Geometry

实数比较

```
const db EPS = 1e-9;
inline int sign(db a) { return a < -EPS ? -1 : a > EPS; }
inline int cmp(db a, db b){ return sign(a-b); }
```

点

```cpp
struct P {
    db x, y;
    P() {}
    P(db _x, db _y) : x(_x), y(_y) {}
    P operator+(P p) { return {x + p.x, y + p.y}; }
    P operator-(P p) { return {x - p.x, y - p.y}; }
    P operator*(db d) { return {x * d, y * d}; }
    P operator/(db d) { return {x / d, y / d}; }

    bool operator<(P p) const {
        int c = cmp(x, p.x);
        if (c) return c == -1;
        return cmp(y, p.y) == -1;
    }

    bool operator==(P o) const{
        return cmp(x,o.x) == 0 && cmp(y,o.y) == 0;
    }
```

点积/叉积

```
db dot(P p) { return x * p.x + y * p.y; }
db det(P p) { return x * p.y - y * p.x; }
```

例题

- 有n个点，求锐角三角形的个数/总面积(Beijing Regional 18)。
- n<=2000，没有三点共线。

其他函数

```
db distTo(P p) { return (*this-p).abs(); }
db alpha() { return atan2(y, x); }
void read() { cin>>x>>y; }
void write() {cout<<"("<<x<<","<<y<<")"<<endl;}
db abs() { return sqrt(abs2());}
db abs2() { return x * x + y * y; }
P rot90() { return P(-y,x);}
P unit() { return *this/abs(); }
int quad() const { return sign(y) == 1 || (sign(y) == 0 && sign(x) >= 0); }
```

旋转

```
P rot(db an){ return {x*cos(an)-y*sin(an),x*sin(an) + y*cos(an)}; }
```

线/半平面

```
struct L{ //ps[0] -> ps[1]
    P ps[2];
    P& operator[](int i) { return ps[i]; }
    P dir() { return ps[1] - ps[0]; }
    bool include(P p) { return sign((ps[1] - ps[0]).det(p - ps[0])) > 0; }
    L push(){ // push eps outward
        const double eps = 1e-6;
        P delta = (ps[1] - ps[0]).rot90().unit() * eps;
        return {ps[0] - delta, ps[1] - delta};
    }
};
```

crossop

```
#define cross(p1,p2,p3) ((p2.x-p1.x)*(p3.y-p1.y)-(p3.x-p1.x)*(p2.y-p1.y))
#define crossOp(p1,p2,p3) sign(cross(p1,p2,p3))
```

直线平行

```
bool chkLL(P p1, P p2, P q1, P q2) {
    db a1 = cross(q1, q2, p1), a2 = -cross(q1, q2, p2);
    return sign(a1+a2) != 0;
}
```

直线交点

```
P isLL(P p1, P p2, P q1, P q2) {
    db a1 = cross(q1, q2, p1), a2 = -cross(q1, q2, p2);
    return (p1 * a2 + p2 * a1) / (a1 + a2);
}
```

线段相交

```
bool intersect(db l1,db r1,db l2,db r2){
    if(l1>r1) swap(l1,r1); if(l2>r2) swap(l2,r2);
    return !( cmp(r1,l2) == -1 || cmp(r2,l1) == -1 );
}

bool isSS(P p1, P p2, P q1, P q2){
    return intersect(p1.x,p2.x,q1.x,q2.x) && intersect(p1.y,p2.y,q1.y,q2.y) &&
    crossOp(p1,p2,q1) * crossOp(p1,p2,q2) <= 0 && crossOp(q1,q2,p1)
            * crossOp(q1,q2,p2) <= 0;
}

bool isSS_strict(P p1, P p2, P q1, P q2){
    return crossOp(p1,p2,q1) * crossOp(p1,p2,q2) < 0 && crossOp(q1,q2,p1)
            * crossOp(q1,q2,p2) < 0;
}
```

点在线段上判定

```
bool isMiddle(db a, db m, db b) {
    return sign(a - m) == 0 || sign(b - m) == 0 || (a < m != b < m);
}

bool isMiddle(P a, P m, P b) {
    return isMiddle(a.x, m.x, b.x) && isMiddle(a.y, m.y, b.y);
}

bool onSeg(P p1, P p2, P q){
    return crossOp(p1,p2,q) == 0 && isMiddle(p1, q, p2);
}

bool onSeg_strict(P p1, P p2, P q){
    return crossOp(p1,p2,q) == 0 && sign((q-p1).dot(p1-p2)) * sign((q-p2).dot(p1-p2)) < 0;
}
```

