Homework #08

1a

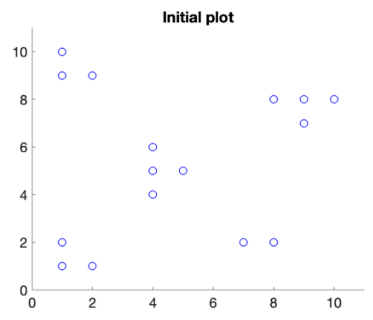
```
X=[ 4, 9, 8, 7, 1, 1, 1, 2, 1, 5, 4, 10, 9, 8, 2, 4;
5, 8, 8, 2, 9, 10, 1, 1, 2, 5, 6, 8, 7, 2, 9, 4];
```

Parameters:

```
% neighborhood kernel:
K_nearest=0.8;
K_other=0.1;

% learning rate:
eta=0.1;
```

Initial plot:

```
set(0,'defaulttextfontsize',18); set(0,'defaultaxesfontsize',18);
figure; clf; hold on; title('Initial plot')
xlim([0 11]); ylim([0 11]);

% plot datapoints
scatter(X(1,:),X(2,:),100,'bo');
hold off
```

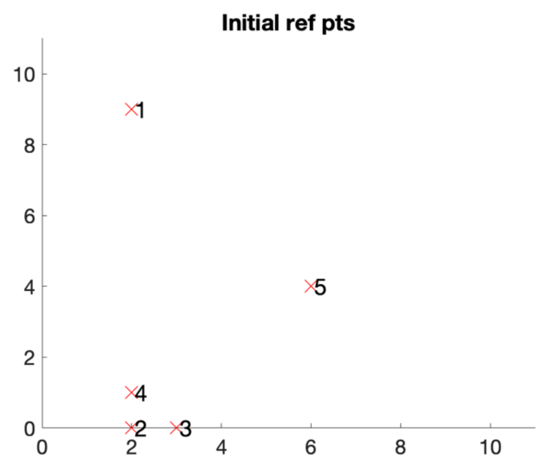


Looks like 5ish clusters from the plot, we can try n=5

Reference pts:

```
N_grid=5;
Xref=X(:, 6:6+N_grid-1);
for j=1:1:length(Xref)
    Xref(:,j)=Xref(:,j)+[1;-1];
end

% plot reference points:
```

```
figure; clf; hold on; title('Initial ref pts')
xlim([0 11]); ylim([0 11]);

href=scatter(Xref(1,:),Xref(2,:),200,'rx');
for ii=1:N_grid
  h_ref_text(ii)=text(Xref(1,ii)+0.07,Xref(2,ii),num2str(ii),'FontSize',20);
end
hold off
```



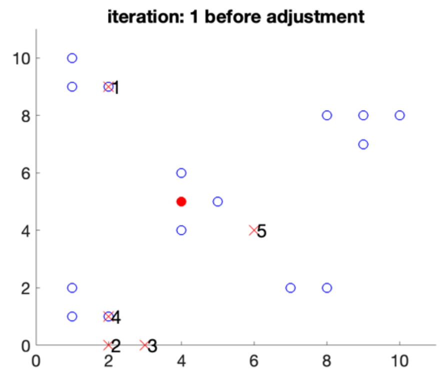
First two iterations/datapoints in detail:

```
iepoch = 1 % TF said just 1st epoch only is ok
```

