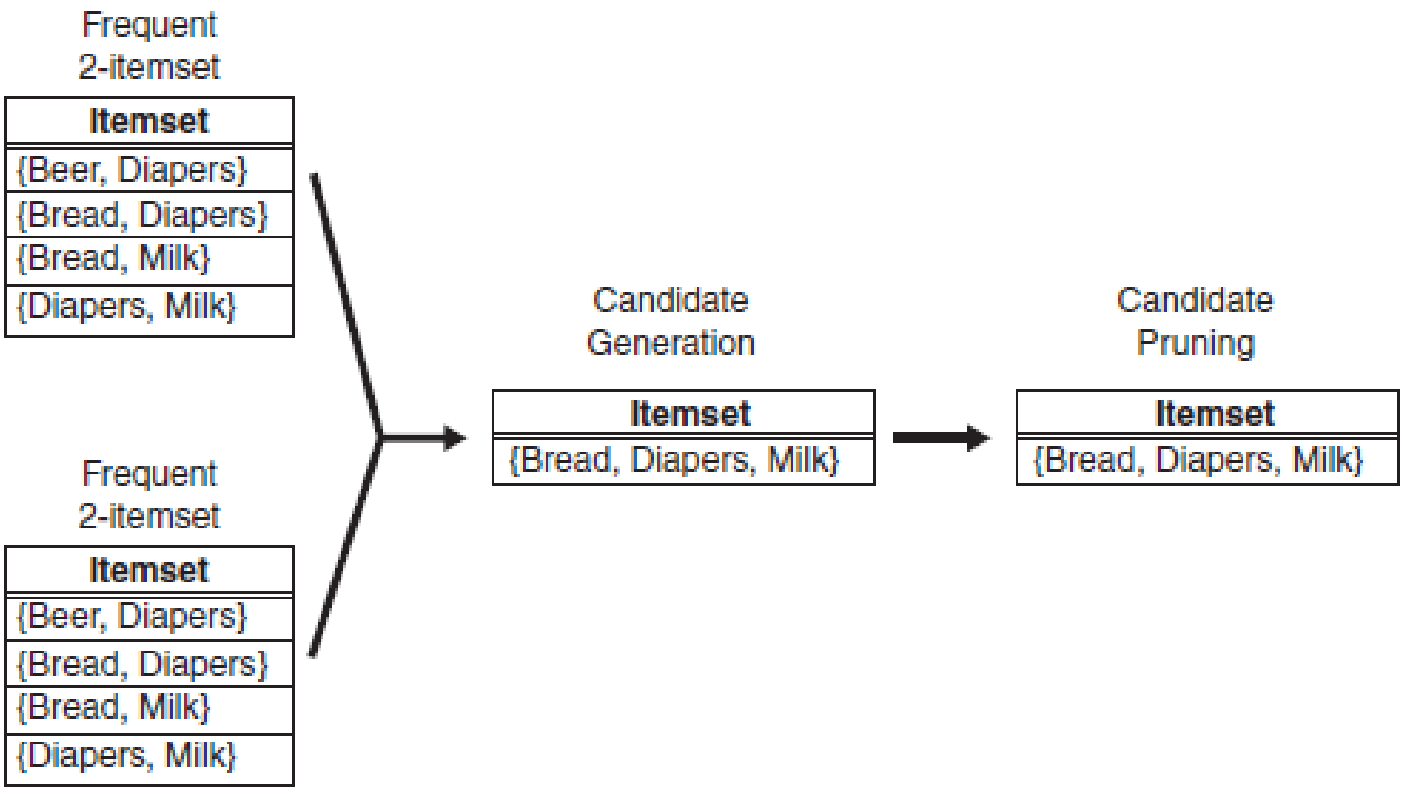


Figure 6.8. Generating and pruning candidate k-itemsets by merging pairs of frequent $(k-1)$-itemsets.

Candidate Generation: $F_{k-1} \times F_{k-1}$ Method

- Merge two frequent (k-1)-itemsets if their first (k-2) items are identical

- F_3 = {ABC,ABD,ABE,ACD,BCD,BDE,CDE}
  - Merge(**AB**C, **AB**D) = **AB**CD
  - Merge(**AB**C, **AB**E) = **AB**CE
  - Merge(**AB**D, **AB**E) = **AB**DE

  - Do not merge(**A**BD,**A**CD) because they share only prefix of length 1 instead of length 2

Candidate Pruning

- Let F_3 = {ABC,ABD,ABE,ACD,BCD,BDE,CDE} be the set of frequent 3-itemsets

- C_4 = {ABCD,ABCE,ABDE} is the set of candidate 4-itemsets generated (from previous slide)

- Candidate pruning
  - Prune ABCE because ACE and BCE are infrequent
  - Prune ABDE because ADE is infrequent

- After candidate pruning: C_4 = {ABCD}

Alternate F_{k-1} x F_{k-1} Method

- Merge two frequent (k-1)-itemsets if the last (k-2) items of the first one is identical to the first (k-2) items of the second.

- F_3 = {ABC,ABD,ABE,ACD,BCD,BDE,CDE}
  - Merge(A**BC**, **BC**D) = A**BC**D
  - Merge(A**BD**, **BD**E) = A**BD**E
  - Merge(A**CD**, **CD**E) = A**CD**E
  - Merge(B**CD**, **CD**E) = B**CD**E

Candidate Pruning for Alternate $F_{k-1} \times F_{k-1}$ Method

- Let F_3 = {ABC,ABD,ABE,ACD,BCD,BDE,CDE} be the set of frequent 3-itemsets

- C_4 = {ABCD,ABDE,ACDE,BCDE} is the set of candidate 4-itemsets generated (from previous slide)
- Candidate pruning
  – Prune ABDE because ADE is infrequent
  – Prune ACDE because ACE and ADE are infrequent
  – Prune BCDE because BCE
- After candidate pruning: C_4 = {ABCD}

Illustrating Apriori Principle

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Items (1-itemsets)

Itemset	Count
{Bread,Milk}	3
{Bread,Beer}	2
{Bread,Diaper}	3
{Milk,Beer}	2
{Milk,Diaper}	3
{Beer,Diaper}	3

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

If every subset is considered,

$$\binom{6}{1} + \binom{6}{2} + \binom{6}{3}$$

$$6 + 15 + 20 = 41$$

With support-based pruning,

$$6 + 6 + 1 = 13$$

Triplets (3-itemsets)

Itemset	Count
{Bread, Diaper, Milk}	2

Use of F_{k-1}xF_{k-1} method for candidate generation results in only one 3-itemset. This is eliminated after the support counting step.

Support Counting of Candidate Itemsets

- Scan the database of transactions to determine the support of each candidate itemset
  - Must match every candidate itemset against every transaction, which is an expensive operation

TID	*Items*
1	Bread, Milk
2	Beer, Bread, Diaper, Eggs
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Bread, Coke, Diaper, Milk

Itemset
{ Beer, Diaper, Milk}
{ Beer,Bread,Diaper}
{Bread, Diaper, Milk}
{ Beer, Bread, Milk}

Support Counting of Candidate Itemsets

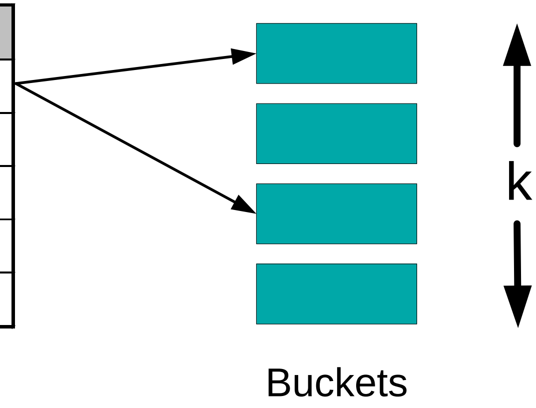
- To reduce number of comparisons, store the candidate itemsets in a hash structure
  - Instead of matching each transaction against every candidate, match it against candidates contained in the hashed buckets

Transactions

N

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Hash Structure

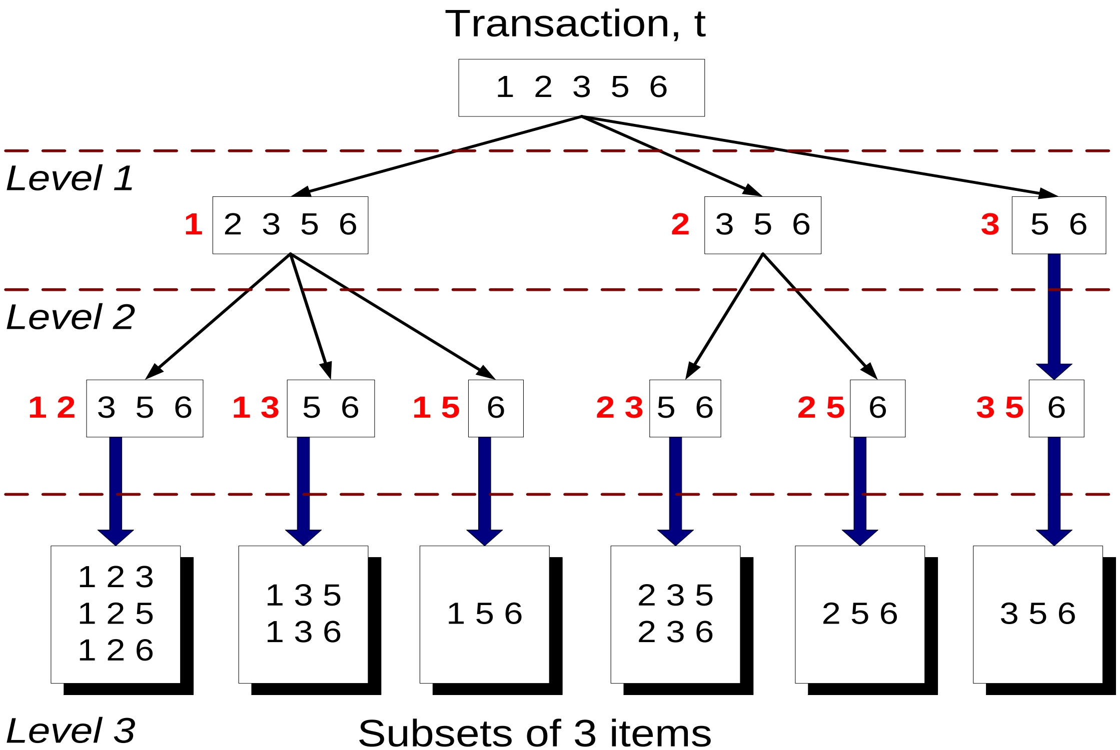


Support Counting: An Example

Suppose you have 15 candidate itemsets of length 3:

{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6}, {2 3 4}, {5 6 7}, {3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}

How many of these itemsets are supported by transaction (1,2,3,5,6)?



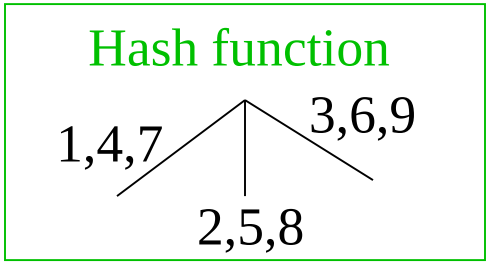
Support Counting Using a Hash Tree

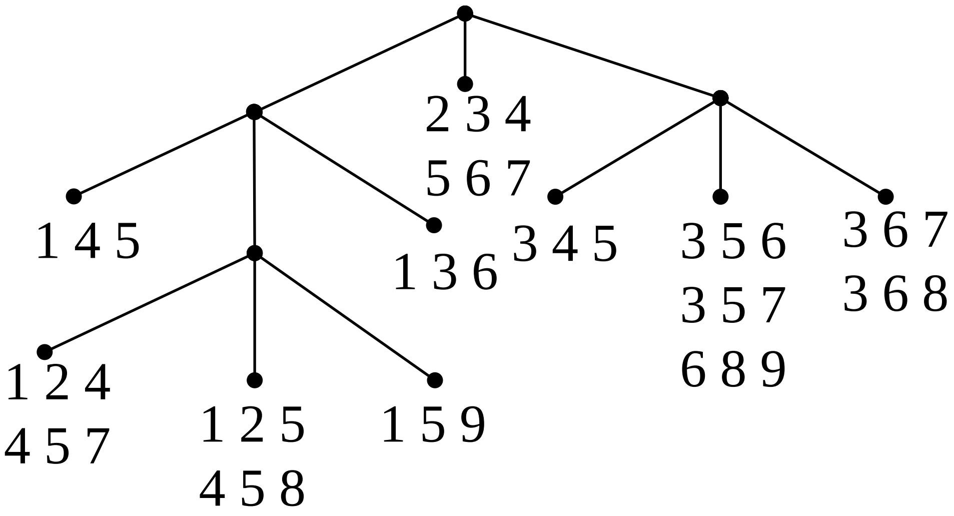
Suppose you have 15 candidate itemsets of length 3:

{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6}, {2 3 4}, {5 6 7}, {3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}

You need:

- **Hash function**
- **Max leaf size: max number of itemsets stored in a leaf node (if number of candidate itemsets exceeds max leaf size, split the node)**





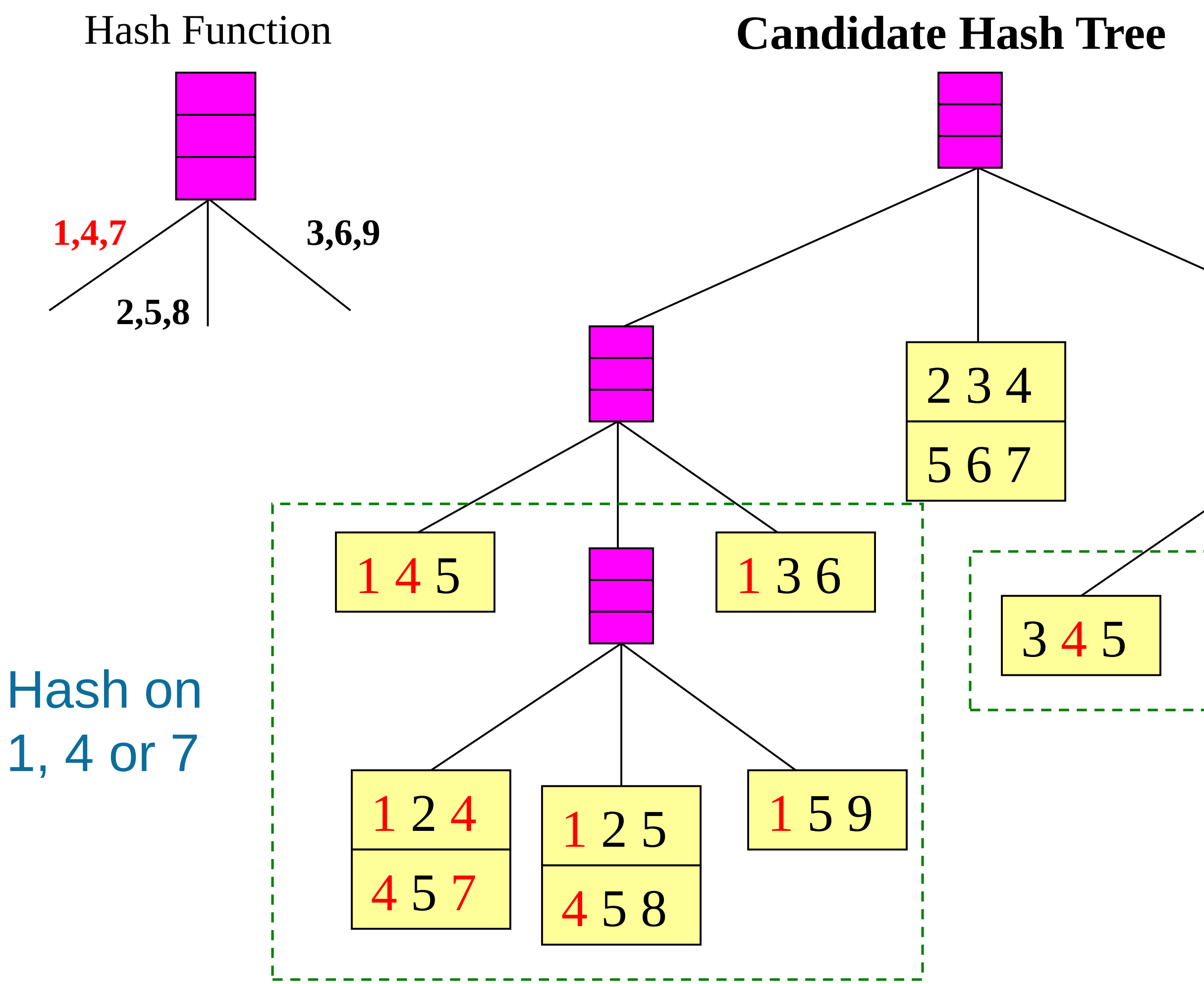
Support Counting Using a Hash Tree

Candidate 3-itemsets:

{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6},

{2 3 4}, {5 6 7},

{3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}



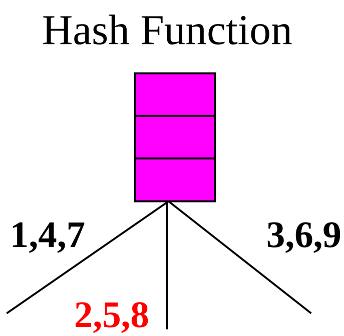
Support Counting Using a Hash Tree

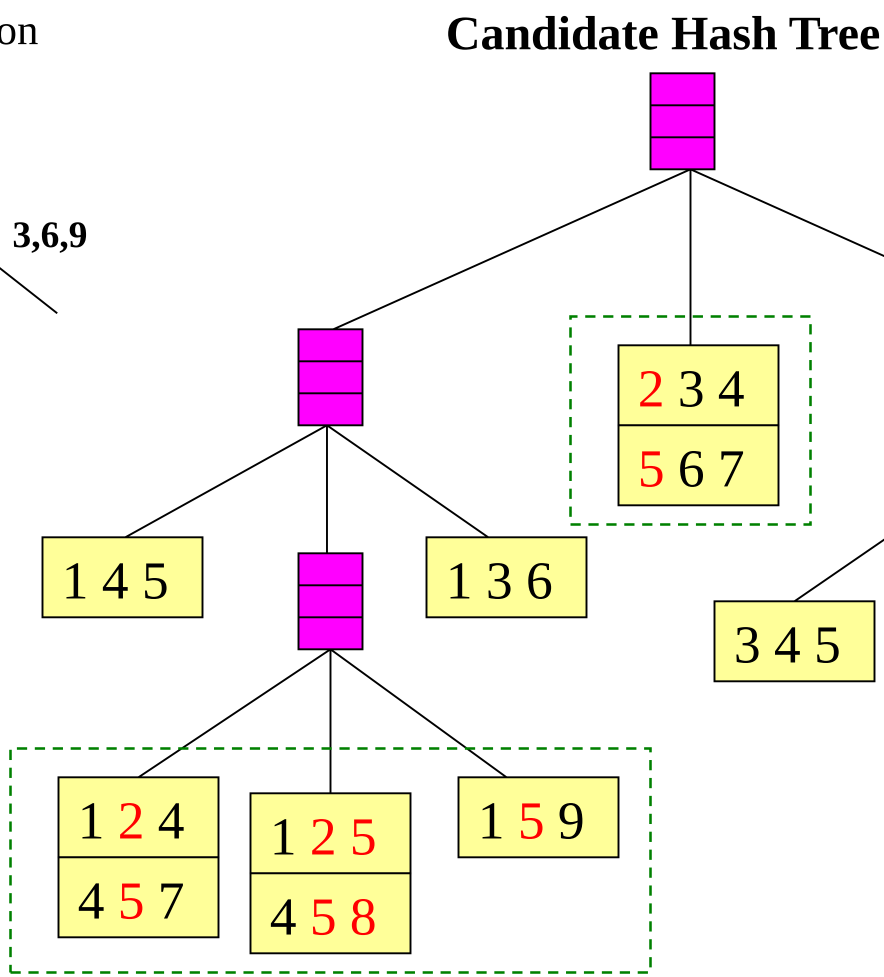
Candidate 3-itemsets:

