Theory

When a fluid flows through a pipe, the layer of liquid in direct contact with the pipe wall experiences friction, which slows it down[1]. In the case of laminar flow, the fluid moves in parallel layers, with each layer exerting a drag force on its neighboring layers, resulting in a velocity gradient across the pipe. The viscous force opposing the flow can be described by the formula:

$$F = \eta \frac{dx}{dv} S$$

where η is the coefficient of viscosity, S is the area of the layer, and $\frac{dv}{dx}$ is the velocity gradient. If both the area and velocity gradient are taken as unity, the force is directly proportional to the viscosity coefficient.

The velocity of the fluid is highest at the center of the pipe and lowest near the walls due to frictional effects. The rate of flow depends not only on the viscosity but also on the pipe's geometry and the pressure difference between its ends. Poiseuille's law relates the volume V of fluid flowing through a horizontal tube in time t to the tube's radius r, length l, fluid viscosity η, and pressure difference p:

$$V = \frac{\pi p r^4 t}{8 l \eta}$$

From this, the viscosity coefficient can be determined as:

$$\eta = \frac{\pi r^4 t p}{8 l V}$$

The pressure difference across the pipe is calculated using:

$$p = \rho g h$$

where ρ is the fluid density, g is the acceleration due to gravity, and h is the average height difference of the water column.

Experimental procedure

The capillary tube was arranged horizontally and connected to the water bottle. Distilled water was added to bottle A. Care was taken to ensure there were no air bubbles in the rubber tube B, as the presence of air could affect the measurements.

At the start of the experiment, the initial height h_1 of the water column was measured. The clamp on tube B was opened, allowing water to flow from bottle A to basin D for a measured time period t. The final height h_2 of the water column was then recorded after closing the clamp.

1 Tallinn University of Technology, Division of Physics (2021): POISEUILLE' METHOD

The volume of water that flowed out was measured using a beaker. The temperature of the water was set to be the temperature of the laboratory's room — $23^{\circ}C$. Using the measured values for flow time, water volume, column heights, and temperature, the viscosity coefficient and its combined uncertainty were calculated according to the theoretical relationships.

Measurements and Formula Application

Determination of the coefficient of viscosity

Measured quantity	**The number and unit of measure**	**Absolute error**
h_1	$120cm$	$\pm 0.05cm$
h_2	$117.5cm$	$\pm 0.05cm$
$\frac{(h_1+h_2)}{2}$	$118.75cm$	$\pm 0.35\ cm$
The length l of the capillary	809	$\pm 1mm$
The volume V of out flowed water	45	$\pm 2ml$
The radius r of the capillary	0.45	$\pm 0.005mm$
The duration t of flow	$305s$	$\pm 1s$
The temperature of the water	$23^{\circ}C$	
η	$\approx 0.00157 Pa \cdot s$	

Preparing measurements in the SI form

$$h = \frac{(h_1 + h_2)}{2} = 118.75cm = 1.875m$$

$$r = 0.45 \pm 0.5mm = 0.00045 \pm 0.00005m$$

$$l = 809 \pm 1mm = 0.809 \pm 0.001m$$

$$t = 305s$$

$$V = 45 \pm 2ml = 4.5 \times 10^{-5} m^3 = 0.000045 m^3$$

Calculating pressure

$$p = \rho g h = 1000 \times 9.8 \times 1.1875 = 11637.5 Pa$$

Calculating viscosity

$$\eta = \frac{\pi \times (0.00045)^4 \times 305 \times 11637.5}{8 \times 0.809 \times 0.000045}$$

$$\eta \approx 0.00157 Pa \cdot s = 1.57 mPa \cdot s$$

Uncertainties

The combined relative uncertainty for η is calculated as:

$$\frac{U_c}{\eta} = 4\frac{U_r}{r} + \frac{U_t}{t} + \frac{U_h}{h} + \frac{U_l}{l} + \frac{U_V}{V}$$

And thus the combined relative uncertainty is:

$$\frac{U_c}{\eta} = 4 \times 0.111 + 0.00328 + 0.00295 + 0.00124 + 0.0444 \approx 0.496$$

The absolute uncertainty is calculated as:

$$U_\eta = \eta \frac{U_\eta}{\eta}$$

Which is:

$$U_\eta = 0.00157 \times 0.496 \approx 0.00078 Pa \cdot s$$

Questions

1. **Define the coefficient of viscosity.**
   The coefficient of viscosity (usually denoted as η) is a physical quantity that characterizes the internal friction in a fluid. It is proportional to the force acting between two parallel layers of fluid with unit area when there is a unit velocity gradient between them. It quantifies the resistance of the fluid to shear or flow.
2. **Define the units of viscosity in SI-and CGS system.**
   In the SI system, viscosity is measured in pascal-seconds(Pa ·s). In the CGS system, the unit of viscosity is the Poise (P), where $1P = 0.1 Pa \cdot s$
3. **What does the velocity gradient $10\frac{1}{s}$ mean?**
   The velocity gradient $\frac{dv}{dx}$ represents the rate of change of fluid velocity with respect to the distance between fluid layers. A value of 10^{-1}means that the velocity changes by 10 meters per second for every meter of separation between the layers, reflecting the shear rate in the fluid.

4. **What kind of directions acquires the coefficient of viscosity which has an influence on conceivable plane layer of liquid in the flowing fluid?**
The coefficient of viscosity relates to the shear forces acting tangentially to the plane layers of the fluid. These forces oppose the relative motion between adjacent layers and act parallel to the surfaces of the layers.

5. **How does the coefficient of viscosity of liquid and gas depend on temperature? Why?**
For liquids, the coefficient of viscosity decreases with increasing temperature because higher temperature reduces intermolecular forces, facilitating easier flow. In gases, viscosity increases with temperature as the increased molecular speed enhances momentum transfer between layers.

6. **Why is the capillary tube used in the given method?**
The capillary tube provides a narrow and uniform channel for fluid flow, ensuring laminar flow conditions. It allows precise measurement of flow volume and pressure drop, which are essential for calculating the fluid's viscosity using Poiseuille's equation.

7. **Find the pressure in the ocean 1 km below the water surface.**
Using the formula $p = \rho g h$ where $\rho = 1000 kg/m^3$, $g = 9.8 m/s^2$, and $h = 1000 m$, the pressure is calculated as:

$$p = 1000 \times 9.8 \times 1000 = 9.8 \times 10^6 Pa$$

8. **Why can't there be any air bubbles in the fluid during measurements?**
Air bubbles disrupt the continuous flow of liquid and create irregularities in the pressure and volume measurements, leading to inaccurate determination of viscosity.

9. **Why must the size of the water bottle A and the tube B be large?**
A large water bottle and tube volume help maintain a nearly constant water level and pressure difference during the experiment, ensuring steady flow conditions necessary for accurate viscosity measurement.

10. **Why is the pressure of atmosphere not taken into account?**
Atmospheric pressure acts equally on both the inlet and outlet of the capillary tube, thus it cancels out and does not affect the pressure difference driving the flow.

11. **Why must the water level stay in the water bottle A?**
Maintaining the water level in the bottle ensures continuous supply of fluid, preventing air entry and guaranteeing a stable flow during the measurement.

12. **If the capillary tube is extended, why does water stop flowing after a certain length?**
When the tube length increases, the viscous resistance grows until it balances the pressure difference, halting the flow as the driving force is insufficient to overcome friction.

13. **Why is it necessary in this experiment to take a large diminishing of water level in water bottle A?**
A significant drop in the water level produces a measurable pressure difference that drives the flow through the capillary, allowing accurate determination of viscosity.