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NRC: 2359

Homework 1

1. Transform the following numbers from one base to another

Decimal to binary

a) 10_{10}
$$\begin{array}{r} 10 \div 2 = 5 \\ 5 \div 2 = 2 \text{ R } 1 \\ 2 \div 2 = 1 \text{ R } 0 \\ 1 \div 2 = 0 \text{ R } 1 \end{array}$$

 $= 1010_2$

b) 1369_{10}
$$\begin{array}{r} 1369 \div 2 = 684 \text{ R } 1 \\ 684 \div 2 = 342 \text{ R } 0 \\ 342 \div 2 = 171 \text{ R } 0 \\ 171 \div 2 = 85 \text{ R } 1 \\ 85 \div 2 = 42 \text{ R } 1 \\ 42 \div 2 = 21 \text{ R } 0 \\ 21 \div 2 = 10 \text{ R } 1 \\ 10 \div 2 = 5 \text{ R } 0 \\ 5 \div 2 = 2 \text{ R } 1 \\ 2 \div 2 = 1 \text{ R } 0 \\ 1 \div 2 = 0 \text{ R } 1 \end{array}$$

 $= 1010101001_2$

c) 9234876_{10}
$$\begin{array}{r} 9234876 \div 2 = 4617438 \text{ R } 0 \\ 4617438 \div 2 = 2308719 \text{ R } 0 \\ 2308719 \div 2 = 1154359 \text{ R } 1 \\ 1154359 \div 2 = 577179 \text{ R } 1 \\ 577179 \div 2 = 288589 \text{ R } 1 \\ 288589 \div 2 = 144294 \text{ R } 1 \\ 144294 \div 2 = 72147 \text{ R } 0 \\ 72147 \div 2 = 36073 \text{ R } 1 \\ 36073 \div 2 = 18036 \text{ R } 1 \\ 18036 \div 2 = 9018 \text{ R } 0 \\ 9018 \div 2 = 4509 \text{ R } 0 \\ 4509 \div 2 = 2254 \text{ R } 1 \\ 2254 \div 2 = 1127 \text{ R } 0 \\ 1127 \div 2 = 563 \text{ R } 1 \\ 563 \div 2 = 281 \text{ R } 1 \\ 281 \div 2 = 140 \text{ R } 1 \\ 140 \div 2 = 70 \text{ R } 0 \\ 70 \div 2 = 35 \text{ R } 0 \\ 35 \div 2 = 17 \text{ R } 1 \\ 17 \div 2 = 8 \text{ R } 1 \\ 8 \div 2 = 4 \text{ R } 0 \\ 4 \div 2 = 2 \text{ R } 0 \\ 2 \div 2 = 1 \text{ R } 0 \\ 1 \div 2 = 0 \text{ R } 1 \end{array}$$

 $= 10001100111010011011100_2$

d) 49263749_{10}
$$\begin{array}{r} 49263749 \div 2 = 24631874 \text{ R } 1 \\ 24631874 \div 2 = 12315937 \text{ R } 0 \\ 12315937 \div 2 = 6157968 \text{ R } 1 \\ 6157968 \div 2 = 3078984 \text{ R } 0 \\ 3078984 \div 2 = 1539492 \text{ R } 0 \\ 1539492 \div 2 = 769746 \text{ R } 0 \\ 769746 \div 2 = 384873 \text{ R } 0 \\ 384873 \div 2 = 192436 \text{ R } 1 \\ 192436 \div 2 = 96218 \text{ R } 0 \\ 96218 \div 2 = 48109 \text{ R } 0 \\ 48109 \div 2 = 24054 \text{ R } 1 \\ 24054 \div 2 = 12027 \text{ R } 0 \\ 12027 \div 2 = 6013 \text{ R } 1 \\ 6013 \div 2 = 3006 \text{ R } 1 \\ 3006 \div 2 = 1503 \text{ R } 0 \\ 1503 \div 2 = 751 \text{ R } 1 \\ 751 \div 2 = 375 \text{ R } 1 \\ 375 \div 2 = 187 \text{ R } 1 \\ 187 \div 2 = 93 \text{ R } 1 \\ 93 \div 2 = 46 \text{ R } 1 \\ 46 \div 2 = 23 \text{ R } 0 \\ 23 \div 2 = 11 \text{ R } 1 \\ 11 \div 2 = 5 \text{ R } 1 \\ 5 \div 2 = 2 \text{ R } 1 \\ 2 \div 2 = 1 \text{ R } 0 \\ 1 \div 2 = 0 \text{ R } 1 \end{array}$$

