Learning and distinguishing between active versions of prominent decision strategies

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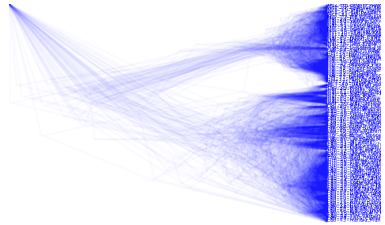
If humans are active learners, then...

1. ...how can they learn a heuristic that is made up of building blocks on the fly when the goal is to make good decisions?

2. ...how can we use that fact to test different decision making strategies in an active learning context?



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Overview

1. **Heuristics from Approximate Bayesian Computation**

   Collaborators:  Meder  Speekenbrink

   - How can a heuristic emerge from small building blocks?
   - Can a ABC method recover and/or outperform TTB?

2. **Distinguishing active versions of decision strategies**

   Collaborators:  Parpart  Speekenbrink  Love

   - How to distinguish between active versions of strategies?
   - How does the used strategy depend on the environment?

PART 1: Can heuristics emerge/grow adaptively?

- Heuristics are made of smaller building blocks
- Different combinations of blocks produce the heuritsic toolbox
- Definition of a heuristic:

  "Ignores information to be faster and/or more accurate."

  "Exhibits a starting rule, a search rule, and a stopping rule."

PART 1: Problem statement

Problem

Given a number of binary cues $C_1, C_2, \ldots, C_n$ –some of which might be invalid– predict a binary outcome y on every trial while at the same time learning a heuristic structure that consitsts of a starting, a search, and a stopping rule.

Take-The-Best cannot deal with that, because...

1. it is not clear how cue validities are learned
2. it is not clear how information is ignored
3. it cannot make decision and learn (evolve over time)
4. it is impossible to compute all valdities exactly on the fly

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PART 1: Approximate Bayesian Computation

How do you calculate a posterior when both analytical inference and sampling methods are intractable?

Requirements: model $\mathcal{M}$, prior $\pi(\theta)$, data $\mathcal{D} = (\mathbf{X}, \mathbf{y})$

1. Sample a proposal θ^* from your prior $\pi(\theta)$
2. Plug the proposal in your model $\mathcal{M}$ to get a model $\mathcal{M}^*$
3. Simulate $\mathbf{y}^* = \mathcal{M}^*(\mathbf{X})$
4. Calculate distance $\delta = \sum_i^n |y_i - y_i^*|$
5. If $\delta < \epsilon \rightarrow$ accept proposal
   else $\rightarrow$ reject proposal

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PART 1: Approximately Bayesian Computed TTB

How can TTB emerge from smaller buidling blocks?

Requirements: urn containing the cues, for each cue: an urn containing the weights
urn has a pre-assigned number of k balls for each class

1. Sample a cue c^* from the cue urn without replacement
2. Sample a weight w^* for c^* from c^*'s weight urn
3. Encounter a situation y_i
4. If $y^* == y_i \rightarrow$ put two more balls into w^* and c^*
   elseif $y^* == 0 \rightarrow$ go back to 1
   elseif $y^* \neq y_i \rightarrow$ reject proposal, start again

