P8130 Final Project

Abstract

Introduction (brief context and background of the problem)

Methods (data description and statistical methods)

Results

Conclusions/Discussion

```
# import necessary datasets
library(tidyverse)
library(ggplot2)
library(GGally)
library(PerformanceAnalytics)
library(performance)
library(MASS)
library(leaps)
library(modelr)
library(olsrr)
```

Read in dataset

```
cdi = read_csv("./cdi.csv") %>%
  janitor::clean_names()
cdi %>%
  group_by(cty, state)
```

```
## # A tibble: 440 x 17
## # Groups:   cty, state [438]
##       id cty     state  area    pop pop18 pop65  docs  beds crimes hsgrad bagrad
##    <dbl> <chr>   <chr> <dbl>  <dbl> <dbl> <dbl> <dbl> <dbl>  <dbl>  <dbl>  <dbl>
##  1     1 Los_An~ CA     4060 8.86e6  32.1   9.7 23677 27700 688936   70     22.3
##  2     2 Cook    IL      946 5.11e6  29.2  12.4 15153 21550 436936   73.4   22.8
##  3     3 Harris  TX     1729 2.82e6  31.3   7.1  7553 12449 253526   74.9   25.4
##  4     4 San_Di~ CA     4205 2.50e6  33.5  10.9  5905  6179 173821   81.9   25.3
##  5     5 Orange  CA      790 2.41e6  32.6   9.2  6062  6369 144524   81.2   27.8
##  6     6 Kings   NY       71 2.30e6  28.3  12.4  4861  8942 680966   63.7   16.6
##  7     7 Marico~ AZ     9204 2.12e6  29.2  12.5  4320  6104 177593   81.5   22.1
##  8     8 Wayne   MI      614 2.11e6  27.4  12.5  3823  9490 193978   70     13.7
##  9     9 Dade    FL     1945 1.94e6  27.1  13.9  6274  8840 244725   65     18.8
## 10    10 Dallas  TX      880 1.85e6  32.6   8.2  4718  6934 214258   77.1   26.3
## # ... with 430 more rows, and 5 more variables: poverty <dbl>, unemp <dbl>,
## #   pcincome <dbl>, totalinc <dbl>, region <dbl>
```

```
## no missing value
cdi %>%
  dplyr::select(everything()) %>%
  summarise_all(funs(sum(is.na(.)))) %>%
  knitr::kable()
```

id	cty	state	area	pop	pop18	pop65	docs	beds	crimes	hsgrad	bagrad	poverty	unemp	pcincome	totalinc	region
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Data cleaning

First, some normalization for better comparison

```
cdi =
  cdi %>%
  mutate(crm_1000 = crimes/pop*1000,  # as indicated by the project prompt
         docs_1000 = docs/pop*1000,  # every 1000 people how many doctors
         beds_1000 = beds/pop*1000, # similar as above
         pop_density = pop/area,  # how many people per square miles
         northeast = ifelse(region == 1, 1, 0),  # region as dummy varaible
         northcentral = ifelse(region == 2, 1, 0),
         south = ifelse(region == 3, 1, 0)) %>%
  dplyr::select(-id, -crimes,-area, -docs, -beds, -totalinc, -region)
```

Data Exploration

```
## summary statistics, tentative, NOT FINAL
sum_cdi =
  cdi %>%
  dplyr::select(-c(cty, state))
summary(sum_cdi)
```

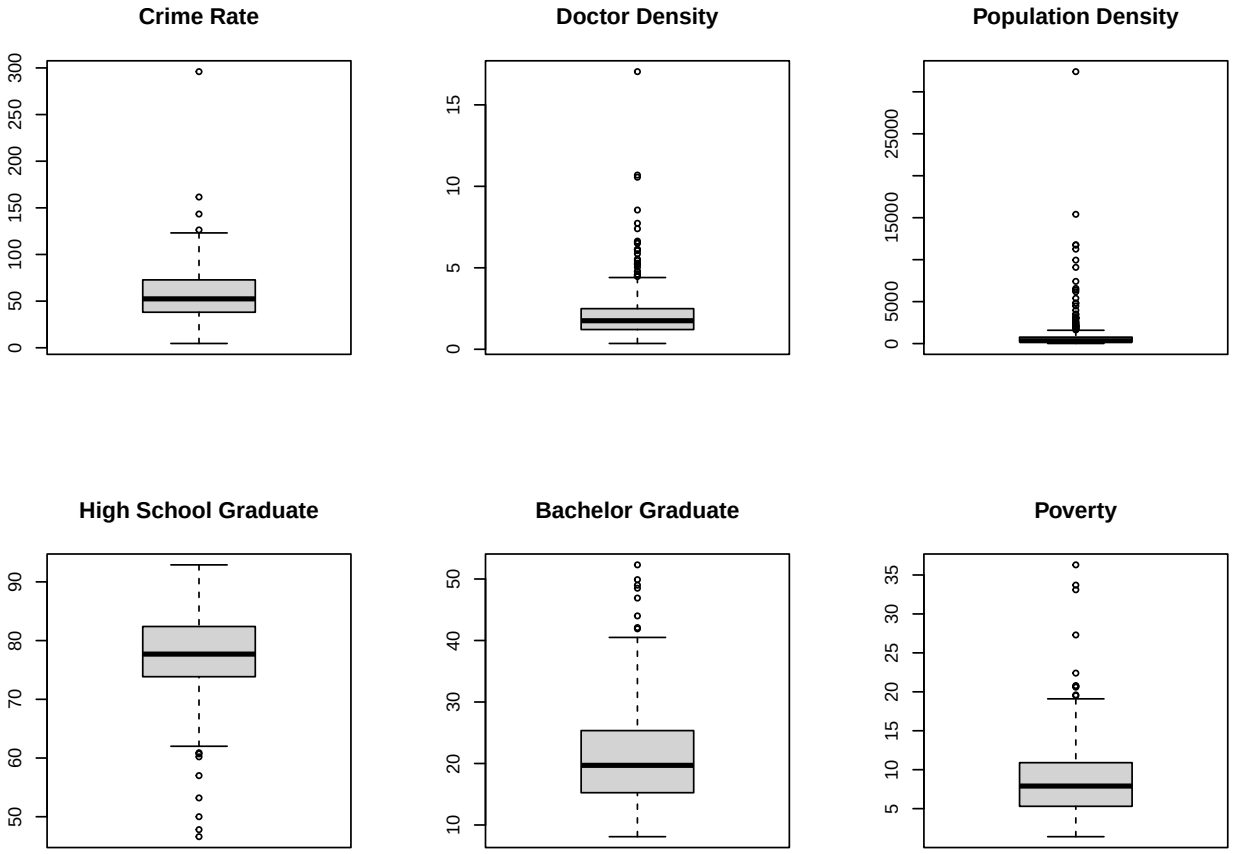
```
##       pop              pop18           pop65            hsgrad
##  Min.   : 100043   Min.   :16.40   Min.   : 3.000   Min.   :46.60
##  1st Qu.: 139027   1st Qu.:26.20   1st Qu.: 9.875   1st Qu.:73.88
##  Median : 217280   Median :28.10   Median :11.750   Median :77.70
##  Mean   : 393011   Mean   :28.57   Mean   :12.170   Mean   :77.56
##  3rd Qu.: 436064   3rd Qu.:30.02   3rd Qu.:13.625   3rd Qu.:82.40
##  Max.   :8863164   Max.   :49.70   Max.   :33.800   Max.   :92.90
##      bagrad         poverty           unemp           pcincome
##  Min.   : 8.10   Min.   : 1.400   Min.   : 2.200   Min.   : 8899
##  1st Qu.:15.28   1st Qu.: 5.300   1st Qu.: 5.100   1st Qu.:16118
##  Median :19.70   Median : 7.900   Median : 6.200   Median :17759
##  Mean   :21.08   Mean   : 8.721   Mean   : 6.597   Mean   :18561
##  3rd Qu.:25.32   3rd Qu.:10.900   3rd Qu.: 7.500   3rd Qu.:20270
##  Max.   :52.30   Max.   :36.300   Max.   :21.300   Max.   :37541
##     crm_1000         docs_1000         beds_1000       pop_density
##  Min.   :  4.601   Min.   : 0.3559   Min.   : 0.1649   Min.   :   13.26
##  1st Qu.: 38.102   1st Qu.: 1.2127   1st Qu.: 2.1972   1st Qu.:  192.34
##  Median : 52.429   Median : 1.7509   Median : 3.3287   Median :  335.91
##  Mean   : 57.286   Mean   : 2.1230   Mean   : 3.6493   Mean   :  888.44
##  3rd Qu.: 72.597   3rd Qu.: 2.4915   3rd Qu.: 4.5649   3rd Qu.:  756.55
##  Max.   :295.987   Max.   :17.0377   Max.   :19.6982   Max.   :32403.72
##    northeast       northcentral        south
##  Min.   :0.0000   Min.   :0.0000   Min.   :0.0000
##  1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000
##  Median :0.0000   Median :0.0000   Median :0.0000
##  Mean   :0.2341   Mean   :0.2455   Mean   :0.3455
##  3rd Qu.:0.0000   3rd Qu.:0.0000   3rd Qu.:1.0000
##  Max.   :1.0000   Max.   :1.0000   Max.   :1.0000
```

```
mean_crm = mean(sum_cdi$crm_1000)
cdi_state = cdi %>%
```

