Sheet 09

Ex 1

a)

$$a_k^{(l+1)}(tx) = \max\left(0, \textstyle\sum_j a_j^{(l)}(tx) \cdot w_{jk}^{(l)}\right)$$

$$= \max\left(0, \textstyle\sum_j t\, a_j^{(l)}(x) \cdot w_{jk}^{(l)}\right)$$

$$= t \cdot \max\left(0, \textstyle\sum_j a_j^{(l)}(x)\, w_{jk}^{(l)}\right)$$

$$= t \cdot a_k^{(l+1)}(x) \quad \square$$

Ex 2

$$\tilde{x} = \mu + t(x-\mu)$$

$$\tilde{x} - x = (t-1)(x-\mu)$$

$$R_i = [\nabla f(\tilde{x})]_i \cdot (x_i - \tilde{x}_i)$$

$$= \frac{\tilde{x}_i - \mu_i}{\|\tilde{x}-\mu\|} \cdot (x_i - \tilde{x}_i)$$

$$= \frac{t(x_i-\mu_i)}{\|t(x-\mu)\|} \cdot (t-1)(x_i-\mu_i)$$

$$= \frac{(x_i-\mu_i)^2}{\|x-\mu\|} \cdot (t-1) \quad (1)$$

Using constraint $\sum_i R_i = f(x) = \|x-\mu\| - 1$

$$\sum_i \frac{(x_i-\mu_i)^2}{\|x-\mu\|} \cdot (t-1) = \|x-\mu\| - 1$$

$$\Leftrightarrow \frac{\|x-\mu\|^2}{\|x-\mu\|} \cdot (t-1) = \|x-\mu\| - 1$$

$$\Leftrightarrow (t-1) = \frac{\|x-\mu\| - 1}{\|x-\mu\|}$$

with (1)

$$\frac{(x_i-\mu_i)^2}{\|x-\mu\|} \cdot \frac{\|x-\mu\| - 1}{\|x-\mu\|} = \frac{(x_i-\mu_i)^2}{\|x-\mu\|^2} \left(\|x-\mu\| - 1\right) = R_i$$

Ex 3

(a) $y = \min(a_1, a_2)$ $\quad R_3 = \frac{a_3 1}{a_3 1} y = y = \min(a_1, a_2)$

$R_4 = 0$

$R_1 = \frac{a_1 1}{a_1 1} R_3 + \frac{a_1 1}{a_1 1} R_4 = y = \min(a_1, a_2)$

$R_2 = 0$

(b) $y = \min(a_1, a_2)$ $\quad R_3 = 0$

$R_4 = \frac{a_4 1}{a_4 1} y = y$

$R_1 = 0$

$R_2 = \frac{a_2 1}{a_2 1} R_3 + \frac{a_2 1}{a_2 1} R_4 = R_4 = y$

(c) $y = \min(a_1, a_2)$ $\quad R_3 = \frac{a_3 0{,}5}{a_3 0{,}5} y = y$

$R_4 = 0 \quad R_5 = 0$

$R_1 = \frac{a_1 1}{a_1 1 + a_2 1} R_3 + 0 + 0 = \frac{a_1}{a_1 + a_2} y$

$R_2 = \frac{a_2 1}{a_1 + a_2} R_3 + 0 + 0 = \frac{a_2}{a_1 + a_2} y$