

Exercício: 1.6.1

$$w(P) = w(Q) = w(R) = F$$

①. Idempotência :  $P \wedge P \leftrightarrow P$

$$\text{Eval}(P \wedge P \leftrightarrow P, w)$$

$$= \bar{w}((P \wedge P) \leftrightarrow P)$$

$$= \bar{\varphi} \leftrightarrow (\bar{w}(P \wedge P), \bar{w}(P))$$

$$= \varphi \leftrightarrow (\varphi \wedge (\bar{w}(P), \bar{w}(P)), \bar{w}(P))$$

$$= \varphi \leftrightarrow (\varphi \wedge (w(P), w(P)), w(P))$$

$$= \varphi \leftrightarrow (\varphi \wedge (F, F), w(P))$$

$$= \varphi \leftrightarrow (F, w(P))$$

$$= \varphi \leftrightarrow (F, F)$$

$$= \textcircled{T}$$



②. Absorção:  $P \wedge (P \vee Q) \leftrightarrow P$

$$\begin{aligned} & \text{Eval}(P \wedge (P \vee Q) \leftrightarrow P, \omega) \\ &= \bar{\omega}((P \wedge (P \vee Q)) \leftrightarrow P) \\ &= \varphi^{\leftrightarrow}(\bar{\omega}(P \wedge (P \vee Q)), \bar{\omega}(P)) \\ &= \varphi^{\leftrightarrow}(\varphi^{\wedge}(\bar{\omega}(P), \bar{\omega}(P \vee Q)), \bar{\omega}(P)) \\ &= \varphi^{\leftrightarrow}(\varphi^{\wedge}(\omega(P), \varphi^{\vee}(\bar{\omega}(P), \bar{\omega}(Q))), \bar{\omega}(P)) \\ &= \varphi^{\leftrightarrow}(\varphi^{\wedge}(\omega(P), \varphi^{\vee}(\omega(P), \omega(Q))), \omega(P)) \\ &= \varphi^{\leftrightarrow}(\varphi^{\wedge}(\omega(P), \varphi^{\vee}(F, F)), \omega(P)) \\ &= \varphi^{\leftrightarrow}(\varphi^{\wedge}(\omega(P), F), \omega(P)) \\ &= \varphi^{\leftrightarrow}(\varphi^{\wedge}(F, F), \omega(P)) \\ &= \varphi^{\leftrightarrow}(F, F) \\ &= \textcircled{T} \end{aligned}$$

③. Terceira excluído:  $P \vee \neg P$

$$\text{Eval}(P \vee \neg P, \omega)$$

$$\begin{aligned} &= \bar{\omega}(P \vee \neg P) \\ &= \bar{\omega}(P \vee (\neg P)) \\ &= \varphi^{\vee}(\bar{\omega}(P), \omega(\neg P)) \\ &= \varphi^{\vee}(\omega(P), \varphi^{\neg}(\bar{\omega}(P))) \\ &= \varphi^{\vee}(\omega(P), \varphi^{\neg}(F)) \\ &= \varphi^{\vee}(\omega(P), T) \\ &= \varphi^{\vee}(F, T) \\ &= \textcircled{T} \end{aligned}$$



④. Contraposição:  $(P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P)$

$$\text{Eval } ((P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P), w)$$

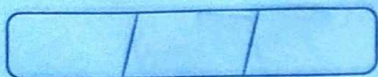
$$\begin{aligned} &= \bar{w}((P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P)) \\ &= \varphi \leftrightarrow (\bar{w}(P \rightarrow Q), \bar{w}(\neg Q \rightarrow \neg P)) \\ &= \varphi \leftrightarrow (\varphi \rightarrow (\bar{w}(P), \bar{w}(Q)), \varphi \rightarrow (\bar{w}(\neg Q), \bar{w}(\neg P))) \\ &= \varphi \leftrightarrow (\varphi \rightarrow (w(P), w(Q)), \varphi \rightarrow (\varphi \neg(\bar{w}(Q)), \varphi \neg(\bar{w}(P)))) \\ &= \varphi \leftrightarrow (\varphi \rightarrow (F, F), \varphi \rightarrow (\varphi \neg(F), \varphi \neg(F))) \\ &= \varphi \leftrightarrow (T, \varphi \rightarrow (T, T)) \\ &= \varphi \leftrightarrow (T, T) \\ &= (T) \end{aligned}$$

⑤. Exportação:  $(P \rightarrow (Q \rightarrow R)) \leftrightarrow (P \wedge Q \rightarrow R)$

$$\text{Eval } ((P \rightarrow (Q \rightarrow R)) \leftrightarrow (P \wedge Q \rightarrow R), w)$$

$$\begin{aligned} &= \bar{w}((P \rightarrow (Q \rightarrow R)) \leftrightarrow (P \wedge Q \rightarrow R)) \\ &= \varphi \leftrightarrow (\bar{w}(P \rightarrow (Q \rightarrow R)), \bar{w}(P \wedge Q \rightarrow R)) \\ &= \varphi \leftrightarrow (\varphi \rightarrow (\bar{w}(P), \bar{w}(Q \rightarrow R)), \varphi \rightarrow (\bar{w}(P \wedge Q), \bar{w}(R))) \\ &= \varphi \leftrightarrow (\varphi \rightarrow (w(P), \varphi \rightarrow (\bar{w}(Q), \bar{w}(R))), \varphi \rightarrow (\varphi \wedge (\bar{w}(P), \bar{w}(Q)), w(R))) \\ &= \varphi \leftrightarrow (\varphi \rightarrow (F, \varphi \rightarrow (F, F)), \varphi \rightarrow (\varphi \wedge (F, F), F)) \\ &= \varphi \leftrightarrow (\varphi \rightarrow (F, T), \varphi \rightarrow (F, F)) \\ &= \varphi \leftrightarrow (T, T) \\ &= (T) \end{aligned}$$





