Project CME211 - Writeup

Paula Charles

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1 Introduction

In this project, we aim at solving 2D heat equations. To do so, we use sparse matrix solvers in C++. We use the finite- difference method to discretize the system into a grid of points. We then solve this system using our matrix solvers and vizualize the results. This enables us to visualize heat maps of systems that are in contact with hot and cold streams, and for which the temperature is not uniform.

This solution is only able to solve simplified periodic systems.

2 CG solver implementation

To implement the CG solver, we use 2 classes:

- The Class SparseMatrix that handles sparse matrices. It initializes a matrix as a COO matrix, which can then be modified thanks to its method AddEntry(). It has then some more methods that can be used for our 2D heat problem: a method that converts the COO matrix into a CSR matrix (that is more efficient for calculations) and a get_CGSolver() method that uses the files CGSolver.hpp/cpp to get a solution. A pseudo-code for this solver is available below.

  To implement the Conjugate gradient code, we used several functions to prevent redundancy. These functions can be found in the matvecops.cpp file and enable us to make basic vector / matrix calculations.

- The Class HeatEquation2D that is the center of the heat equation and returns the solution files. It has mainly 2 methods. A first method, SetUp() to set up the system, get the matrix and vectors that constitute the numerical equation. It takes an input file as input (see the specifications of that input file in the User Guide). The matrix created is a COO matrix and its size is the number of unknowns of the system: every grid point except the boundary ones (which are known) and the periodic ones.

A second method, Solve(), solves the equations, by using the class SparseMatrix method get_CGSolver() and returns the solution files (see User Guide for more description of these files).

```
Data: matrix A, vector b, vector x, double tol
Result: Number of iterations to reach convergence (-1 if not
        convergent), updated vector of solutions
Initialization: u0 = x;
success = 0; // it will tell us if algorithm converges
r0 = b - A * u0;
L2norm_r0 = L2norm(r0);
p0 = r0;
niter = 0; // number of iterations
niter_max = number of rows;
while niter < niter_max do
    niter = niter + 1;
    alpha = transp(r0)*r0 / ( transp(p0)*A*p0 );
    u0 = u0 + alpha*p0;
    r1 = r0 - alpha*A*p0;
    L2norm_r1 = L2norm(r1);
    if L2norm_r1/L2norm_r0 < tol then
        success = 1;
        break;
    else
        beta = transp(r1)*r1 / transp(r0)*r0;
        p0 = r1 + beta*p0;
        r0 = r1;
    end
end
x = u0; // update value of the vector of solutions
if success = 0 then
    return -1;
else
    return niter;
end
```

3 Users guide

To compile the code, one needs to create an input file. This input file needs:

- In the first line, separated by spaces: the length, the width and the grid size. The length and width should be multiples of the grid size (otherwise, an error will be created).

- In the second line, separated by spaces: the cold temperature and the hot temperature.

Once that input is created, we need to compile the code and for that, we can use the makefile file: we only need to write ***make*** in the command.

Then, we need to run the files: we do that thanks to the command:

./main [name of your input file] [generic name of your solution files].

If the CG solver converges, it will print in how many iterations this solver converges and will create solution files for the vector of unknowns every 10 iterations. These solution files are called after the generic name you gave in the command line. They will for instance be called solution000.txt, solution010.txt and so on if you asked for the generic name "solution".

The files of solutions print out every coordinate of the vector of unknowns. Let's call this vector (u_i,j)i,j. Then, the file of solution prints out: u_1,0, u_1,1, ... u_1,n-1, u_2,0, ..., u_2,n-1, ... u_m-1,n-1. In this file, the first and last rows (i=0 and i=m) are not printed, as they are not unknown (we know their value: the boundary temperatures) The last column is also not printed (j=n), because it is the same as the first (we consider a periodic system).

To see a plot representing the value of this vector across the system, we can use a python file thanks to the command:

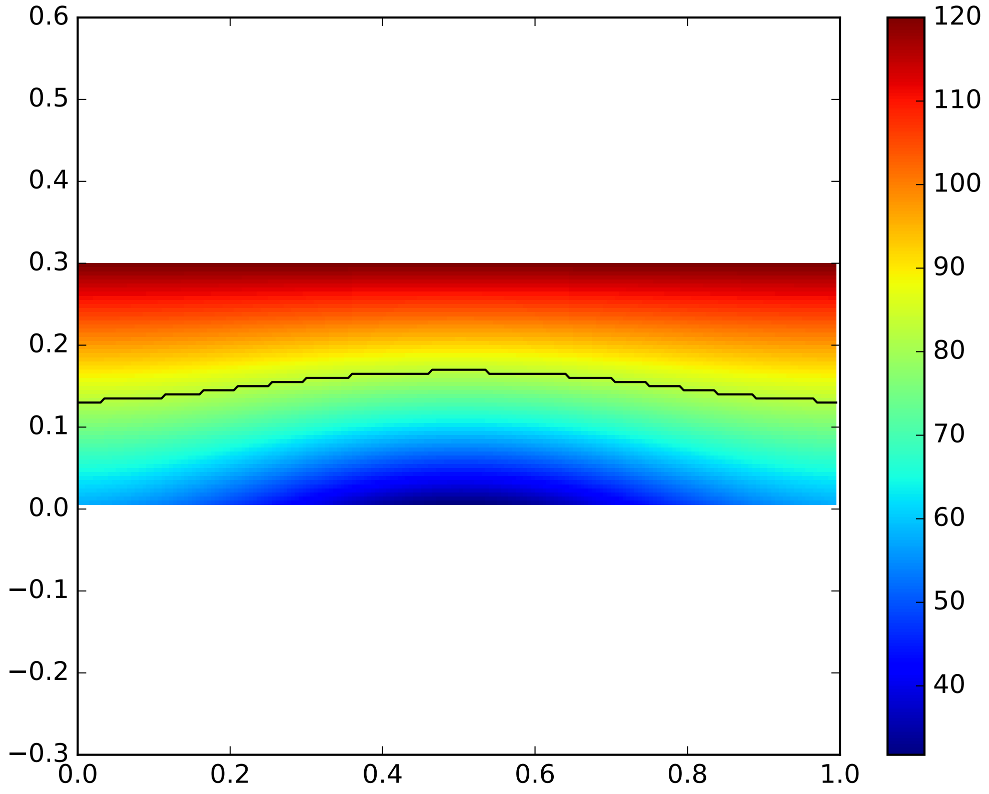
python3 postprocess.py input1.txt solution110.txt

It will create a pdf that can be seen. Please find on the next page an example of such a file. It will also print the mean temperature of the system represented by the solution file.

References

[1] CME 211: Project Part 2 (Handout) , Andreas Santucci, December 2021. Stanford CA.

[2] CME 211: Project Part 1 (Handout) , Andreas Santucci, November 2021. Stanford CA.



plot.pdf

Figure 1: An example of a pseudo color plot with an isoline to represent the location of the mean temperature