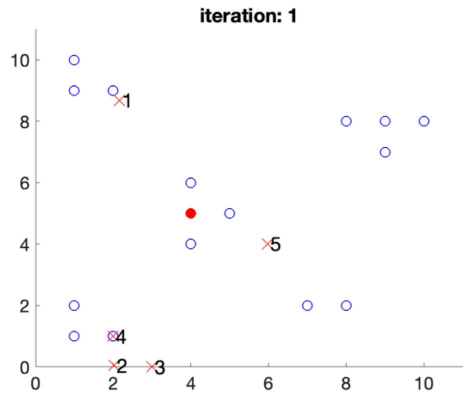
```
 iepoch = 1
```

```

iter=0; % counter of data points being examined
for n=1:1:2
    iter = iter + 1;
    disp(['Iteration: ' num2str(iter)])

    % initialize plot
    set(0,'defaulttextfontsize',18); set(0,'defaultaxesfontsize',18);
    figure; clf; hold on; title(['iteration: ' num2str(iter) ' before
adjustment'])
    xlim([0 11]); ylim([0 11]);

    % plot datapoints
    scatter(X(1,:),X(2,:),100,'bo');

    % draw current data point in red:
    h_current_data=scatter(X(1,n),X(2,n),100,'ro','MarkerFaceColor','r');
    % plot reference points:
    href=scatter(Xref(1,:),Xref(2,:),200,'rx');
    for ii=1:N_grid

h_ref_text(ii)=text(Xref(1,ii)+0.07,Xref(2,ii),num2str(ii),'FontSize',20);
    end
    hold off

    % name current data pt in red
    disp(['current datapoint in red is: ' num2str(n)])

    % find which reference point is nearest to current data point:
    % out of the 4 reference pts
    [Dmin,I]=min([norm(X(:,n)-Xref(:,1)),norm(X(:,n)-Xref(:,2))...
                  ,norm(X(:,n)-Xref(:,3)),norm(X(:,n)-Xref(:,4))]);
    disp(['nearest ref point is: ' num2str(I)])

    % adjust the nearest reference point using the learning rate:
    Xref(:,I)=Xref(:,I)+eta*K_nearest*(X(:,n)-Xref(:,I));
    x0=Xref(:,I); dx=K_nearest*(X(:,n)-Xref(:,I)); dx=0.6*dx/norm(dx);
    disp(['nearest ref point adjusted by:'])
    disp(eta*K_nearest*(X(:,n)-Xref(:,I)))

    % adjust the other reference points using the specified kernel:
    nearest_neighbors_grid_space=[I-1,I+1];
    nearest_neighbors_grid_space(nearest_neighbors_grid_space==0)=N_grid;
    nearest_neighbors_grid_space(nearest_neighbors_grid_space==N_grid+1)=1;
    i1=0;
    for nn=nearest_neighbors_grid_space
      i1=i1+1;
      Xref(:,nn)=Xref(:,nn)+eta*K_other*(X(:,n)-Xref(:,nn));
      x0=Xref(:,nn); dx=K_nearest*(X(:,n)-Xref(:,nn)); dx=0.3*dx/norm(dx);
      disp(['neighbor ref point ' num2str(nn) ' adjusted by:'])
      disp(eta*K_other*(X(:,n)-Xref(:,nn)))
    end

    disp('output adjusted new ref pts:')

    % initialize plot
    set(0,'defaulttextfontsize',18); set(0,'defaultaxesfontsize',18);
    figure; clf; hold on; title(['iteration: ' num2str(iter)])
    xlim([0 11]); ylim([0 11]);

    % plot datapoints
    scatter(X(1,:),X(2,:),100,'bo');
    % draw current data point in red:
    h_current_data=scatter(X(1,n),X(2,n),100,'ro','MarkerFaceColor','r');

    % plot reference points:
    href=scatter(Xref(1,:),Xref(2,:),200,'rx');
    for ii=1:N_grid

h_ref_text(ii)=text(Xref(1,ii)+0.07,Xref(2,ii),num2str(ii),'FontSize',20);
    end
    hold off

    disp('')

 end
```

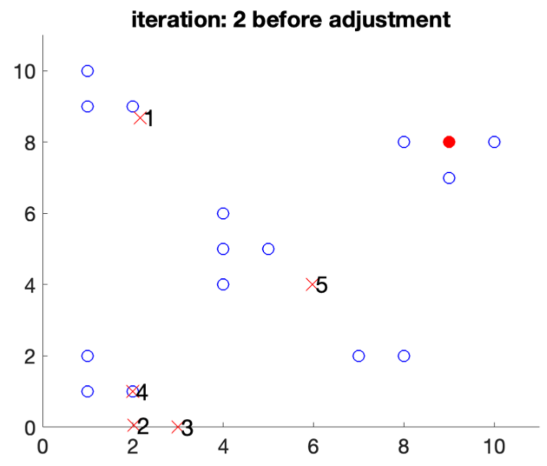
```
Iteration: 1
```



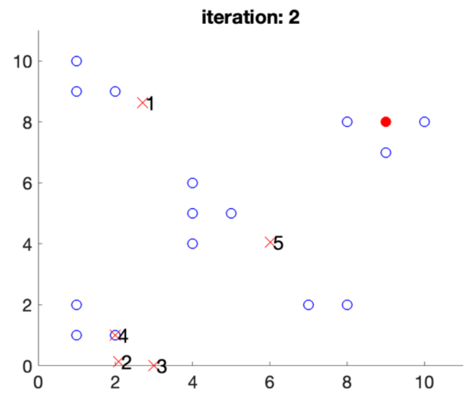
```
current datapoint in red is: 1
nearest ref point is: 1
nearest ref point adjusted by:
    0.1472
   -0.2944
neighbor ref point 5 adjusted by:
   -0.0198
    0.0099
neighbor ref point 2 adjusted by:
    0.0198
    0.0495
output adjusted new ref pts:
```



```
Iteration: 2
```

