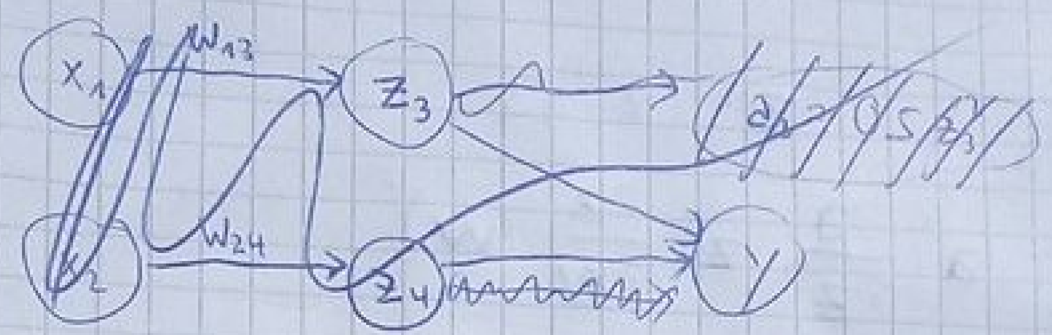
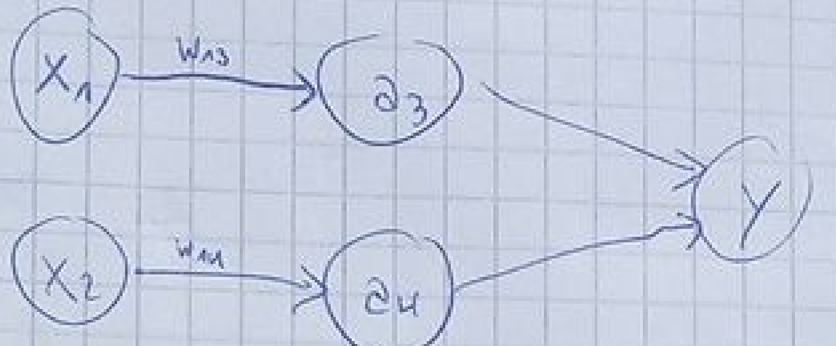
Sheet 7

1. (a)



(1.)(a)



~~forward pass~~

(b) $\ell(y,t) = \frac{1}{2}(y-t)^2$

- $$\frac{\partial \ell}{\partial w_{13}}(y,t) = (y-t)\cdot\frac{\partial}{\partial w_{13}} y$$
$$= (y-t)\cdot\frac{\partial}{\partial w_{13}}(a_3+a_4)$$
$$= (y-t)\cdot\frac{\partial}{\partial w_{13}}(0.5\, z_3^2) \qquad \left(\frac{\partial}{\partial w_{13}} a_4 = 0\right)$$
$$= (y-t)\; z_3 \cdot \frac{\partial}{\partial w_{13}} z_3$$
$$= (y-t)\, z_3\, \frac{\partial}{\partial w_{13}}(x_1 w_{13})$$
$$= (y-t)\, z_3\, x_1 \qquad \left(= (0.5(x_1 w_{13})^2 + 0.5(x_2 w_{24})^2 - t)\, x_1 w_{13} \ldots\right.$$

- $$\frac{\partial \ell}{\partial w_{24}}(y,t) = (y-t)\, z_4\, x_2 \qquad \text{similarly}$$
$$\left(= (0.5(x_1 w_{13})^2 + 0.5(x_2 w_{24})^2 - t)\, x_2 w_{24} x_2\right)$$

(c) Considering ℓ as a function of w_{13}, w_{24}, the chain rule reads

$$\frac{\partial \ell}{\partial v}(w_{13}(v), w_{24}(v)) = \frac{\partial \ell}{\partial w_{13}}(w_{13}(v), w_{24}(v))\frac{\partial w_{13}}{\partial v}(v) + \frac{\partial \ell}{\partial w_{24}}(w_{13}(v), w_{24}(v))\ldots$$

Hence

$$\frac{\partial \ell}{\partial v} = (y-t)\, z_3\, x_1 \frac{1}{1+e^{v}}\cdot e^{v} + (y-t)\, z_4\, x_2 \frac{1}{1+e^{-v}} e^{-v}$$

$$\cancel{\left(= (0.5(x_1 \log(1+e^{v}))^2 + 0.5(x_2 \ldots \right.}$$

$$\cancel{x_1}$$

$$= \left(0.5\, x_1^2 \log^2(1+e^{v}) + 0.5\, x_2^2 \log^2(1+e^{-v})\right)\left(x_1^2 \log(1+e^{v}) - x_2^2 \log(1+e^{-v})\right)$$

2) (a) $\cancel{\frac{\partial f}{\partial w} = \frac{\partial f}{\partial a}}$ $\qquad \mathbf{z} := \left(z^{(l)}\right)_{l=1}^{L}$

$$\frac{\partial f}{\partial \mathbf{w}} = \frac{\partial f}{\partial \mathbf{a}}(\mathbf{a})\, \frac{\partial \mathbf{a}}{\partial \mathbf{z}}(\mathbf{z})\, \frac{\partial \mathbf{z}}{\partial \mathbf{w}}(\mathbf{w})$$

- $\frac{\partial \mathbf{z}}{\partial \mathbf{w}}(\mathbf{w}) =$
- $\cancel{\frac{\partial z_t^{(l)}}{\partial w^{(k,m)}} = \frac{\partial}{\partial w^{(k,m)}} \sum_{l}}$

2(a) - $\frac{\partial z_t^{(l)}}{\partial w_r^{(m,n)}} = \frac{\partial}{\partial w^{(m,n)}} \sum_{k=1}^{K} \sum_{s=-\infty}^{\infty} x_s^{(k)} \cdot w_{t-s}^{(k,l)}$

$$= x_{t-r}^{(m)}\, \delta_{l,n}$$

- $\frac{\partial a_t^{(l)}}{\partial z_s^{(m)}} = \begin{cases} 1 & \text{if } m=l,\ s=t,\ z_s^{(m)} \geq 0 \\ 0 & \text{otherwise} \end{cases}$

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