

Formulation for The Merry Movie Montage

Paula Fermín Cueto

December 2021

Sets:

A	Set of activities (either a movie, a wildcard or no movie). $A = \{\emptyset, *, A, B, C, D, E, F, G\}$
M	Set of movies. $M = \{A, B, C, D, E, F, G\}$
T	Set of time periods. $T = 1 \dots T_{max}$.
G	Set of groups of elves wathing movies. $G = \{1, 2, 3\}$.
P	Set of movie permutations.
P_{all}	Set of movie permutations that must be watched by all teams.

Parameters:

$ P $	Number of permutations of $ M $ movies. $ P = M ! = 7! = 5,040$.
$ P_{all} $	Number of permutations starting with AB . $ P_{all} = M - 2 ! = 5! = 120$.
T_{max}	Upper bound on the length of the longest schedule.
	$T_{max} = \frac{1}{ G }(M - 1)!(2 M - 1) + \frac{2}{ G }(M - 2)! M = 2,880$.
ω_{pk}	Movie in position $k \in 1 \dots M $ of permutation $p \in P$.

Decision variables:

x_{gta}	$= \begin{cases} 1 & \text{if group } g \in G \text{ performs activity } a \in A \text{ at time } t \in T. \\ 0 & \text{otherwise.} \end{cases}$
δ_{pgt}	$= \begin{cases} 1 & \text{if permutation } p \in P \text{ is found in group } g \in G \text{ starting at time } t \in T. \\ 0 & \text{otherwise.} \end{cases}$
γ_{pg}	$= \begin{cases} 1 & \text{if permutation } p \in P \text{ is found in group } g \in G. \\ 0 & \text{otherwise.} \end{cases}$
λ_{pgt}	$= \begin{cases} 1 & \text{if } \delta_{pgt} \text{ is the only permutation } p \in P \text{ counted in team } g \in G. \\ 0 & \text{otherwise.} \end{cases}$
ℓ	Duration of the longest schedule of the three teams.

$$\min_{x, \delta, \gamma, \ell} \quad \ell \tag{1}$$

$$\text{s.t.} \quad \ell \geq \sum_{\substack{t \in T, \\ a \in A \setminus \{\emptyset\}}} x_{gta} \quad \forall g \in G. \tag{2}$$

$$\sum_{a \in A} x_{gta} = 1 \quad \forall g \in G, t \in T. \tag{3}$$

$$x_{gt\emptyset} \leq x_{g,t+1,\emptyset} \quad \forall g \in G, t = 1 \dots T_{max} - 1. \quad (4)$$

$$x_{gtm} + x_{g,t+1,m} \leq 1 \quad \forall g \in G, m \in M, t = 1 \dots T_{max} - 1. \quad (5)$$

$$\sum_{s=t}^{t+|M|-1} x_{gsm} \leq 2 \quad \forall g \in G, m \in M, t = 1 \dots T_{max} - |M| + 1. \quad (6)$$

$$\sum_{s=t}^{t+|M|-1} x_{gs*} \leq 1 \quad \forall g \in G, t = 1 \dots T_{max} - |M| + 1. \quad (7)$$

$$\sum_{t \in T} x_{gt*} \leq 2 \quad \forall g \in G. \quad (8)$$

$$\delta_{pgt} \leq x_{gk\omega_{pk}} \quad \forall g \in G, p \in P, t = 1 \dots T_{max} - |M| + 1, k = p \dots p + |M| - 1. \quad (9)$$

$$\gamma_{pg} \geq \delta_{pgt} \quad \forall g \in G, p \in P, t = 1 \dots T_{max} - |M| + 1. \quad (10)$$

$$\gamma_{pg} \leq \delta_{pgt} + \lambda_{pgt} \quad \forall g \in G, p \in P, t = 1 \dots T_{max} - |M| + 1. \quad (11)$$

$$\sum_{t=1}^{T_{max}-|M|+1} \lambda_{pgt} = T_{max} - |M| \quad \forall g \in G, p \in P. \quad (12)$$

$$\sum_{g \in G} \gamma_{pg} \geq 1 \quad \forall p \in P \setminus P_{all}. \quad (13)$$

$$\gamma_{pg} = 1 \quad \forall p \in P_{all}, g \in G. \quad (14)$$

$$x_{gta} \in \{0, 1\} \quad \forall g \in G, t \in T, a \in A. \quad (15)$$

$$\delta_{pgt} \in \{0, 1\} \quad \forall p \in P, g \in G, t = 1 \dots T_{max} - |M| + 1. \quad (16)$$

$$\lambda_{pgt} \in \{0, 1\} \quad \forall p \in P, g \in G, t = 1 \dots T_{max} - |M| + 1. \quad (17)$$

$$\gamma_{pg} \in \{0, 1\} \quad \forall p \in P, g \in G. \quad (18)$$

$$\ell \in \mathbb{N}. \quad (19)$$