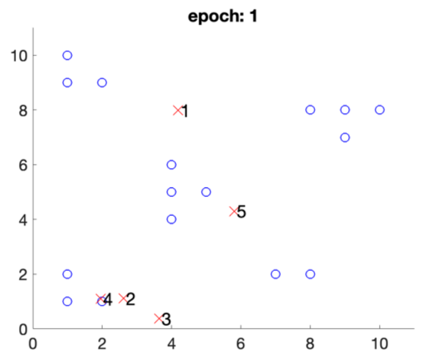


```
current datapoint in red is: 2
nearest ref point is: 1
nearest ref point adjusted by:
    0.5034
   -0.0500
neighbor ref point 5 adjusted by:
    0.0299
    0.0395
neighbor ref point 2 adjusted by:
    0.0691
    0.0787
output adjusted new ref pts:
```



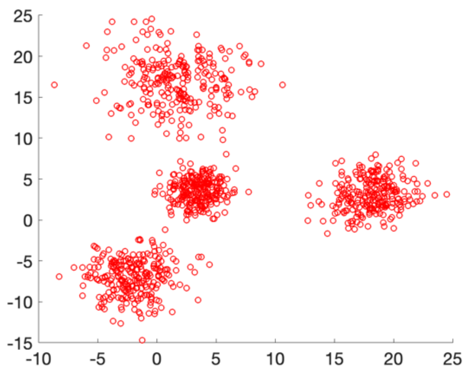
Iterating through all data pts:

```
for iepoch = 1:1:1 % TF said just 1st epoch only is ok
iter=0; % counter of data points being examined
for n=1:length(X(1,:))
    iter = iter + 1;

    % find which reference point is nearest to current data point:
    % out of the 4 reference pts
    [Dmin,I]=min([norm(X(:,n)-Xref(:,1)),norm(X(:,n)-Xref(:,2))...
                    ,norm(X(:,n)-Xref(:,3)),norm(X(:,n)-Xref(:,4)),norm(X(:,n)-
Xref(:,4))]);

     % adjust the nearest reference point using the learning rate:
     Xref(:,I)=Xref(:,I)+eta*K_nearest*(X(:,n)-Xref(:,I));
     x0=Xref(:,I); dx=K_nearest*(X(:,n)-Xref(:,I)); dx=0.6*dx/norm(dx);

     % adjust the other reference points using the specified kernel:
     nearest_neighbors_grid_space=[I-1,I+1];
     nearest_neighbors_grid_space(nearest_neighbors_grid_space==0)=N_grid;
     nearest_neighbors_grid_space(nearest_neighbors_grid_space==N_grid+1)=1;
     i1=0;
     for nn=nearest_neighbors_grid_space
       i1=i1+1;
       Xref(:,nn)=Xref(:,nn)+eta*K_other*(X(:,n)-Xref(:,nn));
       x0=Xref(:,nn); dx=K_nearest*(X(:,n)-Xref(:,nn)); dx=0.3*dx/norm(dx);
     end
 end

     % initialize plot
     set(0,'defaulttextfontsize',18); set(0,'defaultaxesfontsize',18);
     figure; clf; hold on; title(['epoch: ' num2str(iepoch)])
     xlim([0 11]); ylim([0 11]);

     % plot datapoints
     scatter(X(1,:),X(2,:),100,'bo');

     % plot reference points:
     href=scatter(Xref(1,:),Xref(2,:),200,'rx');
     for ii=1:N_grid

h_ref_text(ii)=text(Xref(1,ii)+0.07,Xref(2,ii),num2str(ii),'FontSize',20);
     end
     hold off
 end
```



1b

```
% Matlab
rng(2021); N=250; X=zeros(2,4*N);
for i=1:N; X(:,i)=[5,5]' +(1.5*(randn(2,1)-1)); end;
for i=N+1:2*N; X(:,i)=[0,-5]'+(2.0*(randn(2,1)-1));end;
for i=2*N+1:3*N; X(:,i)=[5,20]'+(3.0*(randn(2,1)-1));end;
for i=3*N+1:4*N; X(:,i)=[20,5]'+(2.0*(randn(2,1)-1));end;
p=randperm(4*N); X=X(:,p);figure(1); clf; scatter(X(1,:),X(2,:),'r')
```



