Moreover, the threshold $\overline{\theta}$ is proportional to the importance of the inefficient-decision component relative to the price-dispersion component ($\frac{1-\gamma}{\gamma}$) in the welfare loss function.

Proof. See Appendix B. □

The intuition behind Proposition 1 is as follows. Consider a special case with $\theta = 0$, where both agents are adjusters and can react to information. In this scenario, full transparency is optimal for social welfare.

However, with a positive mass of non-adjusters ($\theta > 0$), where one type of agent can react to information and the other cannot, providing information can hurt social welfare. Specifically, more information induces price dispersions among adjusters and non-adjusters ($E(\bar{p}^*)^2$), resulting in a larger social welfare loss. Against this cost of information, there is also a welfare benefit from adjusters choosing prices closer to their frictionless level. However, this social benefit of information is reduced with more non-adjusters. Combining the two, the welfare effect of more information can be negative. Therefore, the optimal central bank communication strategy depends on θ and the relative importance of the inefficient-decision component (capturing the benefit of information) and the price-dispersion component (capturing the cost of information) for social welfare ($\frac{1-\gamma}{\gamma}$).

Section 3 introduces an NK model with a micro-founded welfare loss function, yielding results similar to Proposition 1. Furthermore, the NK model is more comprehensive, enabling a joint consideration of monetary policy and central bank communication.

3 The Static Model

We begin our analysis based on a simple two-sector static model with information frictions and, importantly, nominal rigidity. This simple model allows us to derive analytical results.

3.1 The Model Setup

Household There is a representative household whose utility depends positively on consumption and negatively on labor supply. Particularly, we assume a utility function as follows:

$$U(C, \{L_k\}) = log(C) - \sum_{k=1}^{2} L_k,$$

where subindex k denotes a sector k, L_k is the supply of labor to sector k, $C \equiv \left[\sum_{k=1}^{2} n_k^{1/\eta} C_k^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}$ is the constant elasticity of substitution (CES) aggregator that aggregates sectoral-level consumption C_k and $C_k \equiv \left[n_k^{-1/\epsilon} \int_{n_k} C_{k,i}^{(\epsilon-1)/\epsilon} di\right]^{\epsilon/(\epsilon-1)}$ is the CES aggregator that aggregates varieties within a sector to aggregate consumption at the sectoral level. n_k denotes the sector size and ϵ and η measure the within-sector and cross-sector elasticities of substitution, respectively. ϵ is greater than η, reflecting the fact that goods within a sector are more substitutable than goods across sectors.

The consumer's budget constraint consists of labor income ($W_k L_k$), profits from firms ($\sum\limits_{k=1}^{2} \Pi_k$), and a lump-sum transfer/tax (T) from the government. Formally, the budget constraint is:

$$PC = \sum_{k=1}^{2} W_k L_k + \sum_{k=1}^{2} \Pi_k + T,$$

where $P \equiv \left[\sum_{k=1}^{2} n_k P_k^{1-\eta}\right]^{1/(1-\eta)}$ and $P_k \equiv \left[\int_0^1 P_{k,i}^{1-\epsilon} di\right]^{1/(1-\epsilon)}$ are aggregate prices in the economy and sector k, respectively.

The household's side of the model plays two important roles. First, The household's utility function micro-founds the welfare loss function that we rely on to evaluate alternative policies. Discussions of the welfare loss function are reserved for section (3.2). Second, the representative household's optimization results in the demand curves in the goods market. Formally, for a given expenditure, the optimizing consumer demands the variety i within a sector k ($C_{k,i}$), and the aggregate consumption at a sector k (C_k) according to the following equations:

$$C_{k,i} = \left(\frac{P_{k,i}}{P_k}\right)^{-\epsilon} \frac{1}{n_k} C_k \qquad C_k = \left(\frac{P_k}{P}\right)^{-\eta} n_k C. \tag{3.1}$$