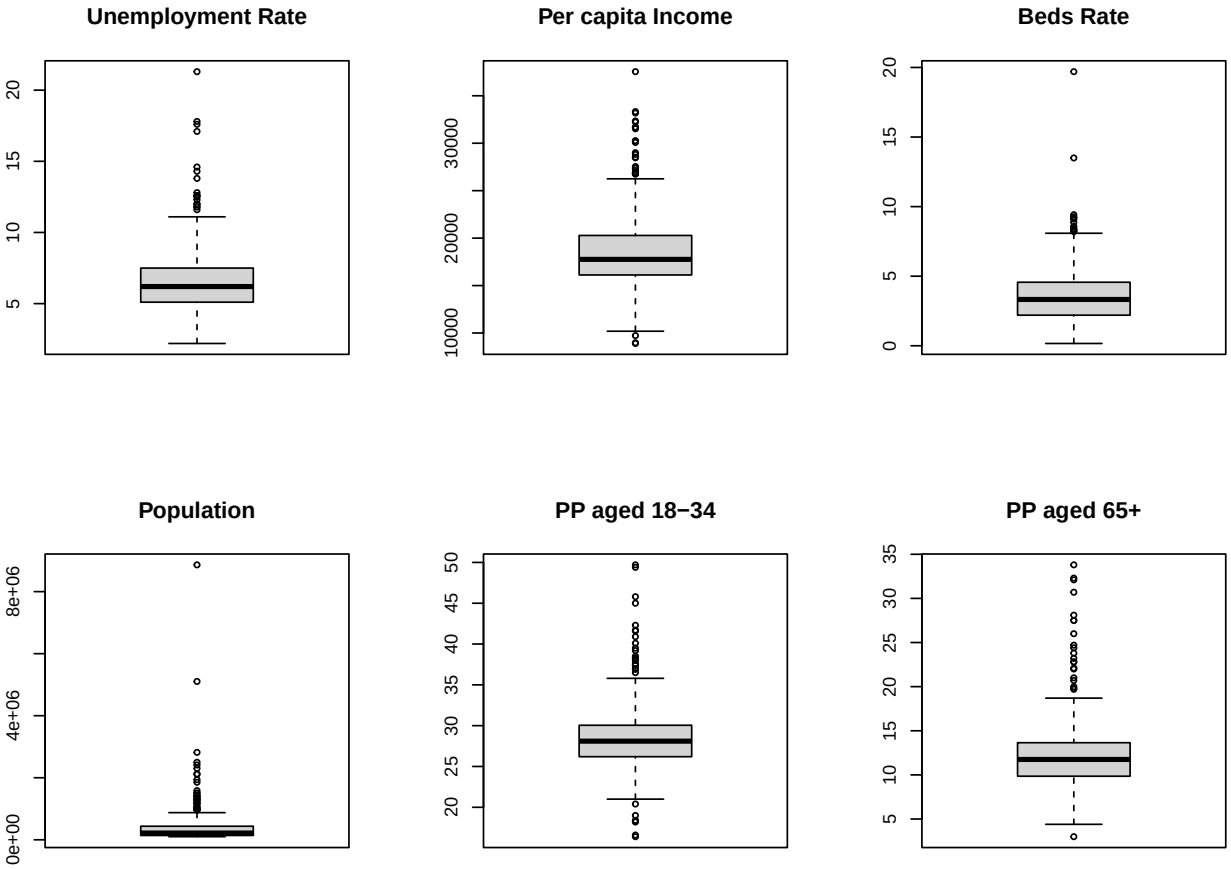
```
  group_by(state) %>%
  summarize(crime_rate = mean(crm_1000)) %>%
  mutate(low_high = ifelse(crime_rate>mean_crm, TRUE,FALSE))


cdi_state %>%
  mutate(state = fct_reorder(state, crime_rate)) %>%
  ggplot(aes(x = state, y = crime_rate))+
  geom_hline(yintercept = mean_crm, color = "red")+
  geom_point(aes(color = low_high),size = 3)+
  ggtitle("Crime Rates in the US States") +
  labs(y = "Crime Rate Per 1000", x = "States") +
  theme(plot.title = element_text(hjust = 0.5),
        axis.text.x = element_text(angle = 90, vjust = 0.5, hjust= 1),
        legend.position = "none")
```

boxplot for each variable

```
par(mfrow=c(2,3))
boxplot(sum_cdi$crm_1000, main='Crime Rate')
boxplot(sum_cdi$docs_1000, main='Doctor Density')
boxplot(sum_cdi$pop_density,main='Population Density' )
boxplot(sum_cdi$hsgrad, main='High School Graduate')
boxplot(sum_cdi$bagrad, main='Bachelor Graduate')
boxplot(sum_cdi$poverty, main='Poverty')
```

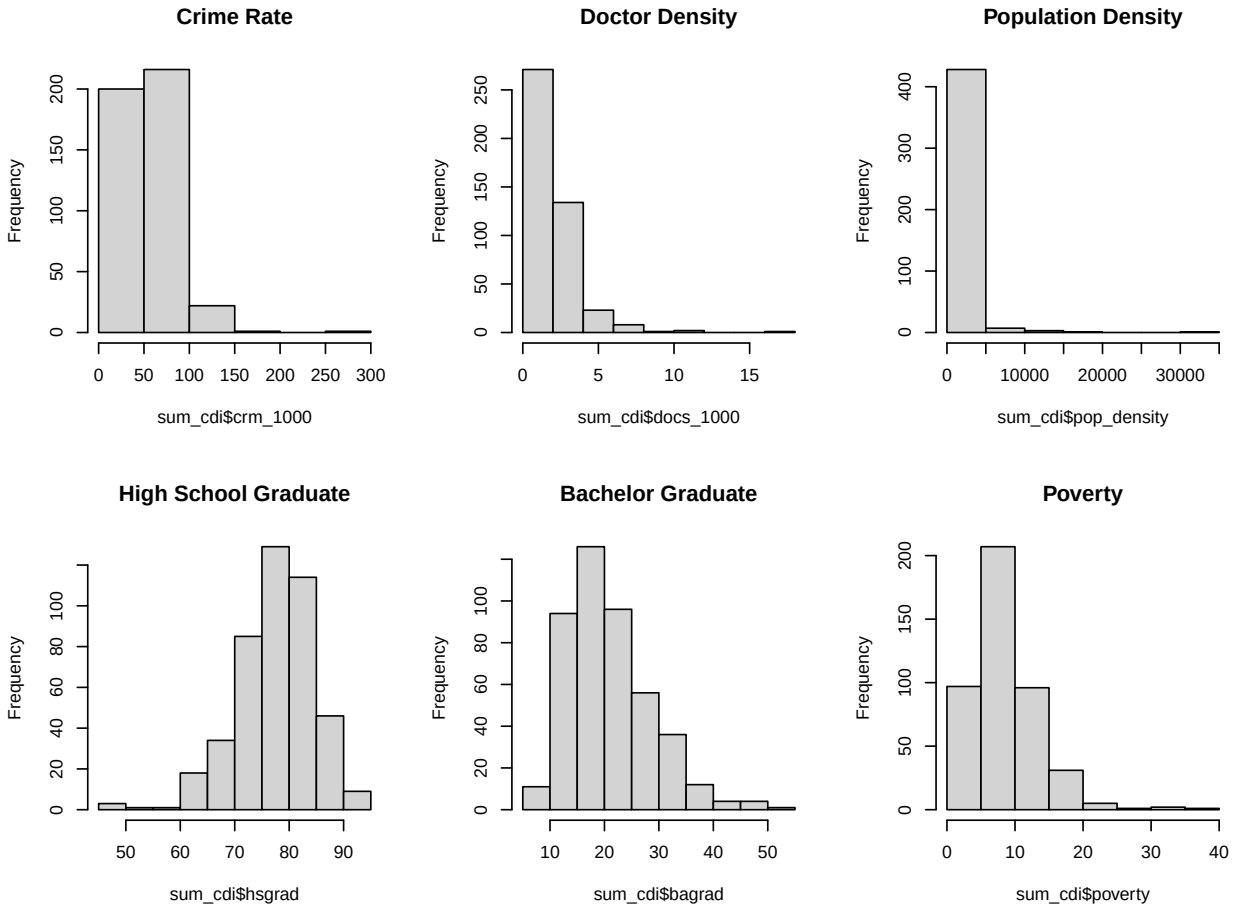



```
par(mfrow=c(2,3))
boxplot(sum_cdi$unemp, main='Unemployment Rate')
boxplot(sum_cdi$pcincome, main='Per capita Income')
boxplot(sum_cdi$beds_1000, main='Beds Rate')
boxplot(sum_cdi$pop, main='Population')
boxplot(sum_cdi$pop18, main='PP aged 18-34')
boxplot(sum_cdi$pop65, main='PP aged 65+')
```

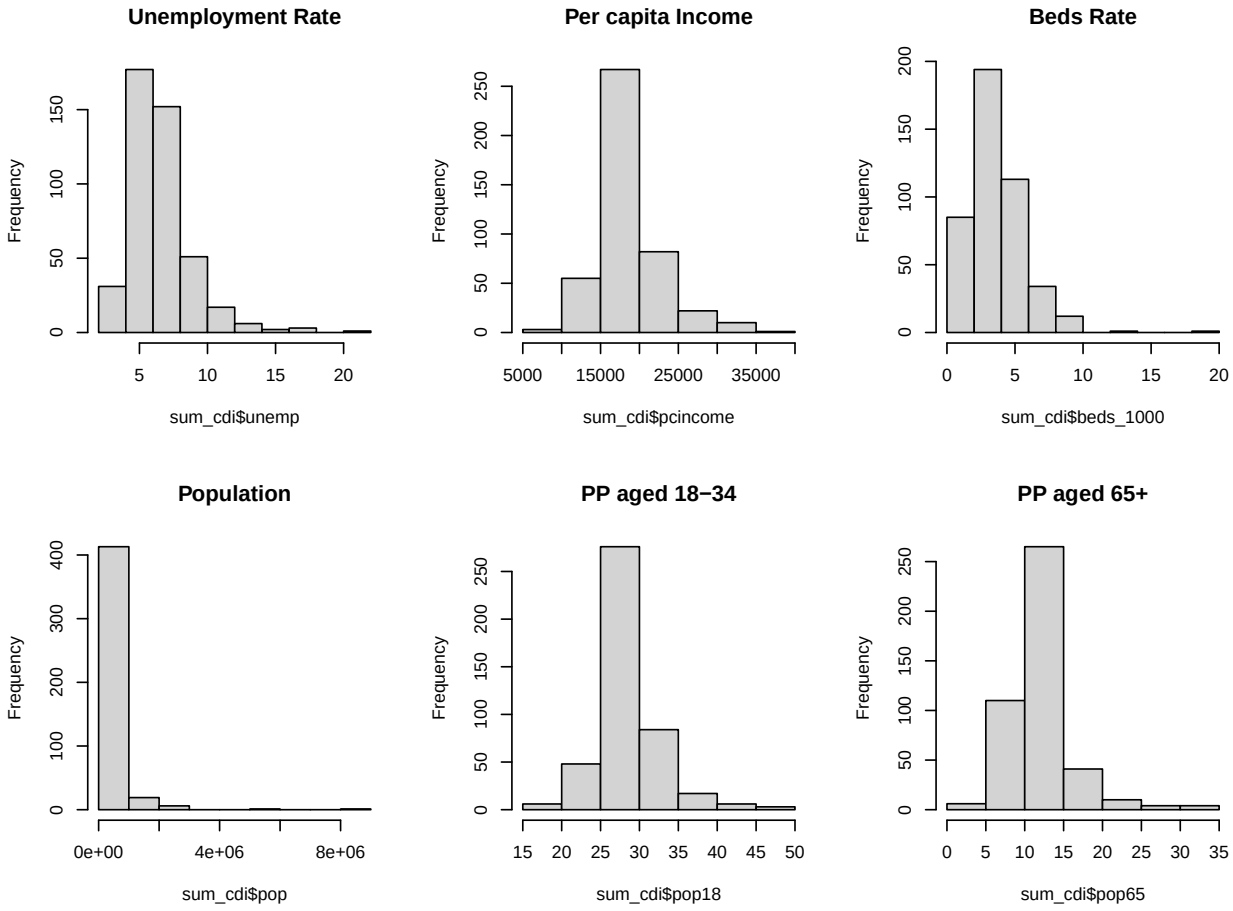


Histogram for each variable

```
par(mfrow=c(2,3))
hist(sum_cdi$crm_1000, main='Crime Rate')
hist(sum_cdi$docs_1000, main='Doctor Density')
hist(sum_cdi$pop_density,main='Population Density' )
hist(sum_cdi$hsgrad, main='High School Graduate')
hist(sum_cdi$bagrad, main='Bachelor Graduate')
hist(sum_cdi$poverty, main='Poverty')
```

