Contents


```
% Paula Burgi
% Midterm 2, Problem 2

%clear
% close all
```

Part a - generate gravity data

parameters

```
n = 40;
m = 40;
L  = 40;
hs = sym('hs');
xs = sym('xs');
sn = linspace(1,40,n)';
xn = linspace(1,40,n)';

% function that represents real pipe height (data)
h1 = (2.*pi.*(xn-(L./4)))./L;
h2 = (2.*pi.*(xn-((3.*L)./4)))./L;
h  = 15 - (5./2).*tanh(h1) + (5/2).*tanh(h2);
% functional form of data
df  = hs./(((sn-xs).^2 + hs.^2).^(3/2));

% turn integral into sum in gravity equation
% sum over all x's a d symbolic variables hsi
d_all = [];
for i = 1:m
    % add 0 if i=1-9 so symvar puts in correct order
    if i < 10
        hsi = ['hs0' num2str(i)];
    else
        hsi = ['hs' num2str(i)];
    end
    % create symbolic variable for each hieght estimate
    sym(hsi);
    % substitute x and hs into gravity equation and sum
    di = subs(df, hs, hsi);
    di = subs(di, xs, i);
    if i == 1
        d_all = di;
    else
        d_all = d_all+di;
    end
end
```

```
% generate gravity anomaly data
for i = 1:m
    % add 0 if i=1-9 so symvar puts in correct order
    if i < 10
        hsi = ['hs0' num2str(i)];
    else
        hsi = ['hs' num2str(i)];
    end
    % let each height variable equal result from given function
    evalc([hsi '=' num2str(h(i))]);
end

% evaluate gravity function after above for-loop assigned values to each
% symbolic variable hsi s.t. they are equal to result from equation 1.25
dnn = eval(subs(d_all));
% add 1% noise
st  = max(dnn).*0.01;
dn  = dnn + randn(40,1).*st;
```

Part b - use LM subroutine to find height of pipe

calculate covar matrix

```
Cd   = eye(m).*(st^2);
Cdi  = inv(sqrt(Cd));

% calculate residual function f to go in LM code
f    = Cdi*(d_all - dn);

% calculate Jacobian
J = [];
for i = 1:m
    % add 0 if i=1-9 so symvar puts in correct order
    if i < 10
        hsi = ['hs0' num2str(i)];
    else
        hsi = ['hs' num2str(i)];
    end
    % add column to jacobian with derivative wrt each hieght variable hsi
    Ji  = diff(f, hsi);
    J   = [J Ji];
end

% initial guess
%var0 = h - 1;
var0 = ones(n,1).*12.5;

% convergence criteria
ep = 1e-2;

% find height values using LM code
[varf, k, Cm, X2] = LMLSQ(f, var0, J, ep);
varf = varf';

disp(['Number of iterations to convergence (non-reg): ' num2str(k)]);
```

```
Number of iterations to convergence (non-reg): 100
```

Part c - plot non-regularized results & chi^2

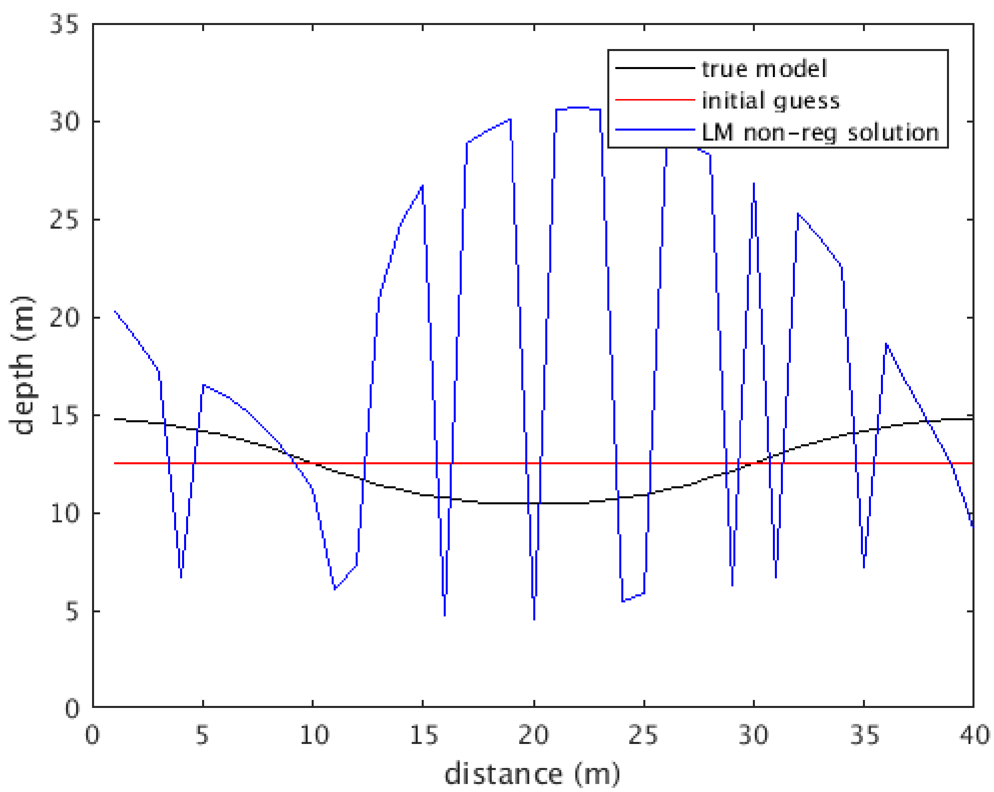
```
figure; hold on; box on;
plot(xn, h, 'k');
plot(xn, var0, 'r');
plot(xn, varf, 'b');
xlabel('distance (m)');
ylabel('depth (m)');
legend('true model', 'initial guess', 'LM non-reg solution');

p_value = chi2cdf(X2,m-n, 'upper')

% The p-value for the the non-regularized result is 0. This implies that it
% is outside the limits of a good p-value.
```

```
p_value =

     0
```



