Homework #10

1

Training datapoint:

```
X=[-1; 2]; y=2;
```

Initial weights:

```
w2=[ -0.2, -0.4;
0.4, -0.4;
0.1, 0.1];
b2=[ -0.5;
0.2;
0.1];
w3=[ 0.1, 0.1, 0.1];
b3=[ -0.1];
```

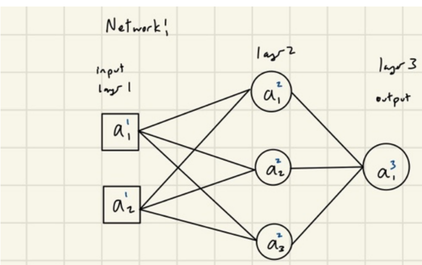
All nodes are sigmoid. Sigmoid activation function:

```
sigmoid = @(zl) 1.0 ./ (1.0+exp(-zl));
```

Cost function:

```
cost = @(y, al) 0.5* ((y-al).^2);
```

1a)



Eqns:

layer 2

$$Z^2 = \begin{bmatrix} -0.2, & -0.4; \\ 0.4, & -0.4; \\ 0.1, & 0.1]; \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -0.5; \\ 0.2; \\ 0.1]; \end{bmatrix}$$

$$a_i^2 = \frac{1}{1 + Z_i^2}$$ sigmoid

layer 3

$$Z^3 = [\ 0.1, \ \ 0.1, \ \ 0.1]\ a^2 + [-0.1]$$

$$a_i^3 = \frac{1}{1 + Z_i^3}$$ sigmoid

```
z2 = w2*X + b2
```

```
z2 = 3×1
   -1.1000
   -1.0000
    0.2000
```

```
a2 = sigmoid(z2)
```

```
a2 = 3×1
    0.2497
    0.2689
    0.5498
```

```
z3 = w3*a2 + b3
```

```
z3 = 0.0069
```

```
a3 = sigmoid(z3)
```

```
a3 = 0.5017
```

```
datapoint_cost = cost(y, a3)
```

```
datapoint_cost = 1.1224
```

1b)

$\delta^L = \frac{\partial C}{\partial z^L} = \frac{\partial C}{\partial a^L}\frac{\partial a^L}{\partial z^L}$

And $\frac{\partial C}{\partial a^L} = \frac{d}{da^L}[\frac{1}{2}(y - a^L)^2] = a^L - y = \nabla_{a^L} C$

And $\frac{\partial a^L}{\partial z^L} = \frac{d}{dz^L}(1 + e^{-z^L})^{-1} = e^{-z^L}(1 + e^{-z^L})^{-2} = \sigma'(z^L)$

So $\delta^L = \nabla_{a^L} C \odot \sigma'(z^L) = (a^L - y) \cdot e^{-z^L}(1 + e^{-z^L})^{-2}$

From the last layer's delta, we can find all other subsequent earlier layer delta using:

$\delta^l = (w^{l+1})^T \delta^{l+1} \odot \sigma'(z^l)$

And then for any layer, we can calculate its gradient and how much to adjust its weights based on its delta using:

$\nabla_{W^l} C = \delta^l (a^{l-1})^T$

1c)

$$\delta_{w^3} = (a^L - y) \cdot \sigma'(z^3)$$

$$= (1.1224 - 2) \cdot \frac{e^{0.0069}}{(1 + e^{0.0069})^2}$$

$$\delta_{w^3} = -0.375$$

$$\delta_{w^2} = (w^{3^T} \delta_{w^3}) \odot \sigma(z^2)$$

$$\delta_{w^2_{3,2}} = (0.1 \times -0.375) \times \frac{e^{0.2}}{(1 + e^{0.2})^2}$$

$$= -0.009 \qquad \nabla_{b^2_3} = -0.009$$

$$\nabla_{w^2_{3,2}} = \delta_{w^2_3} \times X^T = -0.009 \times 2 = -0.018$$

1d)

sigma_prime function:

```
sigma_prime = @(zl) exp(-zl)./ ((1+exp(-zl)).^2);
```

delta L calculation function:

```
calc_delta = @(y, al, zl) (al-y)*sigma_prime(zl);
```