投影/反射/最近点

```
P proj(P p1, P p2, P q) {
    P dir = p2 - p1;
    return p1 + dir * (dir.dot(q - p1) / dir.abs2());
}

P reflect(P p1, P p2, P q){
    return proj(p1,p2,q) * 2 - q;
}

db nearest(P p1,P p2,P q){
    P h = proj(p1,p2,q);
    if(isMiddle(p1,h,p2))
        return q.distTo(h);
    return min(p1.distTo(q),p2.distTo(q));
}
```

线段距离

```
db disSS(P p1, P p2, P q1, P q2){
    if(isSS(p1,p2,q1,q2)) return 0;
    return min(min(nearest(p1,p2,q1),nearest(p1,p2,q2)), min(nearest(q1,q2,p1),nearest(q1,q2,p2)
}
```

夹角

```
db rad(P p1,P p2){
    return atan2l(p1.det(p2),p1.dot(p2));
}
```

JAG2016 I

- 一个矩形的城市，中间有一条河
- 河是两条从上边界到下边界且互相不交的折线
- 给定起点终点，求第一关键字水里距离，第二关键字陆地距离的最短路
- 折线点数不超过 20

多边形面积

```
db area(vector<P> ps){
    db ret = 0; rep(i,0,ps.size()) ret += ps[i].det(ps[(i+1)%ps.size()]);
    return ret/2;
}
```

点包含

```
int contain(vector<P> ps, P p){ //2:inside,1:on_seg,0:outside
    int n = ps.size(), ret = 0;
    rep(i,0,n){
        P u=ps[i],v=ps[(i+1)%n];
        if(onSeg(u,v,p)) return 1;
        if(cmp(u.y,v.y)<=0) swap(u,v);
        if(cmp(p.y,u.y) >0 || cmp(p.y,v.y) <= 0) continue;
        ret ^= crossOp(p,u,v) > 0;
    }
    return ret*2;
}
```

凸包

```
vector<P> convexHull(vector<P> ps) {
    int n = ps.size(); if(n <= 1) return ps;
    sort(ps.begin(), ps.end());
    vector<P> qs(n * 2); int k = 0;
    for (int i = 0; i < n; qs[k++] = ps[i++])
        while (k > 1 && crossOp(qs[k - 2], qs[k - 1], ps[i]) <= 0) --k;
    for (int i = n - 2, t = k; i >= 0; qs[k++] = ps[i--])
        while (k > t && crossOp(qs[k - 2], qs[k - 1], ps[i]) <= 0) --k;
    qs.resize(k - 1);
    return qs;
}
```

```cpp
vector<P> convexHullNonStrict(vector<P> ps) {
    //caution: need to unique the Ps first
    int n = ps.size(); if(n <= 1) return ps;
    sort(ps.begin(), ps.end());
    vector<P> qs(n * 2); int k = 0;
    for (int i = 0; i < n; qs[k++] = ps[i++])
        while (k > 1 && crossOp(qs[k - 2], qs[k - 1], ps[i]) < 0) --k;
    for (int i = n - 2, t = k; i >= 0; qs[k++] = ps[i--])
        while (k > t && crossOp(qs[k - 2], qs[k - 1], ps[i]) < 0) --k;
    qs.resize(k - 1);
    return qs;
}
```

点集直径

```
db convexDiameter(vector<P> ps){
    int n = ps.size(); if(n <= 1) return 0;
    int is = 0, js = 0; rep(k,1,n) is = ps[k]<ps[is]?k:is, js = ps[js] < ps[k]?k:js;
    int i = is, j = js;
    db ret = ps[i].distTo(ps[j]);
    do{
        if((ps[(i+1)%n]-ps[i]).det(ps[(j+1)%n]-ps[j]) >= 0)
            (++j)%=n;
        else
            (++i)%=n;
        ret = max(ret,ps[i].distTo(ps[j]));
    }while(i!=is || j!=js);
    return ret;
}
```

convexcut

```
vector<P> convexCut(const vector<P>&ps, P q1, P q2) {
    vector<P> qs;
    int n = ps.size();
    rep(i,0,n){
        P p1 = ps[i], p2 = ps[(i+1)%n];
        int d1 = crossOp(q1,q2,p1), d2 = crossOp(q1,q2,p2);
        if(d1 >= 0) qs.pb(p1);
        if(d1 * d2 < 0) qs.pb(isLL(p1,p2,q1,q2));
    }
    return qs;
}
```

