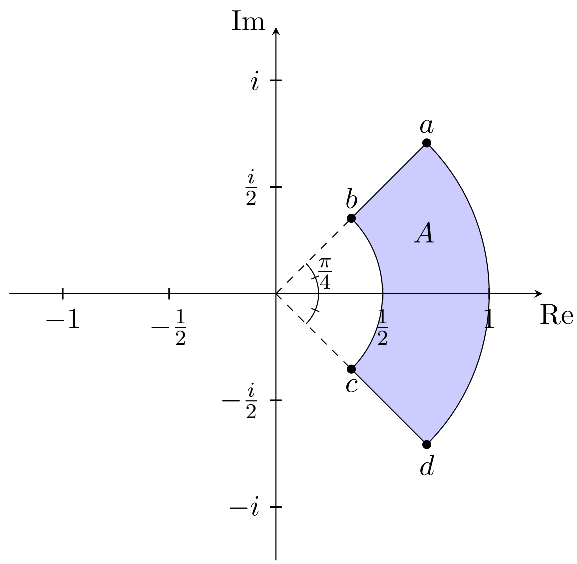
Homework 2 Handed out: Wednesday, September 14, 2022
Due: Wednesday, September 21, 2022 by 11:59pm

Material covered: Outcomes 2.1–2.3.

1. Let A be the annulus sector of points $z = re^{i\theta}$ such that $\frac{1}{2} \leq r \leq 1$ and $-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$ (pictured). For each of the following complex functions $f\colon \mathbb{C} \to \mathbb{C}$, sketch $f(A)$, i.e. the image of A under f. Label the images of the points a, b, c, d (i.e. label $f(a)$, $f(b)$, $f(c)$ $f(d)$).
   a) $f(z) = z^2$
   b) $f(z) = \frac{1}{z}$



2. For the following functions $f\colon \mathbb{C} \to \mathbb{C}$, evaluate the limit

$$\lim_{z \to z_0} f(z)$$

or prove that the limit does not exist. Is f continuous at z_0?

a)

$$f(z) = \begin{cases} \frac{z^5 - z}{z + i} & z \neq -i \\ 0 & z = -i \end{cases}, \qquad z_0 = -i.$$

b)

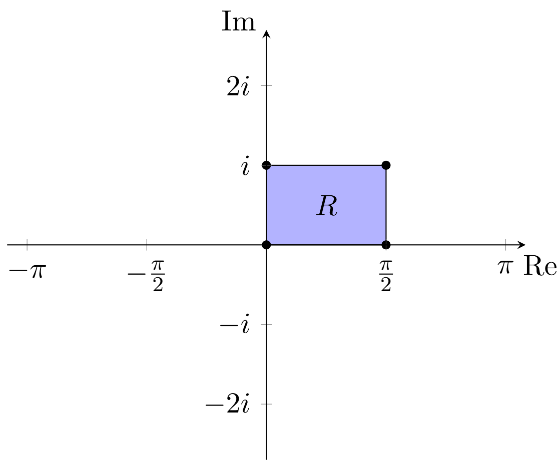
$$f(z) = \begin{cases} \frac{z^2 + \bar{z}^2}{2i|z|} & z \neq 0 \\ 0 & z = 0 \end{cases}, \qquad z_0 = 0.$$

c)

$$f(z) = \begin{cases} \frac{x^2 y}{(x+iy)(x^2+y^2)} & z \neq 0 \\ 0 & z = 0 \end{cases}, \qquad z_0 = 0,$$

where $z = x + iy$, $x, y \in \mathbb{R}$.

3. Sketch the image of the rectangle R shown below under the map $f\colon \mathbb{C} \to \mathbb{C}$, $f(z) = \sin(z)$. Label the images of the corners, i.e. the points $f(0)$, $f(\pi/2)$, $f(\pi/2 + i)$, $f(i)$.



4. Let $z = x + iy$ where $x, y \in \mathbb{R}$. Find the real and imaginary parts of the following expressions in terms of x and y:
   a) $e^{1/z}$
   b) $\cos(z^2)$
5. Find all solutions $z \in \mathbb{C}$ of the following equations:
   a) $e^z = -1$
   b) $(\sin(z))^2 = 4$