Homework #05

1

1a)

So for our least sqs solution, we want to minimize the distance between Ax and b, $|Ax - b|^2$, which is = $(Ax - b)^T(Ax - b)$

So we have an optimization problem and want to look for where the derivative = 0:

$$\frac{d}{dx}(Ax - b)^T(Ax - b)$$

$$= \frac{d}{dx}(x^TA^TAx - b^TAx - x^TA^Tb + b^Tb)$$

$$= 2(A^TAx - A^Tb) = 0$$

Therefore, $A^TAx = A^Tb$ and $x = (A^TA)^{-1}(A^Tb)$

1b)

```
A=[ 4, 1, 3; -4, -2, -5; 3, -1, -4; 0, 4, -1];
b=[ 0; 5; -1; -2];

x=inv(A.'*A)*(A.'*b)
```

```
x = 3×1
   -0.4023
   -0.5666
   -0.0704
```

1c)

```
residual=(A*x)-b
```

```
residual = 4×1
   -2.3868
   -1.9059
    0.6412
   -0.1959
```

Because although this least sqs solution minimizes the distance between Ax and b, it can not get it to 0 since there is no exact solution/intersection since there are more equations than actual unknowns.

1d)

```
disp('norm of residual:')
```

```
norm of residual:
```

```
disp(norm(residual))
```

```
    3.1271
```

```
disp('norm of b:')
```

```
norm of b:
```

```
disp(norm(b))
```

```
    5.4772
```

I would consider the residual large since we can compare the residual as a proportion of the right hand side, b, and it seems to be about 60% (a significant proportion) of the magnitude of b.

1e)

```
x=A\b
```

```
x = 3×1
   -0.4023
   -0.5666
   -0.0704
```

Matlab's function gives the same result

2

2a)

```
A=[-0.4, 0.2, -0.7;
    0.8, -0.7, -0.1];
b=[-1;
    0.5];

[U, sigma, V]=svd(A)
```

```
U = 2×2
   -0.5001   -0.8659
    0.8659   -0.5001
sigma = 2×3
    1.1684         0         0
         0    0.6817         0
V = 3×3
    0.7641   -0.0788   -0.6403
   -0.6044    0.2595   -0.7532
    0.2255    0.9625    0.1506
```

$$V = \begin{bmatrix} 0.7641 & -0.0788 & -0.6403 \\ -0.6044 & 0.2545 & -0.7532 \\ 0.2755 & 0.9625 & 0.1506 \end{bmatrix} \times \Sigma^{+} = \begin{bmatrix} \frac{1}{1.1684} & 0 \\ 0 & \frac{1}{0.6817} \\ 0 & 0 \end{bmatrix} \times U^{T} = \begin{bmatrix} -0.5001 & 0.8659 \\ -0.8659 & -0.5001 \end{bmatrix}$$

$$= V = \begin{bmatrix} 0.7641 & -0.0788 & -0.6403 \\ -0.6044 & 0.2545 & -0.7532 \\ 0.2755 & 0.9625 & 0.1506 \end{bmatrix} \times \begin{bmatrix} -0.428 & +0.7411 \\ -1.2702 & -0.7336 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -0.2270 & 0.6241 \\ -0.0709 & -0.6383 \\ -1.3191 & -0.539 \end{bmatrix}$$

2b)

```
A_pseudo=[-0.2270 0.6241;
    -0.0709 -0.6383;
    -1.3191 -0.5390]
```

```
A_pseudo = 3×2
   -0.2270    0.6241
   -0.0709   -0.6383
   -1.3191   -0.5390
```

```
x=A_pseudo*b
```

```
x = 3×1
    0.5391
   -0.2482
    1.0496
```

2c)

```
disp('using pinv():')
```

```
using pinv():
```

```
x2=pinv(A)*b
```

```
x2 = 3×1
    0.5390
   -0.2482
    1.0496
```

```
disp('using A\b:')
```

```
using A\b:
```

```
x3=A\b
```

```
x3 = 3×1
    0.7500
         0
    1.0000
```

We get the same result if Matlab's pseudoinverse function, pinv() is used, but since there are technically infinite solutions to this sytem, Matlab's A\b will just give a solution that might not be the same as the one gotten using A's pseudoinverse.

However, their norms similar:

```
disp('norm of x from calculated pseudoinverse:')
```

```
norm of x from calculated pseudoinverse:
```

```
disp(norm(x))
```

```
    1.2058
```

```
disp('norm of x from Matlab pinv():')
```

```
norm of x from Matlab pinv():
```

```
disp(norm(x))
```

```
    1.2058
```

```
disp('norm of x from Matlab A\b:')
```

```
norm of x from Matlab A\b:
```

```
disp(norm(x3))
```

```
    1.2500
```

3

3a)

```
A=[-1.3, -0.3; 0.75, -0.52];

[U, sigma, V]=svd(A)
```

```
U = 2×2
   -0.8662    0.4997
    0.4997    0.8662
sigma = 2×2
    1.5008         0
         0    0.6003
V = 2×2
     1     0
     0    -1
```