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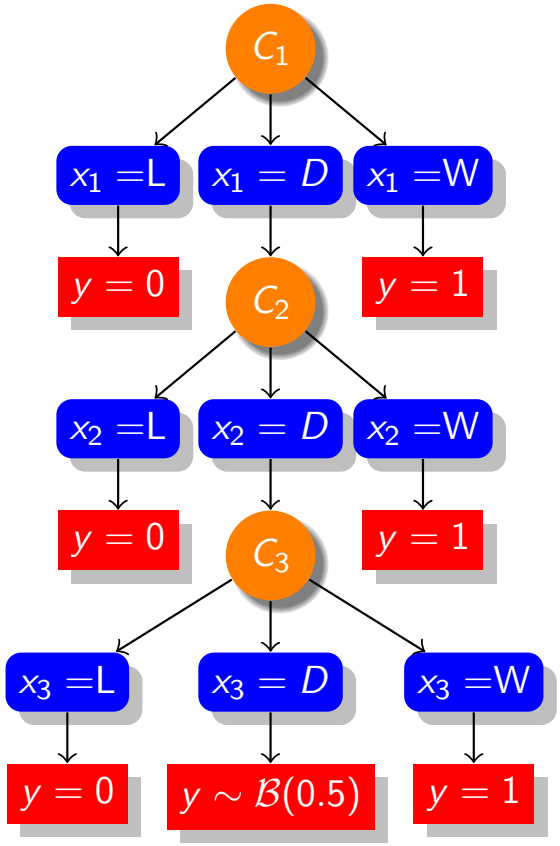
PART 1: Recovering TTB-The set up

- 5 variables x_1, x_2, x_3, x_4, x_5, only the first three are valid
- $p(x_k = W) = p(x_k = D) = p(x_k = L) = \frac{1}{3}$



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PART 1: Recovering TTB: Results

- Results
  1. Can find the most important cues, even with a greedy strategy
  2. Achieve an excellent misclassification rate
  3. Can show mathematically that TTB-ABC corresponds to classic TTB if full set is sampled set
- Current problems
  1. How to avoid local optima (rescaling, jump starts, sampling)?
  2. How to speed up computation (annealing, parallelization)?

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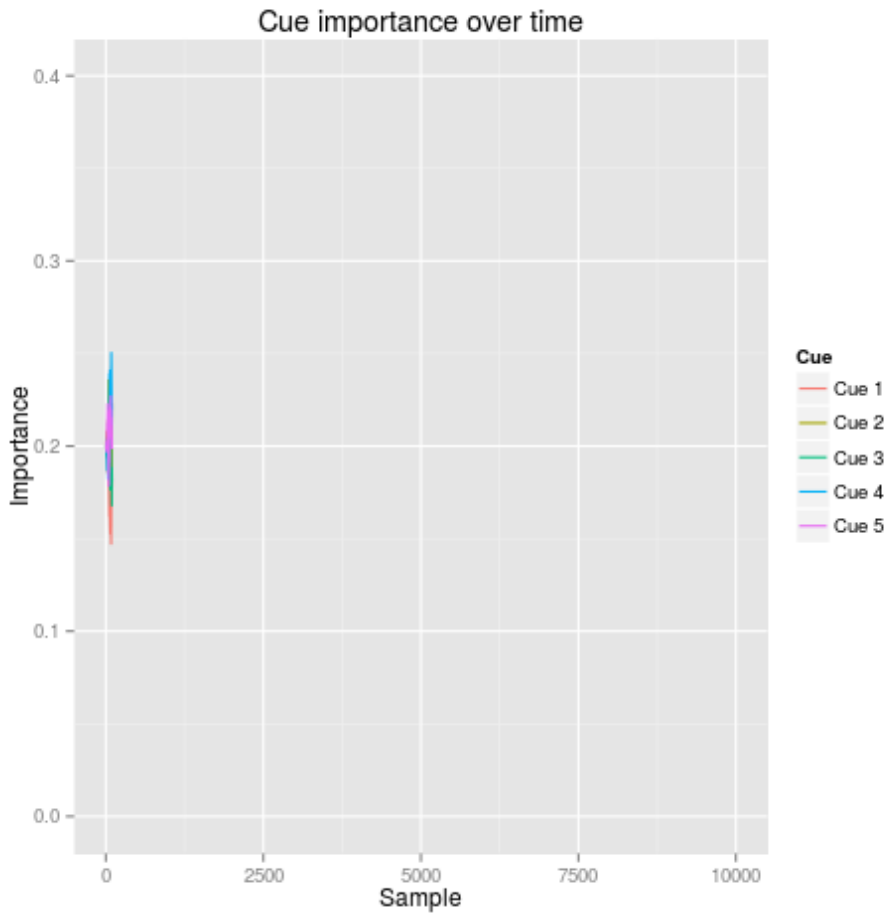
Cue importance over time