Adapted from "self_organizing_maps_library_function_example.m"

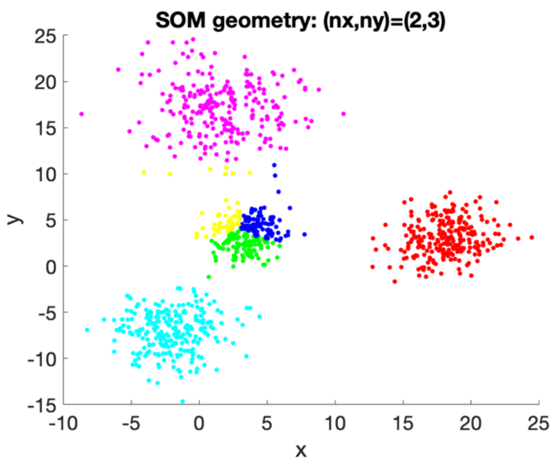
```
nx=2; ny=3;

fprintf(1,' SOM geometry is (nx,ny)=(%d,%d).\n',nx,ny)
```

```
SOM geometry is (nx,ny)=(2,3).
```

```
% specify geometry of SOM:
net = selforgmap([nx ny]);
% train the network to find clusteroids:
net = train(net,X);
% examine network:
view(net);
% classify, Y is a set of vectors (y1,...yn), one per each data point,
% containing cluster assignments for all data points, e.g.,
% assuming four clusters, if y_i=[0,1,0,0], then the
% corresponding i'th data point belongs to cluster 2:
Y = net(X);
% classes is a single vector contain the cluster assignments of all
% data points: if classes(i)=2, point i belongs to cluster 2:
classes = vec2ind(Y);
```

Plot clusters:

```
% plot clusters in different colors:
figure; clf; hold on
color ='rgbcmyrgbcmyrgbcmyrgbcmy';
marker='......xxxxxx++++++oooooo';
Nmaps=nx*ny;
for i=1:1:Nmaps
    plot(X(1,classes==i),X(2,classes==i),[color(i) marker(i)],'MarkerSize',12)
end
xlabel('x');
ylabel('y');
title(sprintf('SOM geometry: (nx,ny)=(%d,%d)',nx,ny));
hold off
```

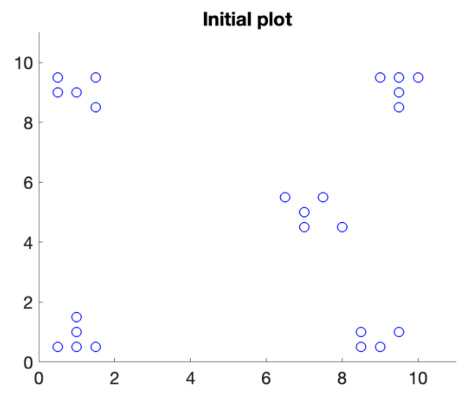


2

```
X=[0.5,1.5,0.5,1,9,7.5,7,9.5,1.5,1,1,8.5,9.5,6.5,8,0.5,1,9,9.5,7,10,1.5,8.5,9.5
;
 9.5,8.5,0.5,1,0.5,5.5,4.5,9.5,9.5,0.5,1.5,0.5,1,5.5,4.5,9,9,9.5,9,5,9.5,0.5,1,8
.5];
```

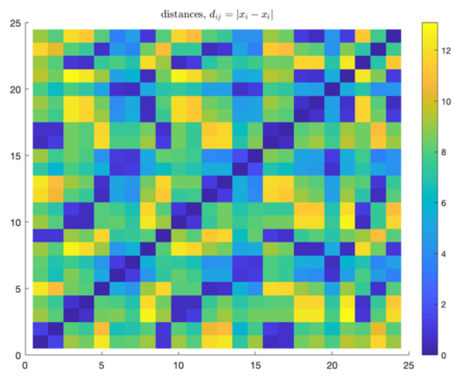
Initial plot:

```
set(0,'defaulttextfontsize',18); set(0,'defaultaxesfontsize',18);
figure; clf; hold on; title('Initial plot')
xlim([0 11]); ylim([0 11]);
color ='rgbcmyrgbcmyrgbcmyrgbcmy';
marker='......xxxxxx++++++oooooo';

% plot datapoints
scatter(X(1,:),X(2,:),100,'bo');
hold off
```