最近点

```cpp
db min_dist(vector<P>&ps,int l,int r){
    if(r-l<=5){
        db ret = 1e100;
        rep(i,l,r) rep(j,l,i) ret = min(ret,ps[i].distTo(ps[j]));
        return ret;
    }
    int m = (l+r)>>1;
    db ret = min(min_dist(ps,l,m),min_dist(ps,m,r));
    vector<P> qs; rep(i,l,r) if(abs(ps[i].x-ps[m].x)<= ret) qs.pb(ps[i]);
    sort(qs.begin(), qs.end(),[](P a,P b) -> bool {return a.y<b.y; });
    rep(i,1,qs.size()) for(int j=i-1;j>=0&&qs[j].y>=qs[i].y-ret;--j)
        ret = min(ret,qs[i].distTo(qs[j]));
    return ret;
}
```

WF2017 A

- 给出一个 n 个点的简单多边形
- 问能放进去的最长线段长度
- $n \le 200$

WF2012 A

- 三维空间中 n 个点，每个点有个速度向量
- 问在运动过程中，最小生成树变化了多少次（只有连续作为最小生成树超过 1e-6 的时间才会被计入）
- $n \leq 50$
- 数据保证在任意长度大于等于 1e-6 的时间范围内，都存在一个时刻最小生成树是唯一的

WF2015 B

- 给两个凸多边形，并分别给出速度向量
- 问相交面积的最大值已经第一个达到最大面积的时刻
- $n \leq 10$

圆与圆的关系

```
int type(P o1,db r1,P o2,db r2){
    db d = o1.distTo(o2);
    if(cmp(d,r1+r2) == 1) return 4;
    if(cmp(d,r1+r2) == 0) return 3;
    if(cmp(d,abs(r1-r2)) == 1) return 2;
    if(cmp(d,abs(r1-r2)) == 0) return 1;
    return 0;
}
```

圆与线的交

```
vector<P> isCL(P o,db r,P p1,P p2){
    if (cmp(abs((o-p1).det(p2-p1)/p1.distTo(p2)),r)>0) return {};
    db x = (p1-o).dot(p2-p1), y = (p2-p1).abs2(), d = x * x - y * ((p1-o).abs2() - r*r);
    d = max(d,0.0); P m = p1 - (p2-p1)*(x/y), dr = (p2-p1)*(sqrt(d)/y);
    return {m-dr,m+dr}; //along dir: p1->p2
}
```

圆交

```
vector<P> isCC(P o1, db r1, P o2, db r2) { //need to check whether two circles are the same
    db d = o1.distTo(o2);
    if (cmp(d, r1 + r2) == 1) return {};
    if (cmp(d,abs(r1-r2))==-1) return {};
    d = min(d, r1 + r2);
    db y = (r1 * r1 + d * d - r2 * r2) / (2 * d), x = sqrt(r1 * r1 - y * y);
    P dr = (o2 - o1).unit();
    P q1 = o1 + dr * y, q2 = dr.rot90() * x;
    return {q1-q2,q1+q2};//along circle 1
}
```

切线

```cpp
vector<P> tanCP(P o, db r, P p) {
    db x = (p - o).abs2(), d = x - r * r;
    if (sign(d) <= 0) return {}; // on circle => no tangent
    P q1 = o + (p - o) * (r * r / x);
    P q2 = (p - o).rot90() * (r * sqrt(d) / x);
    return {q1-q2,q1+q2}; //counter clock-wise
}
```

外切线

```
vector<L> extanCC(P o1, db r1, P o2, db r2) {
    vector<L> ret;
    if (cmp(r1, r2) == 0) {
        P dr = (o2 - o1).unit().rot90() * r1;
        ret.pb({o1 + dr, o2 + dr}), ret.pb({o1 - dr, o2 - dr});
    } else {
        P p = (o2 * r1 - o1 * r2) / (r1 - r2);
        vector<P> ps = tanCP(o1, r1, p), qs = tanCP(o2, r2, p);
        rep(i,0,min(ps.size(),qs.size())) ret.pb({ps[i], qs[i]}); //c1 counter-clock wise
    }
    return ret;
}
```

