Sheet 06

1a) Slaters conditions state that strong duality holds if there exists a solution that satisfies all the constraints and all nonlinear constraints with strict inequalities.

The equalities $\varepsilon_i \geq 0$ is met by choosing a ε_i larger or equal to 0, while the second set of constraints $\forall_i \ \langle \phi(x_i), w \rangle \geq \rho - \varepsilon_i$ can always be satisfied by making ε_i as large as necessary.

b)
$$\mathcal{L} = \frac{1}{2}\|w\|^2 - \rho + \frac{1}{N\nu}\sum_i \varepsilon_i$$
$$+ \sum_i \alpha_i (\rho - \varepsilon_i - \langle \phi(x_i), w \rangle)$$
$$- \sum_i \beta_i (\varepsilon_i)$$

c) $\max\limits_{\alpha,\beta} \min\limits_{w \rho \varepsilon_i} \mathcal{L}$

(1) $\frac{\partial \mathcal{L}}{\partial w} \stackrel{!}{=} 0 \iff 0 = w - \sum_i \alpha_i \phi(x_i) \Rightarrow w = \sum_i \alpha_i \phi(x_i)$

(2) $\frac{\partial \mathcal{L}}{\partial \rho} \stackrel{!}{=} 0 \iff 0 = -1 + \sum_i \alpha_i \Rightarrow \underline{\sum_i \alpha_i = 1}$ *

(3) $\frac{\partial \mathcal{L}}{\partial \varepsilon_i} \stackrel{!}{=} 0 \iff 0 = \frac{1}{N\nu} - \alpha_i - \beta_i \Rightarrow \alpha_i = \frac{1}{N\nu} - \beta_i$ with $\beta_i \geq 0$
$\Rightarrow \underline{0 \leq \alpha_i \leq \frac{1}{N\nu}}$ *

$$\mathcal{L}_{\text{with (1),(2),(3)}} = \underbrace{\frac{1}{2}\|w\|^2 - \sum_i \alpha_i \langle \phi(x_i), w \rangle}_{=0} \underbrace{- \rho + \rho \underbrace{\sum_i \alpha_i}_{=1}} + \sum_i \varepsilon_i \underbrace{\left(\frac{1}{N\nu} - \alpha_i - \beta_i\right)}_{=0}$$

$$= -\frac{1}{2}\sum_{ij} \alpha_i \alpha_j \phi(x_i)^T \phi(x_j) = \underline{-\frac{1}{2}\sum_{ij} \alpha_i \alpha_j k(x_i, x_j)} \ *$$

d) ~~ ~~

$$\max_{\alpha} -\frac{1}{2}\sum_{ij}\alpha_i\alpha_j k(x_i,x_j)$$

$$\Leftrightarrow \min_{\alpha} \underbrace{\frac{1}{2}\sum_{ij}\alpha_i\alpha_j \underbrace{k(x_i,x_j)}_{K_{ij}}}_{\alpha^T K \alpha}$$

Gram matrix

$$\sum_i 1\alpha_i = 1 \quad = \quad 1^T\alpha = 1$$

$$\alpha_1 \geq 0$$
$$\alpha_2 \geq 0$$
$$\vdots$$
$$\alpha_N \geq 0$$
$$\alpha_1 \leq \frac{1}{N\nu}$$
$$\vdots$$
$$\alpha_N \leq \frac{1}{N\nu}$$

$$\Leftrightarrow \begin{pmatrix} -I \\ I \end{pmatrix} \alpha \preceq \begin{pmatrix} 0 \\ \frac{1}{N\nu} \end{pmatrix}$$

elementwise

(e)

$$\langle \phi(x), w \rangle < \langle \phi(x_{sv}), w \rangle$$

$w = \sum_i \alpha_i \phi(x_i)$ see (1c)

$$\sum_i \alpha_i \underbrace{\langle \phi(x), \phi(x_i) \rangle}_{k(x, x_i)} < \sum_i \alpha_i \underbrace{\langle \phi(x_{sv}), \phi(x_i) \rangle}_{k(x_{sv}, x_i)}$$