

# RL 期末考

## 題目

### Selected Topics in Reinforcement Learning (535519)

Final Exam

DATE: 2023/12/26, 18:30-21:30

#### INSTRUCTIONS

1. This examination paper includes **22** questions in **6** pages.
2. This **IS NOT** an **OPEN BOOK** exam.
3. This exam has a total of **110** points.

#### Question 1. (4 points)

- (a) Write the definition of  $V^\pi$  and  $Q^\pi$ . (2 points)
- (b) Prove that  $\mathbb{E}_{s,a \sim \pi}[A^\pi(s, a)] = 0$ . (2 points)

#### Question 2. (6 points)

Let  $Q^\pi(s, a)$  be written in  $V^\pi(s)$  as follows.

$$Q^\pi(s, a) = R_s^a + \gamma \sum_{s' \in S} P_{ss'}^a V^\pi(s')$$

Write the following three expressions in a similar way with the same notation:

- (a)  $V^\pi(s)$  written in  $Q^\pi(s, a)$ . (2 points)
- (b)  $V^\pi(s)$  written in  $V^\pi(s')$ . (2 points)
- (c)  $Q^\pi(s, a)$  written in  $Q^\pi(s', a')$ . (2 points)

#### Question 3. (10 points)

(Contraction mapping) Define a Bellman optimality operator  $T$ :

$$[TV](s) := \max_{a \in A} (R_s^a + \gamma \sum_{s'} P_{ss'}^a V(s'))$$

Prove that Bellman optimality backup operator  $T$  is a  $\gamma$ -contraction.

Hint: for any  $U$  and  $V$ , show that  $\|TU - TV\|_\infty \leq \gamma \|U - V\|_\infty$ .

**Question 4.** (4 points)

Is Q-learning on-policy or off-policy? Explain the reason.

**Question 5.** (4 points)

Describe the UCB (Upper Confidence Bound) algorithm and its principles and objectives.

**Question 6.** (10 points)

Consider a small grid world:

(0,0)	(0,1)	(0,2)
(1,0) S	(1,1) Cliff	(1,2) G

The agent always starts at (1,0). Episodes end when the agent falls off the cliff (1,1) or reaches the goal (1,2). The maximal length of each episode is 10 steps. Reward is given in the following rules:

1.  $r = -10$  when falling off the cliff.
2.  $r = 1$  when reaching the goal.
3. Otherwise,  $r = -1$  every step.

Assume that we train an agent using Q-learning with discount factor  $\gamma = 0.9$  and learning rate  $\alpha = 0.1$ . Initialize the Q-table with value 0, and update after every step.

Q-table:

action/state	(0,0)	(0,1)	(0,2)	(1,0)	(1,1)	(1,2)
up	0	0	0	0	0	0
down	0	0	0	0	0	0
right	0	0	0	0	0	0
left	0	0	0	0	0	0

After two episodes:

- 1: (1,0) to (0,0):  $r = -1$ ; (0,0) to (0,1):  $r = -1$ ; (0,1) to (1,1):  $r = -10$   
 2: (1,0) to (0,0):  $r = -1$ ; (0,0) to (0,1):  $r = -1$ ; (0,1) to (0,2):  $r = -1$ ; (0,2) to (1,2):  $r = 1$

(a) What's the value inside the Q-table after finishing episode 1? (4 points)

- (b) What's the value inside the Q-table after finishing both episodes 1 and 2? (6 points)

**Question 7.** (10 points)

Design an PPO (Proximal Policy Optimization) algorithm by writing the pseudocode of following section 1 and section 2. Also explain your design.

```
for iteration=1,2... do
    // Section 1. Using policy  $\pi_{\theta_{old}}$  to interact with
    // the environment to collect data.
    // Section 2. Optimize  $\theta$ .
     $\theta_{old} = \theta$ 
end for
```

**Question 8.** (3 points)

What's the benefit of the delayed actor update in TD3 (Twin Delayed DDPG)?

**Question 9.** (4 points)

Explain the design purpose of SAC (Soft Actor-Critic) policy objective.

$$J_{\pi}(\phi) = \mathbb{E}_{s_t \sim D} [D_{KL}(\pi_{\phi}(\cdot|s_t) || \frac{\exp(Q_{\theta}(s_t, \cdot))}{Z_{\theta}(s_t)})]$$

**Question 10.** (4 points)

Why can the RND (Random Network Distillation) algorithm alleviate the noisy TV problem?