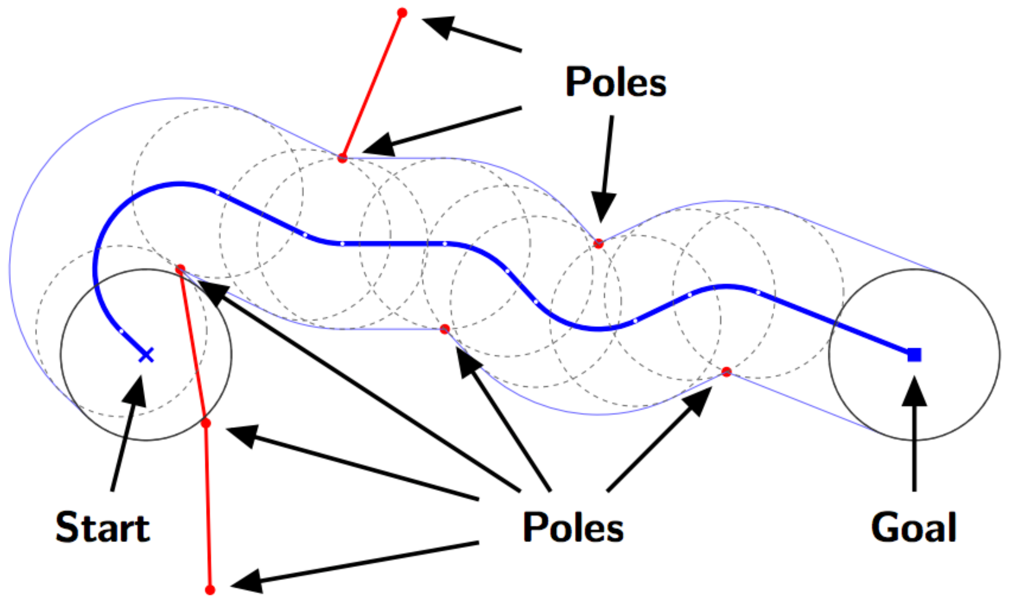
内切线

```
vector<L> intanCC(P o1, db r1, P o2, db r2) {
    vector<L> ret;
    P p = (o1 * r2 + o2 * r1) / (r1 + r2);
    vector<P> ps = tanCP(o1,r1,p), qs = tanCP(o2,r2,p);
    rep(i,0,min(ps.size(),qs.size())) ret.pb({ps[i], qs[i]}); //c1 counter-clock wise
    return ret;
}
```

Cornering at Poles (Tokyo Regional 2014)

给定半径为R的圆的初始位置和目标位置，以及平面上一些障碍点，求最短路距离。

障碍点的个数<=8



Cornering at Poles (Tokyo Regional 2014)

首先将圆缩成点，障碍点扩成圆。

考虑点可能走过的路径

- 圆与圆的公切线
- 点到圆的切线
- 圆弧

建立关键点

- 圆上的切点

建立边

圆与三角形的交

```
db areaCT(db r, P p1, P p2){
    vector<P> is = isCL(P(0,0),r,p1,p2);
    if(is.empty()) return r*r*rad(p1,p2)/2;
    bool b1 = cmp(p1.abs2(),r*r) == 1, b2 = cmp(p2.abs2(), r*r) == 1;
    if(b1 && b2){
        if(sign((p1-is[0]).dot(p2-is[0])) <= 0 &&
            sign((p1-is[0]).dot(p2-is[0])) <= 0)
        return r*r*(rad(p1,is[0]) + rad(is[1],p2))/2 + is[0].det(is[1])/2;
        else return r*r*rad(p1,p2)/2;
    }
    if(b1) return (r*r*rad(p1,is[0]) + is[0].det(p2))/2;
    if(b2) return (p1.det(is[1]) + r*r*rad(is[1],p2))/2;
    return p1.det(p2)/2;
}
```