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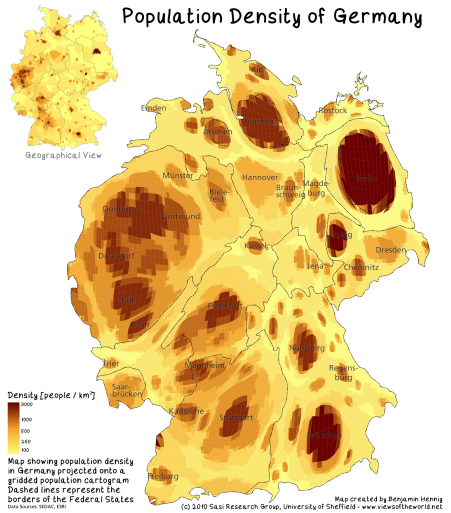
PART 1: Empirical Test: City size data

- Most frequently used data sets within the community
- Predict which of two German cities has the larger population size based on 9 cues
- Take-The-Best is known to perform well
- Cues have been pre-selected



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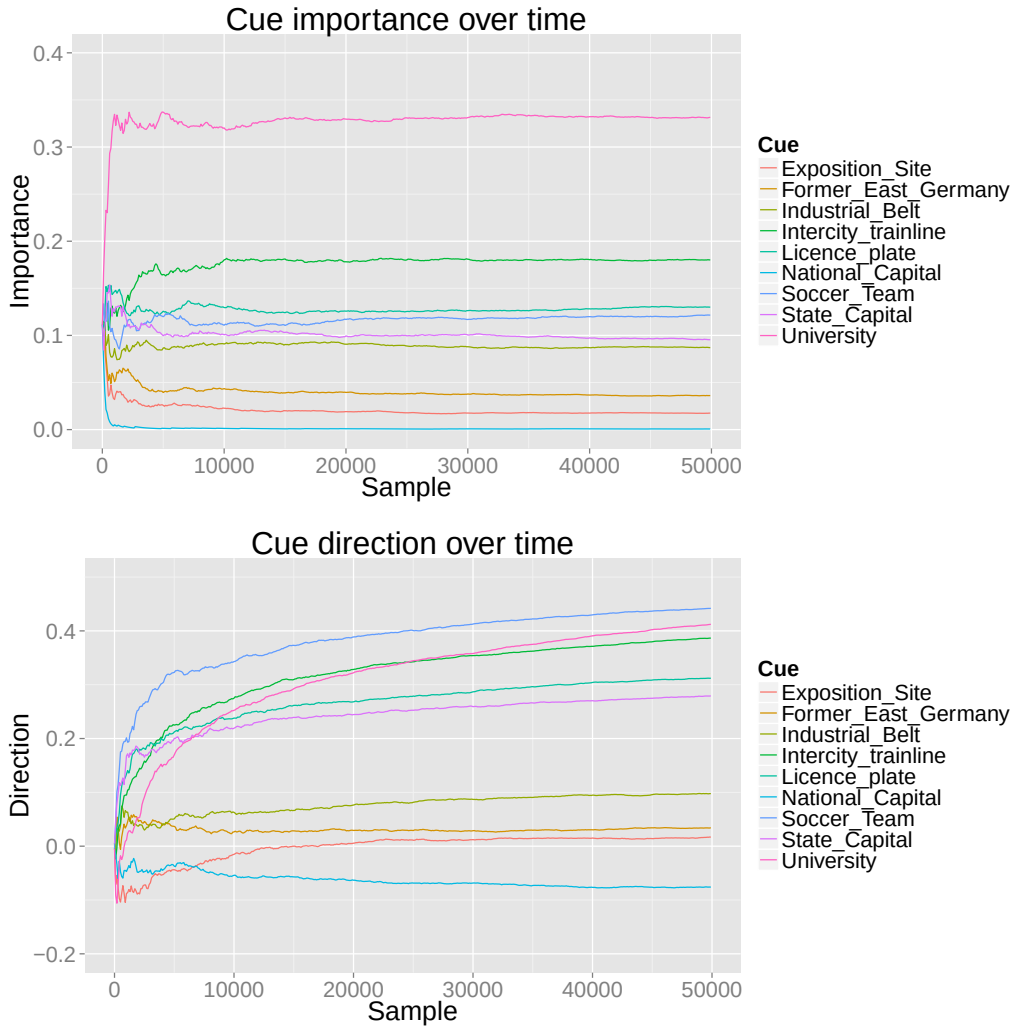
PART 1: Empirical Test: Importance and direction