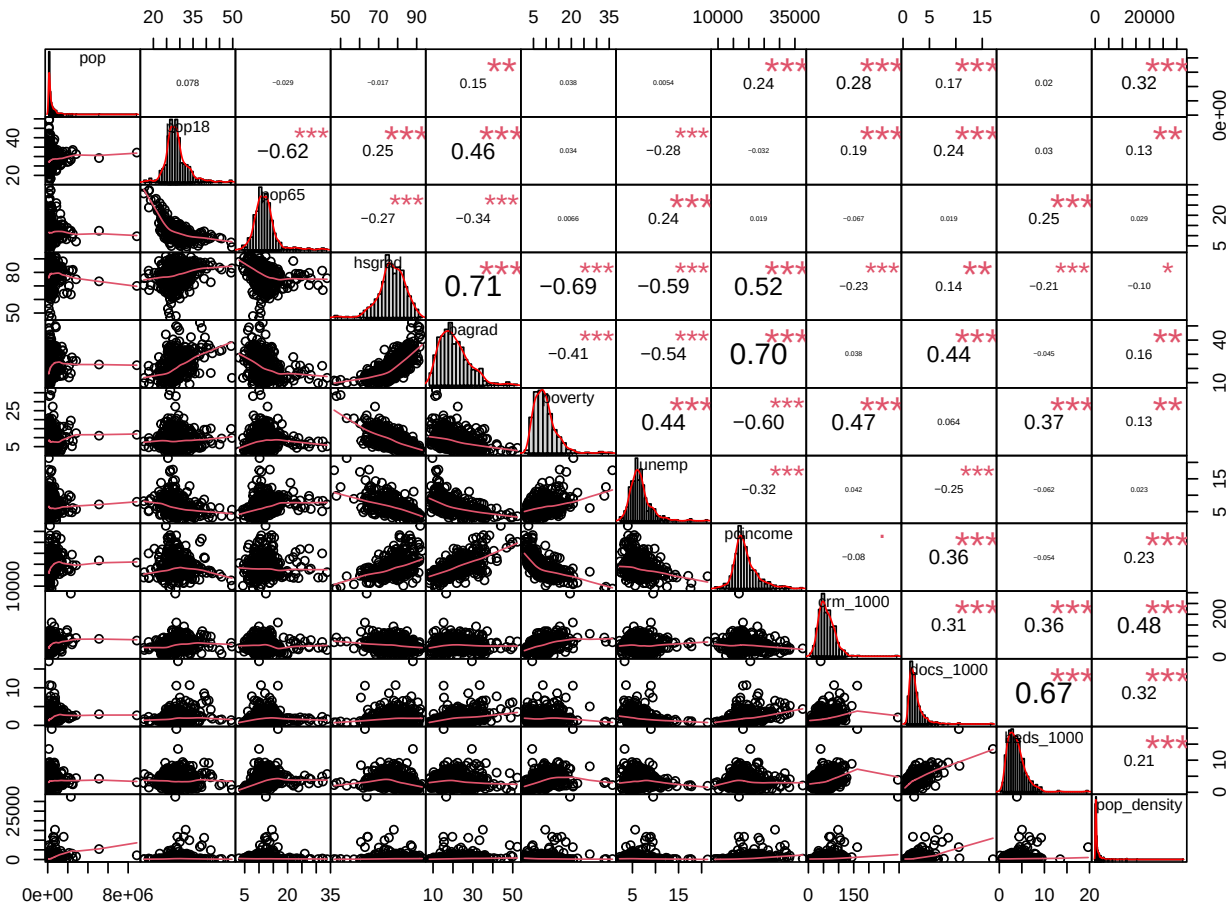



```
par(mfrow=c(2,3))
hist(sum_cdi$unemp, main='Unemployment Rate')
hist(sum_cdi$pcincome, main='Per capita Income')
hist(sum_cdi$beds_1000, main='Beds Rate')
hist(sum_cdi$pop, main='Population')
hist(sum_cdi$pop18, main='PP aged 18-34')
hist(sum_cdi$pop65, main='PP aged 65+')
```



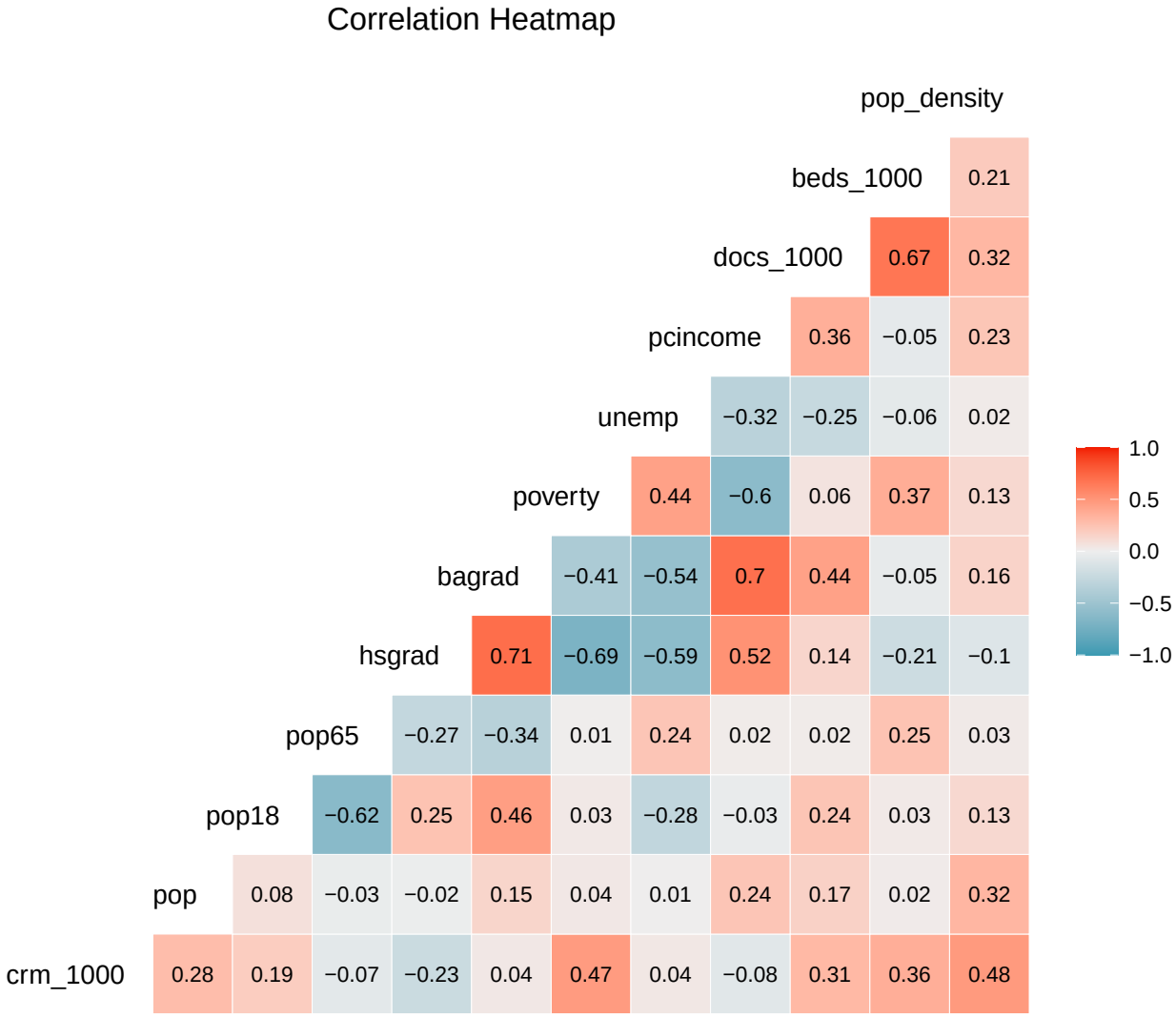
Marginal Correlation and Correlation martix

```
corr_matrix =
  cdi %>%
  dplyr::select(-state, -cty, -northeast, -northcentral, -south) %>%
  chart.Correlation(histogram = TRUE, method = "pearson")
```



Correlation Heatmap

```
cdi %>%
  dplyr::select(-state, -cty, -northeast, -northcentral, -south) %>%
  dplyr::select(crm_1000, everything()) %>%
  ggcorr(label=TRUE, hjust = 0.9, layout.exp = 2, label_size = 3, label_round = 2) +
  ggtitle("Correlation Heatmap") +
  theme(plot.title = element_text(hjust = 0.5))
```



Build Model

Full Model

Let's start with the full model

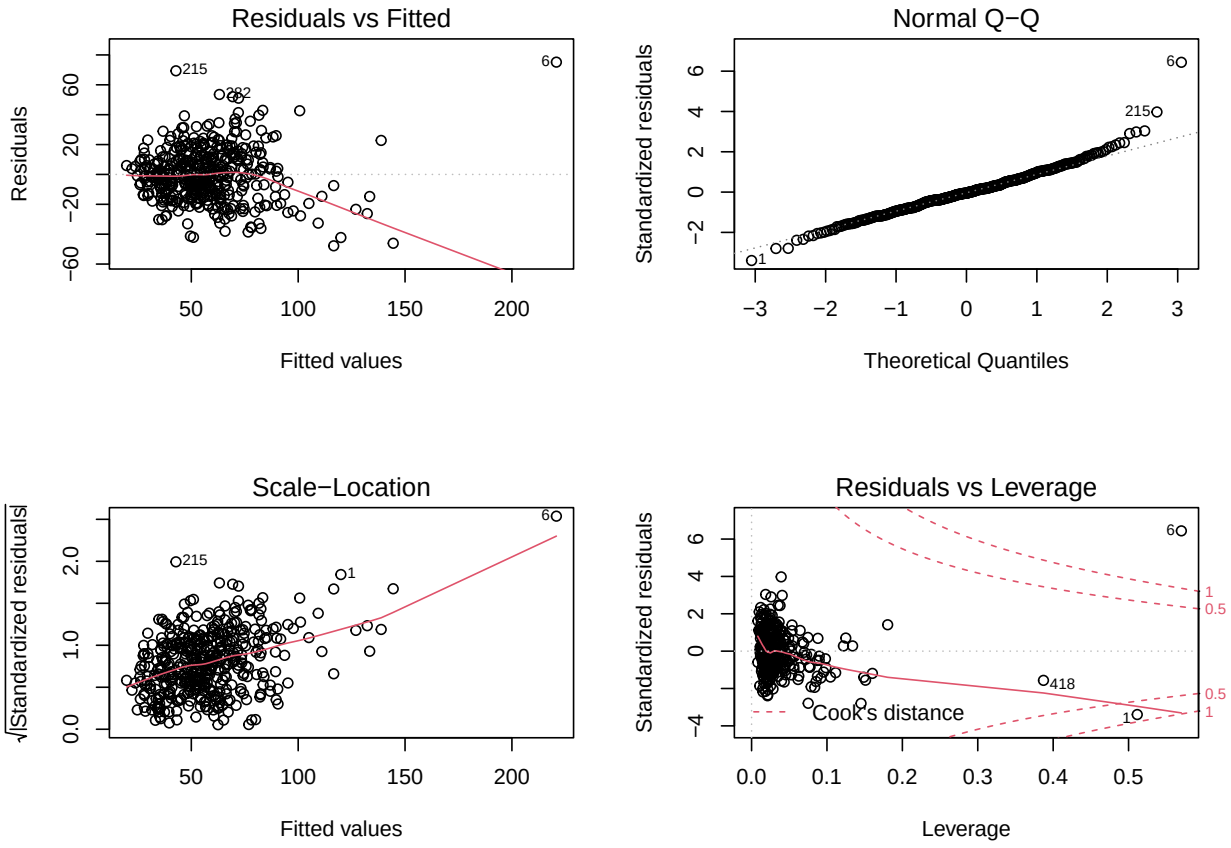
```
mult_fit = lm(crm_1000 ~ ., data = sum_cdi)
summary(mult_fit)
```

```
##
## Call:
## lm(formula = crm_1000 ~ ., data = sum_cdi)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -47.786 -11.422  -0.934  10.200  75.180
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -4.805e+01  2.770e+01  -1.734 0.083592 .
## pop           5.486e-06  1.579e-06   3.474 0.000566 ***
## pop18         6.947e-01  3.305e-01   2.102 0.036150 *
## pop65        -1.998e-01  3.055e-01  -0.654 0.513410
## hsgrad        6.143e-01  2.690e-01   2.284 0.022864 *
```