求多边形与圆的交

多校2018 9E

- 给出一个 n 个点的简单多边形和半径 R
- m 次询问，每一次给出一个半径为 R 的圆，问把这个点给完全移到多边形里的最短距离
- n,m ≤ 200，坐标范围 1e6
- 保证 R 变化 0.1 答案不变

CF 462 C

- 平面上给你n个圆，问将平面分成了多少块。

某个北大冬令营题

- 转转转。
- 求期望。

- 空间中有个圆是障碍，求从s到t的最小距离。

平行/同向/极角序

```
bool parallel(L l0, L l1) { return sign( l0.dir().det( l1.dir() ) ) == 0; }

bool sameDir(L l0, L l1) { return parallel(l0, l1) && sign(l0.dir().dot(l1.dir()) ) == 1; }

bool cmp (P a,  P b) {
    if (a.quad() != b.quad()) {
        return a.quad() < b.quad();
    } else {
        return sign( a.det(b) ) > 0;
    }
}
```

```cpp
bool operator < (L l0, L l1) {
    if (sameDir(l0, l1)) {
        return l1.include(l0[0]);
    } else {
        return cmp( l0.dir(), l1.dir() );
    }
}

bool check(L u, L v, L w) {
    return w.include(isLL(u,v));
}

vector<P> halfPlaneIS(vector<L> &l) {
    sort(l.begin(), l.end());
    deque<L> q;
    for (int i = 0; i < (int)l.size(); ++i) {
        if (i && sameDir(l[i], l[i - 1])) continue;
        while (q.size() > 1 && !check(q[q.size() - 2], q[q.size() - 1], l[i])) q.pop_back();
        while (q.size() > 1 && !check(q[1], q[0], l[i])) q.pop_front();
        q.push_back(l[i]);
    }
    while (q.size() > 2 && !check(q[q.size() - 2], q[q.size() - 1], q[0])) q.pop_back();
    while (q.size() > 2 && !check(q[1], q[0], q[q.size() - 1])) q.pop_front();
    vector<P> ret;
    for (int i = 0; i < (int)q.size(); ++i) ret.push_back(isLL(q[i], q[(i + 1) % q.size()]));
    return ret;
}
```

求两个凸多边形并的周长

最小圆覆盖

```
pair<P,db> min_circle(vector<P> ps){
    random_shuffle(ps.begin(), ps.end());
    int n = ps.size();
    P o = ps[0]; db r = 0;
    rep(i,1,n) if(o.distTo(ps[i]) > r + EPS){
        o = ps[i], r = 0;
        rep(j,0,i) if(o.distTo(ps[j]) > r + EPS){
            o = (ps[i] + ps[j]) / 2; r = o.distTo(ps[i]);
            rep(k,0,j) if(o.distTo(ps[k]) > r + EPS){
                o = circumCenter(ps[i],ps[j],ps[k]);
                r = o.distTo(ps[i]);
            }
        }
    }
    return {o,r};
}
```

多边形重心

凸包操作

```
struct CH{
    int n;

    vector<P> ps, lower, upper;

    P operator[](int i){return ps[i];}

    int find(vector<P>&vec, P dir){
        int l=0,r=vec.size();
        while(l+5<r){
            int L = (l*2+r)/3, R = (l+r*2)/3;
            if(vec[L].dot(dir) > vec[R].dot(dir))
                r=R;
            else
                l=L;
        }
        int ret = l; rep(k,l+1,r) if(vec[k].dot(dir) > vec[ret].dot(dir)) ret = k;
        return ret;
    }

    /*
    ps[0] must be the smallest one!
    */
```

初始化

```
void init(vector<P> _ps){
    ps = _ps; n = ps.size();

    rotate(ps.begin(),min_element(ps.begin(), ps.end()),ps.end());

    int at = max_element(ps.begin(), ps.end()) - ps.begin();

    lower = vector<P>(ps.begin(),ps.begin() + at + 1);

    upper = vector<P>(ps.begin()+at,ps.end()); upper.pb(ps[0]);
}
```

最远点

```
int findFarest(P dir){
    if(dir.y > 0 || dir.y==0 && dir.x > 0){
        return ( (int)lower.size() -1 + find(upper,dir)) % n;
    } else {
        return find(lower,dir);
    }
}
```