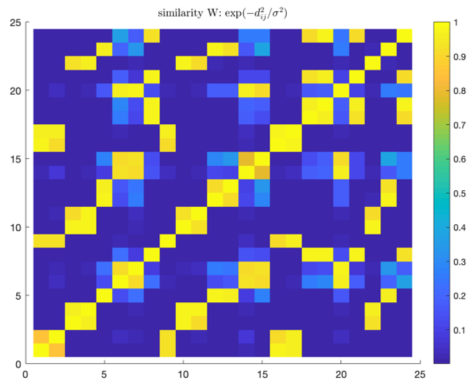


Adapted from "spectral_clustering_example.m"

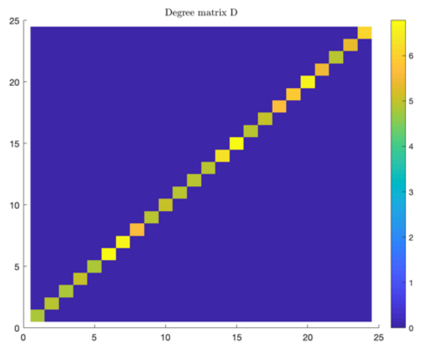
```
 % Calculate distances between all pairs of points:
 Y = pdist(X.','euclid'); % or could use, for example, Y =
pdist(X,'minkowski',1);
 distances=squareform(Y);
 figure; hold on
 imagesc(distances); colorbar
 title('distances, $d_{ij}=|x_i-x_i|$','Interpreter','LaTeX')
 hold off
```



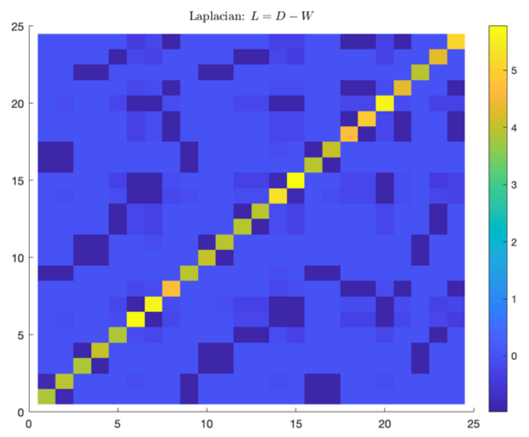
```
 % build similarity matrix from distances (Luxburg 2007):
 dists_std=std(distances(:)); % std of distances
 W=exp(-distances.^2/dists_std^2);
 figure; hold on
 imagesc(W); colorbar
 title('similarity W: $\exp(-d_{ij}^2/\sigma^2)$','Interpreter','LaTeX')
 hold off
```



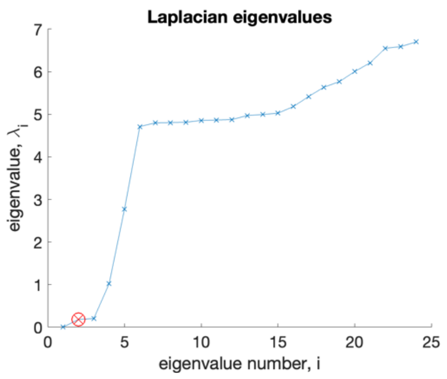
```
% degree matrix is some over rows of adjacency matrix:
D=diag(sum(W));
figure; hold on
imagesc(D); colorbar
title('Degree matrix D','Interpreter','LaTeX')
hold off
```



```
% Laplacian matrix:
L=D-W;
figure; hold on
imagesc(L); colorbar
title('Laplacian: $L=D-W$','Interpreter','LaTeX')
hold off
```



```
% calculate eivenvectors of Laplacian matrix:
[V,D] = eig(L);

figure; hold on
d=diag(D);
plot(d,'-x');
% highlight second smallest eigenvalue:
plot(2,d(2),'ro','markersize',15);
plot(2,d(2),'rx','markersize',15);
xlabel('eigenvalue number, i')
ylabel('eigenvalue, \lambda_i')
title('Laplacian eigenvalues');
hold off
```