**Question 11.** (3 points)

What problem can Double-DQN prevent with respect to DQN?

**Question 12.** (4 points)

What techniques are used for exploration in DQN and DDPG respectively?

**Question 13.** (3 points)

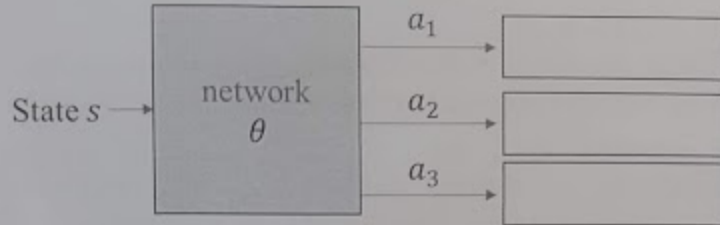
What can be used to reduce variance for the REINFORCE algorithm?

**Question 14.** (4 points)

What is the projection step in C51? Why does C51 need the projection step?

**Question 15.** (4 points)

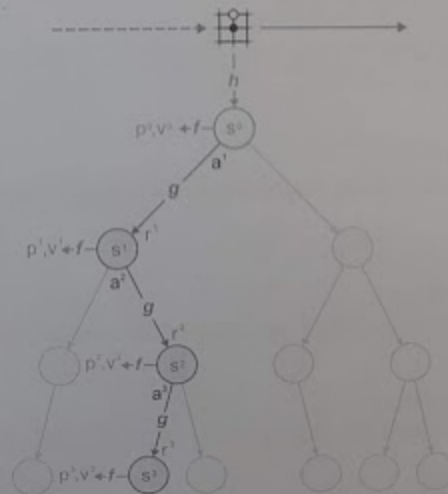
What is the network output of QR-DQN? How are the Q-values calculated for each action?

**Question 16.** (4 points)

In AlphaZero, what are the training targets for the policy network and value network?

**Question 17.** (6 points)

In MuZero, explain the roles of the representation network (h), dynamics network (g), and prediction network (f).

**Question 18.** (3 points)

What is the benefit of MuZero's design compared with AlphaZero?

**Question 19.** (6 points)

How does DQfD (Deep Q-learning from Demonstrations) utilize demonstrations?

**Question 20.** (6 points)

What is the problem of non-stationarity in a multi-agent environment, and why does this non-stationarity occur? Use the example of rock-paper-scissors to illustrate your points.

**Question 21.** (4 points)

What are “Centralized Training” and “Decentralized Execution” in the CTDE approach, and what are the advantages of each?

**Question 22.** (4 points)

Below is the description of IGM. Why do some cooperative value-based MARL methods that use the CTDE framework often need the IGM property?

**Individual-Global-Maximum (IGM)**

- For a joint action-value function  $Q_{joint}: \mathcal{T}^N \times \mathcal{A}^N \rightarrow \mathbb{R}$ , where  $\tau \in \mathcal{T}$  is a joint action-observation histories, if there exist individual action-value function  $[Q^i: \mathcal{T} \times \mathcal{A} \rightarrow \mathbb{R}]_{i=1}^N$ , such that the following holds:

$$\arg \max_a Q_{joint}(\tau, a) = \begin{pmatrix} \arg \max_{a_1} Q^1(\tau^1, a^1) \\ \arg \max_{a_2} Q^2(\tau^2, a^2) \\ \dots \\ \arg \max_{a_N} Q^N(\tau^N, a^N) \end{pmatrix}$$

END OF EXAM

## 我的答案

題號 NO	分數 Score	
1	0	162
2	0	176
3	10	183
4	4	192
5	4	206
6	10	214
7	10	220
8	3	
9	2	
10	4	
11	3	
12	4	
13	0	
14	4	
15	4	
總分 Total		



Q1(a)

 $E_{s,\pi}[V(s)] = E_{s,\pi}[Q^\pi(s,a)]$ , 以  $\pi$  为 policy, state  $s$  的值

 $E_{s,\pi}[Q^\pi(s,a)] = E_{s,\pi}[R(s,a) + \gamma V(s)]$ , 以  $\pi$  为 policy, after state  $(s,a)$  的值

