Homework 9 Handed out: Wednesday, November 9, 2022
Due: Wednesday, November 16, 2022 by 11:59pm

Material covered: Outcomes 8.1–8.5.

1. Recall the Fourier transform pair derived in class:

$$\mathscr{F}\left[e^{-a|x|}\right] = \frac{2a}{k^2 + a^2}$$

   a) Using Fourier transform properties, calculate the Fourier transform:

$$\mathscr{F}\left[\frac{1}{x^2 - 2x + 10}\right]$$

   b) Using your solution to (a) along with Fourier transform properties, calculate the Fourier transform:

$$\mathscr{F}\left[\frac{xe^{3ix} - e^{3ix}}{(x^2 - 2x + 10)^2}\right]$$

2. In this problem, we will use an alternative approach to solve 1(a):

$$\mathscr{F}\left[\frac{1}{x^2 - 2x + 10}\right]$$

   Rewrite the Fourier transform expression as a complex contour integral for an appropriate half-circle, then solve the integral using the residue theorem.

3. The Plancherel Theorem states that for functions $f(x), g(x)$ with Fourier transforms $\hat{f}(k)$, $\hat{g}(k)$,

$$\langle \hat{f}(k), \hat{g}(k) \rangle = 2\pi \langle f(x), g(x) \rangle$$

   where $\langle \dots \rangle$ indicates the functional inner product,

$$\langle f, g \rangle = \int_{-\infty}^{\infty} f(x)\overline{g(x)}dx$$

Prove that

$$\int_{-\infty}^{\infty} \frac{\sin(at)\sin(bt)}{t^2} dt = \pi \min(a, b)$$

for real-valued $a, b > 0$.

4. Let a be a positive integer, and consider the function $f(x) = e^{-x}x^a$ for $x > 0$, $f(x) = 0$ for $x \leq 0$. Find $\mathscr{F}[f(x)]$.

5. *Spectrograms.* Open the Python notebook here and follow the instructions.