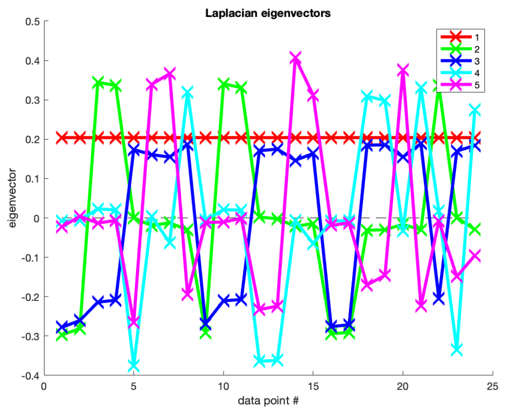
There seems to be a spectral gap around k=5, so we can try that

```
k=5
```

```
k = 5
```

```
figure; hold on
```

```
plot(0*V(:,1),'--k','linewidth',0.5);
for ii=1:k
  hl(ii)=plot(V(:,ii),[color(ii) 'x-']);
  if ii>=2; set(hl,'linewidth',3,'markersize',15); end
  legend_text(ii)=sprintf('%d',ii);
end
title(sprintf('Laplacian eigenvectors',k));
xlabel('data point #')
ylabel('eigenvector')
legend([hl(:)],legend_text(:));
hold off
```



It turns out that $xLx^T = \frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{n} w_{ij}(x_i - x_j)^2$ or the sum of corresponding W elements times the distances of points. So solving to minimize this expression of distances between points, we eventually get that $Lx = \lambda x$ when it's minimized. So the minimum is with x eigenvectors of minimal lamdas. So to make k clusters, we can use the kth minimal lambda's eigenvector, ignoring the first one with eigenvector of just ones.

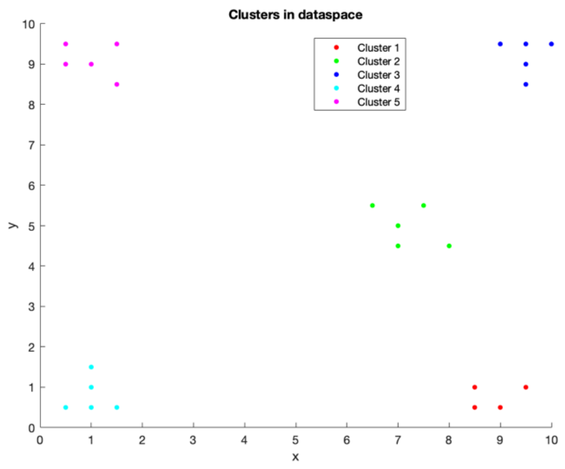
k-means Clustering:

```
opts = statset('Display','final');
[idx,C] = kmeans(V(:,2:k),k,'Distance','sqeuclidean' ...
               ,'Replicates',5,'Options',opts);
```

```
Replicate 1, 1 iterations, total sum of distances = 0.0302072.
Replicate 2, 2 iterations, total sum of distances = 0.0302072.
Replicate 3, 1 iterations, total sum of distances = 0.0302072.
Replicate 4, 1 iterations, total sum of distances = 0.0302072.
Replicate 5, 1 iterations, total sum of distances = 0.0302072.
Best total sum of distances = 0.0302072
```

Plot clusters in dataspace:

```
figure;hold on
for i=1:1:k
  plot(X(1,idx==i),X(2,idx==i),[color(i) marker(i)],'MarkerSize',12)
end
% write legend:
for i=1:1:k
  if i<10
    legends(i,:)=['Cluster ' num2str(i)];
  else
    legends(i,:)=['Cluster' num2str(i)];
  end
end
legend(legends,'Location','best');
title(sprintf('Clusters in dataspace'));
xlabel('x');
ylabel('y');
hold off
```



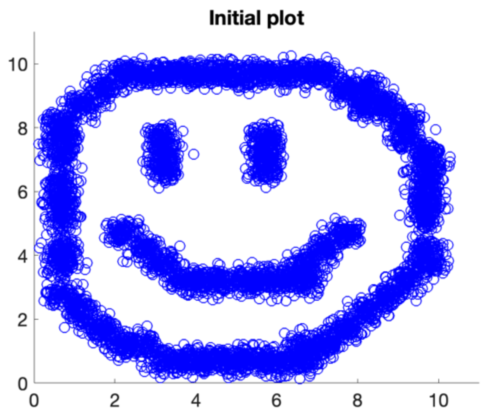
3a

```
clear all
load 'HW08_CURE.mat'
```

Initial plot:

```
set(0,'defaulttextfontsize',18); set(0,'defaultaxesfontsize',18);
figure; clf; hold on; title('Initial plot')
xlim([0 11]); ylim([0 11]);
color ='rgbcmyrgbcmyrgbcmyrgbcmy';
marker='......xxxxxx++++++oooooo';
```

```
% plot datapoints
scatter(X(1,:),X(2,:),100,'bo');
hold off
```



