ML2 Sheet 5 Theory

1 (a)

$$\sum_{ij} c_i c_j \, k(x_i, x_j) \, I(y_i = y_j)$$

$$= \sum_{ij} c_i c_j \sum_l \phi_l(x_i) \phi_l(x_j) \sum_c^C I(y_i = c) \cdot I(y_j = c)$$

$$= \sum_{lc} \sum_i c_i \phi_l(x_i) I(y_i = c) \cdot \sum_j c_j \phi_l(x_j) I(y_j = c)$$

Since i and j run over the ~~t~~ same set we can write

$$= \sum_{lc} \underbrace{\left(\sum_i c_i \phi_l(x_i) I(y_i = c) \right)^2}_{\geq 0} \geq 0$$

b)

$$\Phi_{struct}(x, y) = \begin{pmatrix} \Phi(x) \cdot I(y = 0) \\ \vdots \\ \Phi(x) \cdot I(y = C) \end{pmatrix}$$

2(a)

$$\min_{w} \frac{1}{2}\|w\|^2 + C\sum_{n}^{N} \varepsilon_n$$

$$\max_{\alpha\beta} \min_{w\varepsilon} \mathcal{L}(w, \varepsilon, \alpha, \beta)$$

$$\mathcal{L} = \frac{1}{2}\|w\|^2 + C\sum_n \varepsilon_n + \sum_{ny} \alpha_{ny}(1 - \varepsilon_n - w^T \psi_{ny})$$
$$- \sum_n \beta_n \varepsilon_n$$

$$\frac{\partial \mathcal{L}}{\partial w} \overset{!}{=} 0 \Rightarrow w = \sum_{ny} \alpha_{ny} \psi_{ny} \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial \varepsilon_n} \overset{!}{=} 0 \Rightarrow C - \sum_{y} \alpha_{ny} - \beta_n = 0 \quad (2)$$

(1) in $\mathcal{L}$

$$\frac{1}{2}\sum_{nn'yy'} \alpha_{ny}\alpha_{n'y'} \psi_{n'y'}^T \psi_{ny} + C\sum_n \varepsilon_n$$
$$+ \sum_{ny} \alpha_{ny}\left(1 - \varepsilon_n - \sum_{ny}(\alpha_{ny} \psi_{ny})^T \psi_{ny}\right) - \sum_n \beta_n \varepsilon_n$$

$$= \frac{1}{2}\sum_{nn'yy'} \alpha_{ny}\alpha_{n'y'} \psi_{ny}^T \psi_{n'y'} - \sum_{nn'yy'} \alpha_{ny}\alpha_{n'y'} \psi_{ny}^T \psi_{n'y'}$$
$$+ \underbrace{\sum_n \varepsilon_n \left(C - \sum_{y} \alpha_{ny} - \beta_n\right)}_{=0,\ \text{see (2)}} + \sum_{ny} \alpha_{ny}$$

$$= \max_{\alpha\beta} -\frac{1}{2}\sum_{nn'yy'} \alpha_{ny}\alpha_{n'y'} \psi_{ny}^T \psi_{n'y'} + \sum_{ny} \alpha_{ny}$$

$\boxed{\alpha \geq 0}$ $\cancel{\beta \geq 0}$ From (2) follows $C - \sum_y \alpha_{ny} = \beta_n$

and with $\beta \geq 0 \rightarrow \boxed{C \geq \sum_y \alpha_{ny}}$

ML Sheet 05

(3) (a)

$$1^T \phi(x,y) = x_1 y_1 + x_2 y_2 + x_3 y_3 + 2 y_1 y_2 + 2 y_2 y_3$$

with $x = (1,-1,1)^T$

$$= y_1 + -y_2 + y_3 + 2 y_1 y_2 + 2 y_2 y_3$$

$$\max_{y_1 y_2 y_3} \left\{ y_1 + 2 y_1 y_2 - y_2 + 2 y_2 y_3 + y_3 \right\}$$

$$= \max_{y_1} \Bigg\{ y_1 + \underbrace{\max_{y_2} \Big\{ 2 y_1 y_2 - y_2 + \underbrace{\max_{y_3} \big\{ 2 y_2 y_3 + y_3 \big\}}_{1} \Big\}}_{2} \Bigg\}_{3}$$

b)

1	$y_3 = -1$	$y_3 = 1$
$y_2 = -1$	$\boxed{1}$	-1
$y_2 = 1$	-1	$\boxed{3}$

2	$y_2 = -1$	$y_2 = 1$
$y_1 = -1$	$\boxed{4}$	0
$y_1 = 1$	0	$\boxed{4}$

3	$y_1 = -1$	$y_1 = 1$
	3	$\boxed{5}$

$$(y_1, y_2, y_3) = (1, 1, 1)$$