HOMEWORK 2: Length methods

Reference: Lectures 8-13 **Due: 2018 September 21**

Problem 1. As discussed in class, a strategy for designing inversem methods is to minimize a combination of the data and model resolution spreads and the trace of the model error covariance matrix:

$$\tilde{G} = \underset{\tilde{G}}{\operatorname{argmin}} \ \alpha \, \text{spread}(R_d) + \beta \, \text{spread}(R_m) + \gamma \, \text{Tr}(C_m)$$

Take $\alpha = 1$, $\beta = 0$, and $\gamma = \epsilon^2$ and recover the damped least squares estimator discussed in class. To simplify things, you can take $C_d = I$. You will need to recall the definition of spread and make use of the rules of matrix differentiation.

Problem 2. Revisit the vertical seismic profiling problem from homework 1. Implement the weighted damped least squares estimator this time. Take C_d to be the identity matrix to begin with, and use the second derivative matrix for W_m. Decrease the number of data points n while adjusting the ϵ parameter accordingly to preserve a solution. At what n do things break down, and what ϵ is needed to retain a solution? Next, try assigning very small weights to data from the flat region (where the 'true' slowness is 2). Note what happens in that region for small and large values of ϵ.

Problem 3. Go back to the purely overdetermined vertical seismic profiling problem from homework 1. This time, rewrite the code to impose the constraint that the model slowness is identically equal to 2 in the flat region (where the 'true' slowness is 2). Show the results of the constrained inversion.

Problem 4. The pseudoinverse was defined by Moore and Penrose as one having the following properties:

1. $G\tilde{G}G = G$
2. $\tilde{G}G\tilde{G} = \tilde{G}$
3. $(G\tilde{G})^T = G\tilde{G}$
4. $(\tilde{G}G)^T = \tilde{G}G$

Show that the SVD psuedoinverse as defined in class has these properties.