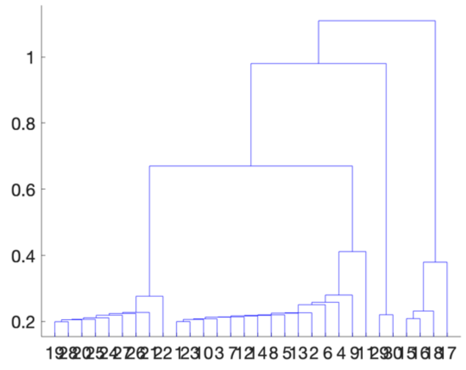
Cute :)

Single:

```
Y=pdist(X');
distances=squareform(Y)
```

```
distances = 7200×7200
        0    0.2556    0.2975    0.5334    0.4553    0.2999    0.5421 ⋯
   0.2556         0    0.2673    0.5753    0.5156    0.2619    0.6627
   0.2975    0.2673         0    0.3081    0.2528    0.0088    0.4161
   0.5334    0.5753    0.3081         0    0.0807    0.3141    0.1938
   0.4553    0.5156    0.2528    0.0807         0    0.2600    0.1811
   0.2999    0.2619    0.0088    0.3141    0.2600         0    0.4243
   0.5421    0.6627    0.4161    0.1938    0.1811    0.4243         0
   0.5863    0.6366    0.3697    0.0617    0.1313    0.3757    0.1768
   0.2061    0.3657    0.4973    0.7371    0.6579    0.4984    0.7266
   0.7689    0.6407    0.4740    0.4494    0.4900    0.4704    0.6427
     ⋮
```

```
Z=linkage(Y,'single');
dendrogram(Z);
```

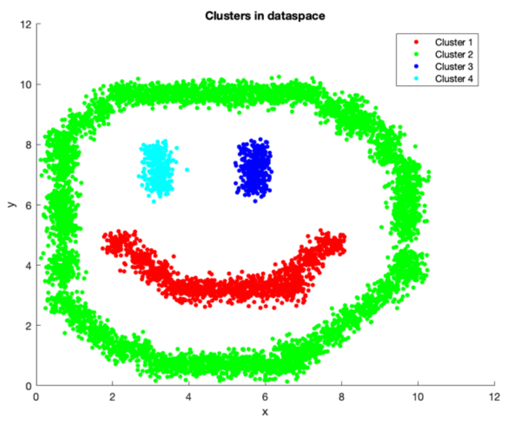


Seems like a pretty big jump @4 clusters

```
k=4
```

```
k = 4
```

```
idx=cluster(Z,'maxclust',k)';
figure;hold on
for i=1:1:k
  plot(X(1,idx==i),X(2,idx==i),[color(i) marker(i)],'MarkerSize',12)
end
% write legend:
for i=1:1:k
  if i<10
    legends(i,:)=['Cluster ' num2str(i)];
  else
    legends(i,:)=['Cluster' num2str(i)];
  end
end
legend(legends,'Location','best');
title(sprintf('Clusters in dataspace'));
xlabel('x');
ylabel('y');
hold off
```

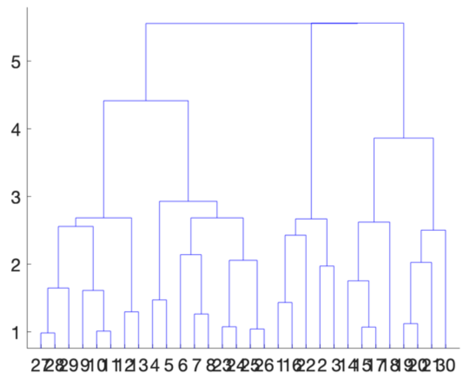


Centroid:

```
Z=linkage(Y,'centroid');
```

```
Warning: Non-monotonic cluster tree -- the centroid linkage is probably not
appropriate.
```

```
dendrogram(Z);
```