{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6},

{2 3 4}, {5 6 7},

{3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}





Hash on
2, 5 or 8

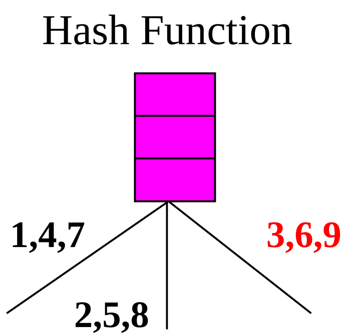
Support Counting Using a Hash Tree

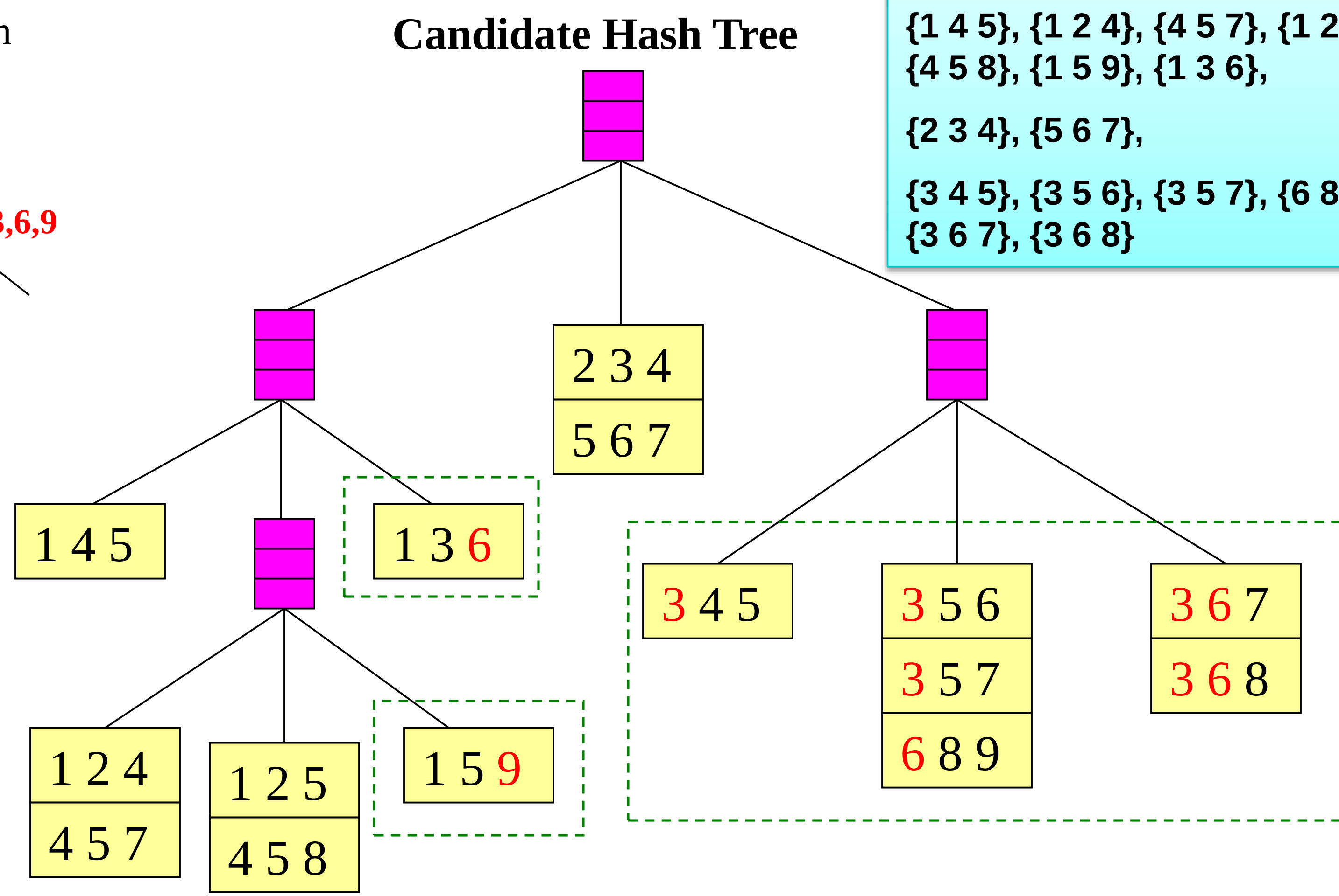
Candidate 3-itemsets:

{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6},

{2 3 4}, {5 6 7},

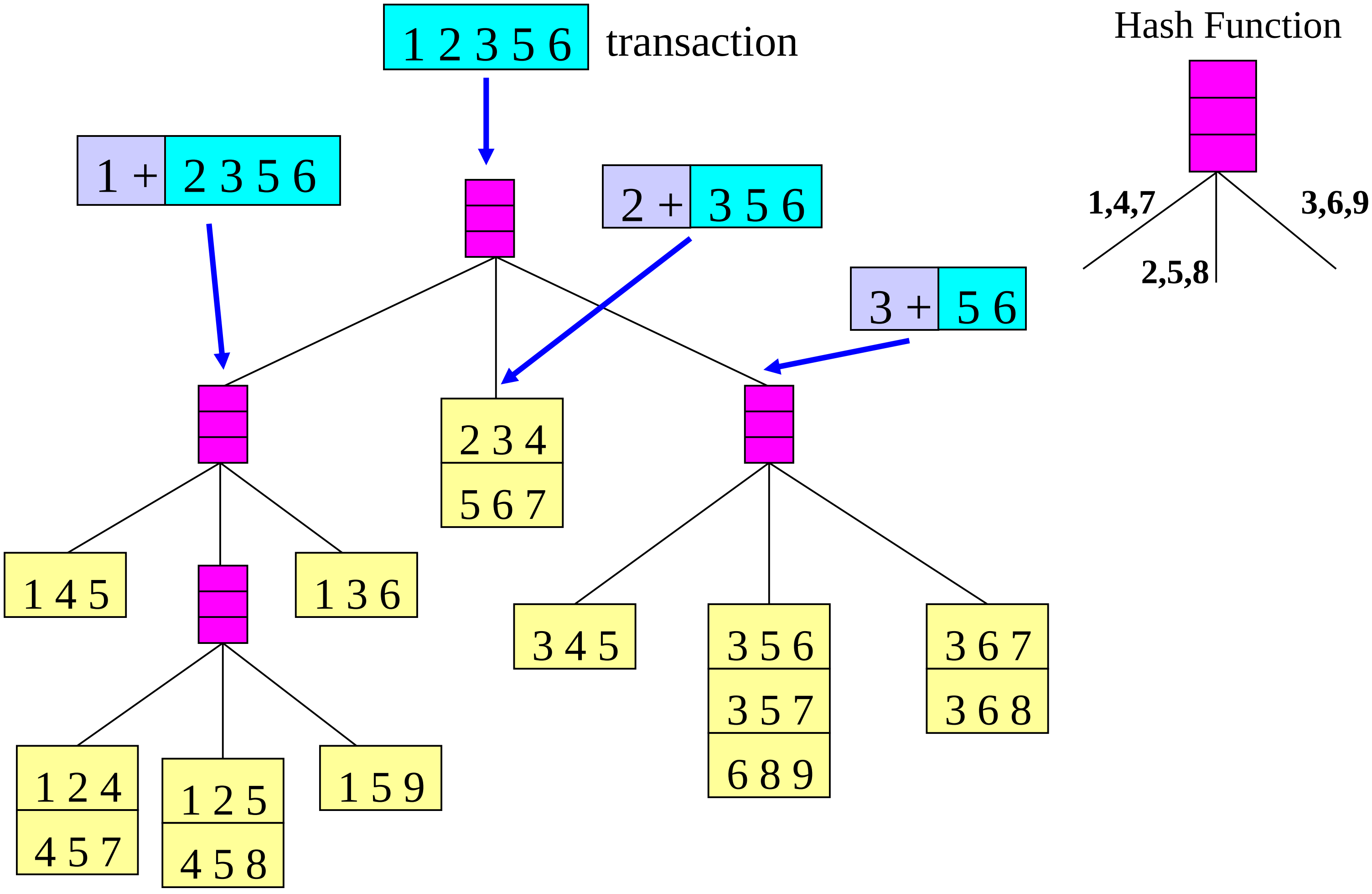
{3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}



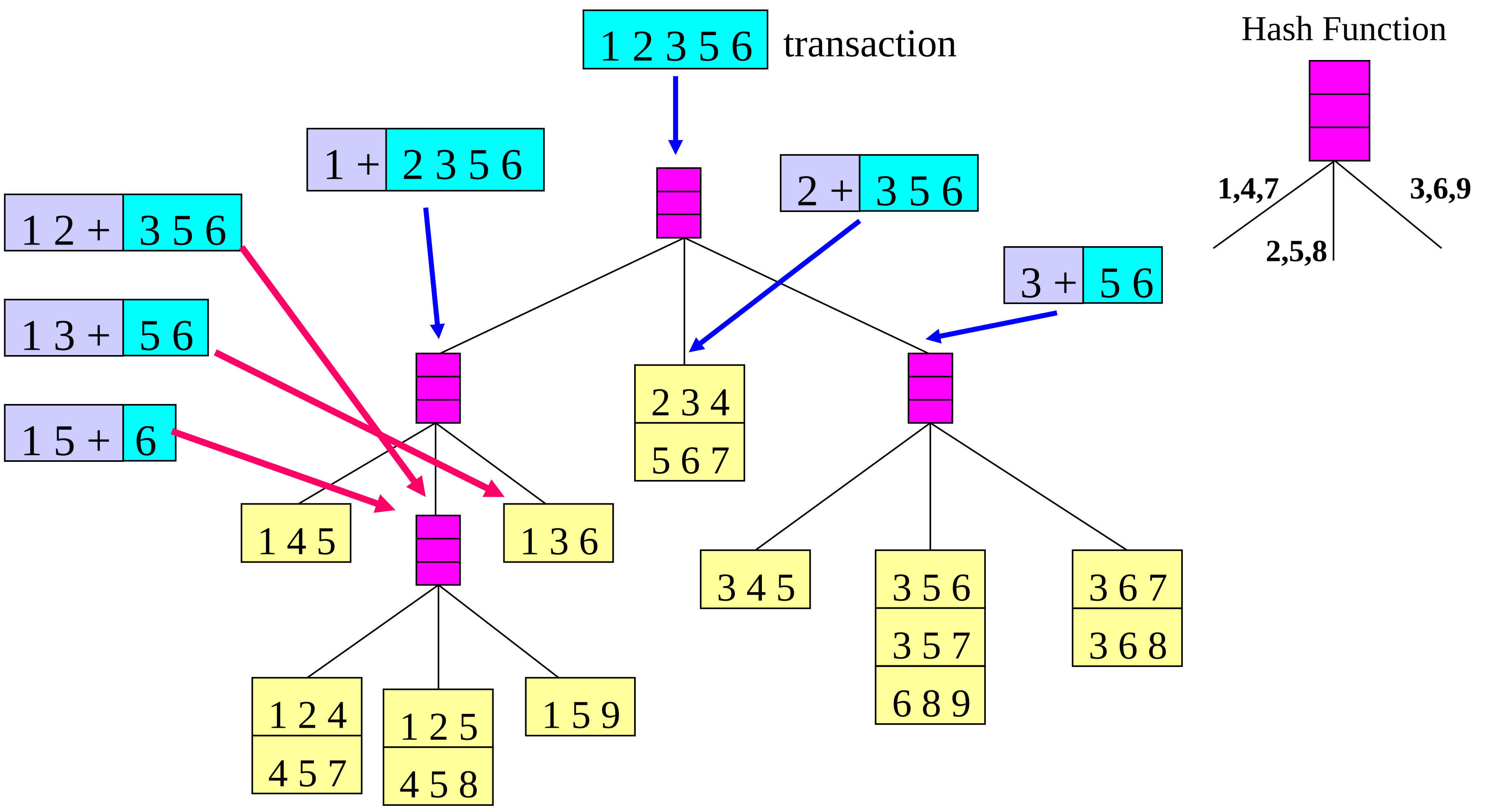


Hash on
3, 6 or 9

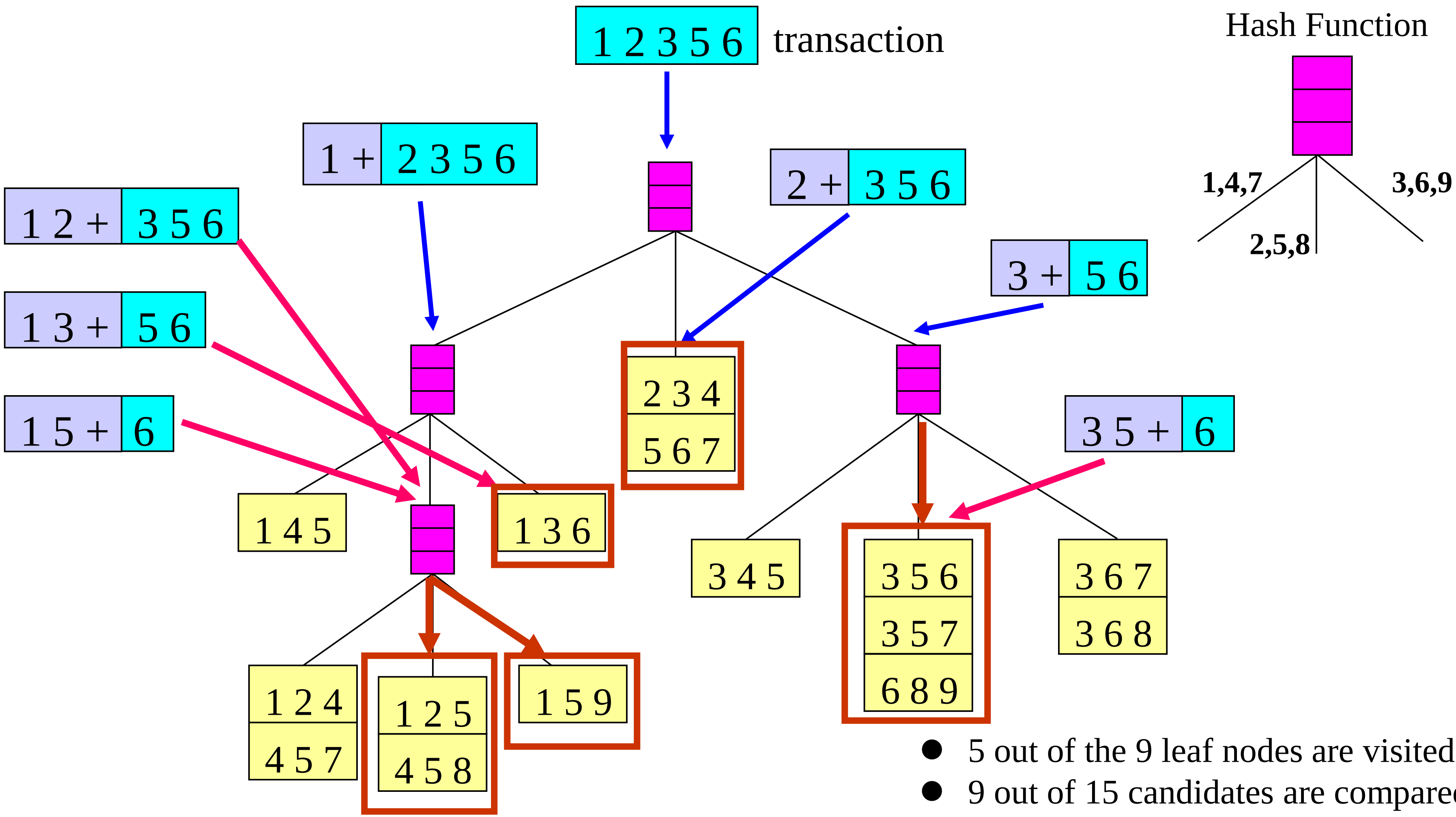
Support Counting Using a Hash Tree



Support Counting Using a Hash Tree



Support Counting Using a Hash Tree



- 5 out of the 9 leaf nodes are visited
- 9 out of 15 candidates are compared against the transaction

Some Efficient and Scalable Frequent Itemset Mining Methods

Data Mining: Concepts and Techniques, 2nd ed.

By Jiawei Han and Micheline Kamber

Source: Revised from Jiawei Han and Micheline Kamber's slides

DHP: Reduce the Number of Candidates

- A *k*-itemset whose corresponding hashing bucket count is below the threshold cannot be frequent
  - Candidates: a, b, c, d, e
  - Hash entries: {ab, ad, ae} {bd, be, de} …
  - Frequent 1-itemset: a, b, d, e
  - ab is not a candidate 2-itemset if the sum of count of {ab, ad, ae} is below support threshold
- J. Park, M. Chen, and P. Yu. An effective hash-based algorithm for mining association rules. In *SIGMOD'95*

Create hash table H_2 using hash function $h(x, y) = ((order\ of\ x) \times 10 + (order\ of\ y))\ mod\ 7$ →

H_2

bucket address	0	1	2	3	4	5	6
bucket count	2	2	4	2	2	4	4
bucket contents	{I1, I4} {I3, I5}	{I1, I5} {I1, I5}	{I2, I3} {I2, I3} {I2, I3} {I2, I3}	{I2, I4} {I2, I4}	{I2, I5} {I2, I5}	{I1, I2} {I1, I2} {I1, I2} {I1, I2}	{I1, I3} {I1, I3} {I1, I3} {I1, I3}

CHARM: Mining by Exploring Vertical Data Format

- Vertical format: $t(AB) = \{T_{11}, T_{25}, \ldots\}$
  - tid-list: list of trans.-ids containing an itemset
- Deriving closed patterns based on vertical intersections
  - $t(X) = t(Y)$: X and Y always happen together
  - $t(X) \subset t(Y)$: transaction having X always has Y
- Using diffset to accelerate mining
  - Only keep track of differences of tids
  - $t(X) = \{T_1, T_2, T_3\}$, $t(XY) = \{T_1, T_3\}$
  - Diffset $(XY, X) = \{T_2\}$
- Eclat/MaxEclat (Zaki et al. @KDD'97), VIPER(P. Shenoy et al.@SIGMOD'00), CHARM (Zaki & Hsiao@SDM'02)

Example

itemset	*TID_set*
I1	{T100, T400, T500, T700, T800, T900}
I2	{T100, T200, T300, T400, T600, T800, T900}
I3	{T300, T500, T600, T700, T800, T900}
I4	{T200, T400}
I5	{T100, T800}

minsup = 2

Intersect each pair of F_1

itemset	*TID_set*
{I1,I2}	{T100, T400, T800, T900}
{I1,I3}	{T500, T700, T800, T900}
{I1,I4}	{T400}
{I1,I5}	{T100, T800}
{I2,I3}	{T300, T600, T800, T900}
{I2,I4}	{T200, T400}
{I2,I5}	{T100, T800}
{I3,I5}	{T800}

- **Generate C_3 from $F_2 \times F_2$ (similar to Apriori)**
- **Intersect corresponding TID_sets**

itemset	*TID_set*
{I1,I2,I3}	{T800, T900}
{I1,I2,I5}	{T100, T800}

Bottleneck of Frequent-pattern Mining

- Multiple database scans are costly
- Mining long patterns needs many passes of scanning and generates lots of candidates
  - To find frequent itemset $i_1 i_2 \ldots i_{100}$
    - # of scans: 100
    - # of Candidates: $\binom{100}{1} + \binom{100}{2} + \ldots + \binom{100}{100} = 2^{100} - 1 = 1.27*10^{30}$!
- Bottleneck: candidate-generation-and-test
- Can we avoid candidate generation?