求交点

```
P get(int l,int r,P p1,P p2){
    int sl = crossOp(p1,p2,ps[l%n]);
    while(l+1<r){
        int m = (l+r)>>1;
        if(crossOp(p1,p2,ps[m%n]) == sl)
            l = m;
        else
            r = m;
    }

    return isLL(p1,p2,ps[l%n],ps[(l+1)%n]);
}

vector<P> getIS(P p1,P p2){
    int X = findFarest((p2-p1).rot90());
    int Y = findFarest((p1-p2).rot90());
    if(X > Y) swap(X,Y);
    if(crossOp(p1,p2,ps[X]) * crossOp(p1,p2,ps[Y]) < 0){
        return {get(X,Y,p1,p2),get(Y,X+n,p1,p2)};
    } else {
        return {};
    }
}
```

点包含

```
bool contain(P p){
    if(p.x < lower[0].x || p.x > lower.back().x) return 0;
    int id = lower_bound(lower.begin(), lower.end(),(P){p.x,-INF}) - lower.begin();
    if(lower[id].x == p.x){
        if(lower[id].y > p.y) return 0;
    } else {
        if(crossOp(lower[id-1],lower[id],p) < 0) return 0;
    }
    id = lower_bound(upper.begin(), upper.end(),(P){p.x,INF},greater<P>()) - upper.begin();
    if(upper[id].x == p.x){
        if(upper[id].y < p.y) return 0;
    } else {
        if(crossOp(upper[id-1],upper[id],p) < 0) return 0;
    }
    return 1;
}
```

求切线

动态加点的凸壳

```
void insert(int x,ll y) {
    cw=0;
    point q=point(x,y);
    ite pr,nt,ppr,nnt,Pr;
    if (hul.size()&&x>=hul.begin()->x&&x<=hul.rbegin()->x) {
        pr=hul.lower_bound(point(x));
        if (pr->x==x) {
            if (pr->y>y) {
                ll &p=const_cast<ll&>(pr->y);
                p=y;
            }
        } else {
            --pr;
            if (getk(pr,q)>=pr->k) return;
            else pr=hul.insert(q).first;
        }
    } else pr=hul.insert(q).first;
    Pr=pr;--pr;
    if (Pr!=hul.begin()) while (1) {
        if (pr==hul.begin()) break;
        --(ppr=pr);
        if (getk(ppr,q)<=ppr->k) hul.erase(pr); else break;
        pr=ppr;
    }
    nt=Pr;++nt;
    if (nt!=hul.end()) while (1) {
        ++(nnt=nt);
        if (nnt==hul.end()) break;
        if (getk(nnt,q)>=nt->k) hul.erase(nt); else break;
        nt=nnt;
    }
    nnt=Pr;nt=nnt++;
    double &p=const_cast<double&>(nt->k);
    p=(nnt==hul.end()?inf:getk(nt,nnt));
    ppr=nt;pr=ppr--;
    if (pr!=hul.begin()) {
        double &p=const_cast<double&>(ppr->k);
        p=getk(ppr,pr);
    }
}
```

```cpp
ll query(int k) {
    if (hul.empty()) return Inf;
    cw=1;
    ite pr=hul.lower_bound(point(0,0,-k));
    return 1ll*k*pr->x+pr->y;
}
```

JSOI2018

- 一个点集的领地为它的凸包（包括边界）
- 给出两个点集 A,B 以及 q 组询问
- 每组询问给出一个向量，问把 A 沿着这个向量平移后，领地是否会和 B 有交
- 点集大小, $q \le 100000$

杂题

- 平面上给出 n 条线段
- 每条线段上选一个点，结果为这些点加起来
- 问在横坐标大于等于 X 的时候纵坐标最小值是多少
- 输出方案
- $n \le 100000$

