Modelo de segunda ordem:

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \sum_{j=i}^{k} \beta_{ij} x_i x_j + \varepsilon \tag{1}$$

Equações para A_3:

$$A_3^{hipo} = \beta_0^{hipo} + \sum_{i=1}^{k} \beta_i^{hipo} x_i + \sum_{i=1}^{k} \sum_{j=i}^{k} \beta_{ij}^{hipo} x_i x_j + \varepsilon \tag{2}$$

$$A_3^{hiper} = \beta_0^{hiper} + \sum_{i=1}^{k} \beta_i^{hiper} x_i + \sum_{i=1}^{k} \sum_{j=i}^{k} \beta_{ij}^{hipo} x_i x_j + \varepsilon \tag{3}$$

No ponto eutetóide:

$$A_3^{hipo} = A_3^{hiper} \Rightarrow A_3^{hipo} - A_3^{hiper} = 0 \tag{4}$$

Logo:

$$\beta_0^{hipo} - \beta_0^{hiper} + \sum_{i=1}^{k} \left(\beta_i^{hipo} - \beta_i^{hiper} \right) x_i + \sum_{i=1}^{k} \sum_{j=i}^{k} \left(\beta_{ij}^{hipo} - \beta_{ij}^{hiper} \right) x_i x_j = 0 \tag{5}$$

Definamos:

$$\beta_i^{eut} = \beta_i^{hipo} - \beta_i^{hiper} \tag{6}$$

$$\beta_{ij}^{eut} = \beta_{ij}^{hipo} - \beta_{ij}^{hiper} \tag{7}$$

Substituindo as definições acima na equação 5, temos:

$$\beta_0^{eut} + \sum_{i=1}^{k} \beta_i^{eut} x_i + \sum_{i=1}^{k} \sum_{j=i}^{k} \beta_{ij}^{eut} x_i x_j = 0 \tag{8}$$

Reescrevendo 8 de modo a separar x_1 (fração de carbono):

$$\beta_0^{eut} + \beta_1^{eut} x_1 + \sum_{i=2}^{k} \beta_i^{eut} x_i + \beta_{11}^{eut} {x_1}^2 + \sum_{j=2}^{k} \beta_{1j}^{eut} x_1 x_j + \sum_{i=2}^{k} \sum_{j=i}^{k} \beta_{ij}^{eut} x_i x_j = 0$$

$$\beta_{11}^{eut} {x_1}^2 + \left(\beta_1^{eut} + \sum_{j=2}^{k} \beta_{1j}^{eut} x_j \right) x_1 + \left(\beta_0^{eut} + \sum_{i=2}^{k} \beta_i^{eut} x_i + \sum_{i=2}^{k} \sum_{j=i}^{k} \beta_{ij}^{eut} x_i x_j \right) = 0 \tag{9}$$

Dividindo todos os termos em 9 por $2\beta_{11}^{eut}$ e definindo:

$$\alpha_i = \frac{\beta_i^{hipo} - \beta_i^{hiper}}{2\beta_{11}} = \frac{1}{2} \frac{\beta_i^{hipo} - \beta_i^{hiper}}{\beta_{11}^{hipo} - \beta_{11}^{hiper}} \tag{10}$$

$$\alpha_{ij} = \frac{\beta_{ij}^{hipo} - \beta_{ij}^{hiper}}{2\beta_{11}} = \frac{1}{2} \frac{\beta_{ij}^{hipo} - \beta_{ij}^{hiper}}{\beta_{11}^{hipo} - \beta_{11}^{hiper}} \tag{11}$$

então obtemos:

$$\frac{{x_1}^2}{2} + \left(\alpha_1 + \sum_{j=2}^{k} \alpha_{1j} x_j \right) x_1 + \left(\alpha_0 + \sum_{i=2}^{k} \alpha_i x_i + \sum_{i=2}^{k} \sum_{j=i}^{k} \alpha_{ij} x_i x_j \right) = 0 \tag{12}$$

A equação 12 pode ser resolvida para x_1 utilizando a fórmula quadrática:

$$x_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{13}$$

em que $a = 1/2$ e b e c são:

$$b = \alpha_1 + \sum_{j=2}^{k} \alpha_{1j} x_j \tag{14}$$

$$c = \alpha_0 + \sum_{i=2}^{k} \alpha_i x_i + \sum_{i=2}^{k} \sum_{j=i}^{k} \alpha_{ij} x_i x_j \tag{15}$$

Usando a substituição $a = 1/2$, a equação 13 torna-se simplesmente:

$$x_1 = -b \pm \sqrt{b^2 - 2c} \tag{16}$$

Em suma:

$$\alpha_i = \frac{1}{2} \frac{\beta_i^{hipo} - \beta_i^{hiper}}{\beta_{11}^{hipo} - \beta_{11}^{hiper}}$$
$$\alpha_{ij} = \frac{1}{2} \frac{\beta_{ij}^{hipo} - \beta_{ij}^{hiper}}{\beta_{11}^{hipo} - \beta_{11}^{hiper}}$$
$$b = \alpha_1 + \sum_{j=2}^{k} \alpha_{1j} x_j$$
$$c = \alpha_0 + \sum_{i=2}^{k} \alpha_i x_i + \sum_{i=2}^{k} \sum_{j=i}^{k} \alpha_{ij} x_i x_j$$
$$x_1 = -b \pm \sqrt{b^2 - 2c}$$

Tabela de correspondência:

x_1	x_2	x_3	x_4	x_5
C	Mn	Si	Cr	Ni

$$\begin{aligned}
b &= \alpha_{C} + \alpha_{CMn}Mn + \alpha_{CSi}Si + \alpha_{CCr}Cr + \alpha_{CNi}Ni && (17)\\
c &= \alpha_0 + \alpha_{Mn}Mn + \alpha_{Si}Si + \alpha_{Cr}Cr + \alpha_{Ni}Ni+\\
&\alpha_{Mn^2}\ Mn^2 + \alpha_{MnSi}\ Mn\ Si + \alpha_{MnCr}\ Mn\ Cr + \alpha_{MnNi}\ Mn\ Ni+\\
&\alpha_{Si^2}\ Si^2 + \alpha_{SiCr}\ Si\ Cr + \alpha_{SiNi}\ Si\ Ni+\\
&\alpha_{Cr^2}\ Cr^2 + \alpha_{CrNi}\ Cr\ Ni+\\
&\alpha_{Ni^2}\ Ni^2 && (18)
\end{aligned}$$