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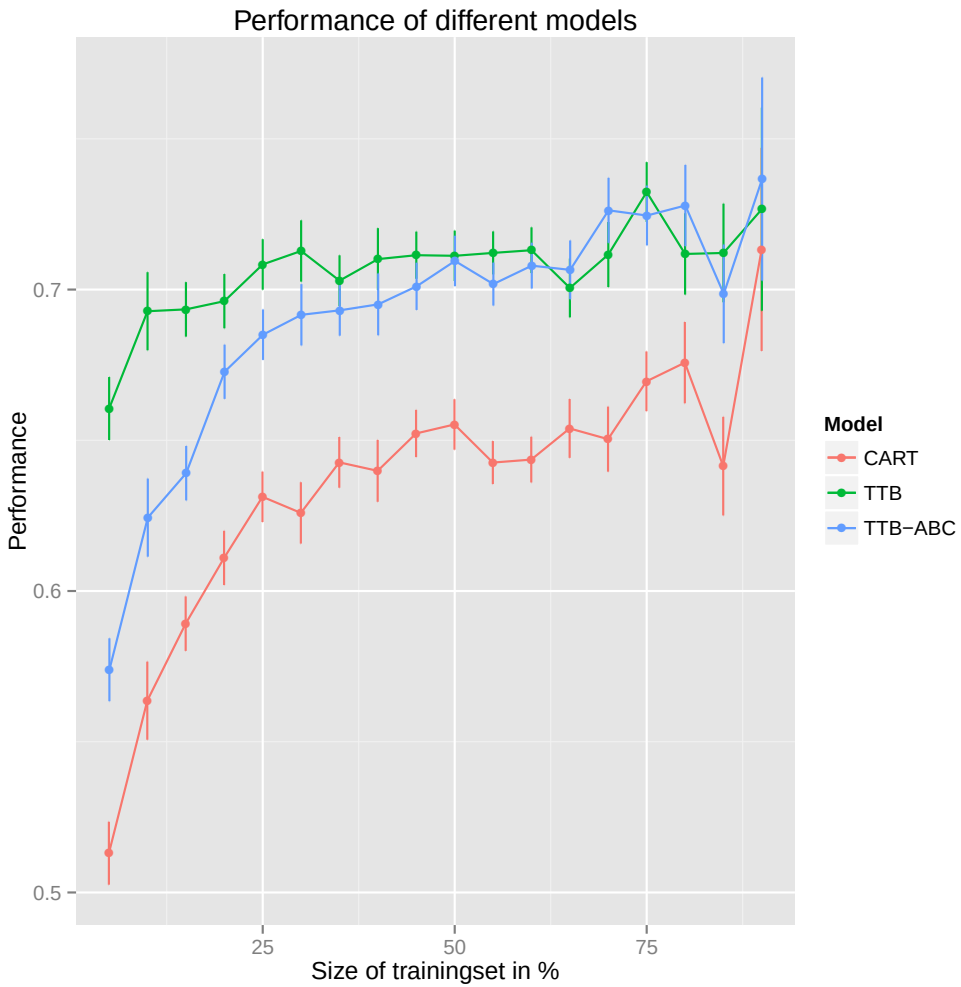
PART 1: Empirical Test: Model comparison



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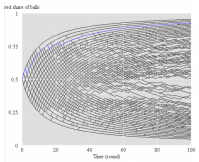
PART 1: Conclusion

TTB can emerge from simple building blocks over time



Relies on simple sampling and approximate computation

Can recover and perform as well as simple decision trees

Speed, global optima, and experimental applications

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PART 2: Can we distinguish different active algorithms?