(b)

$$E_{s,\pi}[A^\pi(s,a)] = E_{s,\pi}[Q^\pi(s,a) - V(s)]$$

$$E_{s,\pi}[A^\pi(s,a)] = E_{s,\pi}[Q^\pi(s,a) - Q^\pi(s,a)] = 0 \quad \#$$

Q2(a)

$$V_\pi(s) = Q_\pi(s, \pi(s))$$

(b)

$$V_\pi(s) = R_s^\pi + \gamma \sum_{s' \in S} P_{ss'}^\pi V_\pi(s')$$

(c)

$$Q^\pi(s,a) = R_s^a + \gamma \sum_{s' \in S} P_{ss'}^a Q_\pi(s', a'), \quad a' = \pi(s') \quad \#$$

Q3

$$\text{known } \max_i \{x_i\} - \max_i \{y_i\} \leq \max_i \{x_i - y_i\}$$

$$\|TU - TV\|_\infty = \left\| \max_{a \in A} \left\{ R_s^a + \gamma \sum_{s'} P_{ss'}^a U(s') \right\} - \max_{a \in A} \left\{ R_s^a + \gamma \sum_{s'} P_{ss'}^a V(s') \right\} \right\|_\infty$$

$$\leq \left\| \max_{a \in A} \left\{ R_s^a + \gamma \sum_{s'} P_{ss'}^a U(s') \right\} - R_s^a - \gamma \sum_{s'} P_{ss'}^a V(s') \right\|_\infty = \left\| \max_{a \in A} \left\{ \gamma \sum_{s'} P_{ss'}^a (U(s') - V(s')) \right\} \right\|_\infty$$

$$\leq \gamma \|U - V\|_\infty$$

$$\therefore \|TU - TV\|_\infty \leq \gamma \|U - V\|_\infty \quad \#$$

Q4

~~off-policy~~, Q-learning 之更新式  $Q(s,a) = (1-\alpha)Q(s,a) + \alpha(R_s + \max_{a'} Q(s',a'))$

只和  $s, a, r, s'$  有關,  $a'$  是採最佳策略, 會取代原始策略, 所以訓練之策略和當前策略不同, 故是 off-policy #

Q5

~~老虎機~~ 採樣  
老虎機: 之勝率  $x_i$ , 已玩次數  $n_i$

每次玩選擇  $\arg \max_i x_i + \frac{\log \sum n_i}{n_i}$

$x_i$  幫我們找出最大勝率的玩,  $\frac{\log \sum n_i}{n_i}$  讓我們玩比較少玩的机会  
提高勝率也兼具探索性(可能可找到  $x_i$  更大的) #

Q6 (a)

	(0,0)	(0,1)	(0,2)	(1,0)	(1,1)	(1,2)
up	0	0	0	-0.1	0	0
down	0	-1	0	0	0	0
right	-0.1	0	0	0	0	0
left	0	0	0	0	0	0

$$Q^+(0,0, \text{up}) = -1 + 0.9 Q(0,0, a') = -1$$

$$Q^+(0,0, \text{right}) = -1 + 0.9 \cdot 0$$



Q6 (b)

	(0,0)	(0,1)	(0,2)	(1,0)	(1,1)	(1,2)
up	0	0	0	-0.19	0	0
down	0	-1	0.1	0	0	0
right	-0.19	-0.1	0	0	0	0
left	0	0	0	0	0	0

$$Q((1,0), \text{up}) = 0.9 \cdot (-0.1) + 0.1 \cdot (-1 + 0.9 \cdot 0) = -0.19$$

$$Q((0,0), \text{right}) = 0.9 \cdot (-0.1) + 0.1 \cdot (-1 + 0.9 \cdot 0) = -0.19$$

$$Q((0,1), \text{right}) = 0.1 \cdot (-1) = -0.1$$

$$Q((0,2), \text{down}) = 0.1 \cdot (1)$$

#

Q7

section 1. 跑  $n$  个 episode, 搜集  $((s_j, a_j, p_j, r_j, s'_j, q_j))$ ; 如下

buffer = []