Mining Frequent Patterns Without Candidate Generation

- Grow long patterns from short ones using local frequent items
  - "abc" is a frequent pattern
  - Get all transactions having "abc": DB|abc
  - "d" is a local frequent item in DB|abc → abcd is a frequent pattern

Frequency pattern tree

Construct FP-tree from a Transaction Database

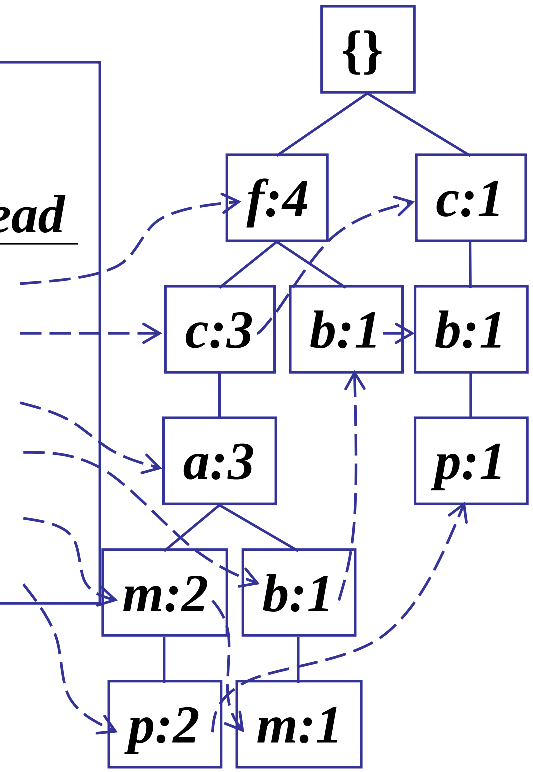
TID	*Items bought*	*(ordered) frequent items*
100	*{f, a, c, d, g, i, m, p}*	*{f, c, a, m, p}*
200	*{a, b, c, f, l, m, o}*	*{f, c, a, b, m}*
300	*{b, f, h, j, o, w}*	*{f, b}*
400	*{b, c, k, s, p}*	*{c, b, p}*
500	*{a, f, c, e, l, p, m, n}*	*{f, c, a, m, p}*

min_support = 3

1. Scan DB once, find frequent 1-itemset (single item pattern)
2. Sort frequent items in frequency descending order, f-list
3. Scan DB again, construct FP-tree

Header Table

Item	*frequency*	*head*
f	*4*	
c	*4*	
a	*3*	
b	*3*	
m	*3*	
p	*3*	



F-list=f-c-a-b-m-p

Benefits of the FP-tree Structure

- Completeness
  - Preserve complete information for frequent pattern mining
  - Never break a long pattern of any transaction
- Compactness
  - Reduce irrelevant info—infrequent items are gone
  - Items in frequency descending order: the more frequently occurring, the more likely to be shared
  - Never be larger than the original database (not count node-links and the *count* field)

Partition Patterns and Databases

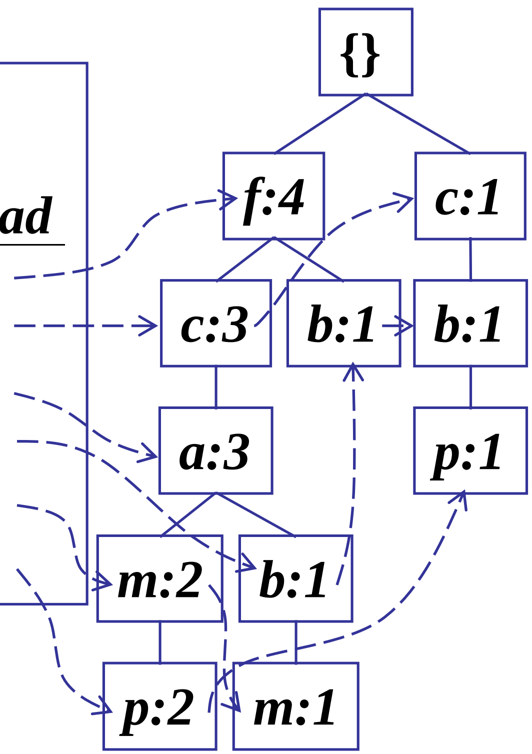
- Frequent patterns can be partitioned into subsets according to f-list
  - F-list=f-c-a-b-m-p
  - Patterns containing p
  - Patterns having m but no p
  - …
  - Patterns having c but no a nor b, m, p
  - Pattern f
- Completeness and non-redundency

Find Patterns Having P From P-conditional Database

- Starting at the frequent item header table in the FP-tree
- Traverse the FP-tree by following the link of each frequent item *p*
- Accumulate all of *transformed prefix paths* of item *p* to form *p*'s conditional pattern base

Header Table

Item	*frequency*	*head*
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



***Conditional* pattern bases**

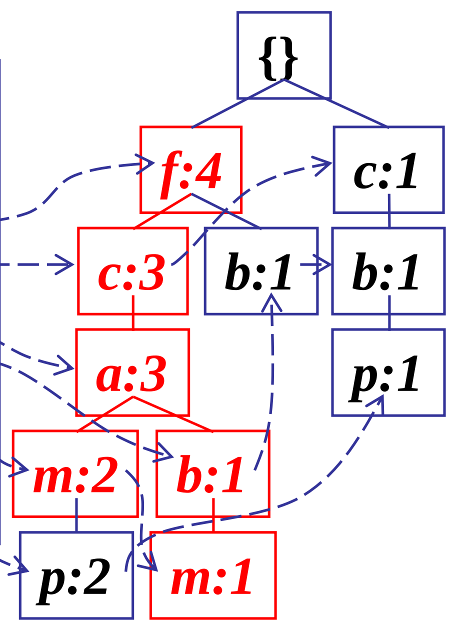
item	*cond. pattern base*
c	*f:3*
a	*fc:3*
b	*fca:1, f:1, c:1*
m	*fca:2, fcab:1*
p	*fcam:2, cb:1*

From Conditional Pattern-bases to Conditional FP-trees

- For each pattern-base
  - Accumulate the count for each item in the base
  - Construct the FP-tree for the frequent items of the pattern base

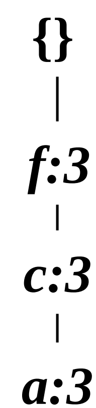
Header Table

Item	*frequency*	*head*
f	*4*	
c	*4*	
a	*3*	
b	*3*	
m	*3*	
p	*3*	



m-conditional **pattern base:**

fca:2, fcab:1



m-conditional **FP-tree**

All frequent patterns relate to *m*

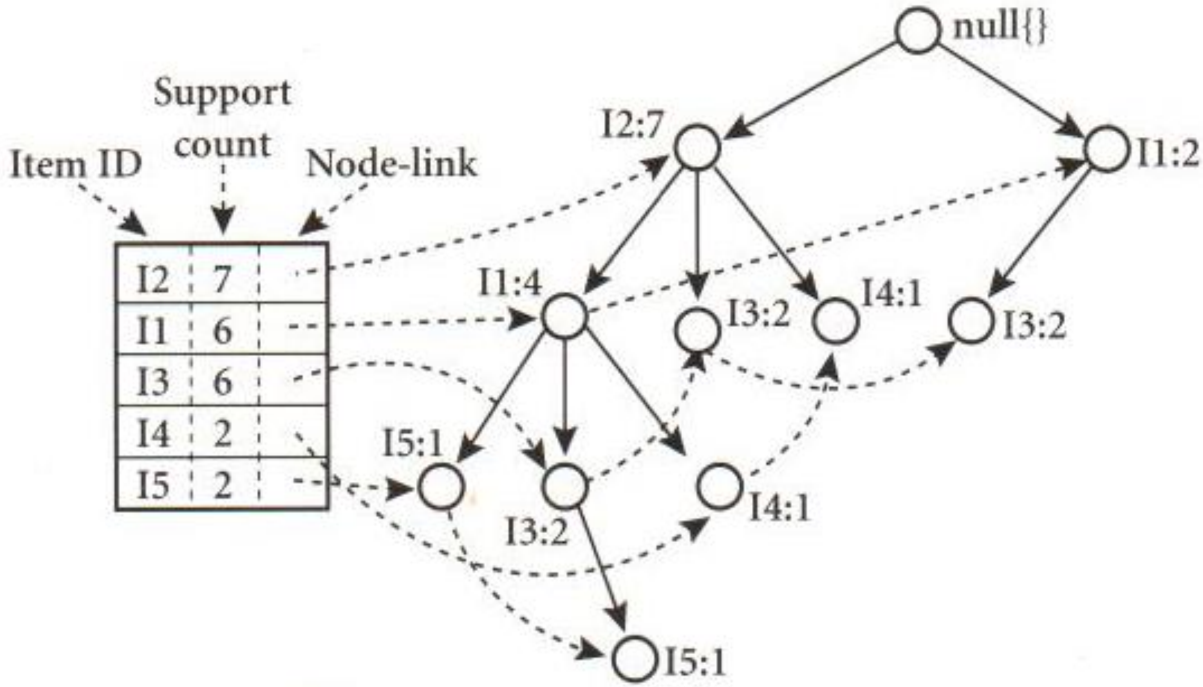
m,

fm, cm, am,

fcm, fam, cam,

fcam

Example



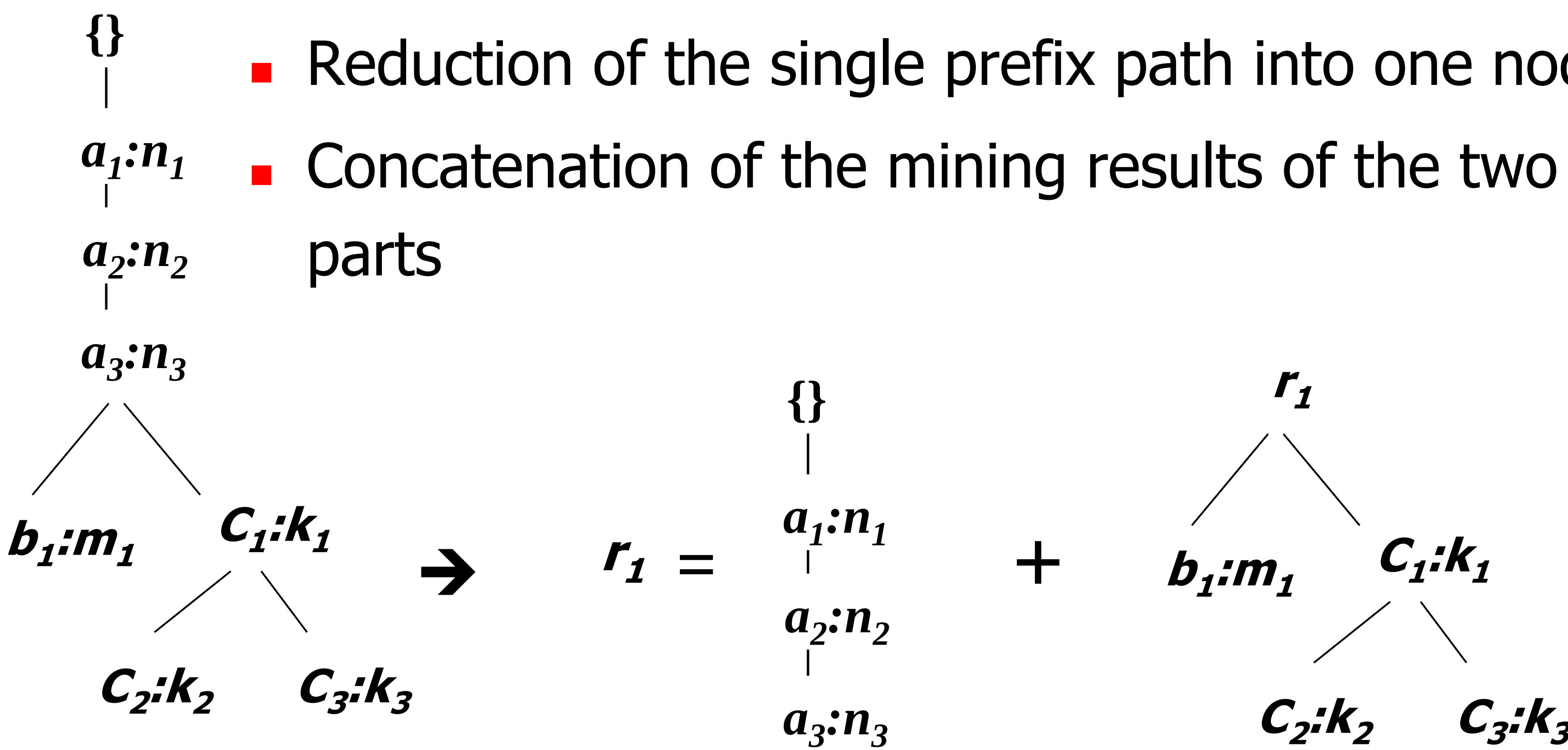
An FP-tree registers compressed, frequent pattern information.

Mining the FP-tree by creating conditional (sub-)pattern bases.

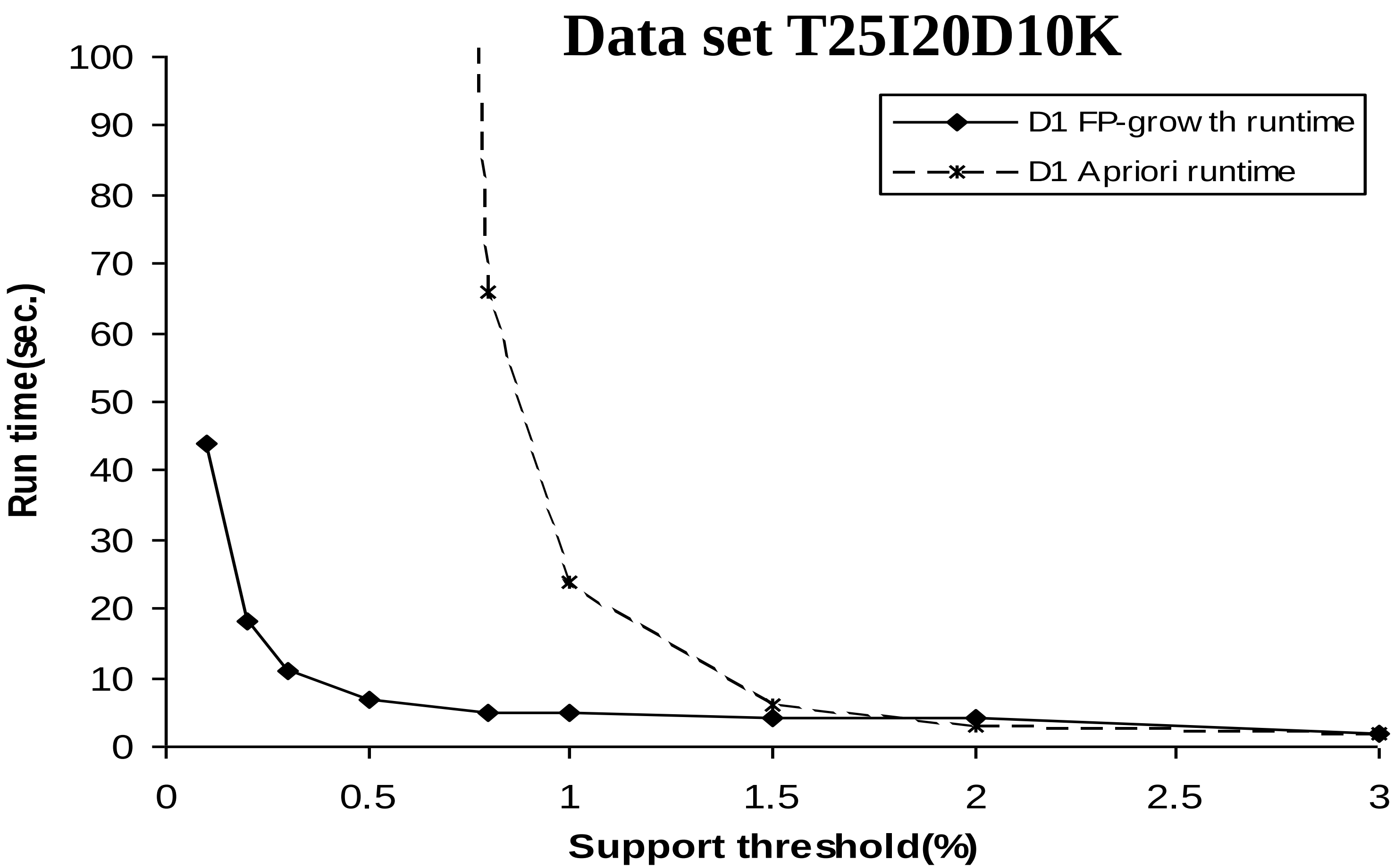
Item	*Conditional Pattern Base*	*Conditional FP-tree*	*Frequent Patterns Generated*
I5	{{I2, I1: 1}, {I2, I1, I3: 1}}	⟨I2: 2, I1: 2⟩	{I2, I5: 2}, {I1, I5: 2}, {I2, I1, I5: 2}
I4	{{I2, I1: 1}, {I2: 1}}	⟨I2: 2⟩	{I2, I4: 2}
I3	{{I2, I1: 2}, {I2: 2}, {I1: 2}}	⟨I2: 4, I1: 2⟩, ⟨I1: 2⟩	{I2, I3: 4}, {I1, I3: 4}, {I2, I1, I3: 2}
I1	{{I2: 4}}	⟨I2: 4⟩	{I2, I1: 4}

A Special Case: Single Prefix Path in FP-tree

- Suppose a (conditional) FP-tree T has a shared single prefix-path P
- Mining can be decomposed into two parts
  - Reduction of the single prefix path into one node
  - Concatenation of the mining results of the two parts



FP-Growth vs. Apriori: Scalability With the Support Threshold



Why Is FP-Growth the Winner?

- Divide-and-conquer:
  - decompose both the mining task and DB according to the frequent patterns obtained so far
  - leads to focused search of smaller databases
- Other factors
  - no candidate generation, no candidate test
  - compressed database: FP-tree structure
  - no repeated scan of entire database
  - basic ops—counting local freq items and building sub FP-tree, no pattern search and matching

Rule Generation

Rule Generation

- Given a frequent itemset L, find all non-empty subsets $f \subset L$ such that $f \to L - f$ satisfies the minimum confidence requirement
  - If {A,B,C,D} is a frequent itemset, candidate rules:

$A \to BCD$,	$B \to ACD$,	$C \to ABD$,	$D \to ABC$
$AB \to CD$,	$AC \to BD$,	$AD \to BC$,	$BC \to AD$,
$BD \to AC$,	$CD \to AB$,		
$ABC \to D$,	$ABD \to C$,	$ACD \to B$,	$BCD \to A$,

- If $|L| = k$, then there are $2^k - 2$ candidate association rules (ignoring $L \to \varnothing$ and $\varnothing \to L$)

Rule Generation

- In general, confidence does not have an anti-monotone property

  $c(ABC \rightarrow D)$ can be larger or smaller than $c(AB \rightarrow D)$

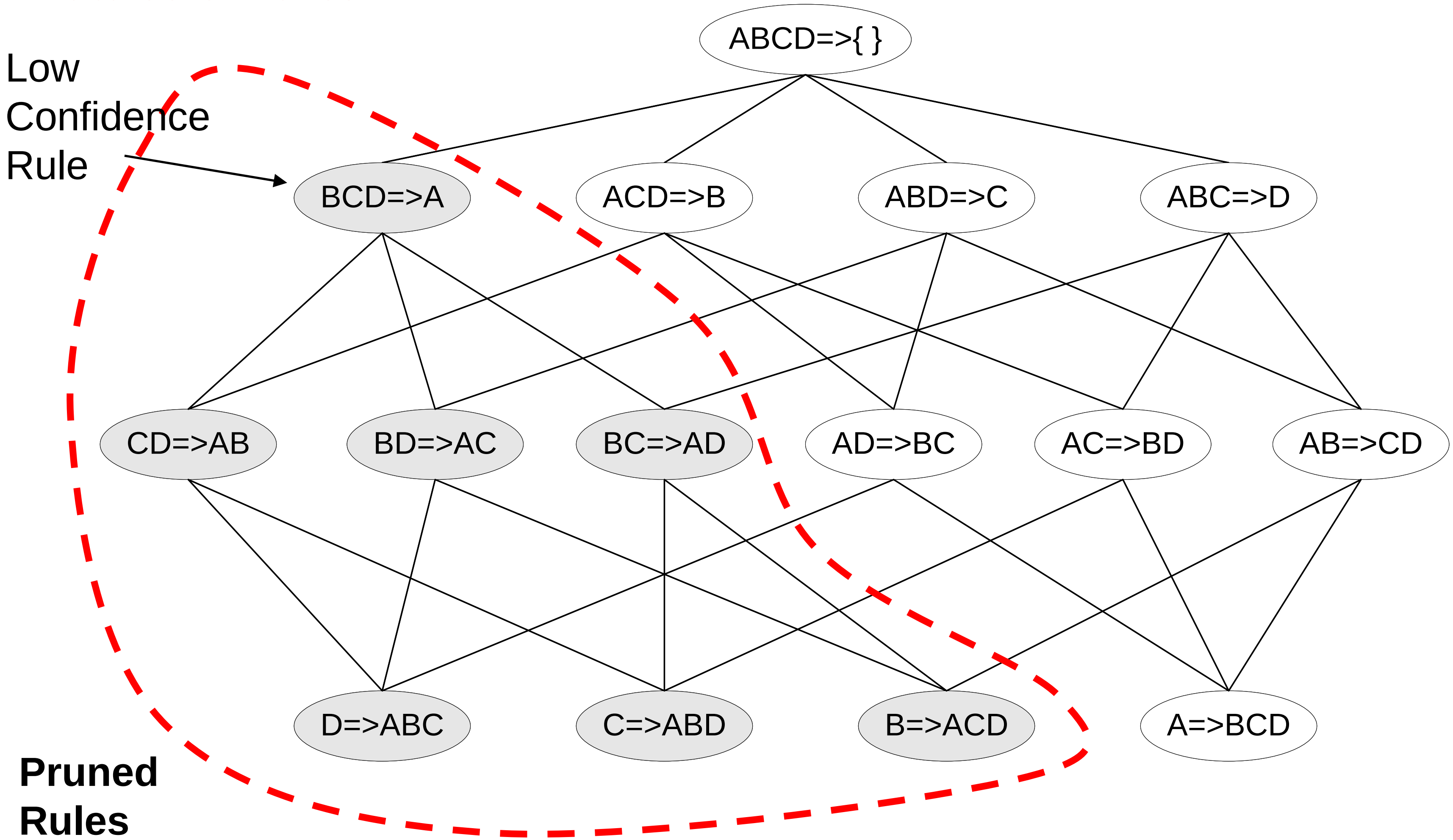
- But confidence of rules generated from the same itemset has an anti-monotone property
  - E.g., Suppose {A,B,C,D} is a frequent 4-itemset:

$$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

  - Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

Rule Generation for Apriori Algorithm

Lattice of rules



Association Analysis: Basic Concepts and Algorithms

Algorithms and Complexity

Factors Affecting Complexity of Apriori

- Choice of minimum support threshold
  - Lowering support threshold results in more frequent itemsets
  - This may increase number of candidates and max length of frequent itemsets
- Dimensionality (number of items) of the data set
  - More space is needed to store support count of each item
  - If number of frequent items also increases, both computation and I/O costs may also increase
- Size of database (number of transactions)
  - Since Apriori makes multiple passes, run time of algorithm may increase with number of transactions
- Average transaction width
  - Transaction width increases with denser data sets
  - This may increase max length of frequent itemsets and traversals of hash tree (number of subsets in a transaction increases with its width)

