

# Problem: Cubes of prime numbers pattern

| Wonderful The mysterious world of prime numbers |               |   |
|---|---------------|---|
| Primes Cubed                                    | Digital Roots |   |
| $2^3 = 8$                                       | →             | $8 = 8$                                   |
| $3^3 = 27$                                      | →             | $2 + 7 = 9$                               |
| $5^3 = 125$                                     | →             | $1+2+5=8$                                 |
| $7^3 = 343$                                     | →             | $3+4+3=10 \mid 1+0=1$                     |
| $11^3 = 1331$                                   | →             | $1+3+3+1=8$                               |
| $13^3 = 2197$                                   | →             | $2+1+9+7=19 \mid 1+9=10 \mid 1+0=1$       |
| $17^3 = 4913$                                   | →             | $4+9+1+3=17 \mid 1+7=8$                   |
| $19^3 = 6859$                                   | →             | $6+8+5+9=28 \mid 2+8=10 \mid 1+0=1$       |
| $23^3 = 12167$                                  | →             | $1+2+1+6+7=17 \mid 1+7=8$                 |
| $29^3 = 24389$                                  | →             | $2+4+3+8+9=26 \mid 2+6=8$                 |
| $31^3 = 29791$                                  | →             | $2+9+7+9+1=28 \mid 2+8=10 \mid 1+0=1$     |
| $37^3 = 50653$                                  | →             | $5+0+6+5+3=19 \mid 1+9=10 \mid 1+0=1$     |
| $41^3 = 68921$                                  | →             | $6+8+9+2+1=26 \mid 2+6=8$                 |
| $43^3 = 79507$                                  | →             | $7+9+5+0+7=28 \mid 2+8=10 \mid 1+0=1$     |
| $47^3 = 103823$                                 | →             | $1+0+3+8+2+3=17 \mid 1+7=8$               |
| $53^3 = 148877$                                 | →             | $1+4+8+8+7+7=35 \mid 3+5=8$               |
| $59^3 = 205379$                                 | →             | $2+0+5+3+7+9=26 \mid 2+6=8$               |
| $61^3 = 226981$                                 | →             | $2+2+6+9+8+1=28 \mid 2+8=10 \mid 1+0=1$   |
| $67^3 = 300763$                                 | →             | $3+0+0+7+6+3=19 \mid 1+9=10 \mid 1+0=1$   |
| $71^3 = 357911$                                 | →             | $3+5+7+9+1+1=26 \mid 2+6=8$               |
| $73^3 = 389017$                                 | →             | $3+8+9+0+1+7=28 \mid 2+8=10 \mid 1+0=1$   |
| $79^3 = 493039$                                 | →             | $4+9+3+0+3+9=28 \mid 2+8=10 \mid 1+0=1$   |
| $83^3 = 571787$                                 | →             | $5+7+1+7+8+7=35 \mid 3+5=8$               |
| $89^3 = 704969$                                 | →             | $7+0+4+9+6+9=35 \mid 3+5=8$               |
| $97^3 = 912673$                                 | →             | $9+1+2+6+7+3=28 \mid 2+8=10 \mid 1+0=1$   |
| $101^3 = 1030301$                               | →             | $1+0+3+0+3+0+1=8$                         |
| $103^3 = 1092727$                               | →             | $1+0+9+2+7+2+7=28 \mid 2+8=10 \mid 1+0=1$ |
| $107^3 = 1225043$                               | →             | $1+2+2+5+0+4+3=17 \mid 1+7=8$             |
| $109^3 = 1295029$                               | →             | $1+2+9+5+0+2+9=28 \mid 2+8=10 \mid 1+0=1$ |
| $113^3 = 1442897$                               | →             | $1+4+4+2+8+9+7=35 \mid 3+5=8$             |
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The above pattern was shared online.

It shows that for every prime number from 5 onwards, that the cube of the number, will have its digits sum to either 1 or 8.

Problem: For how many prime numbers does the pattern hold true? Write a computer program to calculate and find out.

```

def is_prime(n):
    # check all numbers from 2 upto (square root of n) + 1
    for num in range(2, int(n ** 0.5) + 1):
        # if the number is divisible with no remainder, it is not prime
        if n % num == 0:
            return False
    # it must be a prime
    return True

def sum_digits(n):
    total = 0
    while n > 9:
        total = total + (n % 10)
        n = n // 10
    total = total + n
    # if we summed the digits to a new total >9, then sum the digits of the new number
    if total > 9:
        total = sum_digits(total)
    return total

n = 5          # start with the number 5
summation = 8  # the digits of 5^3 sum to 8
prime_count = 3 # 5 is the 3rd prime

# stop if the summation is ever something other than 1 or 8
while summation == 1 or summation == 8:
    if is_prime(n):
        cubed = n*n*n
        summation = sum_digits(cubed)
        # print an update every 1000 numbers
        if prime_count % 1000 == 0:
            print(f"The {prime_count}th prime is {n}. Cubed is {cubed}. Digit summation is {summation}")
        prime_count += 1
    n += 1

```