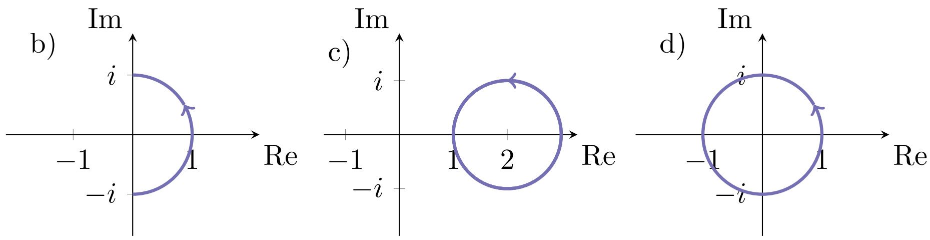
Homework 5 Handed out: Wednesday, October 5, 2022
Due: Wednesday, October 12, 2022 by 11:59pm

Material covered: Outcomes 4.1–4.4.

1. Let $\mathrm{Log}\colon \mathbb{C} \setminus \mathbb{R}_- \to \mathbb{C}$ be the principal branch of the logarithm.
   a) Show that the function F defined by
   $$F\colon \mathbb{C} \setminus \mathbb{R}_- \to \mathbb{C}, \quad F(z) = z\,\mathrm{Log}(z) - z$$
   is an antiderivative for Log on $\mathbb{C} \setminus \mathbb{R}_-$.

   Compute the integral
   $$\int_C \mathrm{Log}(z)\,\mathrm{d}z$$
   for each of the following contours C (pictured below):
   b) C is the right half-circle with center 0 and radius 1, traversed counter-clockwise.
   c) $C = C(2,1)$ is the circle with center 2 and radius 1, traversed counter-clockwise.
   d) $C = C(0,1)$ is the circle with center 0 and radius 1, traversed counter-clockwise.



2. Let
   $$f(z) = \frac{3z^2 - 4}{z(z^2 - 4)}.$$

a) Find a partial fraction decomposition for f.

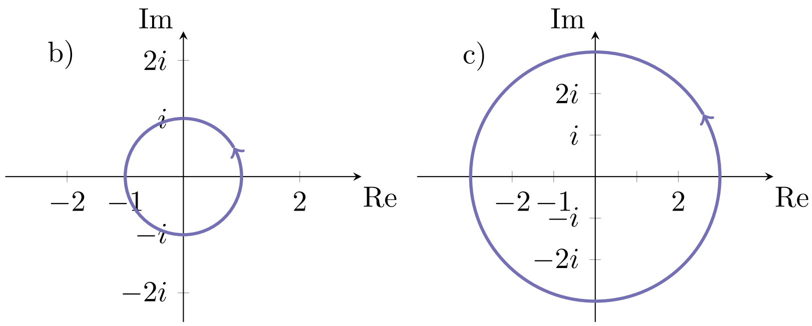
Compute

$$\int_C f(z)\,\mathrm{d}z$$

for the following contours (pictured below):

b) $C = C(0,1)$, the circle with center 0 and radius 1, traversed counter-clockwise.

c) $C = C(0,3)$, the circle with center 0 and radius 3, traversed counter-clockwise.



3. Compute the following integrals ($C(z_0, r)$ denotes a circle with center z_0 and radius r):

   a)
   $$\int_{C(i,1)} \frac{e^{z^2}}{z^2+1}\,\mathrm{d}z$$

   b)
   $$\int_{C(0,4)} \frac{\sin z}{(z-\pi)^4}\,\mathrm{d}z$$

4. Compute the integral

$$I = \int_0^{2\pi} \frac{\mathrm{d}\theta}{(2+\cos\theta)^2}$$

by evaluating an appropriately chosen contour integral, as follows:

a) Show that

$$I = \int_{C(0,1)} \frac{1}{\left(2 + (z + 1/z)/2\right)^2} \frac{\mathrm{d}z}{iz}$$

where $C(0,1)$ is the unit circle centered at the origin.
Hint: Parameterize $C(0,1)$ by $\gamma(\theta) = e^{i\theta}$, $0 \leq \theta \leq 2\pi$.

b) Compute the contour integral in (a) using the Cauchy integral formula.