Compact Representation of Frequent Itemsets

- Some itemsets are redundant because they have identical support as their supersets

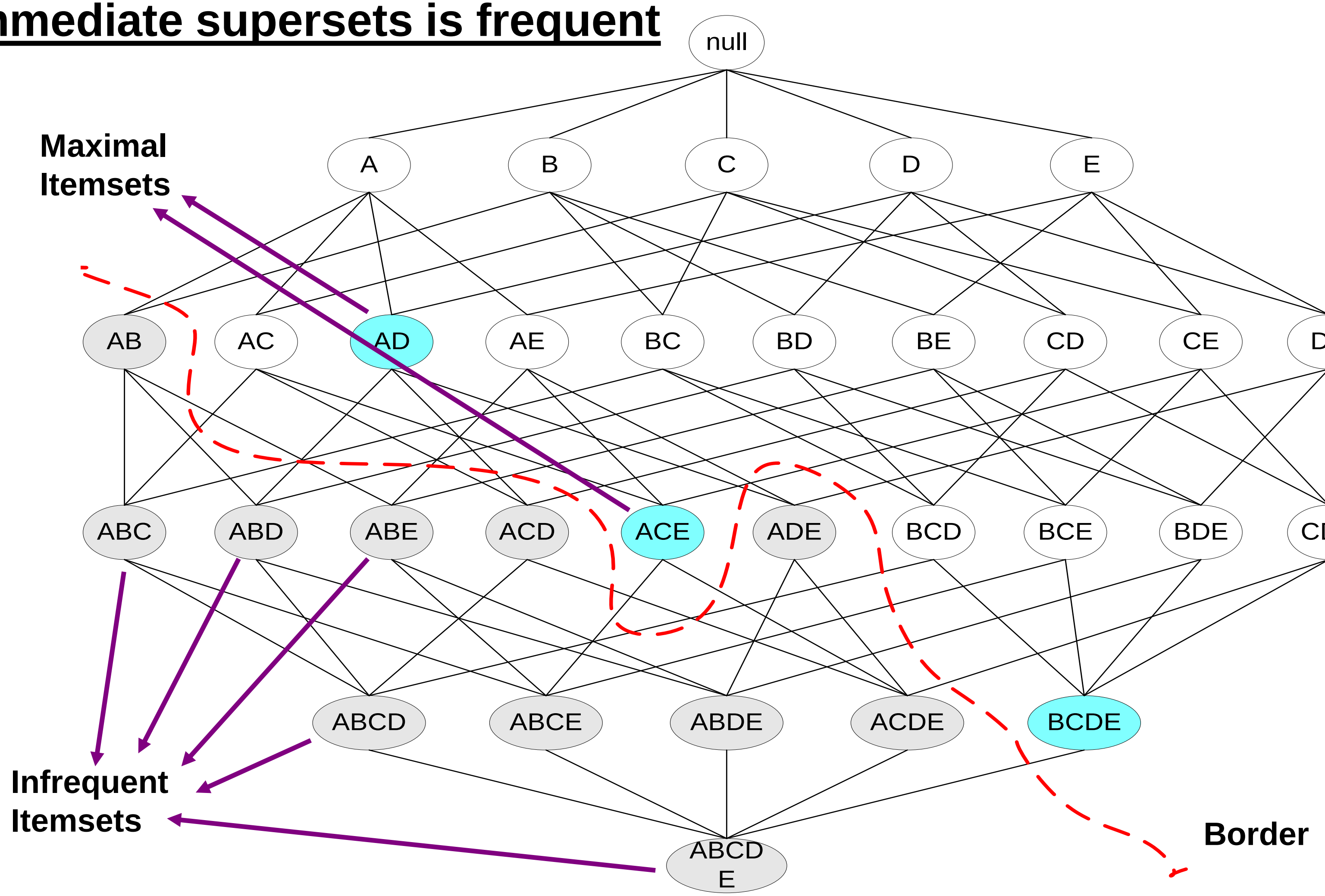
TID	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
1	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**

- Number of frequent itemsets $= 3 \times \sum_{k=1}^{10} \binom{10}{k}$

- Need a compact representation

Maximal Frequent Itemset

An itemset is maximal frequent if it is <u>frequent</u> and <u>none of its immediate supersets is frequent</u>

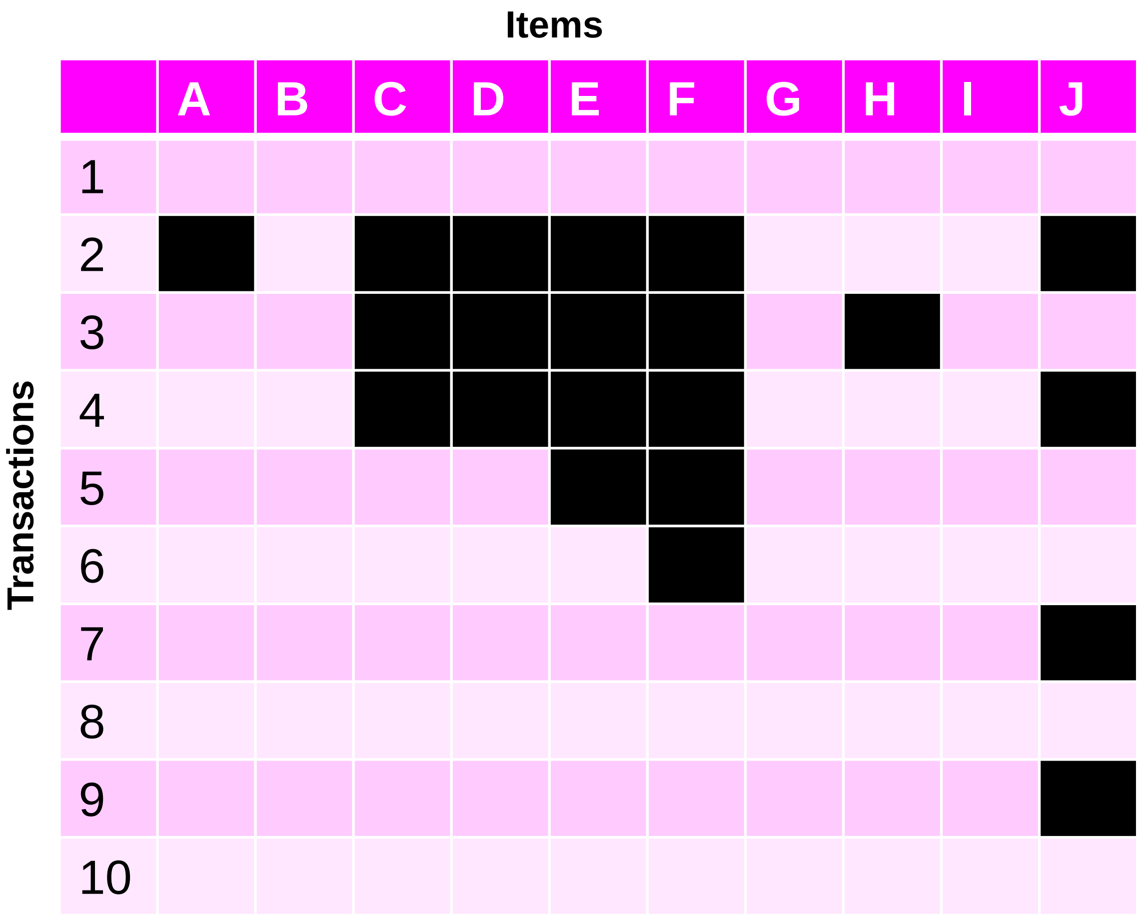


What are the Maximal Frequent Itemsets in this Data?

TID	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
1	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**

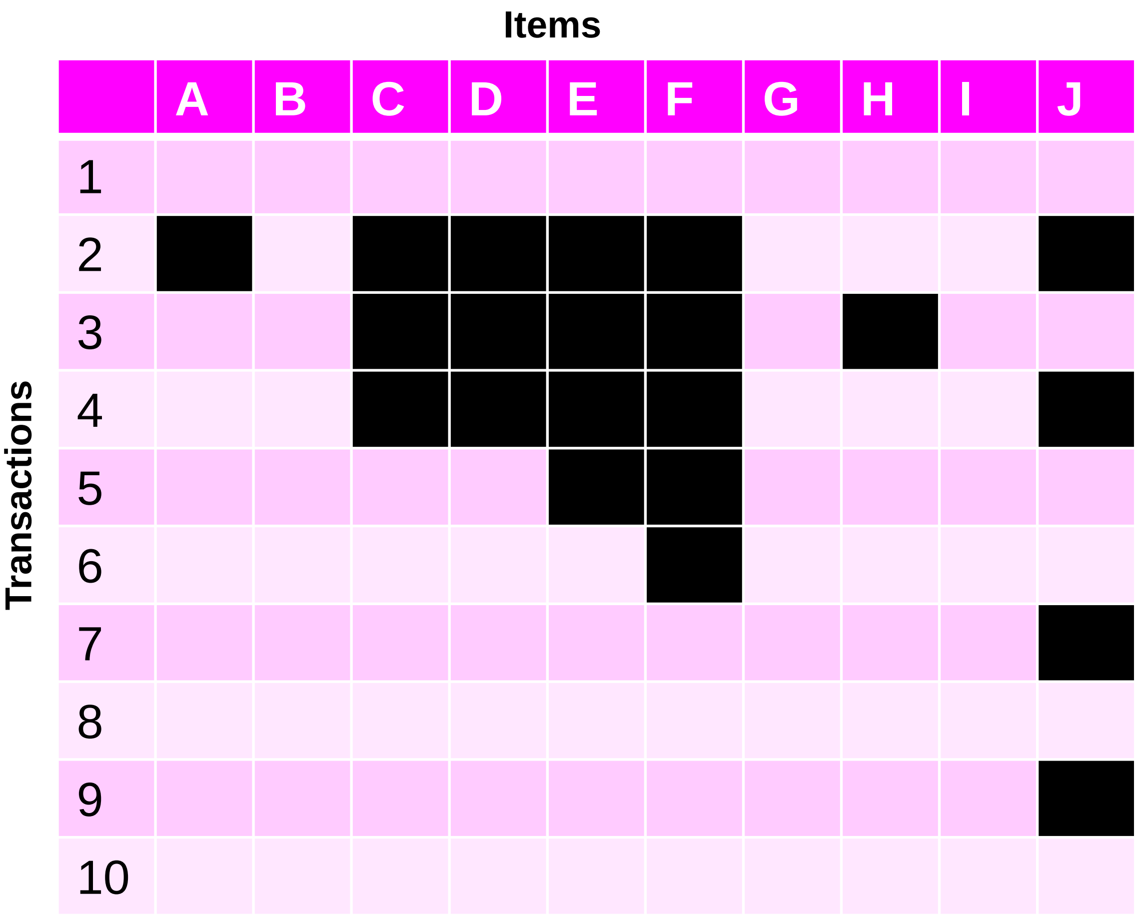
Minimum support threshold = 5

An illustrative example



Support threshold (by count) : 5
Frequent itemsets: ?

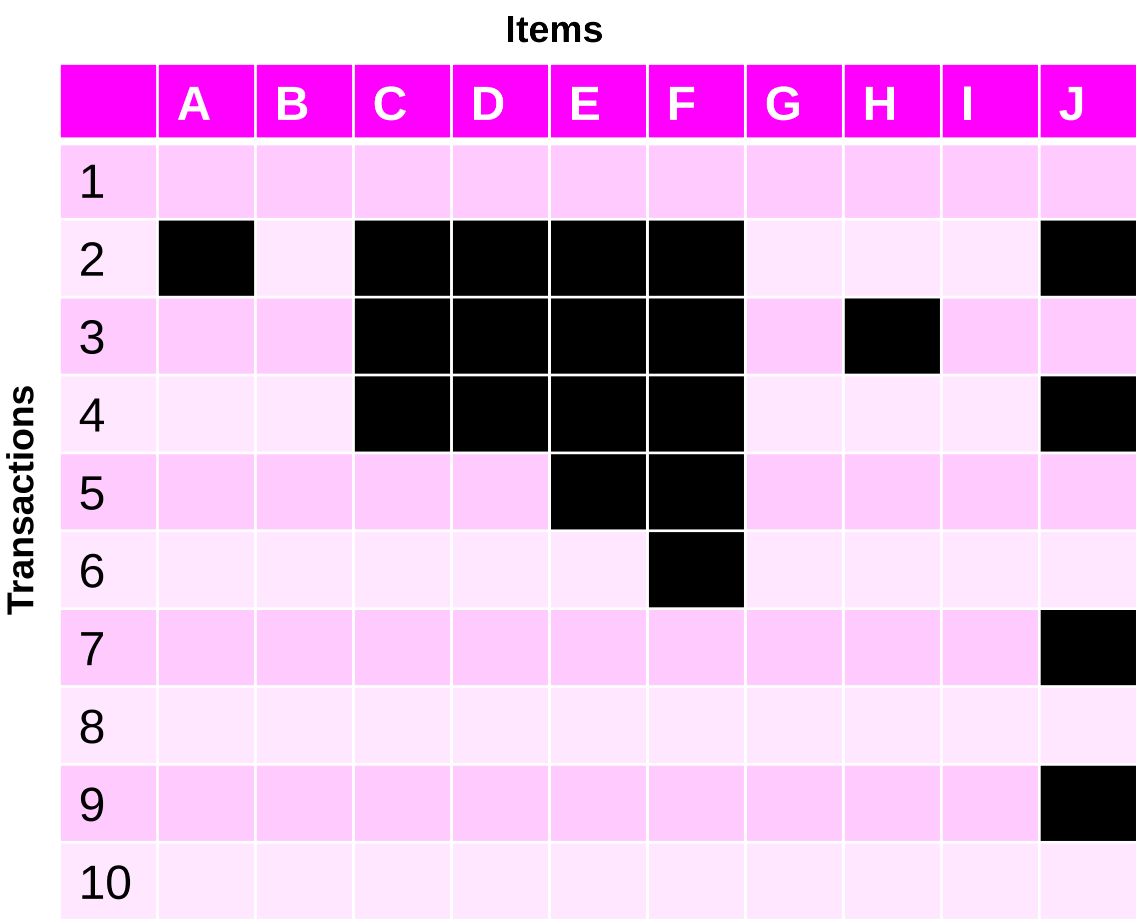
An illustrative example



Support threshold (by count) : 5

Frequent itemsets: {F}

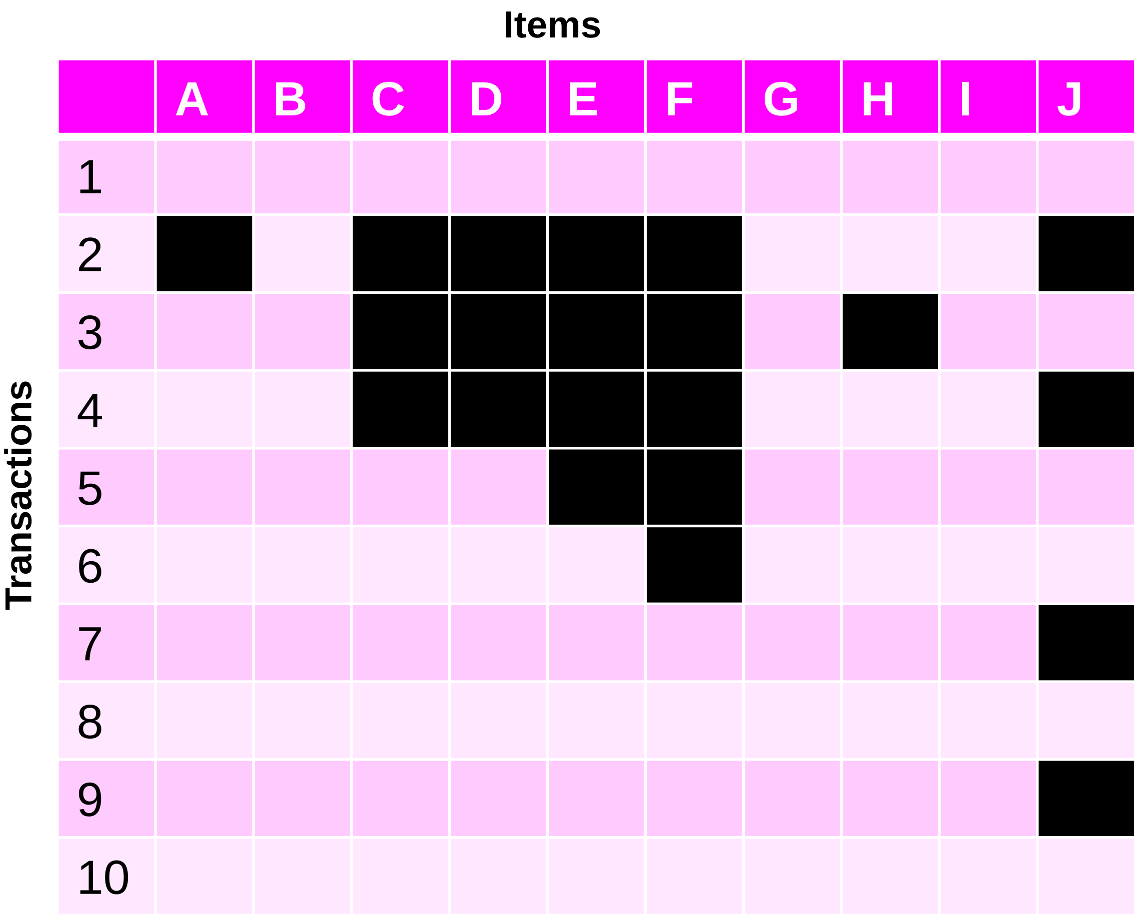
An illustrative example



Support threshold (by count) : 5
Frequent itemsets: {F}

Support threshold (by count): 4
Frequent itemsets: ?

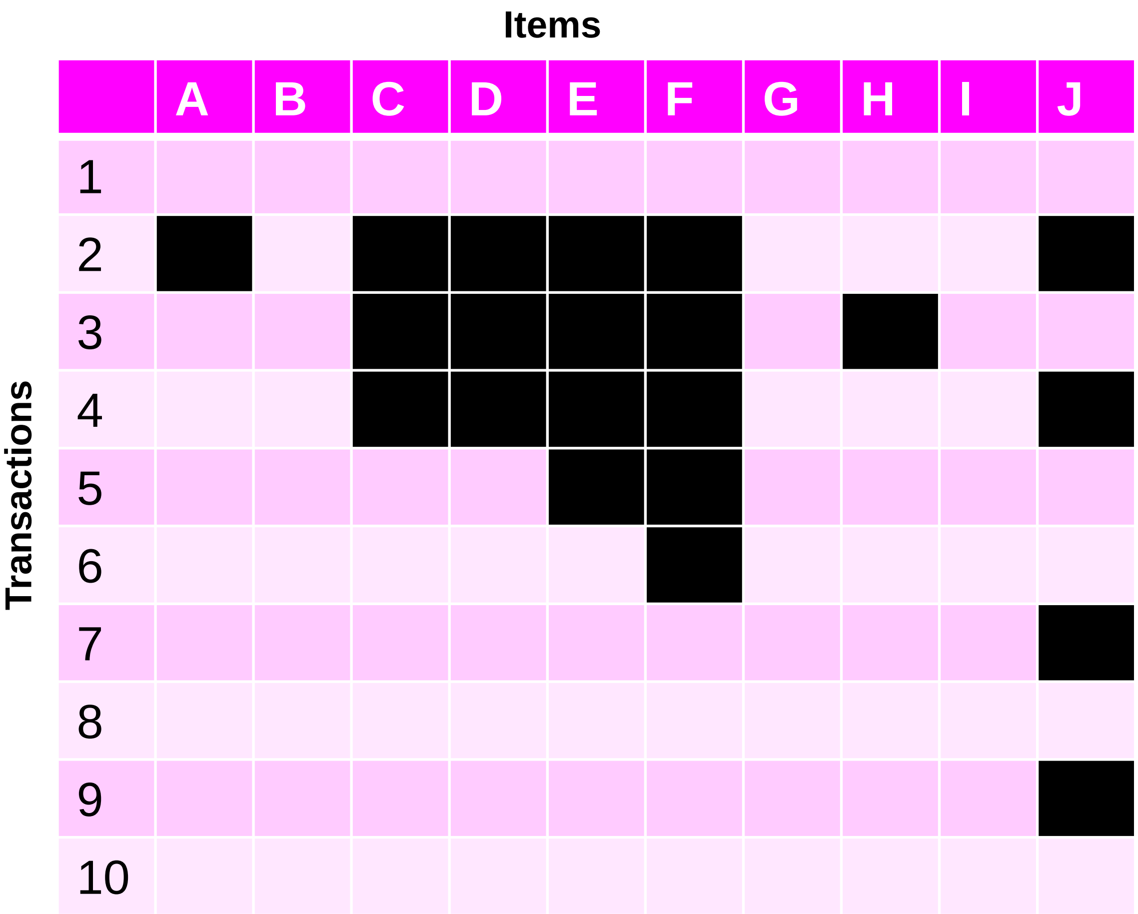
An illustrative example



Support threshold (by count) : 5
Frequent itemsets: {F}

Support threshold (by count): 4
Frequent itemsets: {E}, {F}, {E,F}, {J}

An illustrative example



Support threshold (by count) : 5
Frequent itemsets: {F}

Support threshold (by count): 4
Frequent itemsets: {E}, {F}, {E,F}, {J}

Support threshold (by count): 3
Frequent itemsets: ?