(大小为  $N$ )

for  $ep = 1 \dots n$

env 自动完成一个 episode 得到  $V, \text{data} = ((s_i, a_i, v_i, p_i, r_i, s'_i))_i$

for  $(s_i, a_i, v_i, p_i, r_i, s'_i)$  in data

$\delta_i = r_i + \gamma v_i - v_i$

    for  $i = 0 \dots IV_i$

$\sum_{k=0}^{\infty} \gamma^k \delta_{i+k} = G_i = \delta_i + \gamma \delta_{i+1} + \gamma^2 \delta_{i+2} + \dots$

    buffer append  $(s_i, a_i, v_i, p_i, G_i)$

for  $(s_i, a_i, v_i, p_i, G_i)$  in buffer

$$L_Q = \frac{1}{N} \sum_i (G_i - Q_\phi(s_i, a_i))^2$$

$$A_i = G_i - v_i, p'_{i,0} = \max_a \pi_\theta(a|s_i)$$

$$B_\pi = \frac{1}{N} \sum_i \left( \min \left[ A_i \frac{p'_{i,0}}{p_i}, A_i \cdot \text{clip} \left( \frac{p'_{i,0}}{p_i}, 1-\epsilon, 1+\epsilon \right) \right] \right)$$

$$E_\pi = -\frac{1}{N} \sum_i \sum_a \pi_\theta(a|s_i) \log \pi_\theta(a|s_i)$$

code update  $\theta, \phi$  by loss:  $L_Q - T_a B_\pi - T_b E_\pi$

description

希望  $Q_\phi(s_i, a_i)$  接近  $G_i$ , 故最小化  $L_Q$

希望採取 advantage 大的動作, 故  $A_i > 0$ , 提高  $p'_{i,0}$ , 反之減少

故最大化  $B_\pi$ , clip 是避免  $\pi$  變化太大

希望  $\pi$  可以探索, 故最大化  $\pi$  的 entropy.

loss =  $L_Q - T_a B_\pi - T_b E_\pi$ ,  $T_a, T_b$  為權重

Q2

actor  $\pi_\theta$  一直換策略會導致不穩定, delay actor update 不每回合更新  $\pi_\theta$  緩和該問題, 降低 variance.

的 distribution

Q3. 更新 actor 的參數  $\phi$ , SAC 假設  $\pi_\theta(a|s_t)^\vee$  與  $\exp(Q_\phi(s_t, \cdot))$  很像  
用 DKL 比較兩者差距並試圖最小化它。

Q10. **ICM**  
 ICM 鼓勵模型探索無法預測的情況<sup>s'</sup>

ICM 處理 noisy TV 會不斷 explore 因為 noisy TV 下一狀態<sup>s'</sup>皆是隨機的

RND 則是鼓勵模型探索較少或沒探索過的 state,  
 這讓它在 noisy TV 中不會過度 explore #

Q11 避免 Q-network 過度樂觀,

3

Q12

4 DDQN 是離散策略, 使用  $\epsilon$ -greedy 讓模型訓練時有固定機率採隨機策略。  
 DDPG 是連續策略, 在訓練時, 於 action 中加高斯 noise.

Q13

0 use small learning rate  $\alpha$

Q14

4  $Q_{s1}$  的 Q value 的機率分佈是離散的, 而 sample 出來的 Q value

可能介於兩個 Q value atom 中間, 於是需要 projection 分配回兩個 atom 上。

Q15

① 分布, 以不同百分位的 Q-value 代表

4

② 不同百分點 Q-value 的平均

Q16

是  $z$ ,  $z = \begin{cases} 1, & \text{episode 結束後勝利} \\ 0, & \text{episode 結束後輸了} \end{cases} \rightarrow \text{value}$

policy?