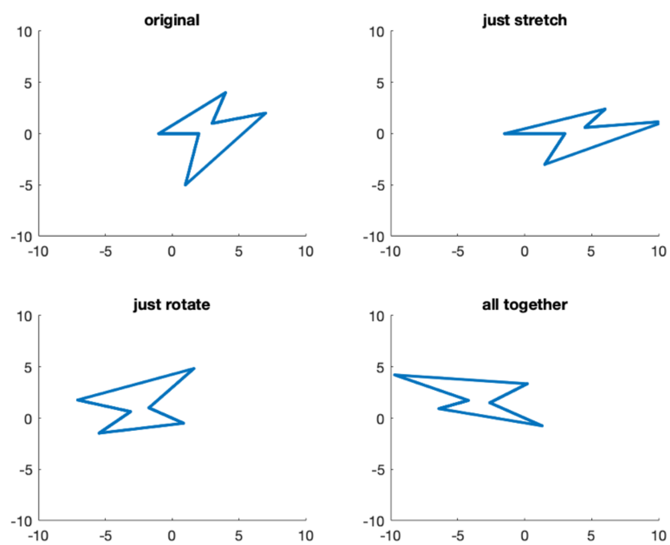
$A = UV^TV\Sigma V^T$

$$A = \begin{bmatrix} -0.8662 & 0.4997 \\ 0.4997 & 0.8662 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1.5008 & 0 \\ 0 & 0.6003 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

3b)

```
Corners=[1, 7, 3, 4,-1, 2;
-5, 2, 1, 4, 0, 0];

% draw lines, made of sets of points, between each two consecutive corners:
R=[0:0.01:1;0:0.01:1];
NR=length(R);
Ncorners=length(Corners(1,:));
start=1;
X=zeros(2, NR);
for j=1:Ncorners
  jm1=j-1;
  % the first line is between the last and first corners:
  if j==1
    jm1=Ncorners;
  end
  X(:,start:start+NR-1)=Corners(:,jm1)+R.*(Corners(:,j)-Corners(:,jm1));
  start=start+length(R(1,:));
end
NX=length(X(1,:));

% rotate and stretch matrices
A_rotate = U*V.';
A_stretch = V*sigma*V.';

figure
subplot(2,2,1)
hold on; xlim([-10 10]); ylim([-10 10]);
title('original')
plot(X(1,:),X(2,:),'.')

subplot(2,2,2)
hold on; xlim([-10 10]); ylim([-10 10]);
title('just stretch')
for i=1:NX
  X_stretch(:,i)=A_stretch*X(:,i);
end
plot(X_stretch(1,:),X_stretch(2,:),'.')
```

```
subplot(2,2,3)
hold on; xlim([-10 10]); ylim([-10 10]);
title('just rotate')
for i=1:NX
  X_rotate(:,i)=A_rotate*X(:,i);
end
plot(X_rotate(1,:),X_rotate(2,:),'.')

subplot(2,2,4)
hold on; xlim([-10 10]); ylim([-10 10]);
title('all together')
for i=1:NX
  X_all(:,i)=A_rotate*A_stretch*X(:,i);
end
plot(X_all(1,:),X_all(2,:),'.')
```



3d)

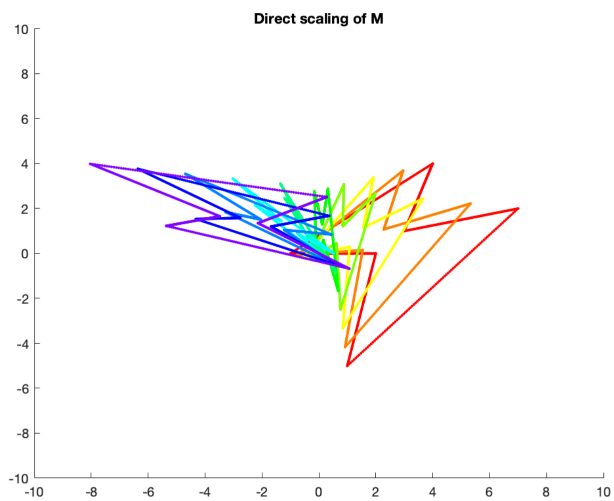
Using direct scaling of M:

```
T=[1, 0, 5;
    0, 1, -0.5;
    0, 0, 1]
```

```
T = 3×3
    1.0000         0    5.0000
         0    1.0000   -0.5000
         0         0    1.0000
```

```
M=T*[A_rotate [0;0]; [0 0 0]]*[A_stretch [0;0]; [0 0 0]];
M=M(1:2,1:2);
```

```
I=eye(2,2);
figure; title('Direct scaling of M')
hold on;xlim([-10 10]); ylim([-10 10]);
color={[1 0 0] [1 0.5 0] [1 1 0] [0.5 1 0] [0 1 0] [0 1 0.5] [0 1 1] [0 0.5 1]
[0 0 1] [0.5 0 1]};
for tn=0:1:9
    t=tn/10;
    for i=1:NX
        X_M1(:, i)=(t.*M + (1-t).*I)*X(:,i);
    end
    plot(X_M1(1,:),X_M1(2,:),'.', 'Color', color{tn+1})

end
```



Using polar decomposition:

```
theta=acos(A_rotate(1,1));
disp(theta)
```

```
    2.6183
```

```
theta_degree=theta*180/pi
```

```
theta_degree = 150.0184
```

```
S=[A_stretch [0;0]; [0 0 0]];

I=eye(3,3);
figure; title('Using polar decomposition')
hold on;xlim([-10 10]); ylim([-10 10]);
```

```
 color={[1 0 0] [1 0.5 0] [1 1 0] [0.5 1 0] [0 1 0] [0 1 0.5] [0 1 1] [0 0.5 1]
[0 0 1] [0.5 0 1]};
 for tn=0:1:9
     t=tn/10;
     for i=1:NX
         R_t_theta=[cos(theta*t), -sin(theta*t), 0;
               sin(theta*t), cos(theta*t), 0;
               0,            0,            0];
         M2=(t.*T + ((1-t).*I))...
             *R_t_theta...
             *(t.*S + ((1-t).*I));
         X_M2(:,i)=M2(1:2, 1:2)*X(:,i);
     end
     plot(X_M2(1,:),X_M2(2,:),'.', 'Color', color{tn+1})

 end
```