An illustrative example

Items

Transactions

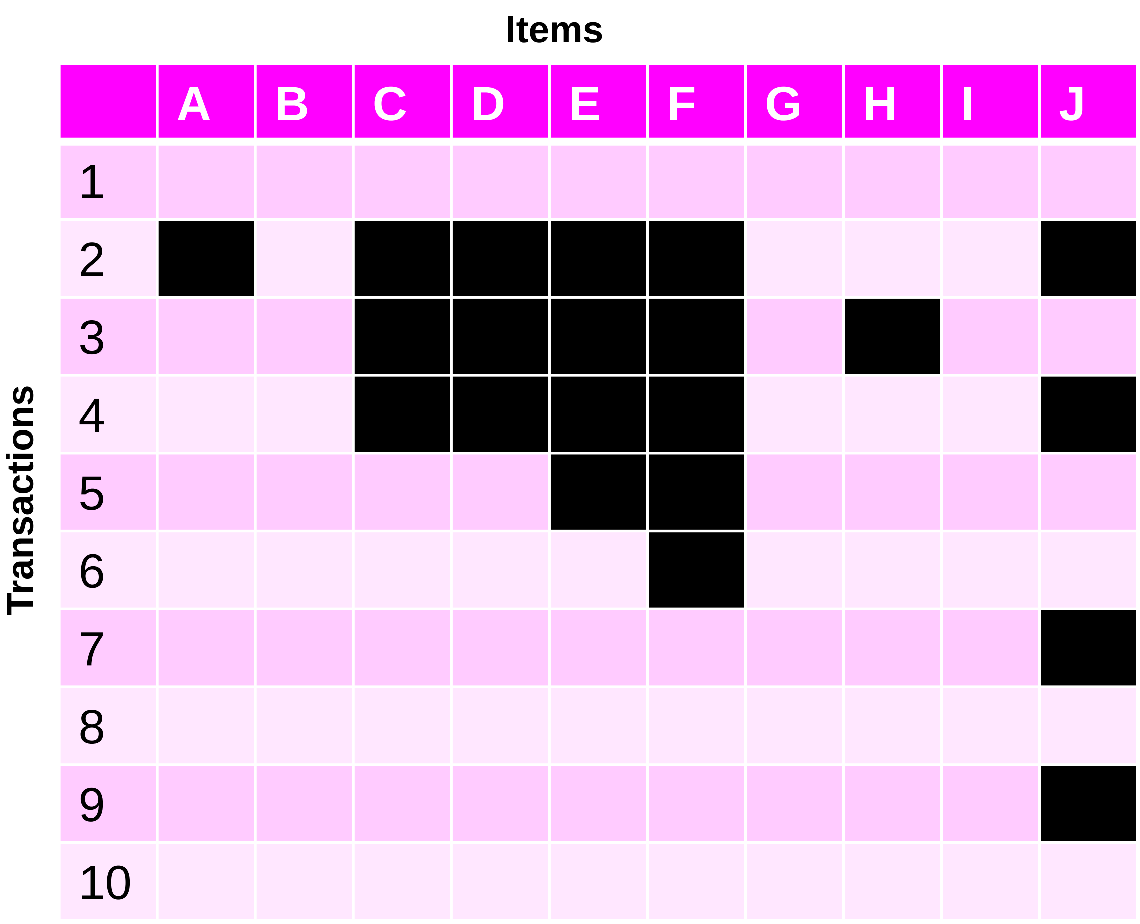
	A	B	C	D	E	F	G	H	I	J
1										
2	■		■	■	■	■				■
3			■	■	■	■		■		
4			■	■	■	■				■
5					■	■				
6						■				
7										■
8										
9										■
10										

Support threshold (by count) : 5
Frequent itemsets: {F}

Support threshold (by count): 4
Frequent itemsets: {E}, {F}, {E,F}, {J}

Support threshold (by count): 3
Frequent itemsets:
All subsets of {C,D,E,F} + {J}

An illustrative example

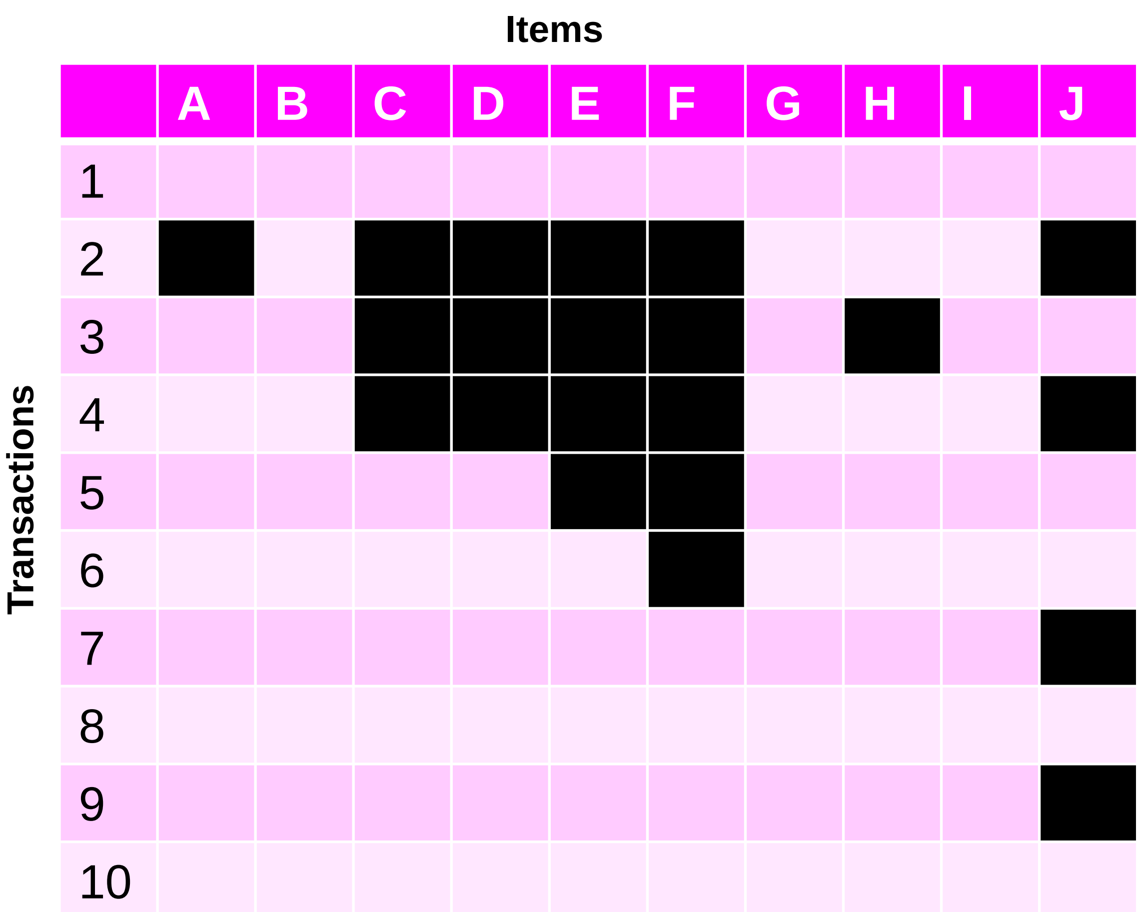


Support threshold (by count) : 5
Frequent itemsets: {F}
Maximal itemsets: ?

Support threshold (by count): 4
Frequent itemsets: {E}, {F}, {E,F}, {J}
Maximal itemsets: ?

Support threshold (by count): 3
Frequent itemsets:
All subsets of {C,D,E,F} + {J}
Maximal itemsets: ?

An illustrative example

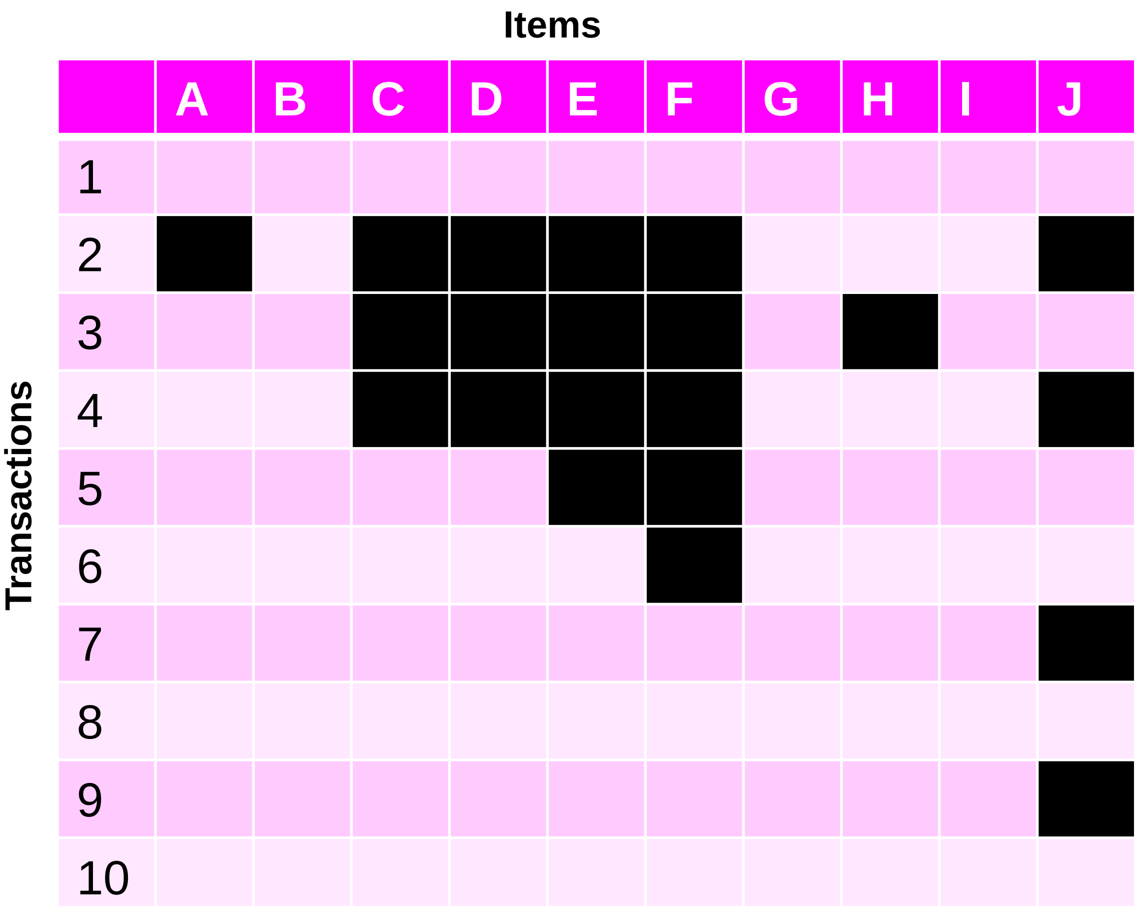


Support threshold (by count) : 5
Frequent itemsets: {F}
Maximal itemsets: {F}

Support threshold (by count): 4
Frequent itemsets: {E}, {F}, {E,F}, {J}
Maximal itemsets: ?

Support threshold (by count): 3
Frequent itemsets:
All subsets of {C,D,E,F} + {J}
Maximal itemsets: ?

An illustrative example

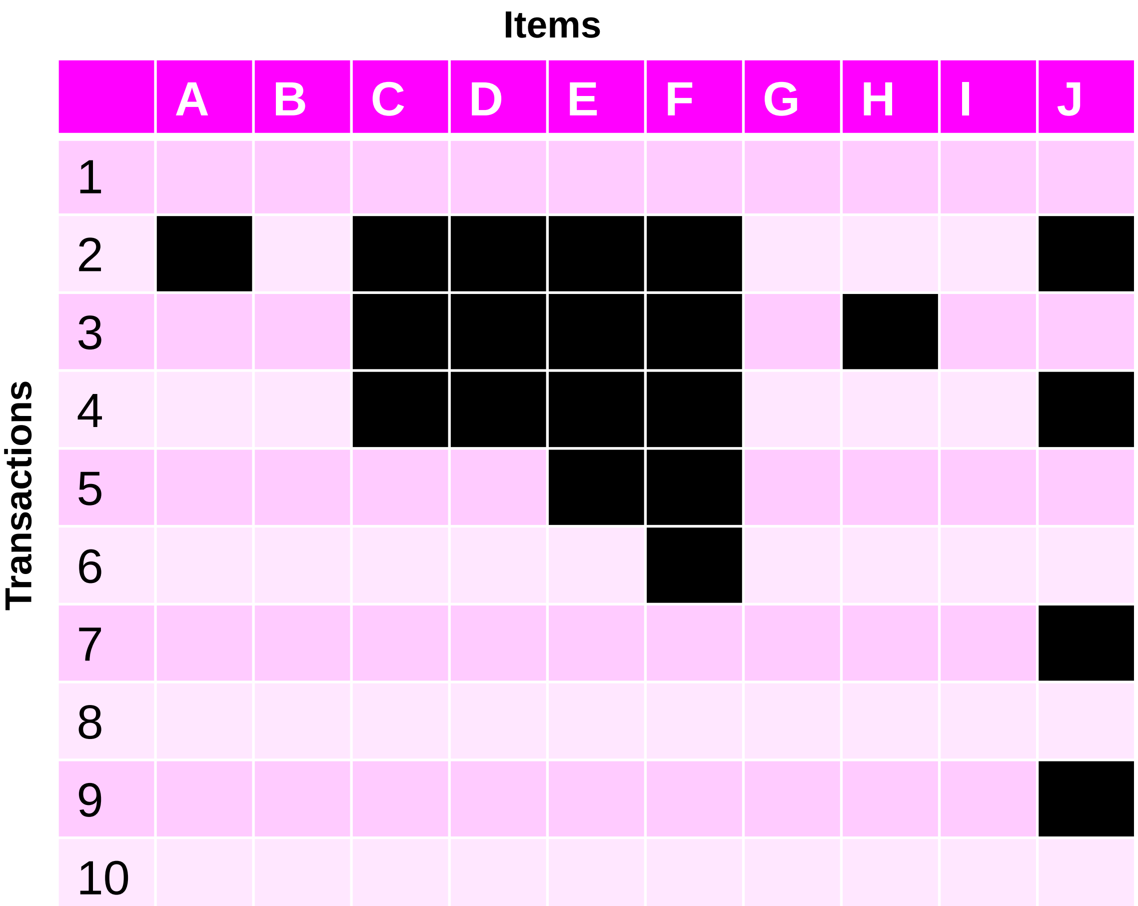


Support threshold (by count) : 5
Frequent itemsets: {F}
Maximal itemsets: {F}

Support threshold (by count): 4
Frequent itemsets: {E}, {F}, {E,F}, {J}
Maximal itemsets: {E,F}, {J}

Support threshold (by count): 3
Frequent itemsets:
All subsets of {C,D,E,F} + {J}
Maximal itemsets: ?

An illustrative example

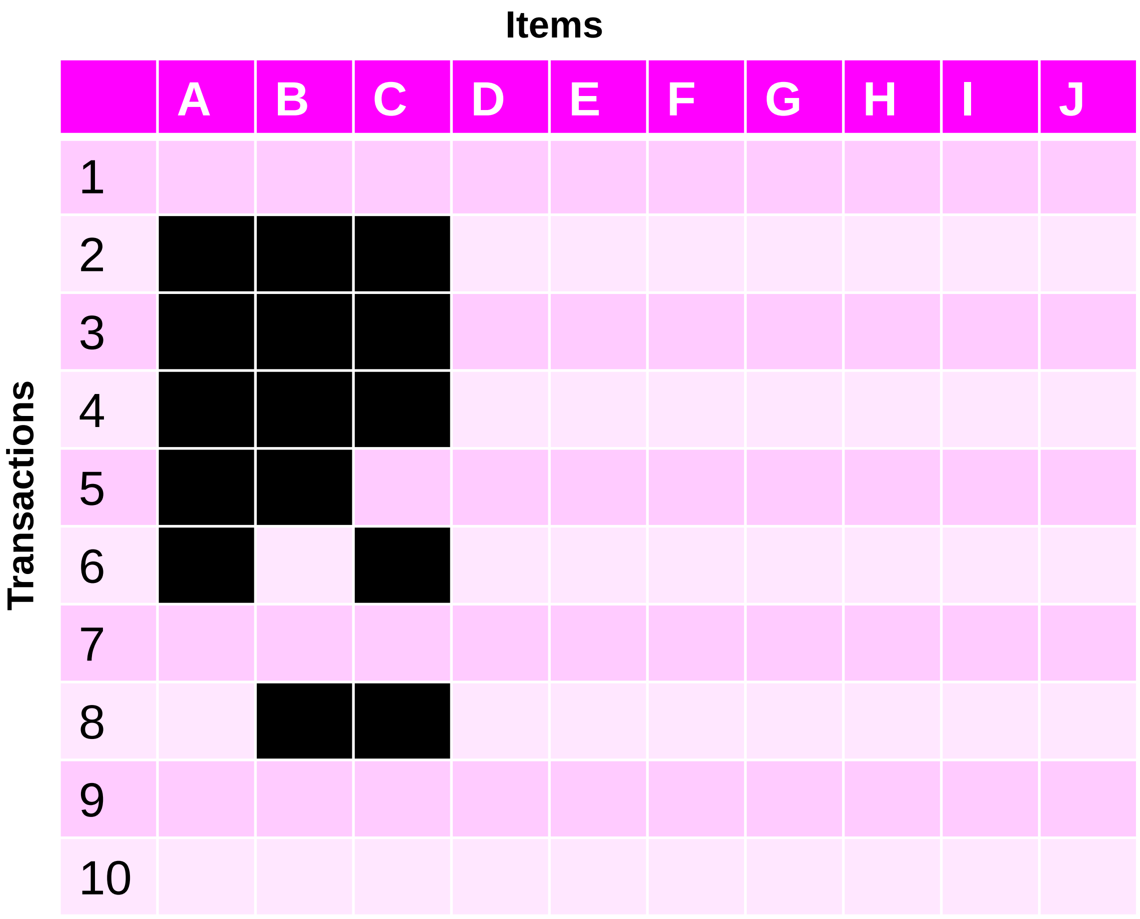


Support threshold (by count) : 5
Frequent itemsets: {F}
Maximal itemsets: {F}

Support threshold (by count): 4
Frequent itemsets: {E}, {F}, {E,F}, {J}
Maximal itemsets: {E,F}, {J}

Support threshold (by count): 3
Frequent itemsets:
All subsets of {C,D,E,F} + {J}
Maximal itemsets:
{C,D,E,F}, {J}

Another illustrative example



Support threshold (by count) : 5
Maximal itemsets: {A}, {B}, {C}

Support threshold (by count): 4
Maximal itemsets: {A,B}, {A,C},{B,C}

Support threshold (by count): 3
Maximal itemsets: {A,B,C}

Closed Itemset

- An itemset X is closed if none of its immediate supersets has the same support as the itemset X.
- X is not closed if at least one of its immediate supersets has support count as X.