Q17

b: 將版面的 feature 取出

g: 預測下一 state

s: 為 state 進行價值估算

Q18

3 可以配學習遊玩規則

Q19

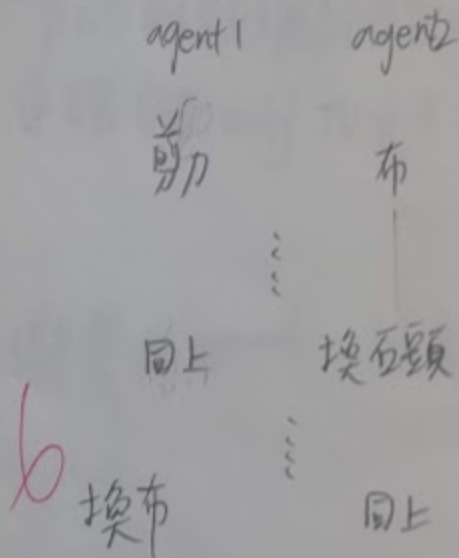
2

總共  $Q(s, a) + I(a_E, a)$  和  $Q(s, a_E)$  的差距

這會讓 demonstration action  $a_E$  高於其它 action  
的 Q-value 一個距離 #

$$I(a_E, a) = \begin{cases} 0 & a = a_E \\ \alpha & a \neq a_E \end{cases} \quad (\alpha > 0)$$

Q20. 每个 agent 會隨著其它 agent 改变动作而改变动作  
造成 non-stationarity, 這讓模型難以收斂  
如剪刀石頭布。



Q21. 是將不同 agent 的模型集中訓練  
Centralized Training, 其好處是避免 non-stationarity.

4 Decentralized Execution, 其好處是避免 action space 过大。  
是 agent 分开決策。

Q22. 這性質讓每个 agent 把配練好等同於  $Q_{joint}$  最大  
因為  $\frac{\partial Q_{joint}}{\partial Q_i} > 0$

## 期末考答題關鍵

### Q1

(a)

$V$ -pi: expected return start from state  $s$  to terminal.

$Q$ -pi: expected return start from state  $s$  and apply action  $a$  to terminal.

(b)

$$E[A] = E[Q - V] = E[Q] - E[V]$$

by definition,  $E[Q] = V$  and  $E[V] = V$ . So,  $E[A] = V - V = 0$

### Q4

Off-policy. Because it uses argmax of  $Q$  for training target.

### Q5

Balance between exploration and exploitation.

### Q8

More accurate critic to stabilize training.

### Q9

Let the policy get close to softmax of  $Q$ , obtaining a multi-modal policy for exploration.

### Q10

Forward prediction model visits the states he can't predict. But states are always hard to predict in noisy TV problem. And RND is just like a pseudo-counter, counting how many times he visits the similar states.

### Q11

Over-optimistic.

### Q12

DQN: epsilon-greedy

DDPG: noise



## Q13

Baseline, actor critic, MC->TD

## Q14

Distribute probability mass to neighboring atoms. The values of the target distribution may not be on the atoms.

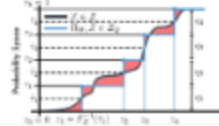
## Q15

Quantile values (not probability).

$$Q\text{-value: } Q(x, a) = \mathbb{E}[Z(x, a)] = \sum_i \frac{1}{N} z_i$$

- e.g.  $N = 4$

$$Q(x, a) = \frac{1}{4} z_1 + \frac{1}{4} z_2 + \frac{1}{4} z_3 + \frac{1}{4} z_4$$



## Q16

policy target: the probability from MCTS

value target: the result of the game (win +1, loss -1)

(有寫到這兩個network在做甚麼就可以)

## Q17

h: convert observations to embedding states.

g: given the current state and action, get the next state and reward.

f: predict the value and policy for the current state.

## Q18

Also learns the dynamics of the environment. Can be applied to Atari games.

## Q19

有描述到以下各點分別可得到的分數

- Pretraining phase (2 points)
- Supervised loss (2 points)
- Replay Buffer/PER with Online Data / Online Training (2 points)

## Q20

有描述到以下各點分別可得到的分數

- 描述 non-stationary 的概念 (2 points)

- 描述原因 (2 points)
- 用 rock-paper-scissors 的例子是否適切 (2 points)
  - 若此例子能很好的順便表達了前兩者，前兩者也可給分

## Q21

有描述到以下各點分別可得到的分數

- Centralized Training 是什麼 (1 point)
- Centralized Training 的優點 (1 point)
- Decentralized Execution 是什麼 (1 point)
- Decentralized Execution 的優點 (1 point)

## Q22

描述到以下相關概念：The global optimal action computed during the centralized training phase is consistent with the actions that would be chosen by the agents acting individually based on their local observations during execution (4 points)