WF2018 G

- 给出一个 n 个点的简单多边形
- 找到最小的 R
- 使得多边形内任何一点到最近顶点的距离小于等于 R
- $n \leq 2000$

CF ???G

- 给你一个凸多边形。
- 每次询问一个点，找到一条经过这个点的直线将整个多边形分成面积相同的两半。
- n,q <=1e5

圆并/交

```
void work(){
    vector<int> cand = {};
    rep(i,0,m){
        bool ok = 1;
        rep(j,0,m) if(i!=j){
            if(rs[j] > rs[i] + EPS && rs[i] + cs[i].distTo(cs[j]) <= rs[j] + EPS){
                ok = 0; break;
            }
            if(cs[i] == cs[j] && cmp(rs[i],rs[j]) == 0 && j < i){
                ok = 0; break;
            }
        }
        if(ok) cand.pb(i);
    }

    rep(i,0,cand.size()) cs[i] = cs[cand[i]], rs[i] = rs[cand[i]];
    m = cand.size();

    db area = 0;
    P wc = 0;
```

```
//work
rep(i,0,m){
    vector<pair<db,int> > ev = {{0,0},{2*PI,0}};

    int cur = 0;

    rep(j,0,m) if(j!=i){
        auto ret = isCC(cs[i],rs[i],cs[j],rs[j]);
        if(!ret.empty()){
            db l = (ret[0] - cs[i]).alpha();
            db r = (ret[1] - cs[i]).alpha();
            l = norm(l); r = norm(r);
            ev.pb({l,1});ev.pb({r,-1});
            if(l > r) ++cur;
        }
    }

    sort(ev.begin(), ev.end());
    rep(j,0,ev.size() - 1){
        cur += ev[j].se;
        if(cur == 0){
            area += calc_area_circle(cs[i],rs[i],ev[j].fi,ev[j+1].fi);
            wc = wc + calc_wc_circle(cs[i],rs[i],ev[j].fi,ev[j+1].fi);
        }
    }
}
```

线段与简单多边

```
bool check(P L, P R, P p){
    //is p strictly belong to L -> R?
    if(L.det(R) > -EPS){ // <= 180 degree
        return L.det(p) > EPS && p.det(R) > EPS;
    }
    //L.det(R) < -EPS
    if(L.det(p) >= 0) return 1;
    if(p.det(R) > EPS) return 1;
    return 0;
}

bool strict_polys_segment(vector<P>&qs,P p1,P p2){
    int n = qs.size();

    P m = (p1+p2)/2;

    if(contain(qs,m) >= 2) return 1;

    rep(i,0,n){
        P q1 = qs[i], q2 = qs[(i+1)%n];
        if(crsSS(p1,p2,q1,q2)) return 1;
        if(onSeg_strict(p1,p2,q1)){
            P q0 = qs[(i+n-1)%n];
            if(check(q2-q1,q0-q1,p1-q1)) return 1;
            if(check(q2-q1,q0-q1,p2-q1)) return 1;
        }
    }

    return 0;
}
```

三维点类

```
struct P3{
    db x,y,z;
    P3 operator+(P3 o){ return {x+o.x,y+o.y,z+o.z}; }
    P3 operator-(P3 o){ return {x-o.x,y-o.y,z-o.z}; }
    db operator*(P3 o){ return x*o.x+y*o.y+z*o.z; }
    P3 operator^(P3 o){ return {y*o.z-z*o.y,z*o.x-x*o.z,x*o.y-y*o.x}; }
    P3 operator*(db o){ return {x*o,y*o,z*o}; }
    P3 operator/(db o){ return {x/o,y/o,z/o}; }

    db abs2(){ return sqr(x) + sqr(y) + sqr(z); }
    db abs(){ return sqrt(abs2()); }

    P3 norm(){ return *this / abs(); }
    bool operator<(P3 o){
        if(cmp(x,o.x) != 0) return x < o.x;
        if(cmp(y,o.y) != 0) return y < o.y;
        return cmp(z,o.z) == -1;
    }
    bool operator==(P3 o){
        return cmp(x,o.x) == 0 && cmp(y,o.y) == 0 && cmp(z,o.z) == 0;
    }
    void read(){
        cin>>x>>y>>z;
    }
    void print(){
        //printf("%lf,%lf,%lf\n",x,y,z);
    }
};
```

距离和交点

```
db disLP(P3 p1,P3 p2,P3 q){
    return ((p2-p1)^(q-p1)).abs() / (p2-p1).abs();
}

db disLL(P3 p1,P3 p2,P3 q1,P3 q2){
    P3 o = (p2-p1) ^ (q2-q1); if(o.abs() <= EPS) return disLP(p1,p2,q1);
    return fabs(o.norm() * (p1-p2));
}

VP isFL(P3 p,P3 o,P3 q1,P3 q2){
    db a = (q2-p)*o, b = (q1-p)*o;
    db d = a - b;
    if(fabs(d) < EPS) return {};
    return {(q1*a-q2*b)/d};
}

VP isFF(P3 p1,P3 o1,P3 p2,P3 o2){
    P3 e = o1 ^ o2, v = o1 ^ e;
    db d = o2 * v; if(fabs(d) < EPS) return {};
    P3 q = p1 + v * (o2 * (p2-p1) / d);
    return {q,q+e};
}
```