- Agent has evolved to obey a certain decision algorithm
- Agent learns with the goal to apply that algorithm
- Definition of active learning:

  "Stepwise selection of observations in order to learn faster."

  "Active learning leads to a banana shaped learning curve."

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PART 2: Problem statement

Problem

Given a number of binary cues $C_1, C_2, \ldots, C_n$ –some of which might be invalid– learn how to predict a binary outcome y by selecting observations as wisely as possible.

Current heuristics cannot deal with that, because...

1. it is not clear how cue validities are learned
2. it is not clear how information is ignored
3. there are no active versions of them
4. active learning has never been utilized for model comparison

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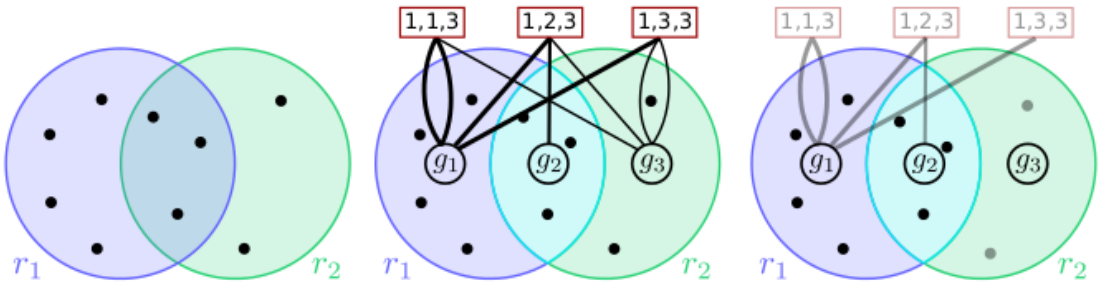
PART 2: Model 1: Active TTB

- **Imagine a scenario without noise:**
  Every observation is deterministic and your only goal is, given your hypothesis space over all cue orders,
  $\mathcal{H} = \{\text{cue-order}_1, \text{cue-order}_2, \ldots, \text{cue-order}_k\}$, to select the observation $s^* = \text{argmax}\{|\mathcal{H}| - |\mathcal{H}|s|\}$
- Unfortunately, observations never come without noise
- We have to find a probabilistic version of the same algorithm



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PART 2: Model 1: Active TTB algorithm

- Put a pseudo-count π over all possible cue orders
- If a cue order makes a correct prediction, then $\pi + +$
- Calculate current entropy over all cue orders $S_0 = \sum_i p_i \log p_i$
- For every possible comparison s calculate $p(y = w)$ and $p(y = L)$ by π-weighted sum over all cue orders
- For every comparison s, calculate posterior expected entropy $\mathbb{E}[S|s] = S(y = W) \times p(y = W) + S(y = L) \times p(y = L)$
- Choose $s^* = \text{argmax}\{S_0 - \mathbb{E}[S|s]\}$, that is the observation with the highest expected information gain
- Method works well a priori

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PART 2: Model 2: Active Logistic Regression

- Given a Bayesian variant of logistic regression:
$$f(x) = \frac{1}{1+\exp(-(\beta_0+\sum_k \beta_k x_k))}$$
- Calculate the current sum of coefficients' uncertainty $S = \sum_k \mathbb{V}(\beta_k)$
- For every comparison s, calculate $p(y = w|s)$ and $p(y = l|s)$
- For every comparison and outcome calculate $S|s, y = w$ and $S|s, y = l$, that is the reduction of variance
- For every comparison s, calculate posterior expected uncertainty
$\mathbb{E}[S|s] = S(y = W) \times p(y = W) + S(y = L) \times p(y = L)$
- Choose $s^* = \text{argmax}\{S_0 - \mathbb{E}[S|s]\}$, that is the observation with the highest expected uncertainty reduction
- Method works well a priori