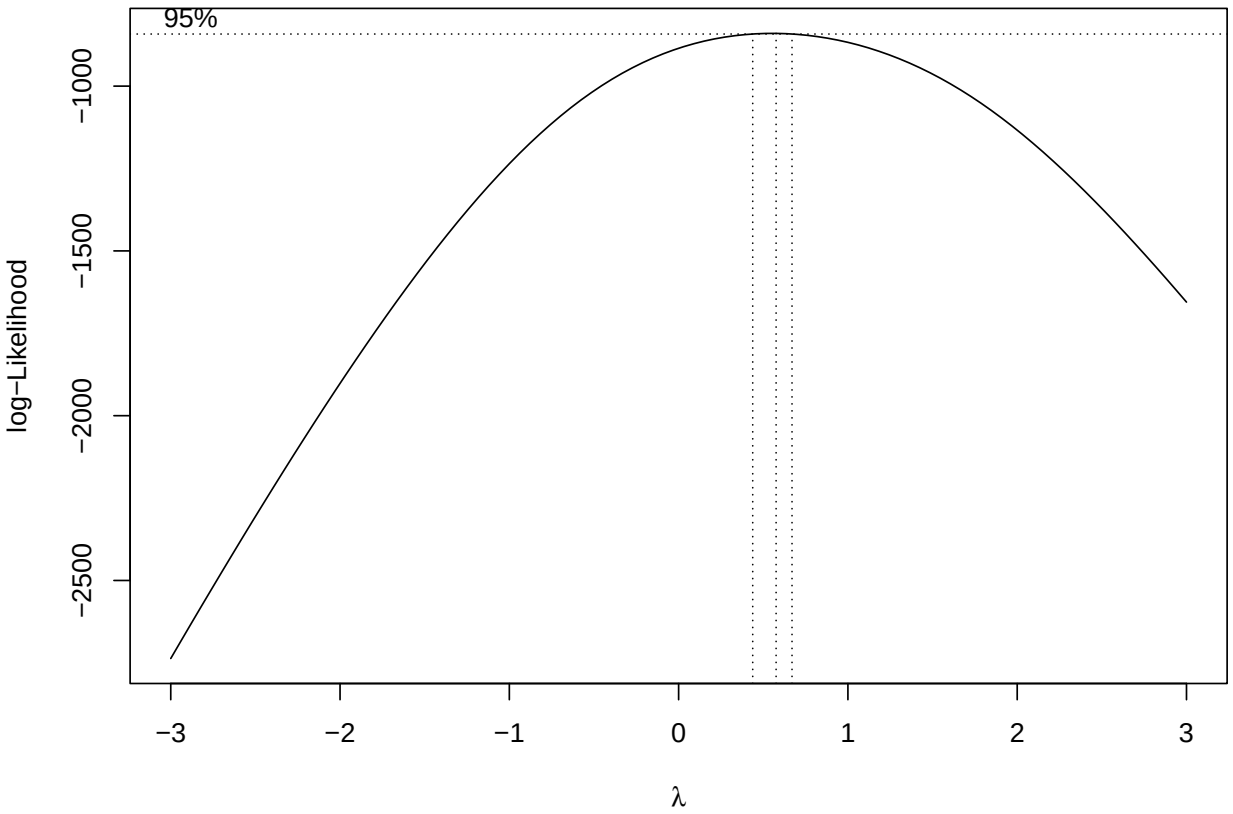
```
## bagrad       -4.835e-01  2.971e-01  -1.628 0.104327
## poverty       1.856e+00  3.864e-01   4.803 2.17e-06 ***
## unemp         6.111e-01  5.314e-01   1.150 0.250812
## pcincome      1.039e-03  4.734e-04   2.195 0.028670 *
## docs_1000    -6.634e-01  1.019e+00  -0.651 0.515556
## beds_1000     3.157e+00  7.939e-01   3.977 8.21e-05 ***
## pop_density   4.901e-03  4.537e-04  10.802  < 2e-16 ***
## northeast    -2.118e+01  3.125e+00  -6.778 4.09e-11 ***
## northcentral -1.220e+01  2.984e+00  -4.089 5.18e-05 ***
## south         6.614e+00  2.863e+00   2.310 0.021353 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 17.81 on 425 degrees of freedom
## Multiple R-squared:  0.589,  Adjusted R-squared:  0.5755
## F-statistic: 43.51 on 14 and 425 DF,  p-value: < 2.2e-16
```

Model diagnostics of the full model

```
par(mfrow=c(2,2))
plot(mult_fit)
```




```
# get the lambda for the transformation
bc_model = boxcox(mult_fit, lambda = seq(-3, 3, by = 0.25))
```



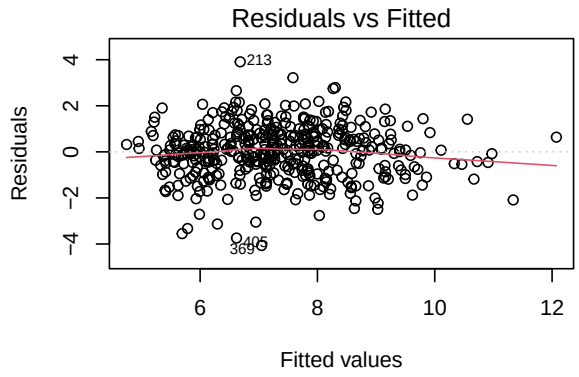

```
lamb = bc_model$x[which.max(bc_model$y)]
lamb
```

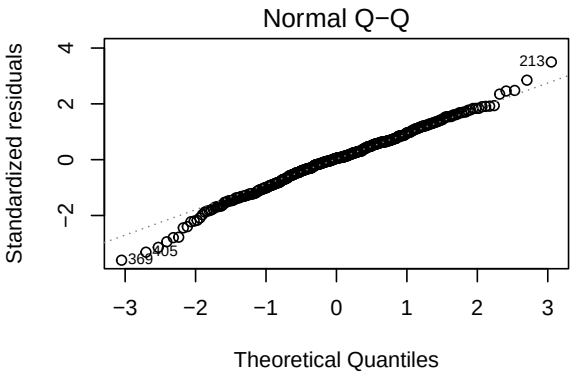
```
## [1] 0.5757576
```

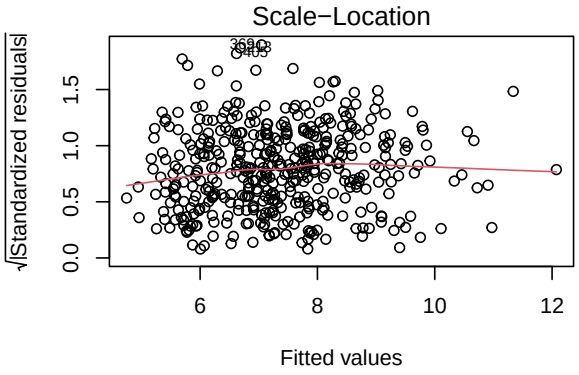
~0.5, thus we applied square root to the Y. Also we get rid of the influential points. The full model is the basis of other models, thus we choose to filter the outliers out at first.

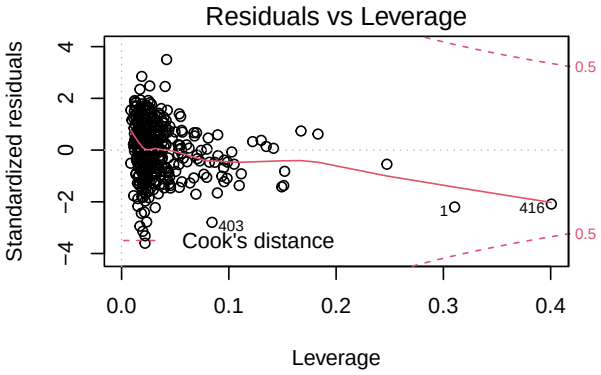
```
sum_cdi_mod = sum_cdi[-c(1,6),]  # filter out outlier and store it as the new dataset
full_trans_fit = lm(sqrt(crm_1000) ~.,data = sum_cdi_mod)  # refit
```

```
# check again
par(mfrow=c(2,2))
plot(full_trans_fit)
```










```
summary(full_trans_fit)
```

```
##
## Call:
## lm(formula = sqrt(crm_1000) ~ ., data = sum_cdi_mod)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.0654 -0.6625  0.0540  0.7183  3.9085
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)   7.644e-02  1.786e+00   0.043 0.965879
## pop           7.281e-07  1.425e-07   5.111 4.87e-07 ***
## pop18         7.584e-02  2.159e-02   3.513 0.000491 ***
## pop65        -2.316e-04  1.965e-02  -0.012 0.990601
## hsgrad        2.583e-02  1.733e-02   1.491 0.136820
## bagrad       -3.462e-02  1.911e-02  -1.812 0.070658 .
## poverty       1.111e-01  2.492e-02   4.457 1.07e-05 ***
## unemp         4.736e-02  3.407e-02   1.390 0.165214
## pcincome      1.058e-04  3.141e-05   3.367 0.000828 ***
## docs_1000    -2.102e-02  6.581e-02  -0.319 0.749576
## beds_1000     2.286e-01  5.101e-02   4.481 9.59e-06 ***
## pop_density   8.083e-05  4.359e-05   1.854 0.064417 .
## northeast    -1.719e+00  2.008e-01  -8.565  < 2e-16 ***
## northcentral -9.851e-01  1.912e-01  -5.151 3.97e-07 ***
## south         3.042e-01  1.835e-01   1.658 0.098155 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.141 on 423 degrees of freedom
```

```
## Multiple R-squared:  0.551,  Adjusted R-squared:  0.5361
## F-statistic: 37.08 on 14 and 423 DF,  p-value: < 2.2e-16
```

```
check_collinearity(full_trans_fit)
```

```
## # Check for Multicollinearity
##
## Low Correlation
##
##          Term  VIF Increased SE Tolerance
##           pop 1.00         1.00      1.00
##         pop18 2.65         1.63      0.38
##         pop65 2.07         1.44      0.48
##        hsgrad 3.28         1.81      0.31
##        bagrad 3.74         1.93      0.27
##       poverty 2.43         1.56      0.41
##         unemp 1.89         1.37      0.53
##      pcincome 1.02         1.01      0.98
##     docs_1000 2.62         1.62      0.38
##     beds_1000 3.16         1.78      0.32
##   pop_density 1.01         1.01      0.99
##     northeast 2.21         1.49      0.45
##  northcentral 2.28         1.51      0.44
##         south 2.46         1.57      0.41
```