Calculate gradient with respect to all weights

```
delta3 = calc_delta(y, a3, z3);
gradient_w3 = delta3*a2.'
```

```
gradient_w3 = 1×3
   -0.0935   -0.1007   -0.2059
```

```
gradient_b3 = delta3
```

```
gradient_b3 = -0.3746
```

```
delta2 = (w3.' * delta3) .* sigma_prime(z2);
gradient_w2 = delta2*X.'
```

```
gradient_w2 = 3×2
    0.0070   -0.0140
    0.0074   -0.0147
    0.0093   -0.0185
```

```
gradient_b2 = delta2
```

```
gradient_b2 = 3×1
   -0.0070
   -0.0074
```

```
-0.0093
```

1e) Direct perturbation approach for w2_12

```
epsilon = 0.002;

epsilon_w2 = w2;
epsilon_w2(1,2) = epsilon_w2(1,2) + epsilon;

epsilon_z2 = epsilon_w2*X + b2;
epsilon_a2 = sigmoid(epsilon_z2);
epsilon_z3 = w3*epsilon_a2 + b3;
epsilon_a3 = sigmoid(epsilon_z3);

epsilon_datapoint_cost = cost(y, epsilon_a3);

disp('direct perturbation approach:')
```

```
direct perturbation approach:
```

```
epsilon_delta = (epsilon_datapoint_cost - datapoint_cost) ./epsilon
```

```
epsilon_delta = -0.0141
```

which is pretty similar to our gradient calculated value:

```
disp('gradient calculation approach:')
```

```
gradient calculation approach:
```

```
disp(gradient_w2(1,2))
```

```
-0.0140
```

1f)

```
eta = 1;
```

1 iteration of steepest-descent

```
new_w2 = w2 - (eta.*gradient_w2);
new_b2 = b2 - (eta.*gradient_b2);
new_w3 = w3 - (eta.*gradient_w3);
new_b3 = b3 - (eta.*gradient_b3);

new_z2 = new_w2*X + new_b2;
new_a2 = sigmoid(new_z2);
new_z3 = new_w3*new_a2 + new_b3;
new_a3 = sigmoid(new_z3)
```

```
new_a3 = 0.6347
```

```
disp('new datapoint cost:')
```

```
new datapoint cost:
```

```
new_datapoint_cost = cost(y, new_a3)
```

```
new_datapoint_cost = 0.9320
```

```
disp('old datapoint cost:')
```

```
old datapoint cost:
```

```
datapoint_cost
```

```
datapoint_cost = 1.1224
```

Yes.

We moved the weights in the direction opposite of the max gradient of the cost, so now the output will in fact have a lower cost.

2

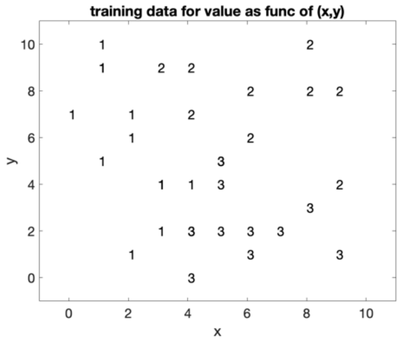
```
close('all')
X=[3,6,1,0,5,5,4,4,3,4,6,3,2,5,4,8,6,8, 9,8,3,6, 1,4,7,2,9,1,4,2,9,1,5;
9,1,5,7,5,5,2,2,2,0,2,4,1,2,9,8,8,10,1,3,4,6,10,7,2,7,4,9,4,6,8,9,4];
y=[2,3,1,1,3,3,3,3,1,3,3,1,1,3,2,2,2, 2,3,3,1,2, 1,2,3,1,2,1,1,1,2,1,3];
```