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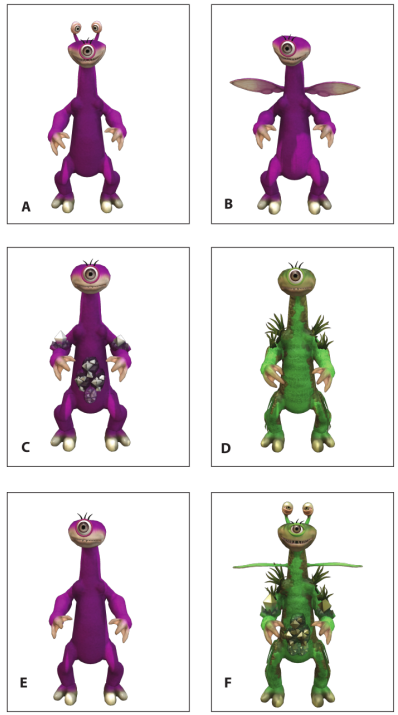
Part 2: Alien Olympics: Design

1. Learn how well aliens perform in Olympics
2. Aliens vary on 4 different features: Wings, Camouflage, Diamonds, and Antennæ
3. 30 learning trials: select 2 out of 4 aliens to compete against each other ($0.5 reward)
4. 10 test trials: select 1 out of 2 aliens for your team (bonus dependent on team)
5. Environment set up with different levels of compensatoriness



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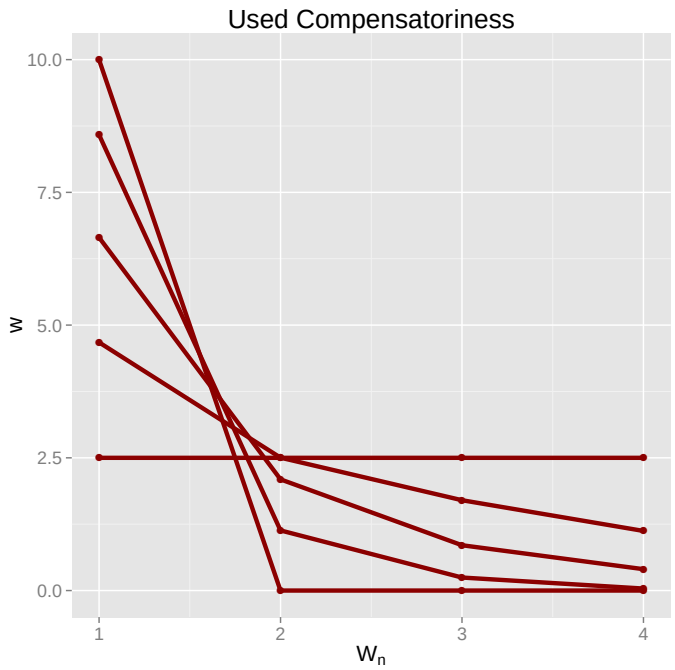
Part 2: Alien Olympics: Results in test set

1. Underlying envrionment obeys logistic regression
2. Weights are generated by a stick-breaking process
   $\beta'_k \sim \text{Beta}(1, \theta)$
   Define $\{\beta'_k\}_{k=1}^4$ as:
   $\beta_k = \beta'_k \prod_{i=1}^{k-1}(1 - \beta'_i)$
3. Allows to trade-off different levels of compensatoriness