 $= 10110111101010010000101_2$

Decimal to binary using 2's complement

a) -20 (+20) 010100 Add 1 \rightarrow 101011
 101011
 $+ 000001$
 $\hline 101100 //$

b) -1025 (+1025) 010000000001 \rightarrow 10111111110
 10111111110
 $+ 000000000001$
 $\hline 10111111111 //$

c) -3925 (+) 01111010101 \rightarrow 100010101010
 100010101010
 $+ 000000000001$
 $\hline 100010101011 //$

d) -104596 (+) 011001100010010100 \rightarrow 10011001110101011
 10011001110101011
 $+ 0000000000000001$
 $\hline 10011001110101100 //$

Unsigned binary to hex

a) $\overset{15}{1}\overset{14}{0}\overset{13}{0}\overset{12}{1}\overset{11}{1}\overset{10}{0}\overset{9}{1}\overset{8}{0}\overset{7}{1}\overset{6}{1}\overset{5}{0}\overset{4}{1}\overset{3}{1}\overset{2}{0}\overset{1}{0}\overset{0}{1}$
 $\text{C F 5 6 6 E D 8 2 9}$

Short: CF566ED829₁₆

Long: Binary \rightarrow decimal \rightarrow Hex

$1 \times 2^{19} + 1 \times 2^{18} + 1 \times 2^{17} + 1 \times 2^{16} + 1 \times 2^{15} + 1 \times 2^{14} + 1 \times 2^{13} + 1 \times 2^{12} + 1 \times 2^{11} + 1 \times 2^{10} + 1 \times 2^9 + 1 \times 2^8 + 1 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 890508335145_{10}$

890508335145_{10}
 $\begin{array}{r} 9 \ 5365670946116 \\ \hline 2 \ 3478548184116 \\ \hline 8 \ 217409261116 \\ \hline 13 \ 13588078116 \\ \hline 14 \ 849754116 \\ \hline 6 \ 53078116 \\ \hline 6 \ 3317116 \\ \hline 5 \ 207116 \\ \hline 15 \ 12 \end{array}$
 $= \text{CF566ED829}_{16}$

b) $\overset{15}{1}\overset{14}{0}\overset{13}{0}\overset{12}{0}\overset{11}{1}\overset{10}{1}\overset{9}{1}\overset{8}{0}\overset{7}{0}\overset{6}{0}\overset{5}{1}\overset{4}{1}\overset{3}{1}\overset{2}{0}\overset{1}{0}\overset{0}{1}\overset{0}{1}\overset{0}{1}\overset{0}{1}$
 $\text{8 7 8 E 3 8 E 3 F 3}$

Short: 878E38E3F3₁₆

Long: $1 \times 2^{19} + 1 \times 2^{18} + 1 \times 2^{17} + 1 \times 2^{16} + 1 \times 2^{15} + 1 \times 2^{14} + 1 \times 2^{13} + 1 \times 2^{12} + 1 \times 2^{11} + 1 \times 2^{10} + 1 \times 2^9 + 1 \times 2^8 + 1 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 582206678003_{10}$

582206678003_{10}
 $\begin{array}{r} 3 \ 36387917375116 \\ \hline 15 \ 2274244885116 \\ \hline 3 \ 142140302116 \\ \hline 14 \ 8868768116 \\ \hline 8 \ 555235116 \\ \hline 2 \ 34702116 \\ \hline 14 \ 2168116 \\ \hline 8 \ 135116 \\ \hline 7 \ 8 \end{array}$
 $= 878E38E3F3_{16}$

$$\begin{array}{r} 845978103518 \\ \underline{3} \quad 105747267918 \\ \quad \underline{6} \quad 13218407818 \\ \quad \quad \underline{6} \quad 1652300918 \\ \quad \quad \quad \underline{1} \quad 206537618 \\ \quad \quad \quad \quad \underline{0} \quad 25817218 \\ \quad \quad \quad \quad \quad \underline{4} \quad 322318 \\ \quad \quad \quad \quad \quad \quad \underline{7} \quad 403318 \\ \quad \quad \quad \quad \quad \quad \quad \underline{1} \quad 50418 \\ \quad \quad \quad \quad \quad \quad \quad \quad \underline{2} \quad 6318 \\ \quad \quad \quad \quad \quad \quad \quad \quad \quad \underline{7} \quad 7 \end{array} = -77017401653811$$

b) $\frac{010101010101111111111110000000}{\begin{smallmatrix} 2 & 5 & 2 & 5 & 7 & 7 & 7 & 7 & 6 & 0 & 0 \end{smallmatrix}}$

Short: $+2525777600_{10}$.