三维凸包

```
VVP convexHull3d(VP _p){
    p = q = _p; n = p.size();
    ret.clear(); eg.clear();
    for(auto&i:q) i = i + (P3){rand_db()*1e-4,rand_db()*1e-4,rand_db()*1e-4};
    for (int i=1;i<n;i++)if (q[i].x<q[0].x)swap(p[0],p[i]),swap(q[0],q[i]);
    for (int i=2;i<n;i++)if (
        (q[i].x-q[0].x)*(q[1].y-q[0].y)>
        (q[i].y-q[0].y)*(q[1].x-q[0].x)) swap(q[1],q[i]),swap(p[1],p[i]);
    wrap(0,1);
    return ret;
}
```

```cpp
void wrap(int a,int b){
    if (eg.find({a,b})==eg.end()){
        int c=-1;
        for (int i=0;i<n;i++)if (i!=a && i!=b){
            if (c==-1 || Volume(q[c],q[a],q[b],q[i])>0)
                c=i;
        }
        if (c!=-1){
            ret.pb({p[a],p[b],p[c]});
            eg.insert({a,b});eg.insert({b,c});eg.insert({c,a});
            wrap(c,b);wrap(a,c);
        }
    }
};
```

```cpp
db Volume(P3 a,P3 b,P3 c,P3 d){
    return ((b-a)^(c-a))*(d-a);
}

db rand_db(){
    return 1.0 * rand() / RAND_MAX;
}
```

NAIPC 2017 B

- 给定三维空间中的 n 个点
- 求最小体积的圆柱覆盖所有点（要求圆柱的至少有一个底面上有大于等于 3 个点）
- n <= 1000

HDU 5846

- There are n points in three dimensional space.

  Consider the convex hull of n points.

  Given a plane, you should work out the area of section of convex hull by the plane.

THUSC 17

- 给你d维空间的d个球，求所有的公切面。

Consider the following procedure that generates a random convex polygon. For a given set of points $P_1, \ldots, P_n$ in 3-dimensional space we uniformly choose a direction and find the projections $Q_1, \ldots, Q_n$ of these points along it. The resulting 2-dimensional polygon is the convex hull of $Q_1, \ldots, Q_n$. In order to generate small enough polygons it's useful to understand what is the average area of the resulting polygon. Find it!

Input

The first line of the input contains one integer number n: $1 \leqslant n \leqslant 50$. Each of the next n lines contains three integer numbers x_i, y_i, z_i. All these numbers don't exceed 50 by absolute value.

Output

Output one real number with six exact digits after the decimal point — the average area of the resulting polygon.

反演

- 这里是反演

两圆相切

- 里面一堆内切的圆，求坐标。
- 两圆不相切呢?

例题

- 若干个圆交于一点，求圆并。

CF 219 E

There are a set of points S on the plane. This set doesn't contain the origin $O(0, 0)$, and for each two distinct points in the set A and B, the triangle OAB has strictly positive area.

Consider a set of pairs of points $(P_1, P_2), (P_3, P_4), \ldots, (P_{2k-1}, P_{2k})$. We'll call the set *good* if and only if:

- $k \geq 2$.
- All P_i are distinct, and each P_i is an element of S.
- For any two pairs (P_{2i-1}, P_{2i}) and (P_{2j-1}, P_{2j}), the circumcircles of triangles $OP_{2i-1}P_{2j-1}$ and $OP_{2i}P_{2j}$ have a single common point, and the circumcircle of triangles $OP_{2i-1}P_{2j}$ and $OP_{2i}P_{2j-1}$ have a single common point.

Calculate the number of good sets of pairs modulo 1000000007 $(10^9 + 7)$.

ZJOI Guard

- 咕咕咕

CF 505 F

- n个点，问有多少种方案选出两个不相交的三角形。
- 假设没有三点共线。
- n<=2000

CF 513 D

- n个点，问是否存在一个三角形，面积恰好为S。
- n<=2000