Práctico 1

Problema 2)

a) $$\bar{Y} = \frac{\sum_{1}^{n} x_i}{n}$$

$$E(W_1) = E\left(\frac{(n-1)}{n}\ \frac{\sum x_i}{n}\right) = \frac{(n-1)}{n}\ \overbrace{E\left(\frac{\sum x_i}{n}\right)}^{E(x)}$$

$$\text{Sesgo} = \frac{(n-1)}{n}\mu - \mu = \mu\left(\frac{n-1-n}{n}\right)$$

$$= \mu\left(\frac{-1}{n}\right)$$ el sesgo tiende a 0 cuando $n \to \infty$

$$E(W_2) = E\left(\frac{\sum x_i}{2n}\right) = \frac{1}{2}E\left(\frac{\sum x_i}{n}\right) = \frac{E(x)}{2}$$

$$\Rightarrow \text{Sesgo} = \frac{E(x)}{2} - E(x) = -\frac{\mu}{2}$$

El sesgo no depende de n

b) ~~$\bar{Y}$ es un estimador de μ, $g(\bar{Y}) = \mu$~~

~~$P\lim(\bar{Y}) = \mu$~~

b)

- $\operatorname*{plim}_{n\to\infty}(W_1) = \operatorname*{plim}_{n\to\infty}\left(\underbrace{\overbrace{\frac{n-1}{n}}^{n}\bar{Y}}_{Y}\right) \Rightarrow \lim_{n\to\infty} \bar{Y} = \mu$

- $\operatorname*{plim}_{n\to\infty}(W_2) = \cancel{\operatorname*{plim}_{n\to\infty}\left(\frac{W_2}{2}\right)} = \operatorname*{plim}_{n\to\infty}\left(\frac{Y}{2}\right) = \dfrac{\lim_{n\to\infty} Y}{2}$

$= \dfrac{\mu}{2}$

c)

$$V(W_1) = \cancel{E}' \, V\left(\frac{n-1}{n} Y\right) = \left(\frac{n-1}{n}\right)^2 \overbrace{V(Y)}^{\sigma^2/n}$$

$$= \frac{(n-1)^2}{n^3}\sigma^2 \Rightarrow V(W_1) \xrightarrow[n\to\infty]{} 0$$

$$V(W_2) = V\left(\frac{Y}{2}\right) = \frac{1}{4}V(Y) = \frac{\sigma^2}{4n}$$

$$V(W_2) \xrightarrow[n\to\infty]{} 0$$

Problema 3)

$$E(GPA/SAT) = 0{,}7 + 0{,}001\,SAT$$

a)

$$\Rightarrow E(GPA/SAT = 800) = 0{,}7 + 0{,}001 \cdot 800 = 1{,}5$$

$$\Rightarrow E(GPA/SAT = 1400) = 0{,}7 + 0{,}001 \cdot 1400 = 2{,}1$$

b)

$$E(SAT) = 1100$$

$$\Rightarrow E(GPA) = E(0{,}7 + 0{,}001\,SAT) = 0{,}7 + 0{,}001\,E(SAT)$$

$$\Rightarrow = 1{,}8$$