We will just use the transformed models for the further model fits

Backward Elimination

```
multi_back = step(full_trans_fit, direction='backward')
```

```
## Start:  AIC=130.27
## sqrt(crm_1000) ~ pop + pop18 + pop65 + hsgrad + bagrad + poverty +
##     unemp + pcincome + docs_1000 + beds_1000 + pop_density +
##     northeast + northcentral + south
##
##                Df Sum of Sq    RSS    AIC
## - pop65         1     0.000 550.67 128.27
## - docs_1000     1     0.133 550.81 128.37
## - unemp         1     2.516 553.19 130.26
## <none>                      550.67 130.27
## - hsgrad        1     2.892 553.56 130.56
## - south         1     3.577 554.25 131.10
## - bagrad        1     4.275 554.95 131.66
## - pop_density   1     4.475 555.15 131.81
## - pcincome      1    14.762 565.43 139.85
## - pop18         1    16.064 566.74 140.86
## - poverty       1    25.858 576.53 148.37
## - beds_1000     1    26.137 576.81 148.58
## - pop           1    34.004 584.68 154.51
## - northcentral  1    34.547 585.22 154.92
## - northeast     1    95.493 646.17 198.31
##
## Step:  AIC=128.27
## sqrt(crm_1000) ~ pop + pop18 + hsgrad + bagrad + poverty + unemp +
##     pcincome + docs_1000 + beds_1000 + pop_density + northeast +
##     northcentral + south
##
##                Df Sum of Sq    RSS    AIC
```

```
## - docs_1000     1     0.133 550.81 126.37
## <none>                      550.67 128.27
## - unemp         1     2.550 553.22 128.29
## - hsgrad        1     2.903 553.58 128.57
## - south         1     3.583 554.26 129.11
## - bagrad        1     4.277 554.95 129.66
## - pop_density   1     4.515 555.19 129.84
## - pcincome      1    14.879 565.55 137.94
## - pop18         1    21.617 572.29 143.13
## - poverty       1    27.010 577.68 147.24
## - beds_1000     1    28.382 579.05 148.28
## - pop           1    34.067 584.74 152.56
## - northcentral  1    34.747 585.42 153.07
## - northeast     1    96.401 647.07 196.93
##
## Step:  AIC=126.37
## sqrt(crm_1000) ~ pop + pop18 + hsgrad + bagrad + poverty + unemp +
##     pcincome + beds_1000 + pop_density + northeast + northcentral +
##     south
##
##                Df Sum of Sq    RSS    AIC
## <none>                      550.81 126.37
## - unemp         1     2.533 553.34 126.38
## - hsgrad        1     3.010 553.82 126.76
## - south         1     3.944 554.75 127.50
## - pop_density   1     4.387 555.19 127.85
## - bagrad        1     4.988 555.79 128.32
## - pcincome      1    14.747 565.55 135.94
## - pop18         1    21.486 572.29 141.13
## - poverty       1    27.234 578.04 145.51
## - pop           1    33.948 584.75 150.57
## - northcentral  1    35.244 586.05 151.54
## - beds_1000     1    52.476 603.28 164.23
## - northeast     1    97.351 648.16 195.66
```

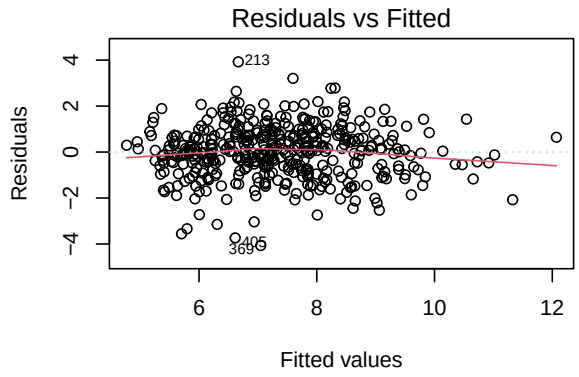
```
multi_back
```

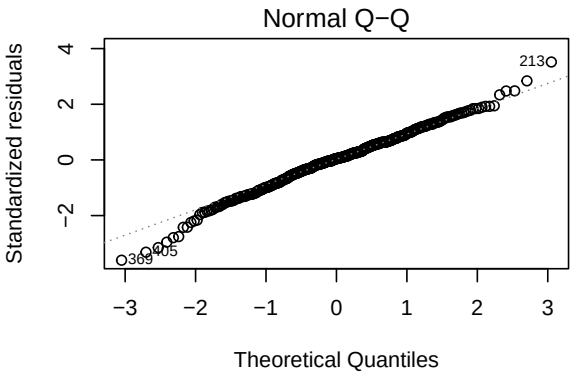
```
##
## Call:
## lm(formula = sqrt(crm_1000) ~ pop + pop18 + hsgrad + bagrad +
##     poverty + unemp + pcincome + beds_1000 + pop_density + northeast +
##     northcentral + south, data = sum_cdi_mod)
##
## Coefficients:
##  (Intercept)           pop         pop18        hsgrad        bagrad
##    9.096e-02     7.261e-07     7.546e-02     2.624e-02    -3.617e-02
##      poverty         unemp      pcincome     beds_1000   pop_density
##    1.115e-01     4.714e-02     1.048e-04     2.172e-01     7.880e-05
##    northeast  northcentral         south
##   -1.711e+00    -9.731e-01     3.142e-01
```

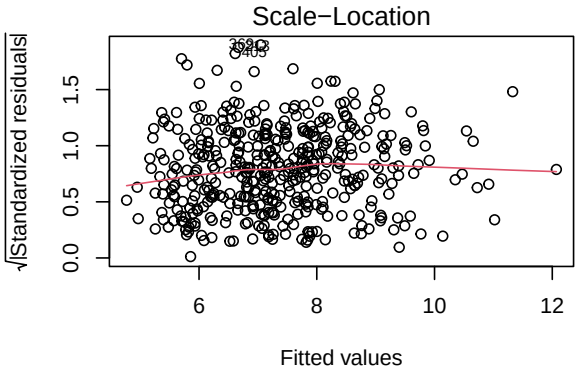
sqrt(crm_1000) ~ pop + pop18 + hsgrad + bagrad + poverty + unemp+ pcincome + beds_1000 + pop_density + northeast + northcentral + south, data = sum_cdi_mod

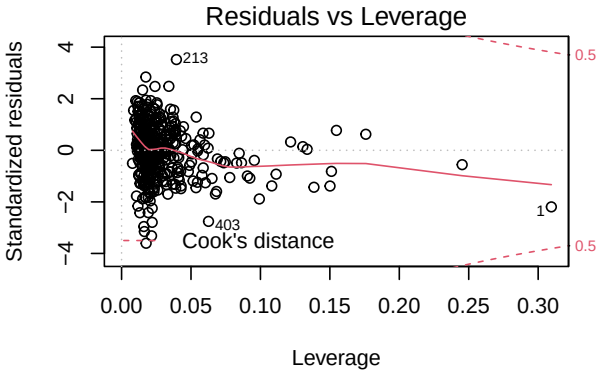
Model Diagnostic

```
par(mfrow = c(2,2))
plot(multi_back)
```










```
check_collinearity(multi_back)
```

```
## # Check for Multicollinearity
##
## Low Correlation
##
##          Term  VIF Increased SE Tolerance
##           pop 1.00         1.00      1.00
##         pop18 1.96         1.40      0.51
##        hsgrad 3.25         1.80      0.31
##        bagrad 3.50         1.87      0.29
##       poverty 2.33         1.53      0.43
##         unemp 1.86         1.36      0.54
##      pcincome 1.03         1.01      0.97
##     beds_1000 1.42         1.19      0.70
##   pop_density 1.01         1.01      0.99
##     northeast 2.15         1.47      0.47
##  northcentral 2.18         1.48      0.46
##         south 2.38         1.54      0.42
```

Forward Selection

```
multi_forward = step(full_trans_fit, direction = 'forward')
```

```
## Start:  AIC=130.27
## sqrt(crm_1000) ~ pop + pop18 + pop65 + hsgrad + bagrad + poverty +
##     unemp + pcincome + docs_1000 + beds_1000 + pop_density +
##     northeast + northcentral + south
```

```
multi_forward
```

```
##
```

```
## Call:
## lm(formula = sqrt(crm_1000) ~ pop + pop18 + pop65 + hsgrad +
##     bagrad + poverty + unemp + pcincome + docs_1000 + beds_1000 +
##     pop_density + northeast + northcentral + south, data = sum_cdi_mod)
##
## Coefficients:
##  (Intercept)           pop         pop18         pop65        hsgrad
##    7.644e-02     7.281e-07     7.584e-02    -2.316e-04     2.583e-02
##       bagrad       poverty         unemp      pcincome     docs_1000
##   -3.462e-02     1.111e-01     4.736e-02     1.058e-04    -2.102e-02
##    beds_1000   pop_density     northeast  northcentral         south
##    2.286e-01     8.083e-05    -1.719e+00    -9.851e-01     3.042e-01
```

sqrt(crm_1000) ~ pop + pop18 + pop65 + hsgrad + bagrad + poverty + unemp + pcincome + docs_1000 + beds_1000 + pop_density + northeast + northcentral + south, data = sum_cdi_mod

Forward selection generated the same result as the full model, thus we will not consider it from now on.