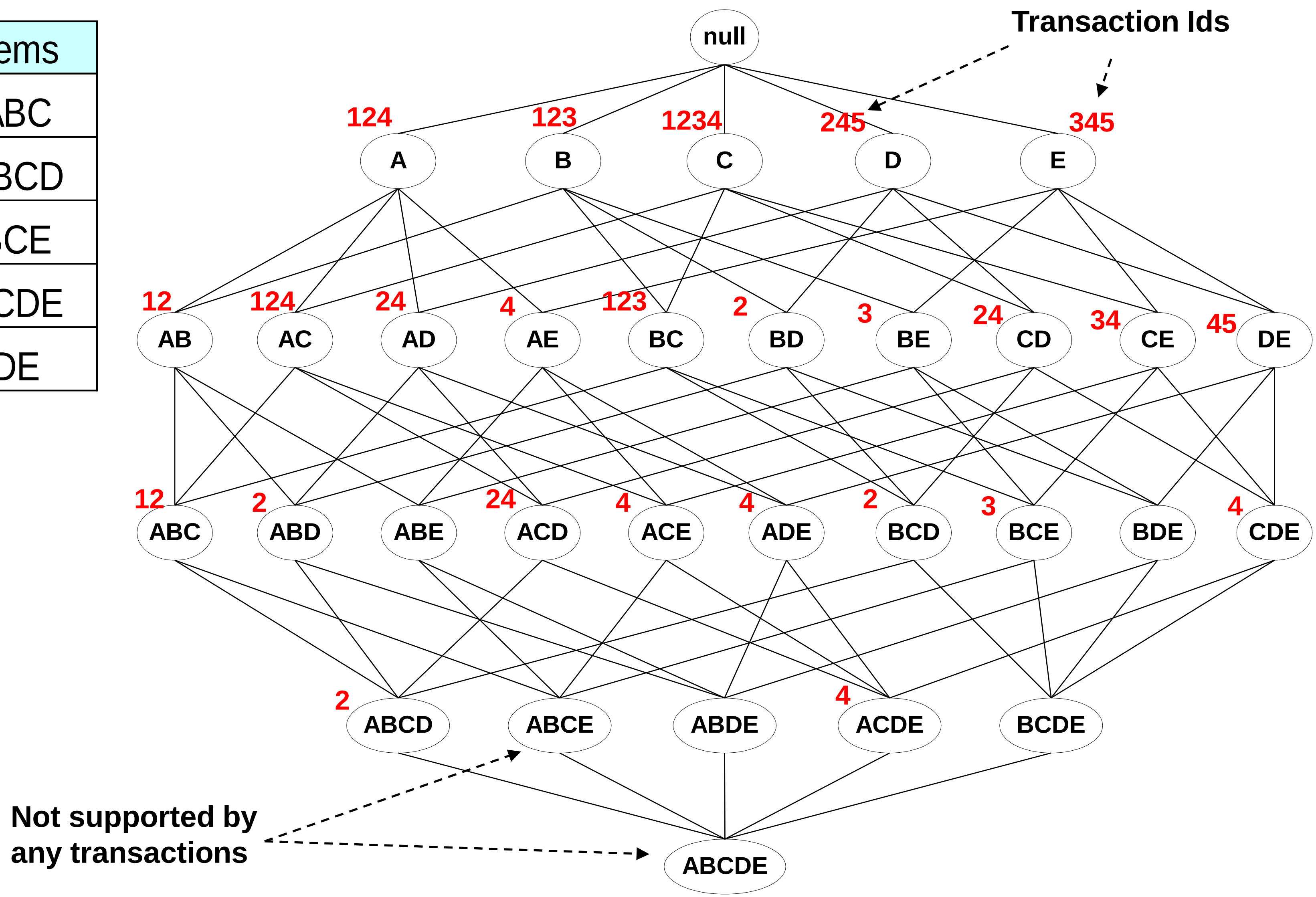
TID	Items
1	{A,B}
2	{B,C,D}
3	{A,B,C,D}
4	{A,B,D}
5	{A,B,C,D}

Itemset	Support
{A}	4
{B}	5
{C}	3
{D}	4
{A,B}	4
{A,C}	2
{A,D}	3
{B,C}	3
{B,D}	4
{C,D}	3

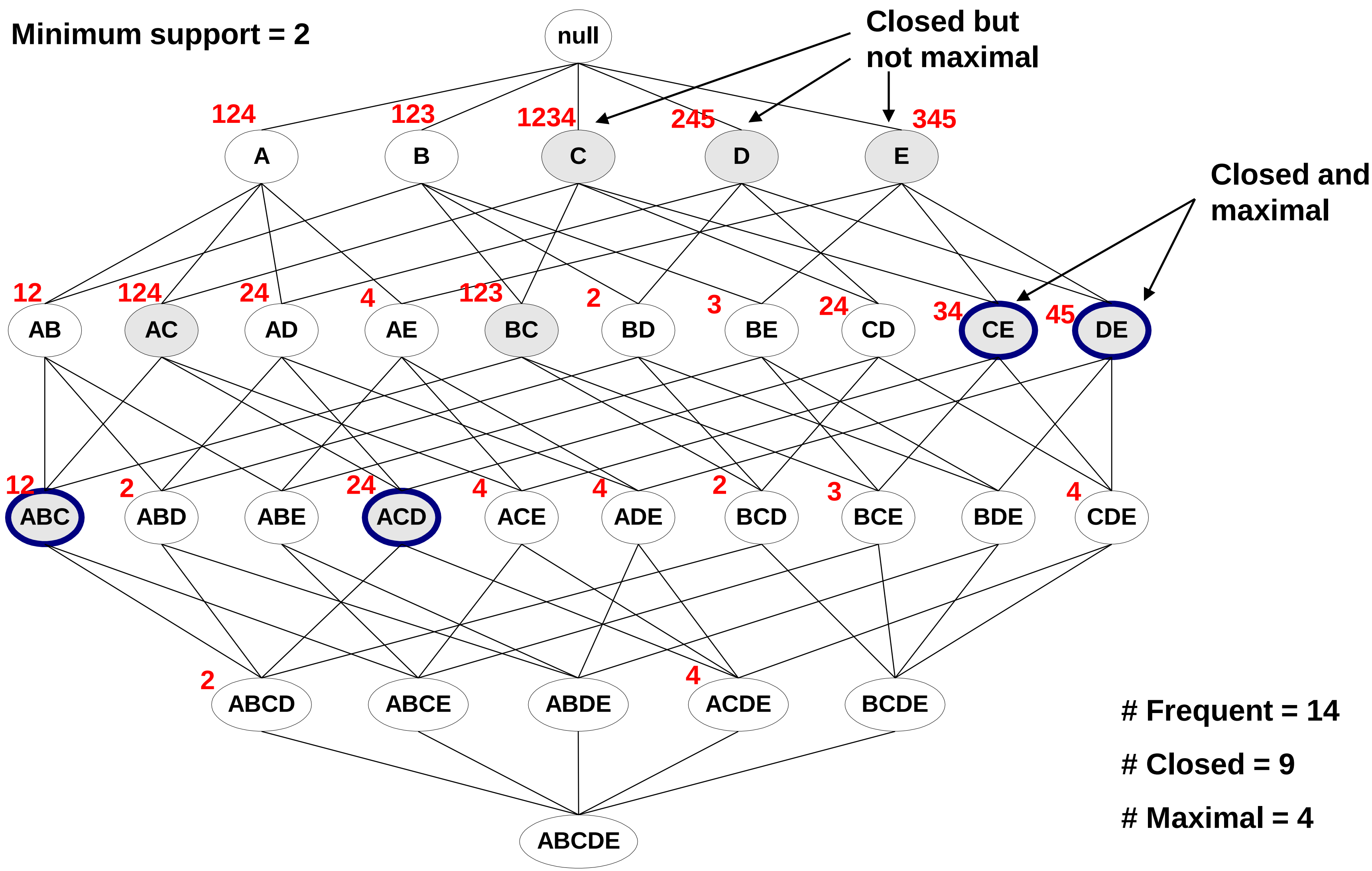
Itemset	Support
{A,B,C}	2
{A,B,D}	3
{A,C,D}	2
{B,C,D}	2
{A,B,C,D}	2

Maximal vs Closed Itemsets

TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



Maximal vs Closed Frequent Itemsets



\# Frequent = 14

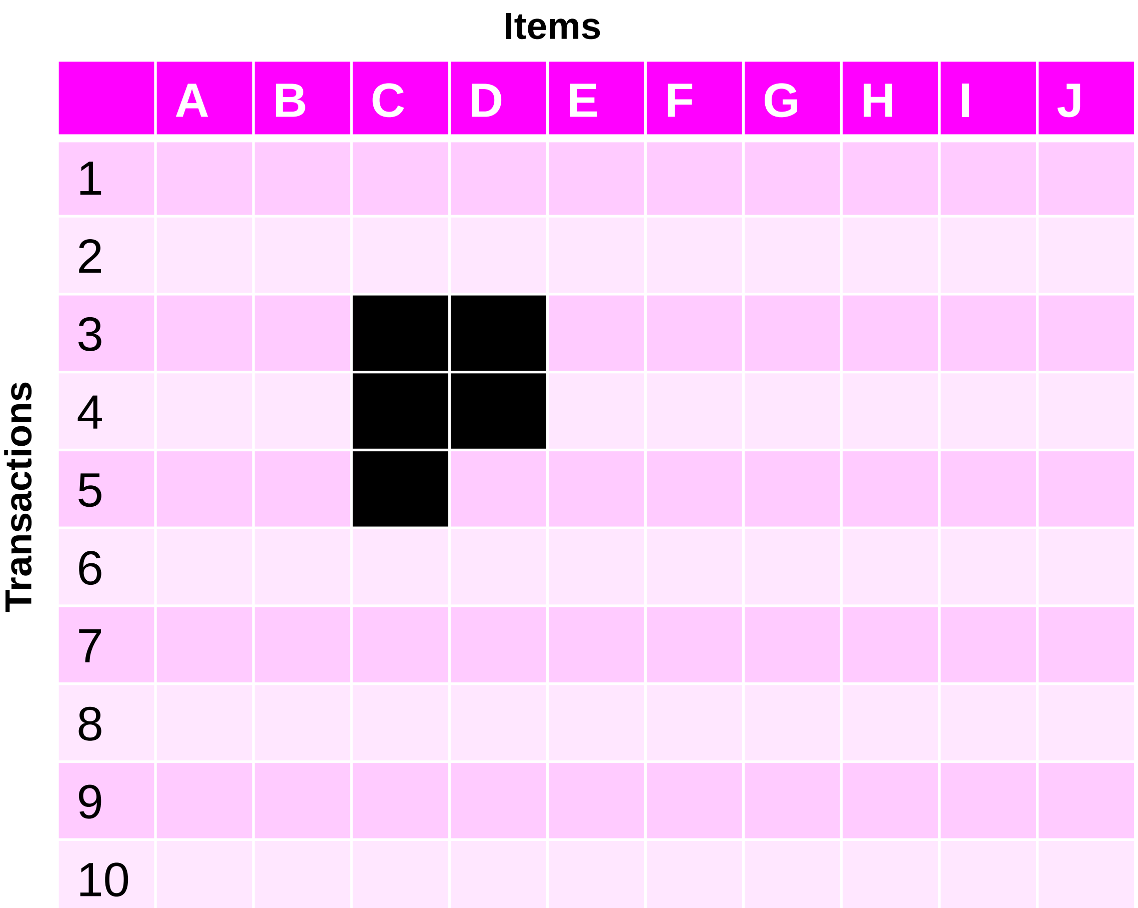
\# Closed = 9

\# Maximal = 4

What are the Closed Itemsets in this Data?

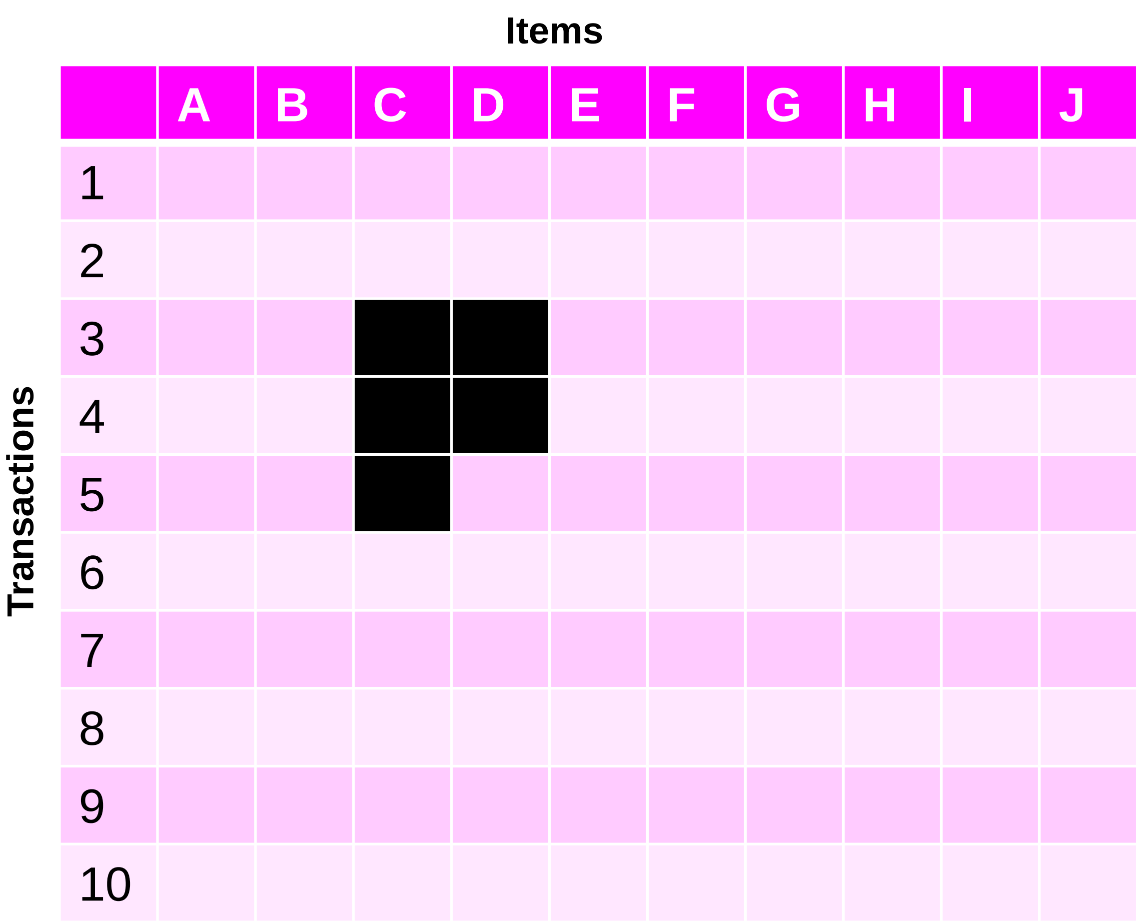
TID	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
1	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**

Example 1



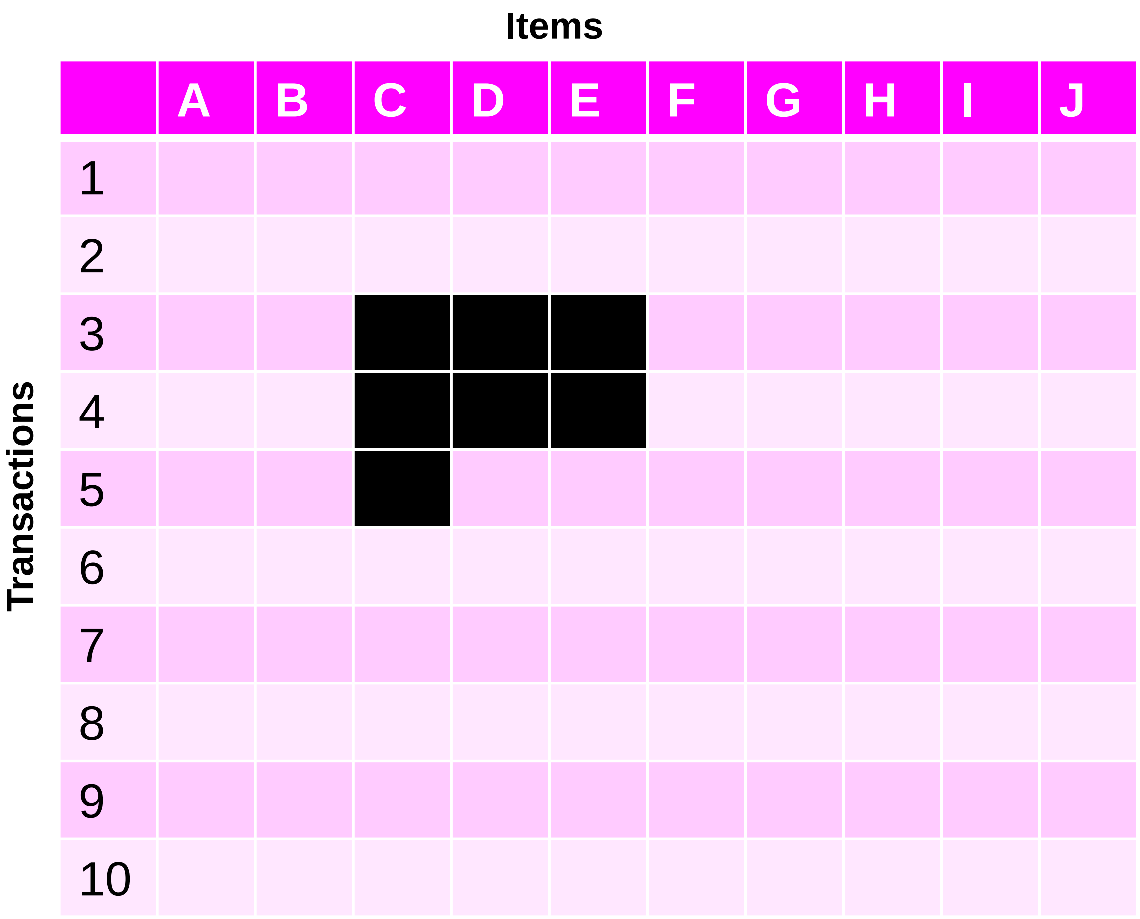
Itemsets	Support (counts)	Closed itemsets
{C}	3	
{D}	2	
{C,D}	2	

Example 1



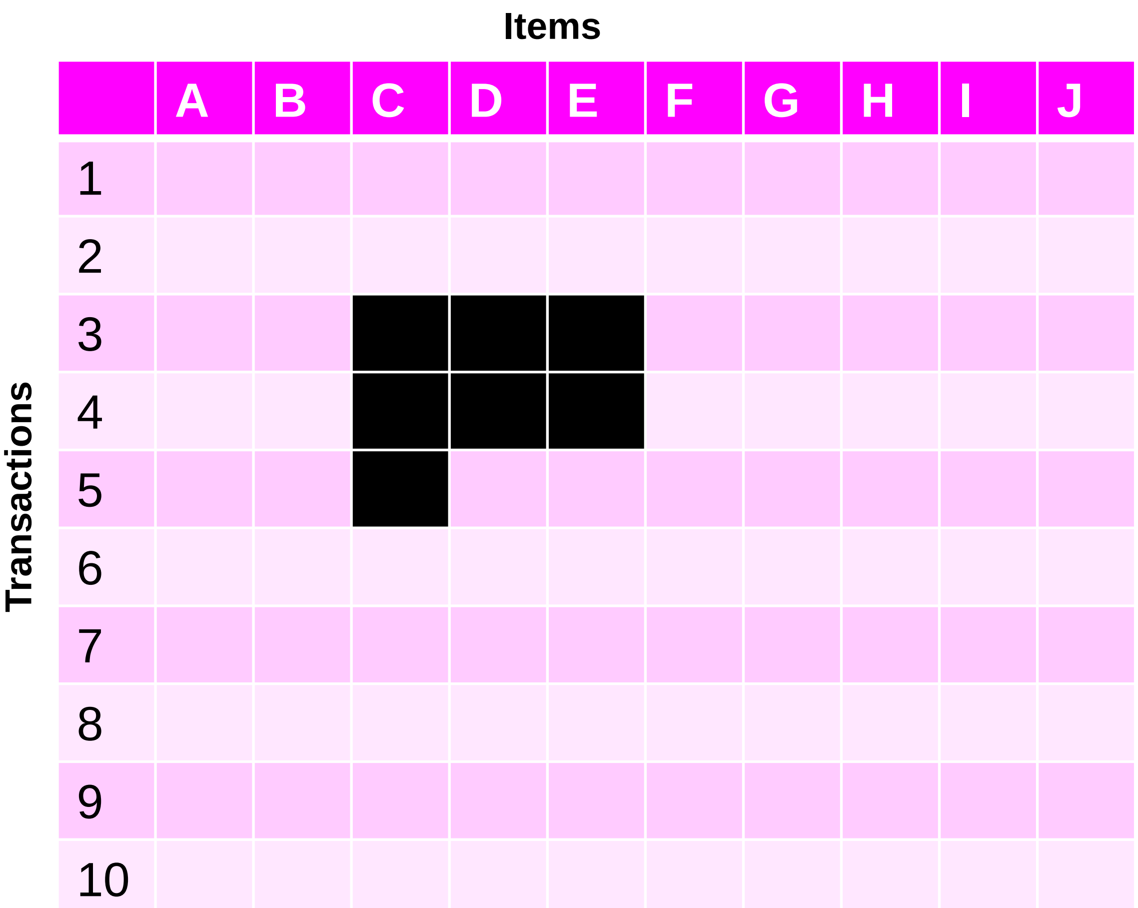
Itemsets	Support (counts)	Closed itemsets
{C}	**3**	✓
{D}	2	
{C,D}	**2**	✓