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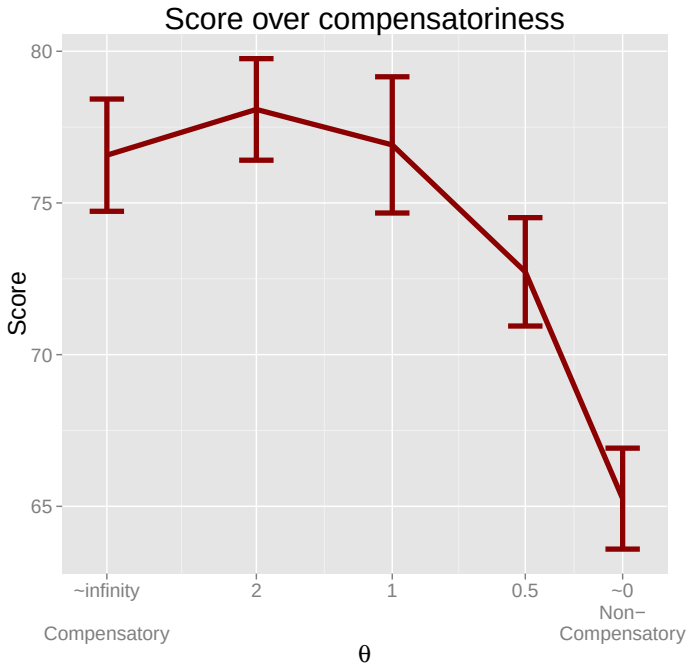
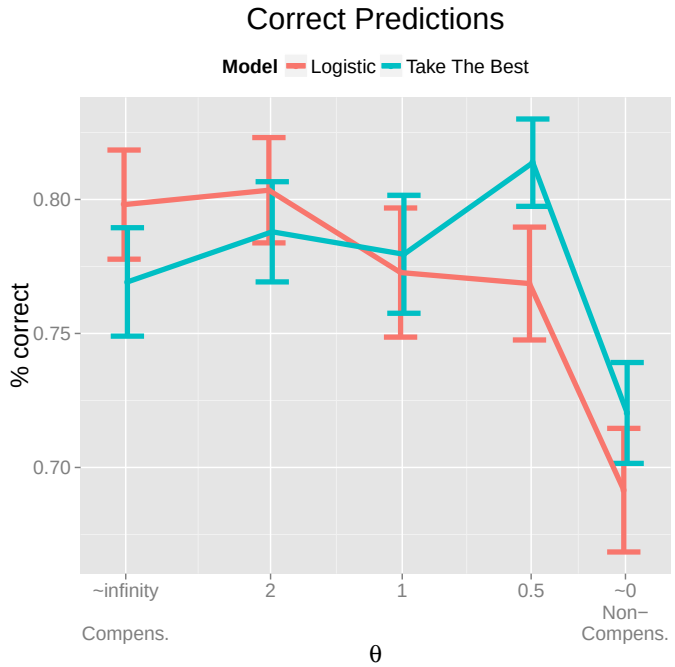
Part 2: Alien Olympics: Results of passive part





