Homework 1 Handed out: Wednesday, September 7, 2022
Due: Wednesday, September 14, 2022 by 11:59pm

Material covered: Outcomes 1.1–1.5.

1. Simplify the following expressions, writing them in the form $x + iy$ where x, y are real numbers:

   a)
   $$(3 + 4i)^2$$

   b)
   $$\frac{1+i}{1-i}$$

   c)
   $$\mathrm{Im}(\overline{1+i}).$$

2. Write the following complex numbers in the form $x + iy$ where x, y are real numbers:

   a)
   $$\sqrt{2}e^{i\frac{\pi}{4}}.$$

   b)
   $$(1+i)^{2022}.$$

   Write the following complex numbers in the form $re^{i\theta}$ where $r > 0$ and $-\pi < \theta < \pi$:

   c)
   $$(1+\sqrt{3}i)^2$$

   d)
   $$(1+\sqrt{3}i)\overline{(1+i)}$$

3. a) Find all third roots of -1.

b) Find all fourth roots of i.

c) Find all fifth roots of 32.

You may write the answers in either Cartesian or polar form.

4. Find all $z \in \mathbb{C}$ that satisfy the equation and sketch the solution set:

a)
$$|z+1| = |z-i|.$$

b)
$$z + \overline{z} = 0.$$

5. Find the roots of the polynomial $p(z)$. Then factor the polynomial.

a)
$$p(z) = z^2 - (1+2i)z - 1 + i.$$

b)
$$p(z) = z^5 - z.$$