Long: $1x^{21} + 1x^{20} + 1x^{19} + 1x^{18} + 1x^{17} + 1x^{16} + 1x^{15} + 1x^{14} + 1x^{13} + 1x^{12} + 1x^{11} + 1x^{10} + 1x^9 + 1x^8 + 1x^7 + 1x^6 + 1x^5 + 1x^4 + 1x^3 + 1x^2 + 1x + 1 = 2864709504_{10}$

$$\begin{array}{r} 2864709504_{10} \\ \underline{0 \ 358088688_{10}} \\ 0 \ 4476108618 \\ \underline{0 \ 4476108618} \\ 0 \ 559513518 \\ \underline{0 \ 559513518} \\ 0 \ 69939118 \\ \underline{0 \ 69939118} \\ 0 \ 8242318 \\ \underline{0 \ 8242318} \\ 0 \ 992318 \\ \underline{0 \ 992318} \\ 0 \ 136518 \\ \underline{0 \ 136518} \\ 0 \ 17018 \\ \underline{0 \ 17018} \\ 0 \ 2118 \\ \underline{0 \ 2118} \\ 0 \end{array} = +2525777600_{10}$$

c) $\frac{00111000111000000111111100000101010}{\begin{smallmatrix} 1 & 6 & 1 & 6 & 0 & 1 & 7 & 7 & 4 & 0 & 5 & 2 \end{smallmatrix}}$

Short: -161601774052_{10} .

Long: $1x^{39} + 1x^{38} + 1x^{37} + 1x^{36} + 1x^{35} + 1x^{34} + 1x^{33} + 1x^{32} + 1x^{31} + 1x^{30} + 1x^{29} + 1x^{28} + 1x^{27} + 1x^{26} + 1x^{25} + 1x^{24} + 1x^{23} + 1x^{22} + 1x^{21} + 1x^{20} + 1x^{19} + 1x^{18} + 1x^{17} + 1x^{16} + 1x^{15} + 1x^{14} + 1x^{13} + 1x^{12} + 1x^{11} + 1x^{10} + 1x^9 + 1x^8 + 1x^7 + 1x^6 + 1x^5 + 1x^4 + 1x^3 + 1x^2 + 1x + 1 = 15267788842_{10}$

$$\begin{array}{r} 15267788842_{10} \\ \underline{0 \ 1908473605_{10}} \\ 0 \ 23855920018 \\ \underline{0 \ 23855920018} \\ 0 \ 2981990018 \\ \underline{0 \ 2981990018} \\ 0 \ 372748718 \\ \underline{0 \ 372748718} \\ 0 \ 46593518 \\ \underline{0 \ 46593518} \\ 0 \ 5824118 \\ \underline{0 \ 5824118} \\ 0 \ 728018 \\ \underline{0 \ 728018} \\ 0 \ 91018 \\ \underline{0 \ 91018} \\ 0 \ 11318 \\ \underline{0 \ 11318} \\ 0 \ 1418 \\ \underline{0 \ 1418} \\ 0 \end{array} = -161601774052_{10}$$

d) $\frac{00101010101000000101010101111000}{\begin{smallmatrix} 1 & 2 & 5 & 2 & 4 & 0 & 5 & 2 & 5 & 3 & 7 & 0 \end{smallmatrix}}$

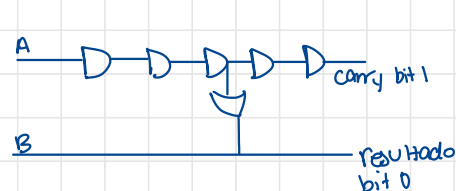
Short: -125240525370_{10} .

Long: $1x^{21} + 1x^{20} + 1x^{19} + 1x^{18} + 1x^{17} + 1x^{16} + 1x^{15} + 1x^{14} + 1x^{13} + 1x^{12} + 1x^{11} + 1x^{10} + 1x^9 + 1x^8 + 1x^7 + 1x^6 + 1x^5 + 1x^4 + 1x^3 + 1x^2 + 1x + 1 = 11450624760_{10}$

$$\begin{array}{r} 11450624760_{10} \\ \underline{0 \ 1431328095_{10}} \\ 0 \ 17891601118 \\ \underline{0 \ 17891601118} \\ 0 \ 2236450118 \\ \underline{0 \ 2236450118} \\ 0 \ 279556218 \\ \underline{0 \ 279556218} \\ 0 \ 34944518 \\ \underline{0 \ 34944518} \\ 0 \ 4368018 \\ \underline{0 \ 4368018} \\ 0 \ 546018 \\ \underline{0 \ 546018} \\ 0 \ 68318 \\ \underline{0 \ 68318} \\ 0 \ 8518 \\ \underline{0 \ 8518} \\ 0 \ 1018 \\ \underline{0 \ 1018} \\ 0 \end{array}$$