⑥. Silogismo:  $(P \rightarrow Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R))$

Eval  $((P \rightarrow Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R)), w)$

$$\begin{aligned} &= \bar{w}((P \rightarrow Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R))) \\ &= \varphi \rightarrow (\bar{w}(P \rightarrow Q), \bar{w}((Q \rightarrow R) \rightarrow (P \rightarrow R))) \\ &= \varphi \rightarrow (\varphi \rightarrow (\bar{w}(P), \bar{w}(Q)), \varphi \rightarrow (\bar{w}(Q \rightarrow R), \bar{w}(P \rightarrow R))) \\ &= \varphi \rightarrow (\varphi \rightarrow (w(P), w(Q)), \varphi \rightarrow (\varphi \rightarrow (\bar{w}(Q), \bar{w}(R)), \varphi \rightarrow (\bar{w}(P), \bar{w}(R)))) \\ &= \varphi \rightarrow (\varphi \rightarrow (F, F), \varphi \rightarrow (\varphi \rightarrow (w(Q), w(R)), \varphi \rightarrow (w(P), w(R)))) \\ &= \varphi \rightarrow (T, \varphi \rightarrow (\varphi \rightarrow (F, F), \varphi \rightarrow (F, F))) \\ &= \varphi \rightarrow (T, \varphi \rightarrow (T, T)) \\ &= \varphi \rightarrow (T, T) \\ &= (T) \end{aligned}$$

⑦. Lei de Frege:  $(P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$

Eval  $((P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R)), w)$

$$\begin{aligned} &= \bar{w}((P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))) \\ &= \varphi \rightarrow (\bar{w}(P \rightarrow (Q \rightarrow R)), \bar{w}((P \rightarrow Q) \rightarrow (P \rightarrow R))) \\ &= \varphi \rightarrow (\varphi \rightarrow (\bar{w}(P), \bar{w}(Q \rightarrow R)), \varphi \rightarrow (\bar{w}(P \rightarrow Q), \bar{w}(P \rightarrow R))) \\ &= \varphi \rightarrow (\varphi \rightarrow (w(P), \varphi \rightarrow (\bar{w}(Q), \bar{w}(R))), \varphi \rightarrow (\varphi \rightarrow (\bar{w}(P), \bar{w}(Q)), \varphi \rightarrow (\bar{w}(P), \bar{w}(R)))) \\ &= \varphi \rightarrow (\varphi \rightarrow (F, \varphi \rightarrow (w(Q), w(R))), \varphi \rightarrow (\varphi \rightarrow (\bar{w}(P), w(Q)), \varphi \rightarrow (w(P), w(R)))) \\ &= \varphi \rightarrow (\varphi \rightarrow (F, \varphi \rightarrow (F, F)), \varphi \rightarrow (\varphi \rightarrow (F, F), \varphi \rightarrow (F, F))) \\ &= \varphi \rightarrow (\varphi \rightarrow (F, T), \varphi \rightarrow (T, T)) \\ &= \varphi \rightarrow (T, T) \\ &= (T) \end{aligned}$$



8. Loi de Pierce:  $((P \rightarrow Q) \rightarrow P) \rightarrow P$

Eval  $((P \rightarrow Q) \rightarrow P) \rightarrow P, w$

$$= \overline{w}(((P \rightarrow Q) \rightarrow P) \rightarrow P)$$

$$= \varphi \rightarrow (\overline{w}((P \rightarrow Q) \rightarrow P), \overline{w}(P))$$

$$= \varphi \rightarrow (\varphi \rightarrow (\overline{w}(P \rightarrow Q)), \overline{w}(P)), w(P))$$

$$= \varphi \rightarrow (\varphi \rightarrow (\varphi \rightarrow (\overline{w}(P), \overline{w}(Q)), w(P)), w(P))$$

$$= \varphi \rightarrow (\varphi \rightarrow (\varphi \rightarrow (F, F), F), F)$$

$$= \varphi \rightarrow (\varphi \rightarrow (T, F), F)$$

$$= \varphi \rightarrow (F, F)$$

$$= (T)$$

9. Enc Falso:  $(P \wedge \neg P) \rightarrow Q$

Eval  $((P \wedge (\neg P)) \rightarrow Q, w)$

$$= \overline{w}((P \wedge (\neg P)) \rightarrow Q)$$

$$= \varphi \rightarrow (\overline{w}(P \wedge (\neg P)), \overline{w}(Q))$$

$$= \varphi \rightarrow (\varphi \wedge (\overline{w}(P), \overline{w}(\neg P)), \overline{w}(Q))$$

$$= \varphi \rightarrow (\varphi \wedge (w(P), \varphi \rightarrow (\overline{w}(P))), \overline{w}(Q))$$

$$= \varphi \rightarrow (\varphi \wedge (w(P), \varphi \rightarrow (F)), w(Q))$$

$$= \varphi \rightarrow (\varphi \wedge (F, T), F)$$

$$= \varphi \rightarrow (F, F)$$

$$= (T)$$