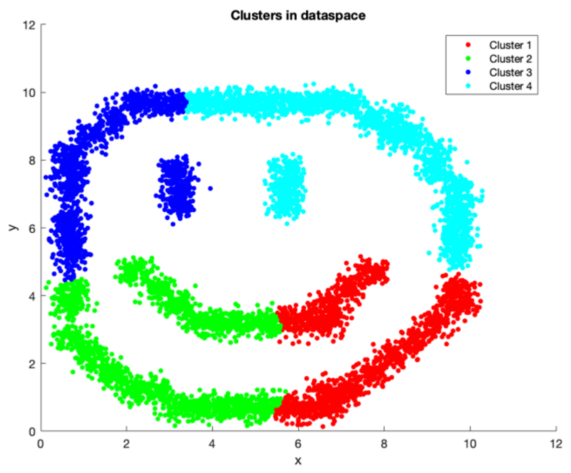
Jump could be @k=4 still kind of

```
k=4
```

```
k = 4
```

```
idx=cluster(Z,'maxclust',k)';
figure;hold on
for i=1:1:k
  plot(X(1,idx==i),X(2,idx==i),[color(i) marker(i)],'MarkerSize',12)
end
% write legend:
for i=1:1:k
  if i<10
```

```
    legends(i,:)=['Cluster ' num2str(i)];
  else
    legends(i,:)=['Cluster' num2str(i)];
  end
end
legend(legends,'Location','best');
title(sprintf('Clusters in dataspace'));
xlabel('x');
ylabel('y');
hold off
```



So centroid isn't going to be great here since clusters within clusters will have similar centroids that are not really differentiable.

But single is better for clusters within clusters as long as they don't have any points or parts touching/too close to each other. Single linkage is by the lowest distance between two points from the two clusters, which is significantly different between these smiley face clusters, and is not affected by centroids of clusters within clusters.

3b

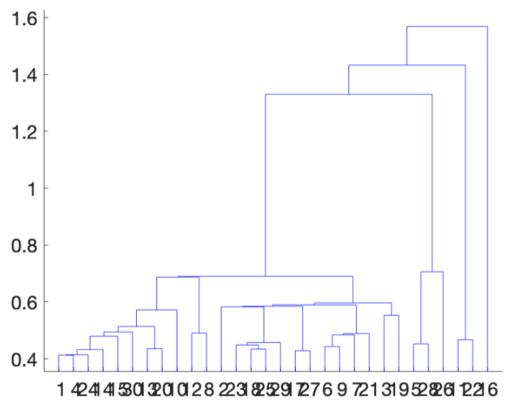
```
clear all
load 'HW08_CURE.mat'
```

Random initial representative points:

```
[M,N]=size(X);
n_reps=300;
rand_cols = randperm(N,n_reps);
rand_reps = X(:,rand_cols);
```

Hierarchical clustering of rand_reps:

```
Y=pdist(rand_reps','euclid');
Z=linkage(Y,'single');
dendrogram(Z);
```



Jump around k=4

```
k=4;
idx_rand_reps = cluster(Z,'maxclust',k);
```

Everyone go find your rand_rep point that's closest:

```
all_dists=zeros([1 n_reps]);
for i=1:N
  for j=1:n_reps
    all_dists(1,j)=norm(X(:,i)-rand_reps(:,j));
  end
  [M,nearest_rand_rep] = min(all_dists);
  idx(i)=idx_rand_reps(nearest_rand_rep);
end
```

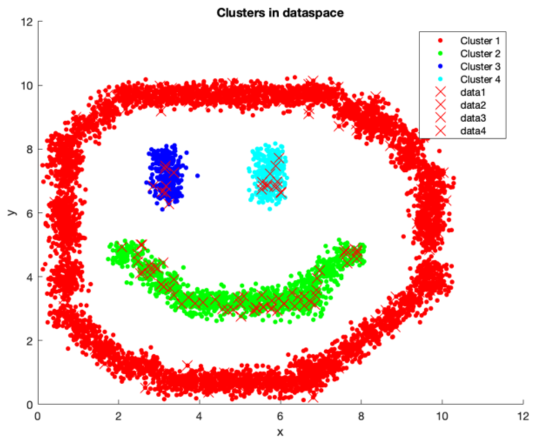
Plot:

```
figure;hold on
color ='rgbcmyrgbcmyrgbcmyrgbcmy';
marker='......xxxxxx++++++oooooo';
for i=1:1:k
  plot(X(1,idx==i),X(2,idx==i),[color(i) marker(i)],'MarkerSize',12)
end
% write legend:
for i=1:1:k
  if i<10
    legends(i,:)=['Cluster ' num2str(i)];
  else
```

```
    legends(i,:)=['Cluster' num2str(i)];
  end
end
legend(legends,'Location','best');
title(sprintf('Clusters in dataspace'));
xlabel('x');
ylabel('y');
```

Plot representatives:

```
for i=1:1:k

plot(rand_reps(1,idx_rand_reps==i),rand_reps(2,idx_rand_reps==i),'rx','MarkerSiz
e',12)
end
hold off
```


Clusters in dataspace
Cluster 1
Cluster 2
Cluster 3
Cluster 4
data1
data2
data3
data4
x
y