Part d - Regularization

alpha (regularization weighting)

```
ai  = 121; %1:10:500;
varfra = [];
for j = 1:length(ai)
    a = ai(j);
    % 2nd order Tikhonov matrix
    L  = [-2 1 zeros(1, n-3) 1; diff(diff(eye(n)));1 zeros(1, n-3) 1 -2];%.*(1/(dn.^2));
```

```
    for i = 1:m
        if i < 10
            hsi = ['hs0' num2str(i)];
        else
            hsi = ['hs' num2str(i)];
        end
        evalc([hsi '=' num2str(var0(i))]);
    end
    d0  = eval(subs(d_all));
    dxx = ones(m,1);
    % augment residual function and jacobian
    fa = [f; a.*L*var0];
    K =  [J; a.*L];

    % calculate model
    [varfr, k, Cm, X2r] = LMLSQ(fa, var0, K, ep);

    varfra = [varfra varfr'];

end

disp(['choice of alpha: ' num2str(a)]);
disp(['Number of iterations to convergence (reg): ' num2str(k)]);
```

```
choice of alpha: 121
Number of iterations to convergence (reg): 9
```

Part e - calculate chi^2

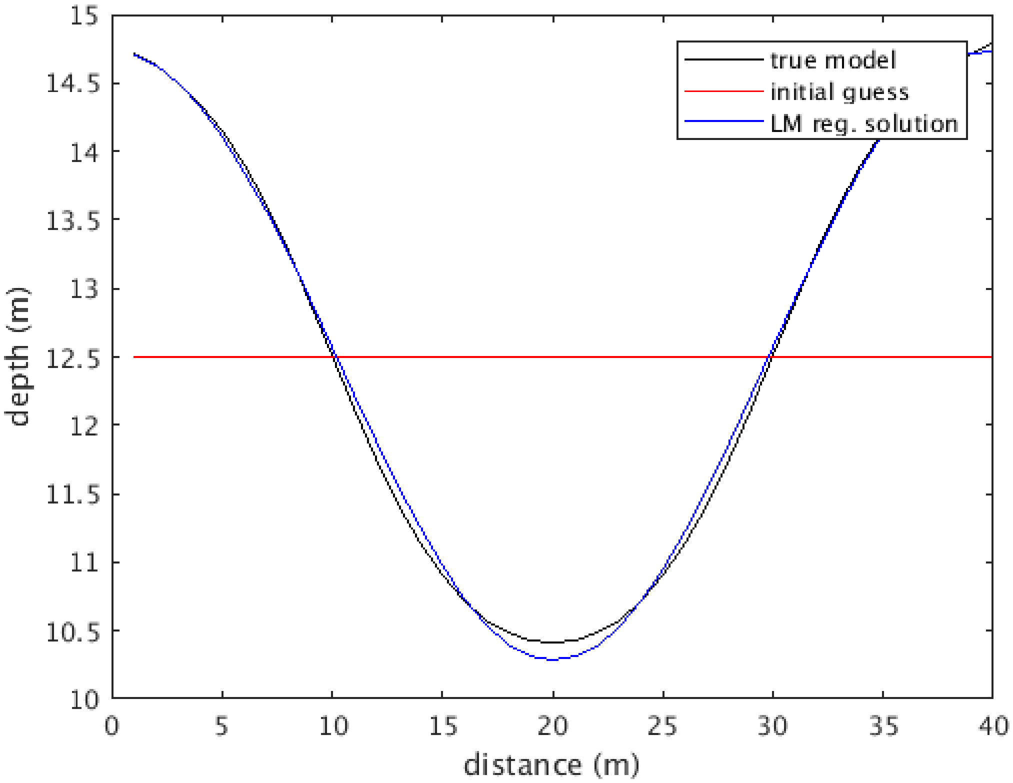
```
% plot
figure; hold on; box on;
plot(xn, h, 'k');
plot(xn, var0, 'r');
plot(xn, varfra, 'b');
xlabel('distance (m)');
ylabel('depth (m)');
legend('true model', 'initial guess', 'LM reg. solution');

% calculate chi^2
p_value = chi2cdf(X2r,n, 'upper')

% with regularization (where DOF = n), the p-value is ~0.96, which means
% the results are acceptable.
```

```
p_value =

    0.9835
```



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