Example 2



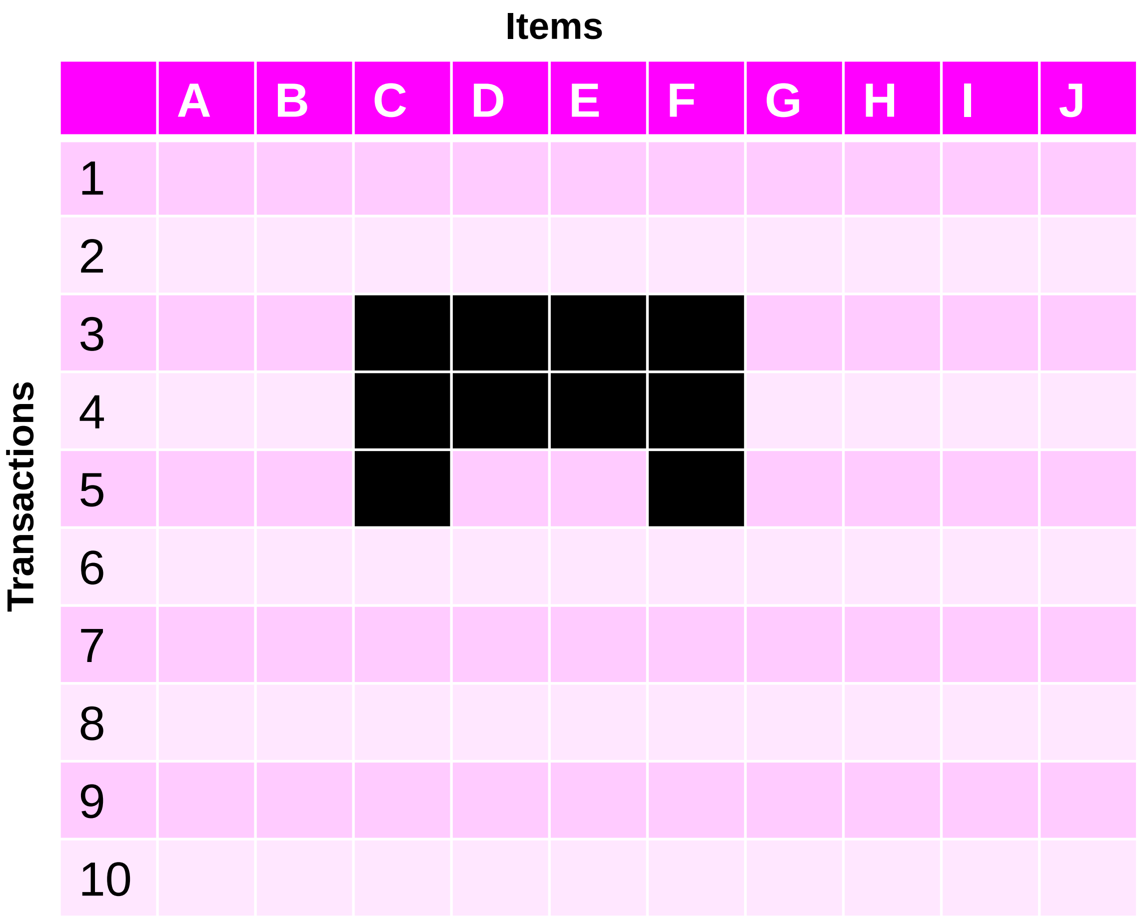
Itemsets	Support (counts)	Closed itemsets
{C}	3	
{D}	2	
{E}	2	
{C,D}	2	
{C,E}	2	
{D,E}	2	
{C,D,E}	2	

Example 2



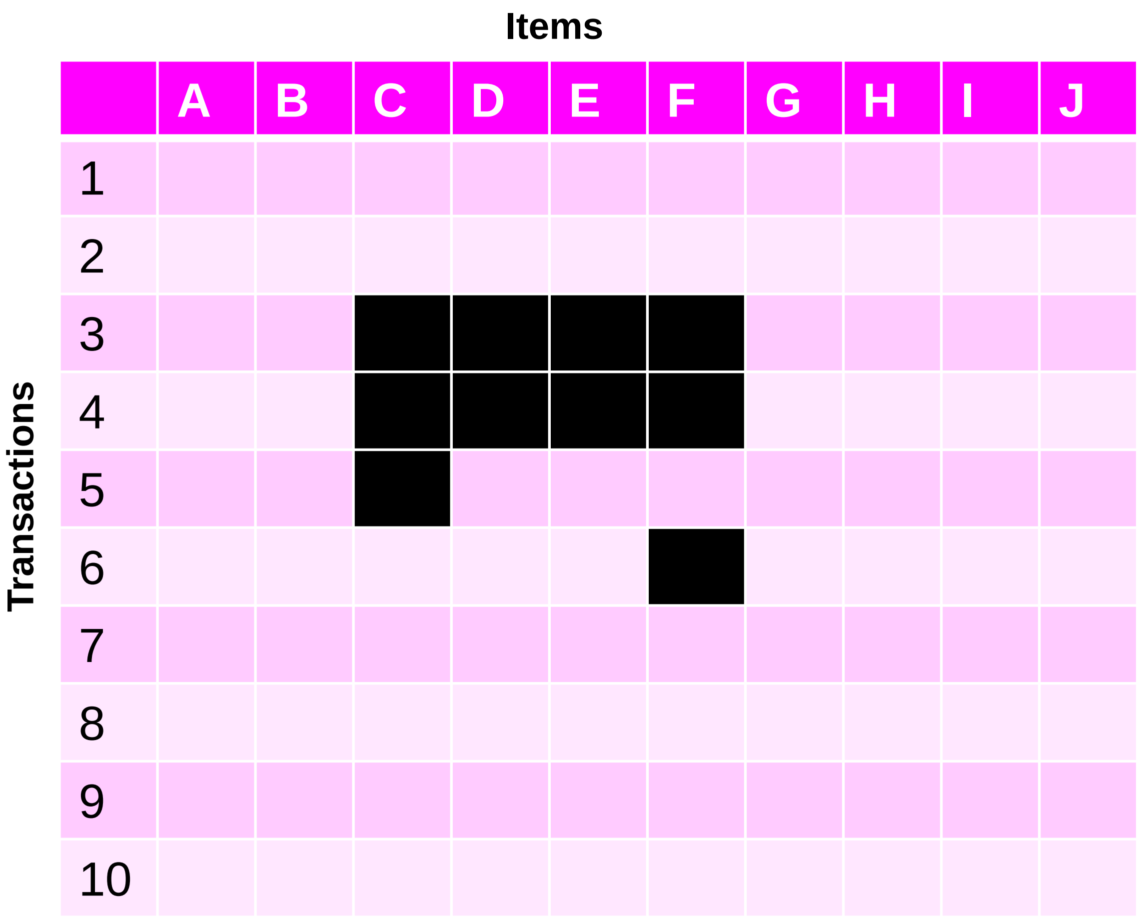
Itemsets	Support (counts)	Closed itemsets
{C}	**3**	✓
{D}	2	
{E}	2	
{C,D}	2	
{C,E}	2	
{D,E}	2	
{C,D,E}	**2**	✓

Example 3



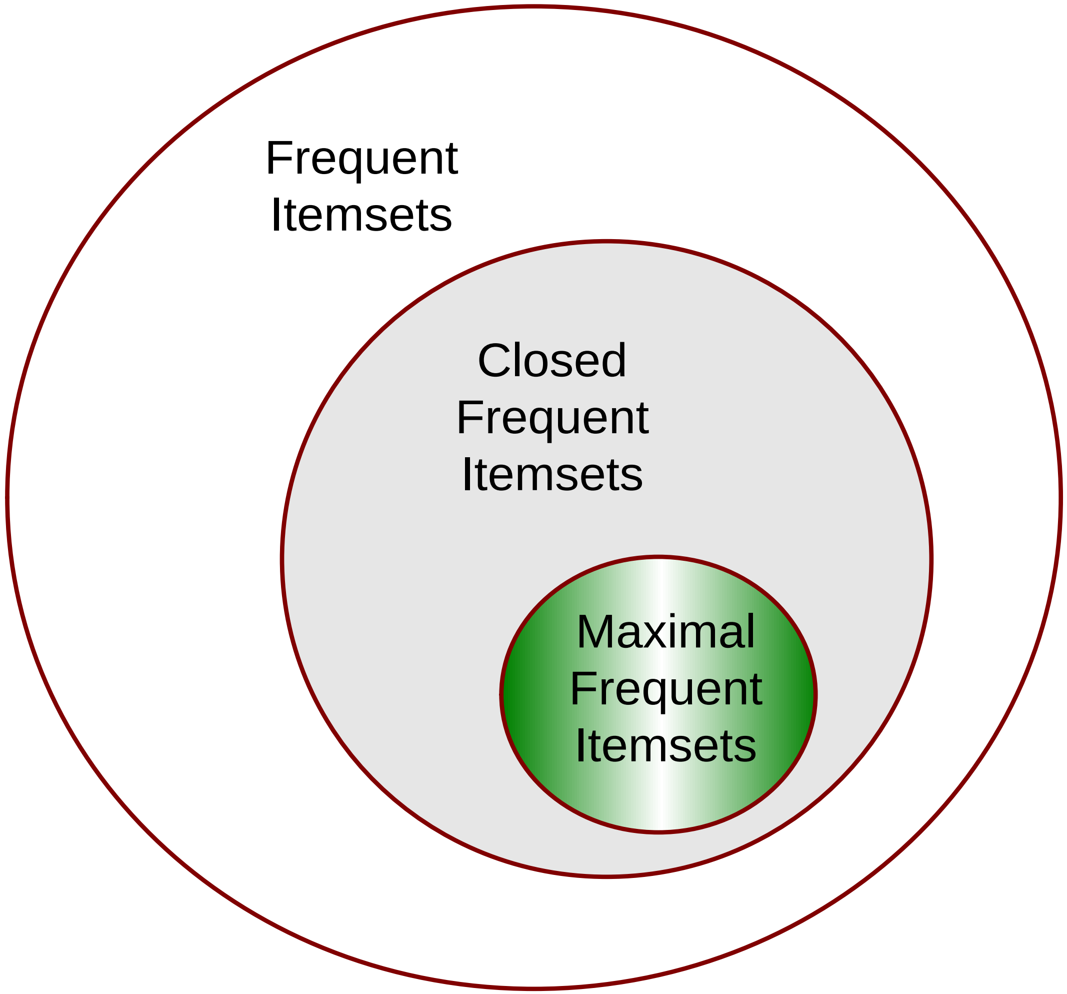
Closed itemsets:
{C,D,E,F}: 2,
{C,F}: 3

Example 4



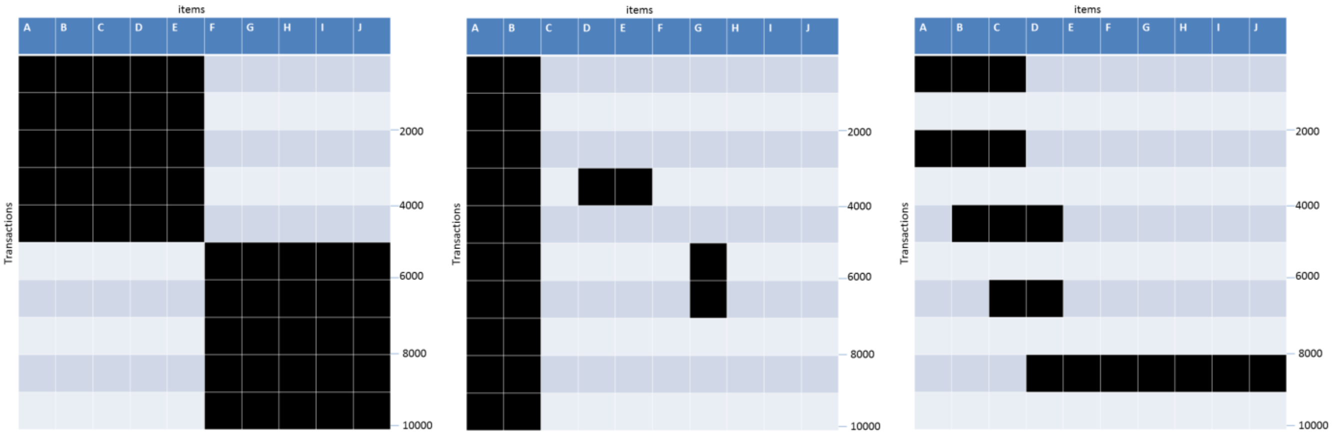
Closed itemsets:
{C,D,E,F}: 2,
{C}: 3,
{F}: 3

Maximal vs Closed Itemsets



Example question

- Given the following transaction data sets (dark cells indicate presence of an item in a transaction) and a support threshold of 20%, answer the following questions



a. **What is the number of frequent itemsets for each dataset? Which dataset will produce the most number of frequent itemsets?**
b. **Which dataset will produce the longest frequent itemset?**
c. **Which dataset will produce frequent itemsets with highest maximum support?**
d. **Which dataset will produce frequent itemsets containing items with widely varying support levels (i.e., itemsets containing items with mixed support, ranging from 20% to more than 70%)?**
e. **What is the number of maximal frequent itemsets for each dataset? Which dataset will produce the most number of maximal frequent itemsets?**
f. **What is the number of closed frequent itemsets for each dataset? Which dataset will produce the most number of closed frequent itemsets?**

Pattern Evaluation

- Association rule algorithms can produce large number of rules

- Interestingness measures can be used to prune/rank the patterns
  - In the original formulation, support & confidence are the only measures used

Computing Interestingness Measure

- Given $X \rightarrow Y$ or $\{X,Y\}$, information needed to compute interestingness can be obtained from a contingency table

Contingency table

	Y	$\overline{Y}$	
X	f_{11}	f_{10}	f_{1+}
$\overline{X}$	f_{01}	f_{00}	f_{o+}
	f_{+1}	f_{+0}	N

f_{11}: support of X and Y
f_{10}: support of X and $\overline{Y}$
f_{01}: support of $\overline{X}$ and Y
f_{00}: support of $\overline{X}$ and $\overline{Y}$

Used to define various measures

- support, confidence, Gini, entropy, etc.

Drawback of Confidence

Customers	Tea	Coffee	…
C1	0	1	…
C2	1	0	…
C3	1	1	…
C4	1	0	…
…			

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea $\rightarrow$ Coffee

Confidence $\cong$ P(Coffee|Tea) = 15/20 = 0.75

Confidence > 50%, meaning people who drink tea are more likely to drink coffee than not drink coffee

So rule seems reasonable

Drawback of Confidence

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea $\rightarrow$ Coffee

Confidence= P(Coffee|Tea) = 15/20 = 0.75

but P(Coffee) = 0.9, which means knowing that a person drinks tea reduces the probability that the person drinks coffee!

$\Rightarrow$ Note that $P(\text{Coffee}|\overline{\text{Tea}}) = 75/80 = 0.9375$

Measure for Association Rules

- So, what kind of rules do we really want?
  - Confidence($X \rightarrow Y$) should be sufficiently high
    - To ensure that people who buy X will more likely buy Y than not buy Y
  - Confidence($X \rightarrow Y$) > support(Y)
    - Otherwise, rule will be misleading because having item X actually reduces the chance of having item Y in the same transaction
    - Is there any measure that capture this constraint?
      - Answer: Yes. There are many of them.

Statistical Independence

- The criterion

  $$\text{confidence}(X \rightarrow Y) = \text{support}(Y)$$

  is equivalent to:
  - $P(Y|X) = P(Y)$
  - $P(X,Y) = P(X) \times P(Y)$

  If $P(X,Y) > P(X) \times P(Y)$: X & Y are positively correlated

  If $P(X,Y) < P(X) \times P(Y)$: X & Y are negatively correlated

Measures that take into account statistical dependence

$$Lift = \frac{P(Y \mid X)}{P(Y)}$$

$$Interest = \frac{P(X,Y)}{P(X)P(Y)}$$

<u>lift</u> is used for <u>rules</u> while <u>interest</u> is used for <u>itemsets</u>

$$PS = P(X,Y) - P(X)P(Y)$$

Piatesky-Shapiro Measure

$$\phi - coefficient = \frac{P(X,Y) - P(X)P(Y)}{\sqrt{P(X)[1-P(X)]P(Y)[1-P(Y)]}}$$

	Y	$\overline{\text{Y}}$	
X	f_{11}	f_{10}	f_{1+}
$\overline{\text{X}}$	f_{01}	f_{00}	f_{o+}
	f_{+1}	f_{+0}	N

Correlation Analysis

$$\frac{f_{11}f_{00} - f_{01}f_{10}}{\sqrt{f_{1+}f_{+1}f_{0+}f_{+0}}}$$

Example: Lift/Interest

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea $\rightarrow$ Coffee

Confidence= P(Coffee|Tea) = 0.75

but P(Coffee) = 0.9

$\Rightarrow$ Lift = 0.75/0.9= 0.8333 (< 1, therefore is negatively associated)

So, is it enough to use confidence/lift for pruning?

Lift or Interest

	Y	$\overline{Y}$	
X	10	0	10
$\overline{X}$	0	90	90
	10	90	100

	Y	$\overline{Y}$	
X	90	0	90
$\overline{X}$	0	10	10
	90	10	100

$$Lift = \frac{0.1}{(0.1)(0.1)} = 10$$

$$Lift = \frac{0.9}{(0.9)(0.9)} = 1.11$$

Statistical independence:

If P(X,Y)=P(X)P(Y) => Lift = 1

There are lots of measures proposed in the literature

#	Measure	Formula
1	ϕ-coefficient	$\frac{P(A,B)-P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$
2	Goodman-Kruskal's (λ)	$\frac{\sum_j \max_k P(A_j,B_k)+\sum_k \max_j P(A_j,B_k)-\max_j P(A_j)-\max_k P(B_k)}{2-\max_j P(A_j)-\max_k P(B_k)}$
3	Odds ratio (α)	$\frac{P(A,B)P(\overline{A},\overline{B})}{P(A,\overline{B})P(\overline{A},B)}$
4	Yule's Q	$\frac{P(A,B)P(\overline{AB})-P(A,\overline{B})P(\overline{A},B)}{P(A,B)P(\overline{AB})+P(A,\overline{B})P(\overline{A},B)} = \frac{\alpha-1}{\alpha+1}$
5	Yule's Y	$\frac{\sqrt{P(A,B)P(\overline{AB})}-\sqrt{P(A,\overline{B})P(\overline{A},B)}}{\sqrt{P(A,B)P(\overline{AB})}+\sqrt{P(A,\overline{B})P(\overline{A},B)}} = \frac{\sqrt{\alpha}-1}{\sqrt{\alpha}+1}$
6	Kappa (κ)	$\frac{P(A,B)+P(\overline{A},\overline{B})-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A)P(B)-P(\overline{A})P(\overline{B})}$
7	Mutual Information (M)	$\frac{\sum_i \sum_j P(A_i,B_j)\log\frac{P(A_i,B_j)}{P(A_i)P(B_j)}}{\min(-\sum_i P(A_i)\log P(A_i), -\sum_j P(B_j)\log P(B_j))}$
8	J-Measure (J)	$\max\Big(P(A,B)\log(\frac{P(B\|A)}{P(B)}) + P(A\overline{B})\log(\frac{P(\overline{B}\|A)}{P(\overline{B})}),$ $P(A,B)\log(\frac{P(A\|B)}{P(A)}) + P(\overline{A}B)\log(\frac{P(\overline{A}\|B)}{P(\overline{A})})\Big)$
9	Gini index (G)	$\max\Big(P(A)[P(B\|A)^2+P(\overline{B}\|A)^2]+P(\overline{A})[P(B\|\overline{A})^2+P(\overline{B}\|\overline{A})^2]$ $-P(B)^2-P(\overline{B})^2,$ $P(B)[P(A\|B)^2+P(\overline{A}\|B)^2]+P(\overline{B})[P(A\|\overline{B})^2+P(\overline{A}\|\overline{B})^2]$ $-P(A)^2-P(\overline{A})^2\Big)$
10	Support (s)	$P(A,B)$
11	Confidence (c)	$\max(P(B\|A),P(A\|B))$
12	Laplace (L)	$\max\Big(\frac{NP(A,B)+1}{NP(A)+2},\frac{NP(A,B)+1}{NP(B)+2}\Big)$
13	Conviction (V)	$\max\Big(\frac{P(A)P(\overline{B})}{P(A\overline{B})},\frac{P(B)P(\overline{A})}{P(B\overline{A})}\Big)$
14	Interest (I)	$\frac{P(A,B)}{P(A)P(B)}$
15	cosine (IS)	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
16	Piatetsky-Shapiro's (PS)	$P(A,B)-P(A)P(B)$
17	Certainty factor (F)	$\max\Big(\frac{P(B\|A)-P(B)}{1-P(B)},\frac{P(A\|B)-P(A)}{1-P(A)}\Big)$
18	Added Value (AV)	$\max(P(B\|A)-P(B),P(A\|B)-P(A))$
19	Collective strength (S)	$\frac{P(A,B)+P(\overline{AB})}{P(A)P(B)+P(\overline{A})P(\overline{B})} \times \frac{1-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A,B)-P(\overline{AB})}$
20	Jaccard (ζ)	$\frac{P(A,B)}{P(A)+P(B)-P(A,B)}$
21	Klosgen (K)	$\sqrt{P(A,B)}\max(P(B\|A)-P(B),P(A\|B)-P(A))$

Comparing Different Measures

10 examples of contingency tables:

Example	f_{11}	f_{10}	f_{01}	f_{00}
E1	8123	83	424	1370
E2	8330	2	622	1046
E3	9481	94	127	298
E4	3954	3080	5	2961
E5	2886	1363	1320	4431
E6	1500	2000	500	6000
E7	4000	2000	1000	3000
E8	4000	2000	2000	2000
E9	1720	7121	5	1154
E10	61	2483	4	7452

Rankings of contingency tables using various measures:

#	ϕ	λ	α	Q	Y	κ	M	J	G	s	c	L	V	I	IS	PS	F	AV	S	ζ	K
E1	1	1	3	3	3	1	2	2	1	3	5	5	4	6	2	2	4	6	1	2	5
E2	2	2	1	1	1	2	1	3	2	2	1	1	1	8	3	5	1	8	2	3	6
E3	3	3	4	4	4	3	3	8	7	1	4	4	6	10	1	8	6	10	3	1	10
E4	4	7	2	2	2	5	4	1	3	6	2	2	2	4	4	1	2	3	4	5	1
E5	5	4	8	8	8	4	7	5	4	7	9	9	9	3	6	3	9	4	5	6	3
E6	6	6	7	7	7	7	6	4	6	9	8	8	7	2	8	6	7	2	7	8	2
E7	7	5	9	9	9	6	8	6	5	4	7	7	8	5	5	4	8	5	6	4	4
E8	8	9	10	10	10	8	10	10	8	4	10	10	10	9	7	7	10	9	8	7	9
E9	9	9	5	5	5	9	9	7	9	8	3	3	3	7	9	9	3	7	9	9	8
E10	10	8	6	6	6	10	5	9	10	10	6	6	5	1	10	10	5	1	10	10	7

Property under Variable Permutation

	B	$\overline{B}$
A	p	q
$\overline{A}$	r	s

	A	$\overline{A}$
B	p	r
$\overline{B}$	q	s

Does M(A,B) = M(B,A)?

