Formulation for The Merry Movie Montage

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Sets:

A	Set of activities (either a movie, a wildcard or no movie). $A = \{\emptyset, *, A, B, C, D, E, F, G\}$
M	Set of movies. $M = \{A, B, C, D, E, F, G\}$
T	Set of time periods. $T = 1 \dots T_{max}$.
G	Set of groups of elves wathing movies. $G = \{1, 2, 3\}$.
P	Set of movie permutations.
P_{all}	Set of movie permutations that must be watched by all teams.

Parameters:

$\lvert P\rvert$	Number of permutations of $\lvert M\rvert$ movies. $\lvert P\rvert = \lvert M\rvert! = 7! = 5,040$.
$\lvert P_{\text{all}}\rvert$	Number of permutations starting with AB. $\lvert P_{\text{all}}\rvert = \lvert M-2\rvert! = 5! = 120$.
T_{max}	Upper bound on the length of the longest schedule.

$$T_{max} = \frac{2}{|G|}(|M|-1)!(|M|-1) + 1 + \frac{|G|-1}{|G|}(|M|-2)! = 2,880.$$

ω_{pk} Movie in position $k \in 1 \dots |M|$ of permutation $p \in P$.

Decision variables:

$$x_{gta} = \begin{cases} 1 & \text{if group } g \in G \text{ performs activity } a \in A \text{ at time } t \in T. \\ 0 & \text{otherwise.} \end{cases}$$

$$\delta_{pgt} = \begin{cases} 1 & \text{if permutation } p \in P \text{ is found in group } g \in G \text{ starting at time } t \in T. \\ 0 & \text{otherwise.} \end{cases}$$

$$\gamma_{pg} = \begin{cases} 1 & \text{if permutation } p \in P \text{ is found in group } g \in G. \\ 0 & \text{otherwise.} \end{cases}$$

$$\lambda_{pgt} = \begin{cases} 1 & \text{if } \delta_{pgt} \text{ is the only permutation } p \in P \text{ counted in team } g \in G. \\ 0 & \text{otherwise.} \end{cases}$$

ℓ Duration of the longest schedule of the three teams.

$$\min_{x,\delta,\gamma,\ell} \quad \ell \tag{1}$$

$$\text{s.t.} \quad \ell \geq \sum_{\substack{t \in T, \\ a \in A \setminus \{\emptyset\}}} x_{gta} \qquad \forall\, g \in G. \tag{2}$$

$$\sum_{a \in A} x_{gta} = 1 \qquad \forall\, g \in G,\ t \in T. \tag{3}$$

$$x_{gt\emptyset} \leq x_{g,t+1,\emptyset} \qquad \forall\ g \in G,\ t = 1 \ldots T_{max} - 1. \tag{4}$$

$$x_{gtm} + x_{g,t+1,m} \leq 1 \qquad \forall\ g \in G,\ m \in M,\ t = 1 \ldots T_{max} - 1. \tag{5}$$

$$\sum_{s=t}^{t+|M|-1} x_{gsm} \leq 2 \qquad \forall\ g \in G,\ m \in M,\ t = 1 \ldots T_{max} - |M| + 1. \tag{6}$$

$$\sum_{s=t}^{t+|M|-1} x_{gs*} \leq 1 \qquad \forall\ g \in G,\ t = 1 \ldots T_{max} - |M| + 1. \tag{7}$$

$$\sum_{t \in T} x_{gt*} \leq 2 \qquad \forall\ g \in G. \tag{8}$$

$$\delta_{pgt} \leq x_{gk\omega_{pk}} \qquad \forall\ g \in G,\ p \in P,\ t = 1 \ldots T_{max} - |M| + 1,\ k = p \ldots p + |M| - 1. \tag{9}$$

$$\gamma_{pg} \geq \delta_{pgt} \qquad \forall\ g \in G,\ p \in P,\ t = 1 \ldots T_{max} - |M| + 1. \tag{10}$$

$$\gamma_{pg} \leq \delta_{pgt} + \lambda_{pgt} \qquad \forall\ g \in G,\ p \in P,\ t = 1 \ldots T_{max} - |M| + 1. \tag{11}$$

$$\sum_{t=1}^{T_{max}-|M|+1} \lambda_{pgt} = T_{max} - |M| \qquad \forall\ g \in G,\ p \in P. \tag{12}$$

$$\sum_{g \in G} \gamma_{pg} \geq 1 \qquad \forall\ p \in P \setminus P_{\text{all}}. \tag{13}$$

$$\gamma_{pg} = 1 \qquad \forall\ p \in P_{\text{all}},\ g \in G. \tag{14}$$

$$x_{gta} \in \{0,1\} \qquad \forall\ g \in G,\ t \in T,\ a \in A. \tag{15}$$

$$\delta_{pgt} \in \{0,1\} \qquad \forall\ p \in P,\ g \in G,\ t = 1 \ldots T_{max} - |M| + 1. \tag{16}$$

$$\lambda_{pgt} \in \{0,1\} \qquad \forall\ p \in P,\ g \in G,\ t = 1 \ldots T_{max} - |M| + 1. \tag{17}$$

$$\gamma_{pg} \in \{0,1\} \qquad \forall\ p \in P,\ g \in G. \tag{18}$$

$$\ell \in \mathbb{N}. \tag{19}$$