Both direction

```
multi_both = step(full_trans_fit, direction = "both")
```

```
## Start:  AIC=130.27
## sqrt(crm_1000) ~ pop + pop18 + pop65 + hsgrad + bagrad + poverty +
##     unemp + pcincome + docs_1000 + beds_1000 + pop_density +
##     northeast + northcentral + south
##
##                Df Sum of Sq    RSS    AIC
## - pop65         1     0.000 550.67 128.27
## - docs_1000     1     0.133 550.81 128.37
## - unemp         1     2.516 553.19 130.26
## <none>                      550.67 130.27
## - hsgrad        1     2.892 553.56 130.56
## - south         1     3.577 554.25 131.10
## - bagrad        1     4.275 554.95 131.66
## - pop_density   1     4.475 555.15 131.81
## - pcincome      1    14.762 565.43 139.85
## - pop18         1    16.064 566.74 140.86
## - poverty       1    25.858 576.53 148.37
## - beds_1000     1    26.137 576.81 148.58
## - pop           1    34.004 584.68 154.51
## - northcentral  1    34.547 585.22 154.92
## - northeast     1    95.493 646.17 198.31
##
## Step:  AIC=128.27
## sqrt(crm_1000) ~ pop + pop18 + hsgrad + bagrad + poverty + unemp +
##     pcincome + docs_1000 + beds_1000 + pop_density + northeast +
##     northcentral + south
##
##                Df Sum of Sq    RSS    AIC
## - docs_1000     1     0.133 550.81 126.37
## <none>                      550.67 128.27
## - unemp         1     2.550 553.22 128.29
## - hsgrad        1     2.903 553.58 128.57
## - south         1     3.583 554.26 129.11
## - bagrad        1     4.277 554.95 129.66
## - pop_density   1     4.515 555.19 129.84
## + pop65         1     0.000 550.67 130.27
```

```
## - pcincome      1    14.879 565.55 137.94
## - pop18         1    21.617 572.29 143.13
## - poverty       1    27.010 577.68 147.24
## - beds_1000     1    28.382 579.05 148.28
## - pop           1    34.067 584.74 152.56
## - northcentral  1    34.747 585.42 153.07
## - northeast     1    96.401 647.07 196.93
##
## Step:  AIC=126.37
## sqrt(crm_1000) ~ pop + pop18 + hsgrad + bagrad + poverty + unemp +
##     pcincome + beds_1000 + pop_density + northeast + northcentral +
##     south
##
##                Df Sum of Sq    RSS    AIC
## <none>                      550.81 126.37
## - unemp         1     2.533 553.34 126.38
## - hsgrad        1     3.010 553.82 126.76
## - south         1     3.944 554.75 127.50
## - pop_density   1     4.387 555.19 127.85
## + docs_1000     1     0.133 550.67 128.27
## - bagrad        1     4.988 555.79 128.32
## + pop65         1     0.000 550.81 128.37
## - pcincome      1    14.747 565.55 135.94
## - pop18         1    21.486 572.29 141.13
## - poverty       1    27.234 578.04 145.51
## - pop           1    33.948 584.75 150.57
## - northcentral  1    35.244 586.05 151.54
## - beds_1000     1    52.476 603.28 164.23
## - northeast     1    97.351 648.16 195.66
```

```
multi_both
```

```
##
## Call:
## lm(formula = sqrt(crm_1000) ~ pop + pop18 + hsgrad + bagrad +
##     poverty + unemp + pcincome + beds_1000 + pop_density + northeast +
##     northcentral + south, data = sum_cdi_mod)
##
## Coefficients:
##  (Intercept)           pop         pop18        hsgrad        bagrad
##    9.096e-02     7.261e-07     7.546e-02     2.624e-02    -3.617e-02
##      poverty         unemp      pcincome     beds_1000   pop_density
##    1.115e-01     4.714e-02     1.048e-04     2.172e-01     7.880e-05
##    northeast  northcentral         south
##   -1.711e+00    -9.731e-01     3.142e-01
```

sqrt(crm_1000) ~ pop + pop18 + hsgrad + bagrad + poverty + unemp + pcincome + beds_1000 + pop_density + northeast + northcentral + south, data = sum_cdi_mod

the same model as the backward selection, we will not focus on this model from now on

Interaction Model

Choose backward selection-generated model as the basis since it's with the highest adjusted r-squared. The choice of interaction is somewhat arbitrary, mainly based on the correlation heatmap and also to avoid high collinearity. After a few tries, we added two more interaction terms as the following

```
multi_interact = lm(sqrt(crm_1000) ~ pop + pop18 + hsgrad + bagrad + poverty + unemp+ pcincome + beds_1000
                    + pop_density + northeast + northcentral + south + pop*bagrad, data = sum_cdi_mod)
summary(multi_interact)
```

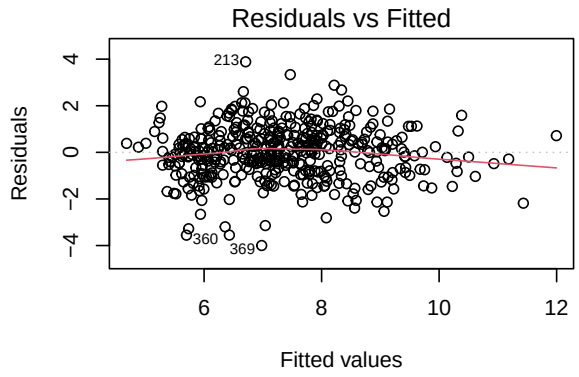
```
##
## Call:
## lm(formula = sqrt(crm_1000) ~ pop + pop18 + hsgrad + bagrad +
##     poverty + unemp + pcincome + beds_1000 + pop_density + northeast +
##     northcentral + south + pop * bagrad, data = sum_cdi_mod)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.9985 -0.6575  0.0414  0.6784  3.8847
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -1.807e-01  1.657e+00  -0.109 0.913202
## pop           2.249e-06  5.585e-07   4.027 6.70e-05 ***
## pop18         7.328e-02  1.840e-02   3.983 8.01e-05 ***
## hsgrad        2.358e-02  1.711e-02   1.378 0.168909
## bagrad       -1.581e-02  1.967e-02  -0.804 0.421894
## poverty       1.120e-01  2.413e-02   4.641 4.63e-06 ***
## unemp         4.273e-02  3.349e-02   1.276 0.202656
## pcincome      1.143e-04  3.101e-05   3.686 0.000257 ***
## beds_1000     2.085e-01  3.401e-02   6.132 1.99e-09 ***
## pop_density   7.027e-05  4.260e-05   1.650 0.099780 .
## northeast    -1.725e+00  1.958e-01  -8.809  < 2e-16 ***
## northcentral -9.747e-01  1.851e-01  -5.266 2.22e-07 ***
## south         3.146e-01  1.787e-01   1.761 0.079001 .
## pop:bagrad   -6.611e-08  2.346e-08  -2.818 0.005064 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.129 on 424 degrees of freedom
## Multiple R-squared:  0.5592, Adjusted R-squared:  0.5456
## F-statistic: 41.37 on 13 and 424 DF,  p-value: < 2.2e-16
```

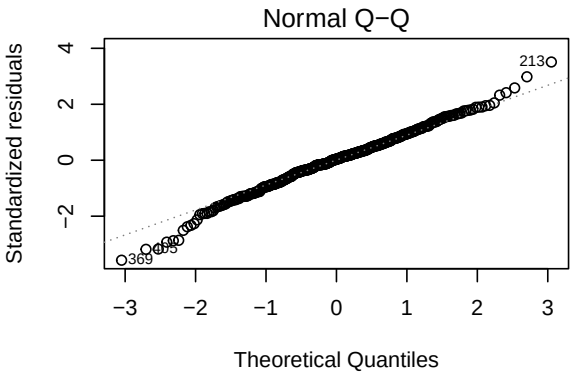
```
anova(multi_back, multi_interact)
```

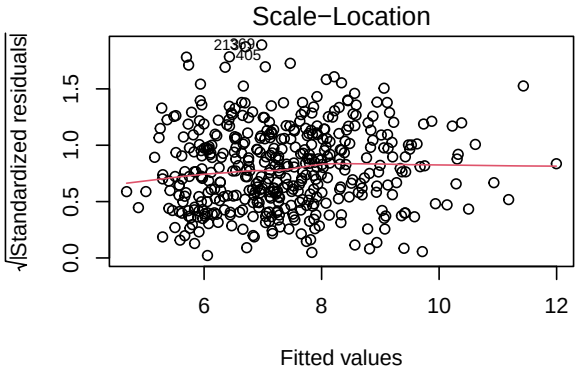
```
## Analysis of Variance Table
##
## Model 1: sqrt(crm_1000) ~ pop + pop18 + hsgrad + bagrad + poverty + unemp +
##     pcincome + beds_1000 + pop_density + northeast + northcentral +
##     south
## Model 2: sqrt(crm_1000) ~ pop + pop18 + hsgrad + bagrad + poverty + unemp +
##     pcincome + beds_1000 + pop_density + northeast + northcentral +
##     south + pop * bagrad
##   Res.Df    RSS Df Sum of Sq      F   Pr(>F)
## 1    425 550.81
## 2    424 540.68  1    10.124 7.9392 0.005064 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

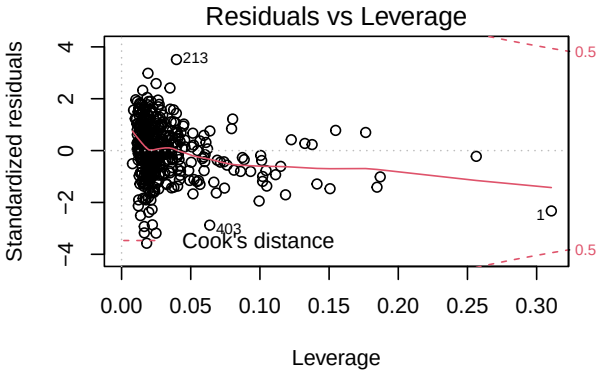
Model diagnostic

```
par(mfrow = c(2,2))
plot(multi_interact)
```










```
check_collinearity(multi_interact)
```

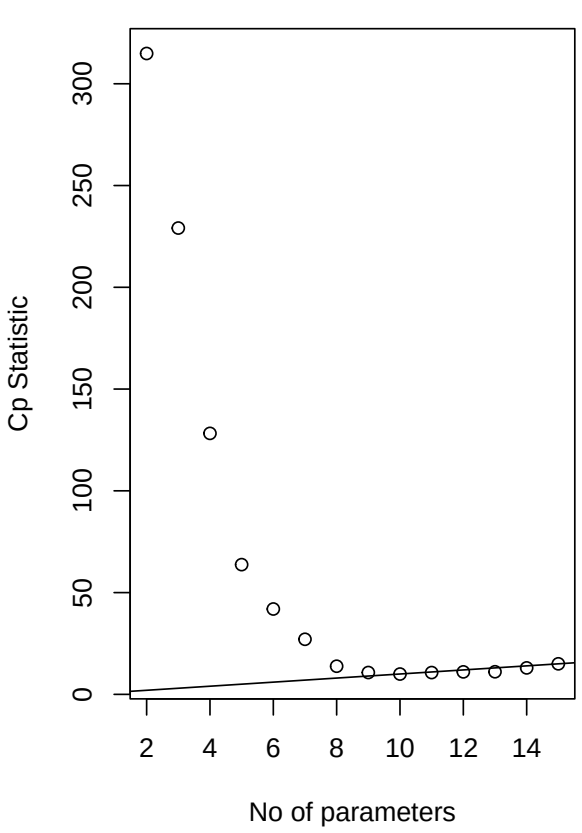
```
## # Check for Multicollinearity
##
## Low Correlation
##
##           Term  VIF Increased SE Tolerance
##            pop 1.00         1.00      1.00
##          pop18 1.72         1.31      0.58
##         hsgrad 2.97         1.72      0.34
##         bagrad 2.78         1.67      0.36
##        poverty 2.33         1.53      0.43
##          unemp 1.72         1.31      0.58
##      pcincome 1.02         1.01      0.98
##     beds_1000 1.43         1.19      0.70
##   pop_density 1.01         1.01      0.99
##     northeast 2.10         1.45      0.48
##  northcentral 2.17         1.47      0.46
##         south 2.34         1.53      0.43
##    pop:bagrad 1.00         1.00      1.00
```

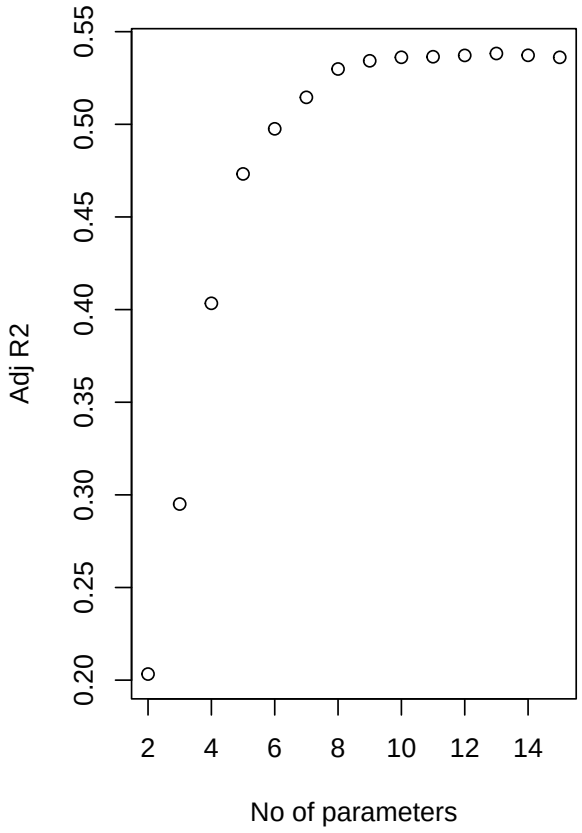
Test based procedures

Model diagnostics