Symmetric measures:

- support, lift, collective strength, cosine, Jaccard, etc

Asymmetric measures:

- confidence, conviction, Laplace, J-measure, etc

Property under Row/Column Scaling

Grade-Gender Example (Mosteller, 1968):

Underlying association should be independent of the relative number of male and female students in the samples.

	Female	Male	
High	2	3	5
Low	1	4	5
	3	7	10

	Female	Male	
High	4	30	34
Low	2	40	42
	6	70	76

2x 10x

Invariant measures:

- odds ratio, etc

Property under Inversion Operation

	A	B	C	D	E	F
Transaction 1 →	1	0	0	1	0	0
.	0	0	1	1	1	0
	0	0	1	1	1	0
.	0	0	1	1	1	0
.	0	1	1	0	1	1
	0	0	1	1	1	0
.	0	0	1	1	1	0
.	0	0	1	1	1	0
	0	0	1	1	1	0
Transaction N →	1	0	0	1	0	0

(a) A, B   (b) C, D   (c) E, F

Example: ϕ-Coefficient

- ϕ-coefficient is analogous to correlation coefficient for continuous variables

	Y	$\overline{Y}$	
X	60	10	70
$\overline{X}$	10	20	30
	70	30	100

	Y	$\overline{Y}$	
X	20	10	30
$\overline{X}$	10	60	70
	30	70	100

$$\phi = \frac{0.6 - 0.7 \times 0.7}{\sqrt{0.7 \times 0.3 \times 0.7 \times 0.3}} = 0.5238$$

$$\phi = \frac{0.2 - 0.3 \times 0.3}{\sqrt{0.7 \times 0.3 \times 0.7 \times 0.3}} = 0.5238$$

ϕ Coefficient is the same for both tables

Property under Null Addition

	B	$\bar{\mathbf{B}}$
A	p	q
$\bar{\mathbf{A}}$	r	s

$\Rightarrow$

	B	$\bar{\mathbf{B}}$
A	p	q
$\bar{\mathbf{A}}$	r	s + k

Invariant measures:

- support, cosine, Jaccard, etc

Non-invariant measures:

- correlation, Gini, mutual information, odds ratio, etc

Different Measures have Different Properties

Symbol	Measure	Inversion	Null Addition	Scaling
ϕ	ϕ-coefficient	Yes	No	No
α	odds ratio	Yes	No	Yes
κ	Cohen's	Yes	No	No
I	Interest	No	No	No
IS	Cosine	No	Yes	No
PS	Piatetsky-Shapiro's	Yes	No	No
S	Collective strength	Yes	No	No
ζ	Jaccard	No	Yes	No
h	All-confidence	No	No	No
s	Support	No	No	No

Simpson's Paradox

Buy HDTV	Buy Exercise Machine		
	Yes	No	
Yes	99	81	180
No	54	66	120
	153	147	300

$$c(\{\text{HDTV} = \text{Yes}\} \rightarrow \{\text{Exercise Machine} = \text{Yes}\}) = 99/180 = 55\%$$

$$c(\{\text{HDTV} = \text{No}\} \rightarrow \{\text{Exercise Machine} = \text{Yes}\}) = 54/120 = 45\%$$

=> Customers who buy HDTV are more likely to buy exercise machines

Simpson's Paradox

Customer Group	Buy HDTV	Buy Exercise Machine		Total
		Yes	No	
College Students	Yes	1	9	10
	No	4	30	34
Working Adult	Yes	98	72	170
	No	50	36	86

College students:

$$c(\{\text{HDTV} = \text{Yes}\} \rightarrow \{\text{Exercise Machine} = \text{Yes}\}) = 1/10 = 10\%$$

$$c(\{\text{HDTV} = \text{No}\} \rightarrow \{\text{Exercise Machine} = \text{Yes}\}) = 4/34 = 11.8\%$$

Working adults:

$$c(\{\text{HDTV} = \text{Yes}\} \rightarrow \{\text{Exercise Machine} = \text{Yes}\}) = 98/170 = 57.7\%$$

$$c(\{\text{HDTV} = \text{No}\} \rightarrow \{\text{Exercise Machine} = \text{Yes}\}) = 50/86 = 58.1\%$$

Simpson's Paradox

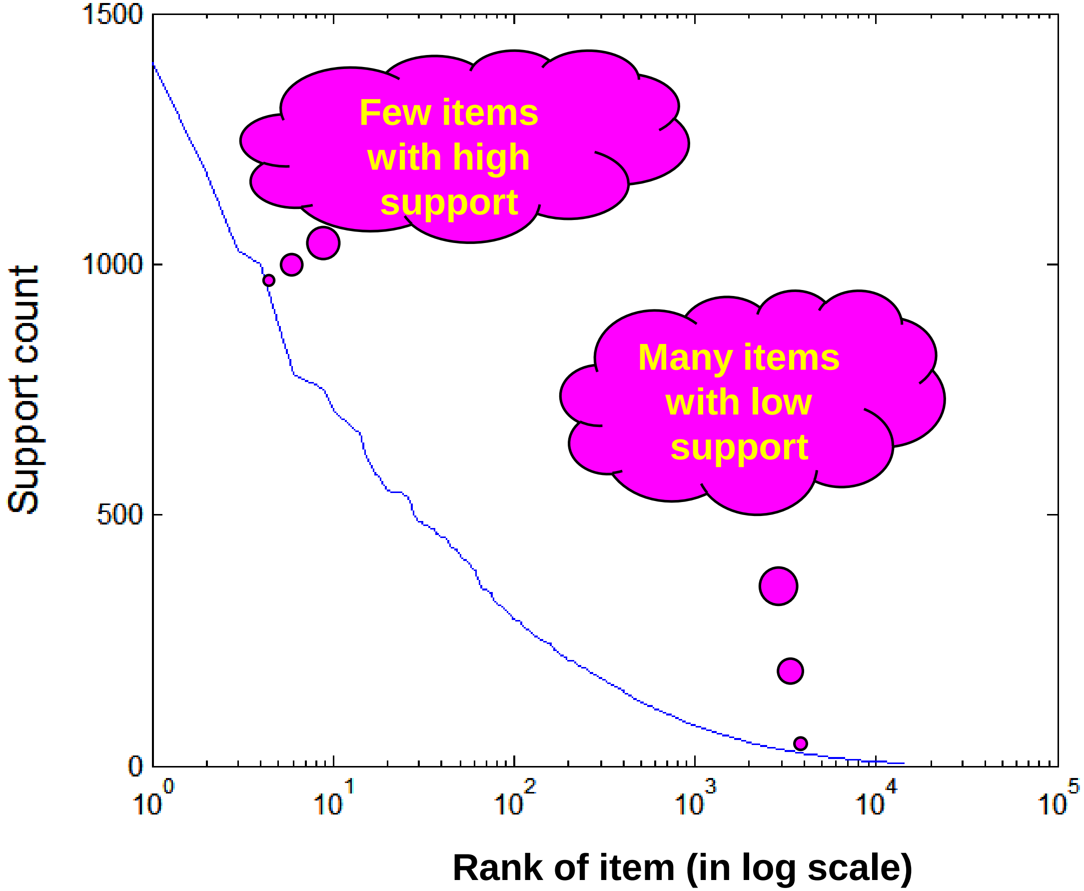
- Observed relationship in data may be influenced by the presence of other confounding factors (hidden variables)
  - Hidden variables may cause the observed relationship to disappear or reverse its direction!

- Proper stratification is needed to avoid generating spurious patterns

Effect of Support Distribution on Association Mining

- Many real data sets have skewed support distribution

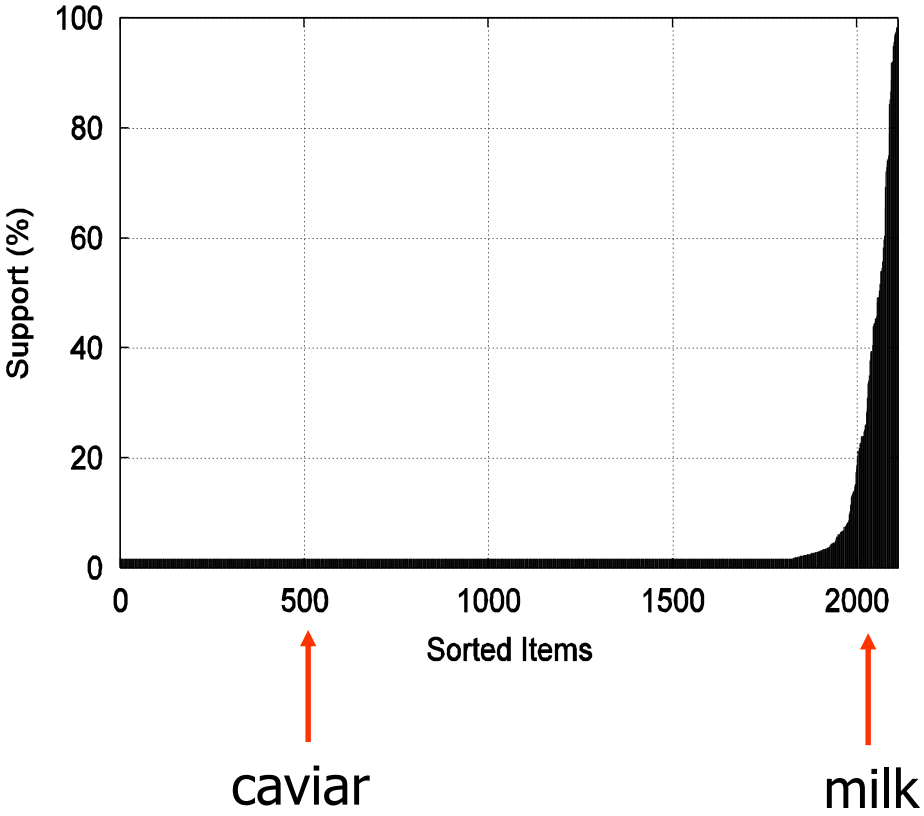
Support distribution of a retail data set



Effect of Support Distribution

- Difficult to set the appropriate *minsup* threshold
  - If *minsup* is too high, we could miss itemsets involving interesting rare items (e.g., {caviar, vodka})
  - If *minsup* is too low, it is computationally expensive and the number of itemsets is very large

Cross-Support Patterns



A cross-support pattern involves items with varying degree of support

- Example: {caviar,milk}

How to avoid such patterns?

A Measure of Cross Support

- Given an itemset, $X = \{x_1, x_2, \ldots, x_d\}$, with d items, we can define a measure of cross support, named support ratio $r(X)$, for the itemset

$$r(X) = \frac{\mathbf{min}\{s(x_1), s(x_2), \ldots, s(x_d)\}}{\mathbf{max}\{s(x_1), s(x_2), \ldots, s(x_d)\}}$$

  where $s(x_i)$ is the support of item x_i

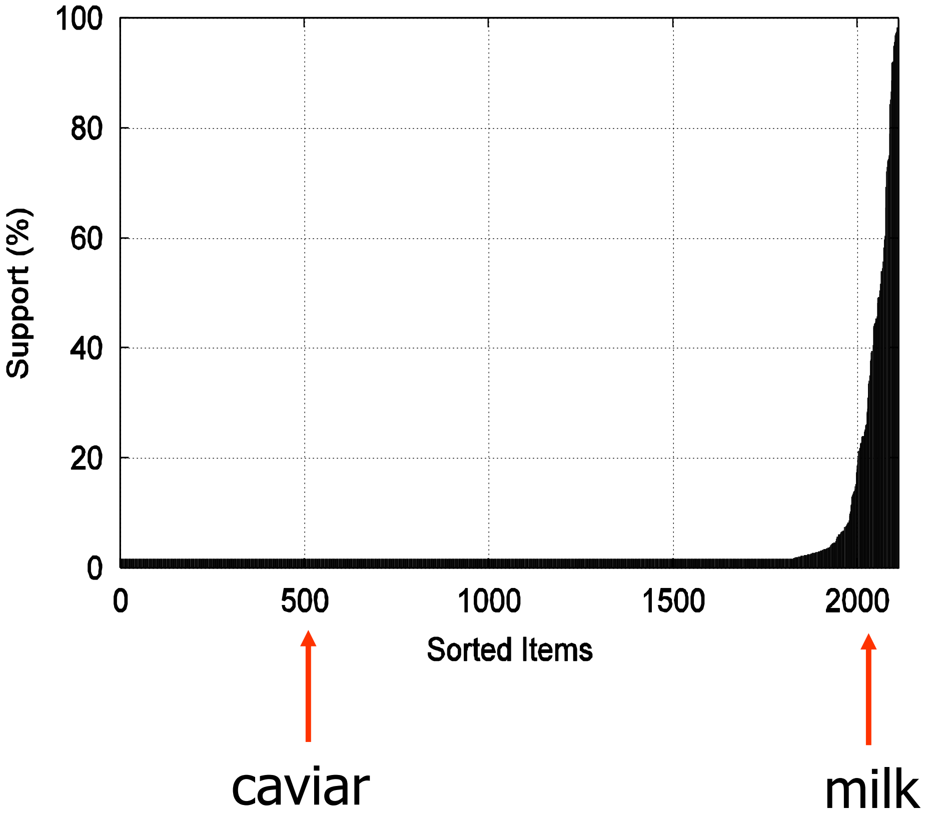
- Given a user-specified threshold h_c, an itemset X is a cross support pattern if $r(X) < h_c$.

Example

p	q	r
0	1	1
1	1	1
1	1	1
1	1	1
1	1	1
1	0	0
1	0	0
1	0	0
1	0	0
1	0	0
1	0	0
1	0	0
1	0	0
1	0	0
1	0	0
1	0	0
1	0	0
1	0	0
1	0	0
1	0	0
1	0	0
1	0	0
1	0	0
1	0	0
1	0	0
1	0	0
0	0	0
0	0	0
0	0	0
0	0	0

- Cross-support patterns can be eliminated
  - {p, q}, {p, r}, and {p, q, r}: $r(X)$ = 0.2
- Interesting patterns with low supports are still pruned by support-based pruning strategy
  - {q, r}
- Confidence pruning?
  - {q} $\rightarrow$ {r}: c = 100%
  - {q} $\rightarrow$ {p}: c = 80%
  - {r} $\rightarrow$ {p}: c = 80%

Confidence and Cross-Support Patterns



Observation:

conf(caviar→milk) is very high

but

conf(milk→caviar) is very low

Therefore,

min(conf(caviar→milk),
conf(milk→caviar))

is also very low

H-Confidence

- To avoid patterns whose items have very different support, define a new evaluation measure for itemsets
  – Known as h-confidence or all-confidence

- Specifically, given an itemset $X = \{x_1, x_2, \ldots, x_d\}$
  – h-confidence is the <u>minimum confidence</u> of any association rule formed from itemset X

  – hconf(X) = min(conf($X_1 \rightarrow X_2$)),

  where $X_1, X_2 \subset X$, $X_1 \cap X_2 = \emptyset, X_1 \cup X_2 = X$

  For example: $X_1 = \{x_1, x_2\}, X_2 = \{x_3, \ldots, x_d\}$

H-Confidence …

- But, given an itemset $X = \{x_1, x_2, \ldots, x_d\}$
  - What is the lowest confidence rule you can obtain from X?
  - Recall conf($X_1 \rightarrow X_2$) = $s(X_1 \cup X_2)$ / support(X_1)
    - The numerator is fixed: $s(X_1 \cup X_2) = s(X)$
    - Thus, to find the <u>lowest confidence rule</u>, we need to find the <u>X_1 with highest support</u>

      anti-monotone property
    - Consider only rules where X_1 is a <u>single item</u>, i.e.,

      $\{x_1\} \rightarrow X - \{x_1\}$, $\{x_2\} \rightarrow X - \{x_2\}$, …, or $\{x_d\} \rightarrow X - \{x_d\}$

$$hconf(X) = \min\left\{\frac{s(X)}{s(x_1)}, \frac{s(X)}{s(x_2)}, \ldots, \frac{s(X)}{s(x_d)}\right\}$$

$$= \frac{s(X)}{\max\{s(x_1), s(x_2), \ldots, s(x_d)\}}$$

Cross Support and H-confidence

- By the anti-monotone property of support

$$s(X) \leq \min\{s(x_1), s(x_2), \dots, s(x_d)\}$$

- Therefore, we can derive a relationship between the h-confidence and cross support of an itemset

$$hconf(X) = \frac{s(X)}{\max\{s(x_1), s(x_2), \dots, s(x_d)\}}$$

$$\leq \frac{\min\{s(x_1), s(x_2), \dots, s(x_d)\}}{\max\{s(x_1), s(x_2), \dots, s(x_d)\}}$$

$$= r(X)$$

Thus, $hconf(X) \leq r(X)$

Cross Support and H-confidence …

- Since, $hconf(X) \leq r(X)$, cross support patterns can be eliminated by ensuring that the h-confidence values for the patterns exceed h_c , a user set threshold
- Notice that

$$0 \leq hconf(X) \leq r(X) \leq 1$$

- An itemset *X* is a hyperclique pattern if and only if $hconf(X) > h_c$, where h_c is a given h-confidence threshold
- H-confidence can be used instead of or in conjunction with support

Properties of Hypercliques

- Hypercliques are itemsets, but not necessarily frequent itemsets
  - Good for finding low support patterns
- H-confidence is anti-monotone
- Can define closed and maximal hypercliques in terms of h-confidence
  - A hyperclique X is closed if none of its immediate supersets has the same h-confidence as X
  - A hyperclique X is maximal if $\mathrm{hconf}(X) > \mathrm{h_c}$ and none of its immediate supersets, Y, have $\mathrm{hconf}(Y) > \mathrm{h_c}$
- Items in a hyperclique cannot have widely different support
  - Allows for more efficient pruning
  - Can be used to find strongly coherent groups of items
    - Words that occur together in documents