1. Non-compensatory conditions produce lower average score
2. Hard to distinguish between models based on test set alone

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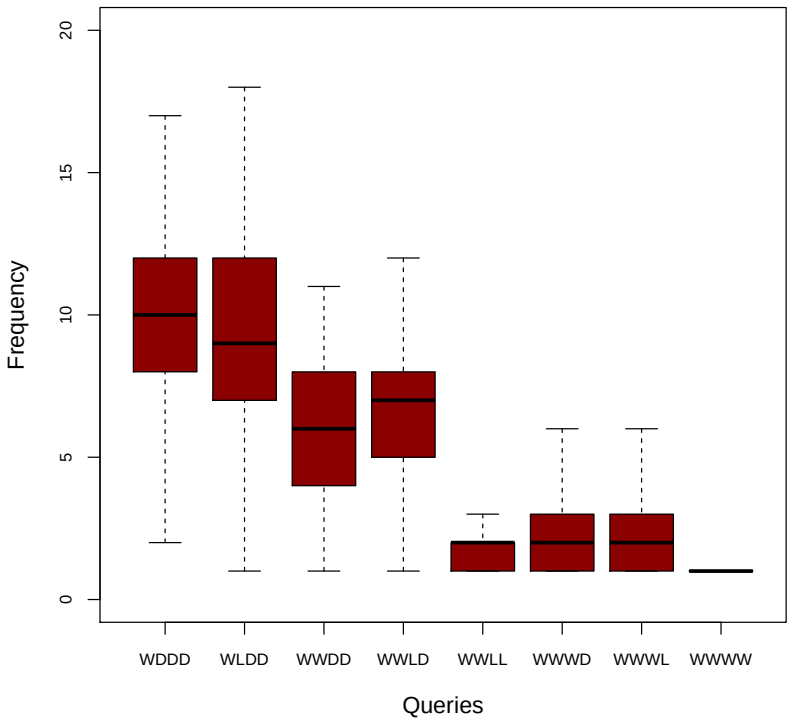
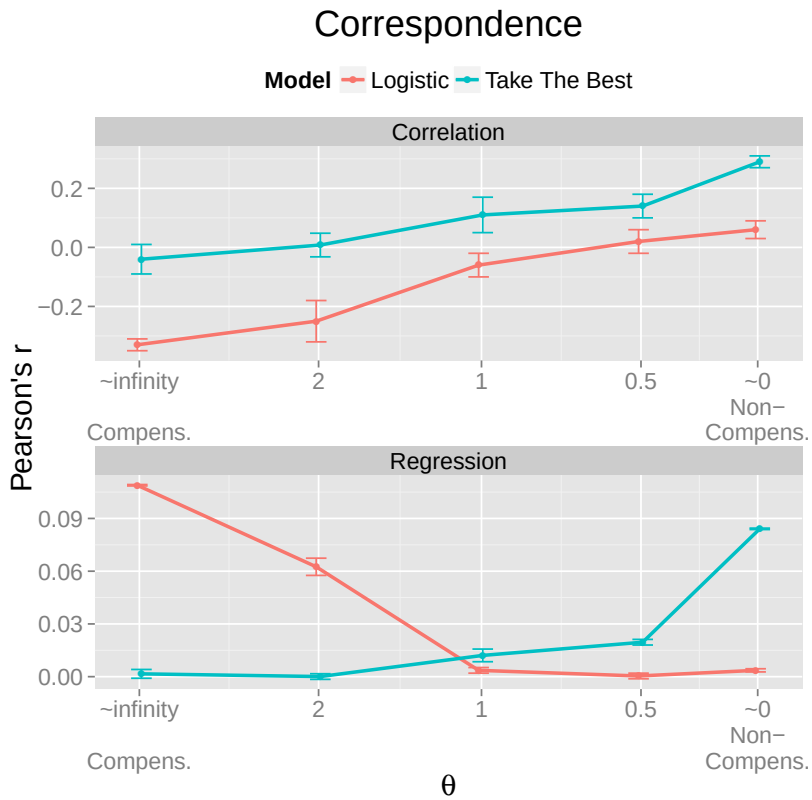
Part 2: Alien Olympics: Results of active part





1. Participants perform simple but sensible queries
2. Can distinguish between active models:
   Only active TTB seems to match behavior well

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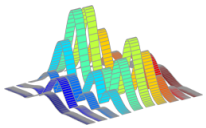
PART 2: Conclusion


Active learning can be used to distinguish different models


It is possible to design active versions of classic models

In a first experiment, active TTB seems to do best

Simpler follow-up, Exploitation scenarios, Non-parametrics

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Overall conclusion

I hope to have convinced you that...

1. ...active learning is an exciting tool to add to our methodological repertoire
2. ...it is both possible and fruitful to design active learning algorithms based on classic models
3. ...if we want to make claims about cognitive strategies, we also have to make claims about how those are acquired

Future steps could involve...

1. ...coming up with more active learning models
2. ...designing additional active learning algorithms
3. ...modeling both exploration and exploitation scenarios

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Thanks you for your attention and to my collaborators

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