2. Boolean circuits

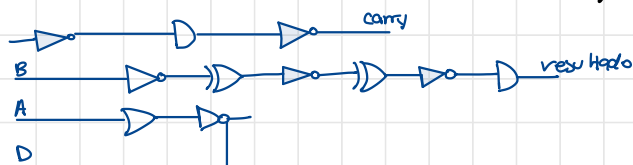
a) Multiplication of two binary numbers of length 2 bits



Truth table

A	B	C	bit 1	bit 0	R
0	0	0	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0
1	1	0	0	1	1
0	0	0	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0
1	1	1	1	0	2

b) Two's complement for a binary number of length 3 bits



Truth table

B	A	D	C	R
0	0	0	0	000
0	0	1	1	111
0	1	0	1	110
0	1	1	1	101
1	0	0	0	000
1	0	1	0	111
1	1	0	0	110
1	1	1	0	101

3. Do the following multiplications in binary

a) 5×8

MD (-5) 0101
MR (8) 1000

$$\begin{array}{r} 1010 \\ + 1011 (-5) \\ \hline 0111 \\ + 1000 \\ \hline 10100 \end{array}$$

0000 1000 0 → 0000 0100 0 → 0000 0010 0 → 0000 0001 0 → 0101 0001 0 → 0010 1000 1 = 00101000 //

PD = PD - MD
0000 0000 0 + 0101 0101 → 0101 0001 0001

b) $11 \times (-10)$

MD (11) 01011
MR (-10) 01010

$$\begin{array}{r} 10100 \\ + 10101 \\ \hline 10101 \\ + 10110 (-10) \\ \hline 01010 \\ + 01001 \\ \hline 10101 \end{array}$$

PD = PD - MD
00000 01010 0 + 10101 10101 → 10101 01010 01010

PD = PD - MD
00000 01010 0 + 10101 10101 → 11010 01010 01010

PD = PD + MD
11010 01010 01010 + 01010 01010 → 01000 01010 1 → 00010

PD = PD - MD
00000 01010 0 + 10101 10101 → 11010 01010 01010

PD = PD + MD
11010 01010 01010 + 01010 01010 → 11100 10010 //

c) 2×3

$$\begin{array}{r} MD (+3) \ 010 \rightarrow \begin{array}{r} 101 \\ + 110 \\ \hline \end{array} \\ MR (+3) \ 011 \rightarrow \begin{array}{r} 100 \\ + 110 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} 010 \\ 011 \end{array} \xrightarrow{PD = PD - MD} \begin{array}{r} 000 \\ 011 \end{array} \xrightarrow{+ 110} \begin{array}{r} 110 \\ 011 \end{array} \xrightarrow{0} \begin{array}{r} 111 \\ 010 \end{array} \xrightarrow{1} \begin{array}{r} 111 \\ 001 \end{array} \xrightarrow{1} \begin{array}{r} 100 \\ 001 \end{array} \xrightarrow{PD = PD + MD} \begin{array}{r} 111 \\ + 010 \\ \hline 2001 \end{array} \rightarrow \begin{array}{r} 001 \\ 100 \\ 001 \end{array} \xrightarrow{1} \begin{array}{r} 000 \\ 110 \\ 000 \end{array} = 000 \ 110_2 (6)$$

d) $(-4) \times (-8)$

$$\begin{array}{l} MD = 0100 \rightarrow \begin{array}{r} 1011 \\ + 1100 \ (-4) \\ \hline 0120 \end{array} \\ MR = 1000 \rightarrow \begin{array}{r} 0111 \\ + 1000 \ (-8) \\ \hline 0111 \end{array} \end{array}$$

$$\begin{array}{r} 1100 \\ 0000 \ 1000 \end{array} \xrightarrow{0} \begin{array}{r} 0000 \\ 0100 \end{array} \xrightarrow{0} \begin{array}{r} 0000 \\ 0010 \end{array} \xrightarrow{0} \begin{array}{r} 0000 \\ 0001 \end{array} \xrightarrow{PD = PD + MD} \begin{array}{r} 0000 \\ + 0100 \\ \hline 0100 \end{array} \rightarrow \begin{array}{r} 0100 \\ 0001 \end{array} \xrightarrow{0} \begin{array}{r} 0010 \\ 0000 \end{array} \xrightarrow{1} \begin{array}{r} 0010 \\ 0000 \end{array} = 0010 \ 0000_2 (32)$$