Input space:

```
x1=[0:0.1:10];
x2=[0:0.1:10];
```

Plot training data:

```
figure;clf
set(0,'defaulttextfontsize',16); set(0,'defaultaxesfontsize',16);
xlim([-1 11])
ylim([-1 11])
xlabel('x')
ylabel('y')
box on
for i=1:length(y)
  text(X(1,i),X(2,i),num2str(y(i)));
end
title('training data for value as func of (x,y)');
```



2a) Classification:

adapted from "neural_networks3_simple_2d_classification.m"

```
% for classification, need to turn labels into matrix format:
T=zeros(max(y),length(y)); for i=1:length(y); T(y(i),i)=1; end

% Create network: specify number of neurons in each layer:
rng(3456);
clear net1
% [e.g., [2 6 2] would create 3 hidden layers with 2,6,2 neurons in each]
net1 = feedforwardnet([3 3]); % 2 hidden layers with 3 neurons each. sure.
disp('net1 = feedforwardnet([3 3]);');
```

```
net1 = feedforwardnet([3 3]);
```

```

% don't divide data into training, testing, validation.
net1.divideFcn='';

% Train network:
net1 = train(net1,X,T);
disp('net1 = train(net1,X,T);');
```

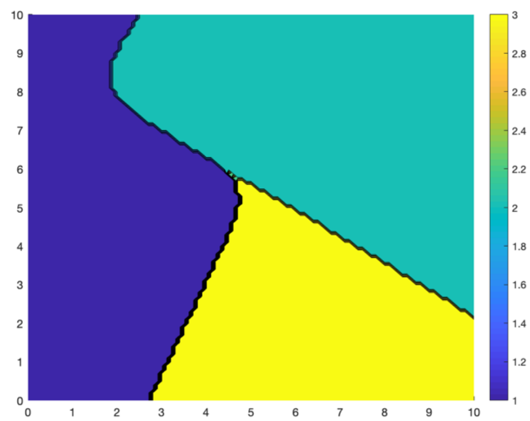
```
net1 = train(net1,X,T);
```

```
view(net1);
```

Contour plot:

```
[X1, X2]=meshgrid(x1, x2);
Xtest = [X1(:).'; X2(:).'];
ytest=net1(Xtest);

% back into labels 1-3
ytest_labels=zeros(1,length(ytest));
for i=1:1:length(ytest)
```

```
    [~,label] = max(ytest(:, i));
    ytest_labels(1,i)=label;
end

figure; hold on
contourf(x1, x2, reshape(ytest_labels, size(X1)) )
colorbar
hold off
```



2b) Regression:

```
% Create network: specify number of neurons in each layer:
rng(3456);
clear net2
```

From trying increasing number of layers, seems like 7 hidden layers did okay so we'll stick with that

```
arch = [3 3 3 3 3 3 3];
% [e.g., [2 6 2] would create 3 hidden layers with 2,6,2 neurons in each]
net2 = feedforwardnet(arch); % 2 hidden layers with 5 neurons each. sure.
disp('net2 = feedforwardnet(arch);');
```

```
net2 = feedforwardnet(arch);
```

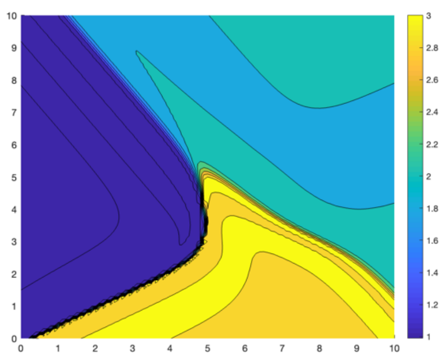
```


% don't divide data into training, testing, validation.
net2.divideFcn='';

% Train network:
net2 = train(net2,X,y);
disp('net2 = train(net2,X,y);');
```

```
net2 = train(net2,X,y);
```

```
view(net2);
```