```
b = regsubsets(sqrt(crm_1000)~ ., data = sum_cdi_mod, nvmax = 15)
rs = summary(b)

# plot of Cp and Adj-R2 as functions of parameters
par(mfrow=c(1,2))
```

```
plot(2:15, rs$cp, xlab="No of parameters", ylab="Cp Statistic")
abline(0,1)

plot(2:15, rs$adjr2, xlab="No of parameters", ylab="Adj R2")
```





Adjusted R-squared based model

```
models_generator = function(predict_num, models){
  predict_intent = summary(models)$which[predict_num, -1]
  predict = names(which(predict_intent == TRUE))
  predictors = paste(predict, collapse = " + ")
  text = paste0("sqrt(crm_1000) ~ ", predictors)
  return(text)
}
adjr2_num = which.max(rs$adjr2)
models_generator(adjr2_num, b)
```

```
## [1] "sqrt(crm_1000) ~ pop + pop18 + hsgrad + bagrad + poverty + unemp + pcincome + beds_1000 + pop_densi
```

Cp based model

```
cp_num = which.min(rs$cp)
models_generator(cp_num, b)
```

```
## [1] "sqrt(crm_1000) ~ pop + pop18 + bagrad + poverty + pcincome + beds_1000 + pop_density + northeast + r
```

Fit both models

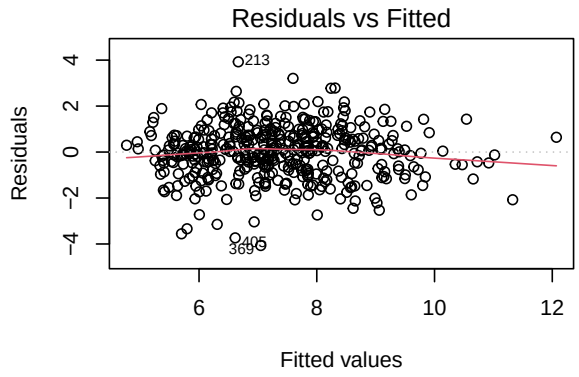
```
# r-adj square
multi_r_adj = lm(sqrt(crm_1000) ~ pop + pop18 + hsgrad + bagrad + poverty + unemp + pcincome + beds_1000 + p
summary(multi_r_adj)
```

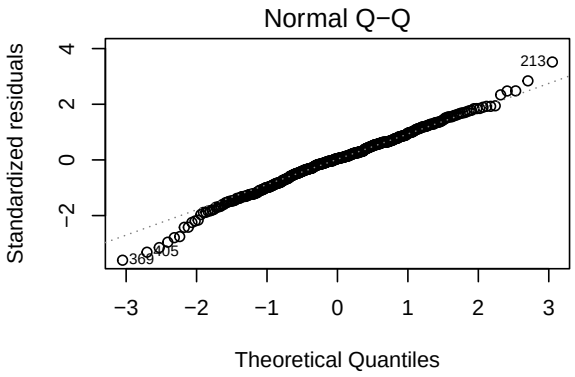
```
##
## Call:
```

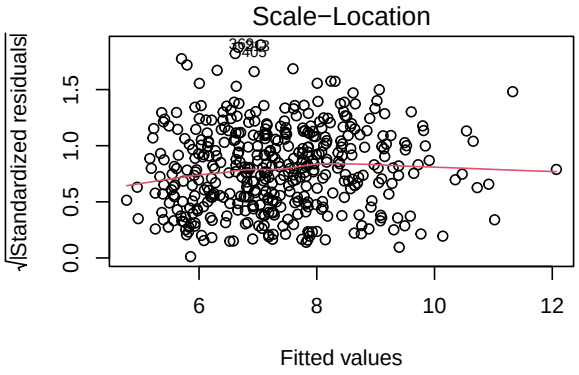
```
## lm(formula = sqrt(crm_1000) ~ pop + pop18 + hsgrad + bagrad +
##     poverty + unemp + pcincome + beds_1000 + pop_density + northeast +
##     northcentral + south, data = sum_cdi_mod)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.0662 -0.6619  0.0502  0.7174  3.9254
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)   9.096e-02  1.667e+00   0.055 0.956516
## pop           7.261e-07  1.419e-07   5.118 4.69e-07 ***
## pop18         7.546e-02  1.853e-02   4.072 5.57e-05 ***
## hsgrad        2.624e-02  1.722e-02   1.524 0.128270
## bagrad       -3.617e-02  1.844e-02  -1.962 0.050439 .
## poverty       1.115e-01  2.432e-02   4.584 6.01e-06 ***
## unemp         4.714e-02  3.372e-02   1.398 0.162867
## pcincome      1.048e-04  3.108e-05   3.373 0.000811 ***
## beds_1000     2.172e-01  3.414e-02   6.363 5.12e-10 ***
## pop_density   7.881e-05  4.283e-05   1.840 0.066502 .
## northeast    -1.711e+00  1.974e-01  -8.667  < 2e-16 ***
## northcentral -9.731e-01  1.866e-01  -5.215 2.88e-07 ***
## south         3.142e-01  1.801e-01   1.744 0.081807 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.138 on 425 degrees of freedom
## Multiple R-squared:  0.5509, Adjusted R-squared:  0.5382
## F-statistic: 43.45 on 12 and 425 DF,  p-value: < 2.2e-16
```

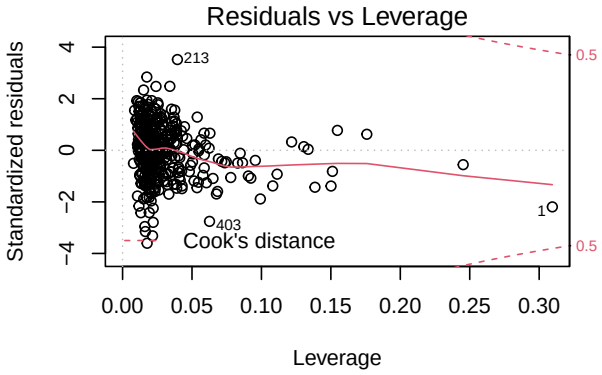
Model diagnostic

```
par(mfrow = c(2,2))
plot(multi_r_adj)
```










```
check_collinearity(multi_r_adj)
```

```
## # Check for Multicollinearity
##
## Low Correlation
##
##          Term  VIF Increased SE Tolerance
##           pop 1.00         1.00      1.00
##         pop18 1.96         1.40      0.51
##        hsgrad 3.25         1.80      0.31
##        bagrad 3.50         1.87      0.29
##       poverty 2.33         1.53      0.43
##         unemp 1.86         1.36      0.54
##      pcincome 1.03         1.01      0.97
##     beds_1000 1.42         1.19      0.70
##   pop_density 1.01         1.01      0.99
##     northeast 2.15         1.47      0.47
##  northcentral 2.18         1.48      0.46
##         south 2.38         1.54      0.42
```

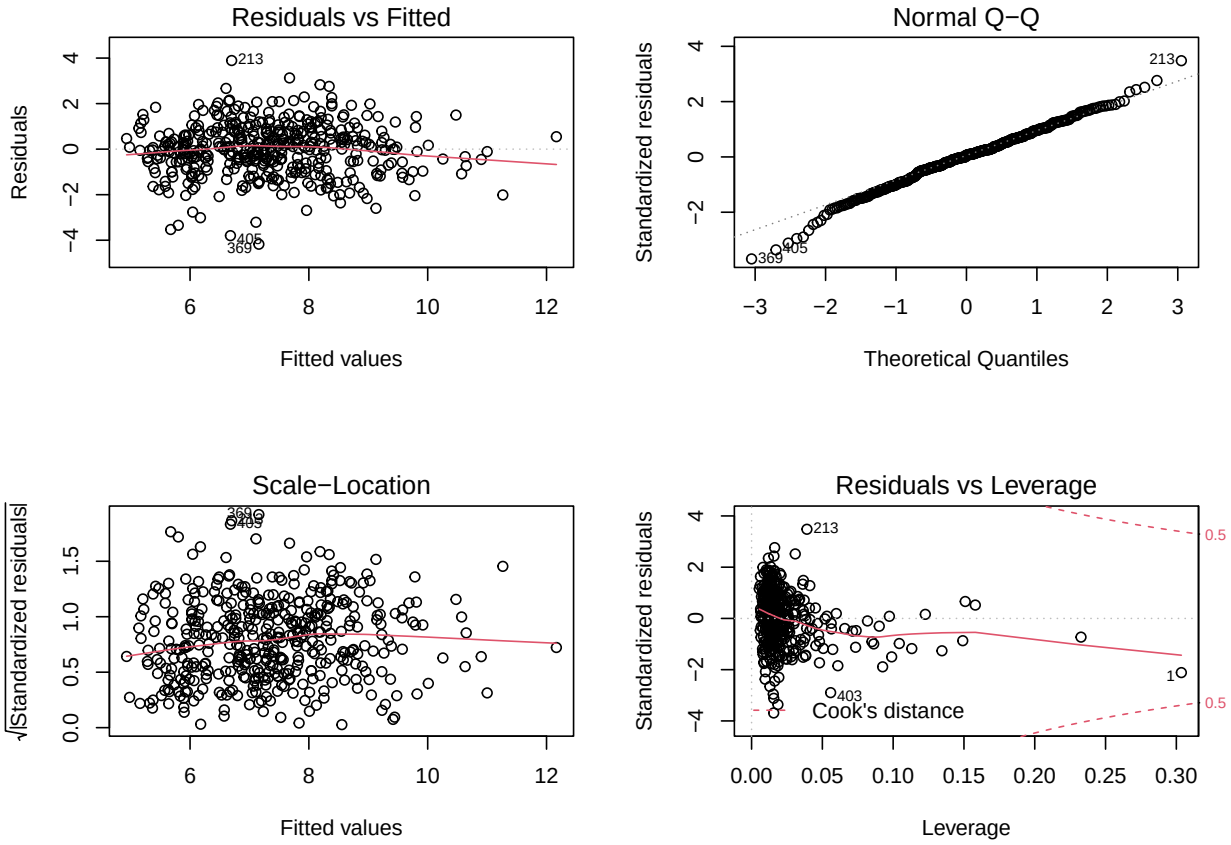
```
# cp value based
multi_cp = lm(sqrt(crm_1000) ~ pop + pop18 + bagrad + poverty + pcincome + beds_1000 + pop_density + northea
summary(multi_cp)
```

```
##
## Call:
## lm(formula = sqrt(crm_1000) ~ pop + pop18 + bagrad + poverty +
##     pcincome + beds_1000 + pop_density + northeast + northcentral,
##     data = sum_cdi_mod)
##
## Residuals:
```

```
##     Min      1Q  Median      3Q     Max
## -4.1762 -0.6227  0.0671  0.7399  3.8919
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.558e+00  7.722e-01   3.313 0.001001 **
## pop           6.984e-07  1.409e-07   4.955 1.04e-06 ***
## pop18         7.619e-02  1.840e-02   4.140 4.19e-05 ***
## bagrad       -2.829e-02  1.420e-02  -1.992 0.046959 *
## poverty       1.034e-01  1.822e-02   5.675 2.57e-08 ***
## pcincome      1.034e-04  2.960e-05   3.494 0.000526 ***
## beds_1000     2.156e-01  3.189e-02   6.760 4.53e-11 ***
## pop_density   7.035e-05  4.245e-05   1.657 0.098194 .
## northeast    -1.888e+00  1.524e-01 -12.383  < 2e-16 ***
## northcentral -1.124e+00  1.445e-01  -7.776 5.63e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.141 on 428 degrees of freedom
## Multiple R-squared:  0.5457, Adjusted R-squared:  0.5362
## F-statistic: 57.13 on 9 and 428 DF,  p-value: < 2.2e-16
```

Model diagnostic

```
par(mfrow = c(2,2))
plot(multi_cp)
```




```
check_collinearity(multi_cp)
```

```
## # Check for Multicollinearity
##
## Low Correlation
```

```
##
##          Term  VIF Increased SE Tolerance
##           pop 1.00         1.00      1.00
##         pop18 1.94         1.39      0.52
##        bagrad 2.11         1.45      0.47
##       poverty 1.36         1.16      0.74
##      pcincome 1.11         1.05      0.90
##     beds_1000 1.26         1.12      0.79
##   pop_density 1.06         1.03      0.94
##     northeast 1.28         1.13      0.78
##  northcentral 1.30         1.14      0.77
```

```
a_row = function(model_data){
  model_data %>%
  broom::glance() %>%
  dplyr::select(adj.r.squared, AIC, BIC)
}
add_in = rbind(
  ols_mallows_cp(full_trans_fit,full_trans_fit),
  ols_mallows_cp(multi_back,full_trans_fit),
  ols_mallows_cp(multi_interact,full_trans_fit),
  ols_mallows_cp(multi_r_adj,full_trans_fit),
  ols_mallows_cp(multi_cp,full_trans_fit))
rmse_add = rbind(
  rmse(full_trans_fit, data = sum_cdi_mod),
  rmse(multi_back, data = sum_cdi_mod),
  rmse(multi_interact, data = sum_cdi_mod),
  rmse(multi_r_adj, data = sum_cdi_mod),
  rmse(multi_cp, data = sum_cdi_mod)
)
rbind(a_row(full_trans_fit),
      a_row(multi_back),
      a_row(multi_interact),
      a_row(multi_r_adj),
      a_row(multi_cp))%>%
  mutate(model = c("Full model", "Backward Selection", "Interaction", "Adj R Based", "Cp Value Based"),
         cp = add_in,
         rmse = rmse_add) %>%
  relocate(model) %>%
  knitr::kable()
```

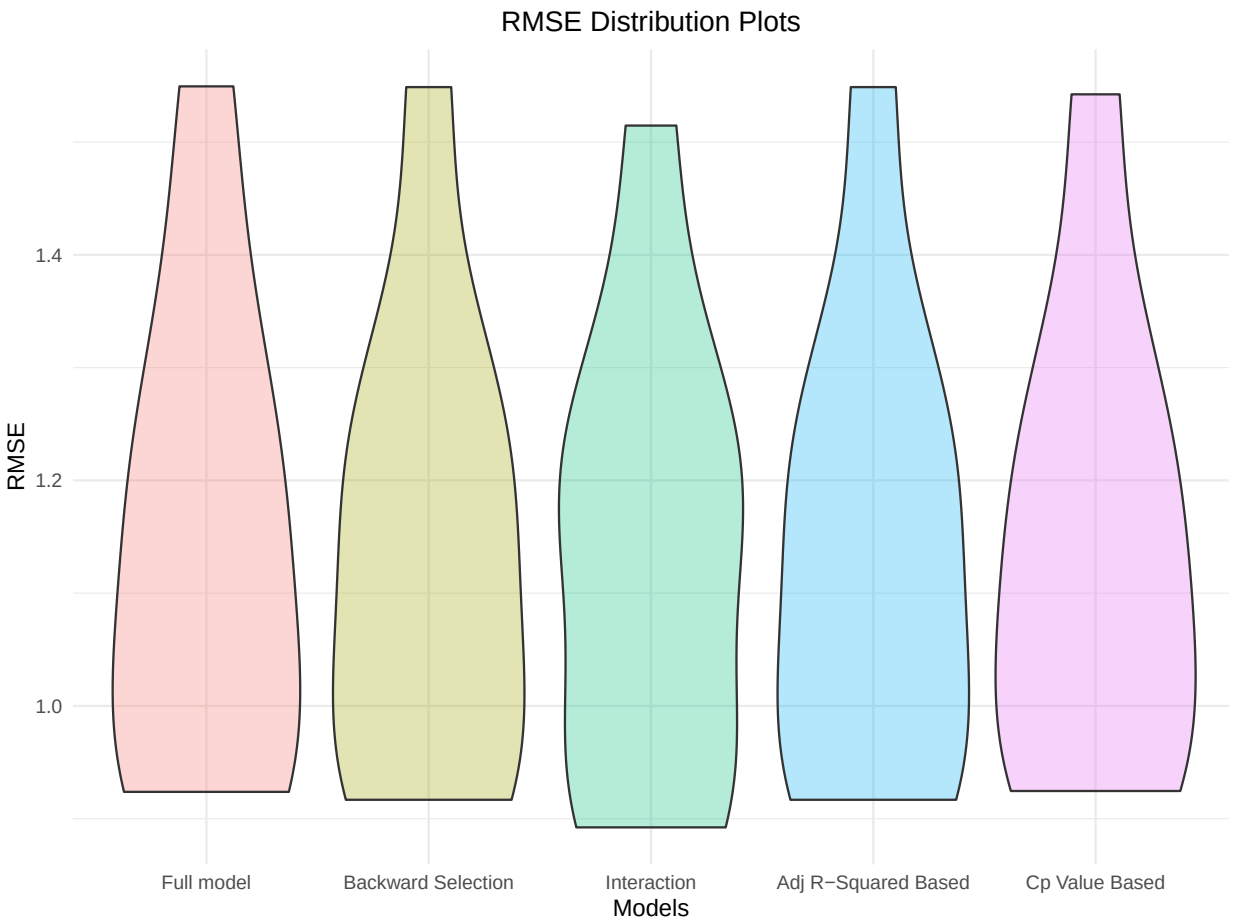
model	adj.r.squared	AIC	BIC	cp	rmse
Full model	0.5361497	1375.258	1440.573	15.000000	1.121268
Backward Selection	0.5382212	1371.363	1428.514	11.102026	1.121404
Interaction	0.5456398	1365.238	1426.471	5.325220	1.111050
Adj R Based	0.5382212	1371.363	1428.514	11.102026	1.121404
Cp Value Based	0.5361687	1370.387	1415.291	9.982508	1.127853

Cross Validation

```
set.seed(1)

cv_df =
  crossv_kfold(sum_cdi_mod, k=10) %>%   # k-fold = 5
  mutate(
    train = map(train, as_tibble),
```

```
    test = map(test, as_tibble)
  )
cv_df =
  cv_df %>%
  mutate(
    full_fit = map(.x = train, ~lm(sqrt(crm_1000) ~., data = .x)),
    back_fit = map(.x = train, ~lm(sqrt(crm_1000) ~ pop + pop18 + hsgrad + bagrad + poverty +
                                     unemp+ pcincome + beds_1000 + pop_density + northeast +
                                     northcentral + south,data = .x)),
    interact_fit = map(.x = train, ~lm(sqrt(crm_1000) ~ pop + pop18 + hsgrad + bagrad + poverty + unemp+ pc
                                       + beds_1000 + pop_density + northeast +
                                         northcentral + south + pop*bagrad, data = .x)),
    adj_fit = map(.x = train, ~lm(sqrt(crm_1000) ~ pop + pop18 + hsgrad + bagrad + poverty + unemp +
                                    pcincome + beds_1000 + pop_density + northeast + northcentral + south,
    cp_fit = map(.x = train, ~lm(sqrt(crm_1000) ~ pop + pop18 + bagrad + poverty + pcincome +
                                   beds_1000 + pop_density + northeast + northcentral, data = .x))
  ) %>%
  mutate(
    rmse_full = map2_dbl(.x = full_fit, .y = test, ~rmse(model = .x, data = .y)),
    rmse_back = map2_dbl(.x = back_fit, .y = test, ~rmse(model = .x, data = .y)),
    rmse_interact = map2_dbl(.x = interact_fit, .y = test, ~rmse(model = .x, data = .y)),
    rmse_adj = map2_dbl(.x = adj_fit, .y = test, ~rmse(model = .x, data = .y)),
    rmse_cp = map2_dbl(.x = cp_fit, .y = test, ~rmse(model = .x, data = .y)),
  )
```

```
cv_df %>%
  dplyr::select(starts_with("rmse")) %>%
  pivot_longer(
    everything(),
    names_to = "model",
    values_to = "rmse",
    names_prefix = "rmse_"
  ) %>%
  mutate(model = fct_relevel(model, "full", "back", "interact", "Adj R-squared", "Cp")) %>%
  ggplot(aes(x = model, y = rmse,))+
  geom_violin(aes(fill = model), alpha = 0.3)+
  scale_x_discrete(labels = c("Full model", "Backward Selection", "Interaction", "Adj R-Squared Based", "Cp
  ggtitle("RMSE Distribution Plots") +
  theme(plot.title = element_text(hjust = 0.5),
        legend.position = "none")  +  # the display of legends is redundant
  labs(y = "RMSE", x = "Models")
```


RMSE Distribution Plots
RMSE
1.4
1.2
1.0
Full model
Backward Selection
Interaction
Adj R−Squared Based
Cp Value Based
Models