Contour plot:

```
[X1, X2]=meshgrid(x1, x2);
Xtest = [X1(:).'; X2(:).'];
ytest2=net2(Xtest);

figure; hold on
contourf(x1, x2, reshape(ytest2, size(X1)) )
colorbar
hold off
```



3

```
clear all
load saved-network-HW-10.mat
% w2, b2, w3, b3, w4, b4
```

Activation functions:

```
tansig = @(zl) ( 2./(1+exp(-2.*zl)) ) -1;
linear = @(zl) zl;
```

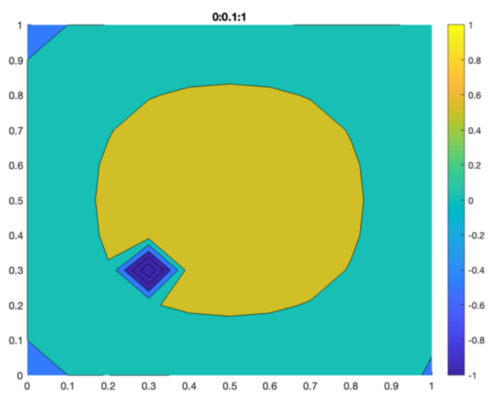
Input space:

```
x1=0:0.1:1;
x2=x1;
```

To make this easier, combine bs into ws and add 1's to input

```
w2 = horzcat(w2, b2);
w3 = horzcat(w3, b3);
```

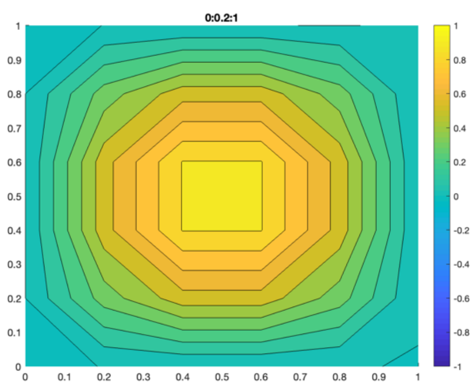
```
w4 = horzcat(w4, b4);

[X1, X2]=meshgrid(x1, x2);
Xtest = [X1(:).'; X2(:).'];
[~, N] = size(Xtest);
Xtest = vertcat(Xtest, ones([1 N]));

% Running network
z2 = w2*Xtest;
a2 = tansig(z2);

[~, N] = size(a2); a2 = vertcat(a2, ones([1 N]));
z3 = w3*a2;
a3 = tansig(z3);

[~, N] = size(a3); a3 = vertcat(a3, ones([1 N]));
z4 = w4*a3;
a4 = linear(z4);

figure; hold on; title('0:0.1:1')
contourf(x1, x2, reshape(a4, size(X1)) )
colorbar; caxis([-1, 1]);
hold off
```



Now with a different input space:

```
x1=0:0.2:1;
x2=x1;

% add row of 1's instead of dealing with separate b's
[X1, X2]=meshgrid(x1, x2);
Xtest = [X1(:).'; X2(:).'];
[~, N] = size(Xtest);
Xtest = vertcat(Xtest, ones([1 N]));
```

```
% Running network
z2 = w2*Xtest;
a2 = tansig(z2);

[~, N] = size(a2); a2 = vertcat(a2, ones([1 N]));
z3 = w3*a2;
a3 = tansig(z3);

[~, N] = size(a3); a3 = vertcat(a3, ones([1 N]));
z4 = w4*a3;
a4 = linear(z4);

figure; hold on; title('0:0.2:1')
contourf(x1, x2, reshape(a4, size(X1)) )
colorbar; caxis([-1, 1]);
hold off
```



There is a local ~0.1 width spot of lowered output with this network that is missed with just 0.2 input increments. This is likely the result of overfitting due to noise in the training data that has slightly lower values (from noise not true signal) in that region.

The problem may be overcome by training with more training data, more dummy data by adding noise to existing data, possibly using a smaller network, or adding a regularization term to account for rising w and b values in the cost.