Firms Each sector k consists of a continuum of firms of mass 1. As in the household's problem, subscript $\{k, i\}$ denotes a firm that produces good i in sector k. The CES aggregators introduced earlier indicate that firms operate in a monopolistic competitive market. Given the monopolistic power, firms optimally set their prices to maximize their profits whenever possible. Formally, the firms' optimization problem can be summarized as follows:

$$\max_{P^*_{k,i}} \mathbb{E}\left\{P^*_{k,i}Y_{k,i} - W_k L_{k,i} | \mathbb{I}_{k,i}\right\} \tag{3.2}$$

subject to the demand function (3.1) and production function

$$Y_{k,i} = A_k L_{k,i}, \tag{3.3}$$

where A_k denotes the productivity in sector k, $\mathbb{E}$ is the expectation operator, and $\mathbb{I}_{k,i}$ indicates the information set of the firm $\{k,i\}$. The log of productivity shocks (a_k) are randomly drawn from normal distributions:

$$a_k \sim N(0, \sigma^2_{k,a}) \ \forall k. \tag{3.4}$$

Solving the firms' optimization problem and log-linearize it delivers the following optimal price ($p^*_{k,i}$) setting rule:

$$p^*_{k,i} = \mathbb{E}[p + \tilde{y} + u_k | \mathbb{I}_{k,i}], \tag{3.5}$$

where variables in small letters denote the log of the underlying variable, and all variables are in log deviations from their initial values. $p = \sum_1^k n_k p_k$ is the aggregate price and $p_k = \int_i p_{k,i} di$ is the sectoral price. We also define the average price among the price-resetting firms in sector k $p^*_k = \int_i p^*_{k,i} di$. $\tilde{y}$ is the output gap, defined as $\tilde{y} = y - y^e$. $y^e \equiv n_1 a_1 + n_2 a_2$ represents the efficient output under flexible prices and perfect information. u_k is proportional to the relative sectoral productivity, particularly, $u_1 = n_2(a_2 - a_1)$ and $u_2 = n_1(a_1 - a_2)$.

Frictions and the Central Bank Communication We now introduce two key frictions: nominal rigidity and information frictions.

In the data, firms adjust prices sluggishly: the median duration of a price is between eight and eleven months (Nakamura and Steinsson 2008 and Dhyne et al. 2006). This empirical evidence suggests that firms do not react to shocks on time, which has important implications for central bank communications– as the primary focus of this study. Incorporating this empirical feature in the model, we assume that only a fraction $(1 - \theta_k)$ of the firms in sector k have the freedom to adjust prices. The remaining firms staggered to the (log) price at the initial level. This assumption implies that in a dynamic framework, the average the price duration is $1/(1 - \theta_k)$. The flexible price economy is a special case in which $\theta_1 = \theta_2 = 0$.

An alternative and isomorphic way to model a firm's infrequent price adjustment is to introduce sticky information, as in Mankiw and Reis (2002). The assumption in the previous paragraph is isomorphic to the following assumption. Only a fraction $(1 - \theta_k)$ of firms in sector k receives signals about the unobserved economic conditions. The remaining firms do not observe any signals; therefore, they do not change their prices.

Nominal rigidity combined with the optimal price-setting rule (3.5) gives rise to the Philips curve in a static setting:

$$p_k = (1 - \theta_k) \int_i \mathbb{E}[p + \tilde{y} + u_k | \mathbb{I}_{k,i}] di. \tag{3.6}$$

Firms set prices in a noisy information environment. Particularly, firms do not observe the productivity a_k. However, firms know the unconditional distribution of a_k, which follows a normal distribution, $N(0, \sigma^2_{k,a})$.

The only signals that firms observe are sent by the central bank. The central bank communicates about a_l by sending noisy signals s_l, for $l \in \{1, 2\}$. In the baseline model, we assume that each firm k, i interprets s_l differently:

$$s_{ki,l} = a_l + e_{ki,l}, \tag{3.7}$$

where $e_{ki,l} \sim N(0, \sigma^2_{l,e})$. The idiosyncratic noise shock $e_{ki,l}$ gives rise to dispersed beliefs. This assumption in the baseline model is made to match the empirical observations that firms' beliefs are dispersed (see e.g., Coibion et al. 2020 and Coibion et al. 2021), and that central bank communications are interpreted differently by firm (Andrade et al. 2019). Our findings are robust to the alternative assumption that the central bank communicates via a public signal with common noise e_l (see Section 5.2).

Firms update their beliefs according to the Bayes' theorem. Formally,

$$E(a_l | \mathbb{I}_{k,i}) = K_l s_{ki,l} \tag{3.8}$$

where $K_l = \frac{\sigma^2_{l,a}}{\sigma^2_{l,e} + \sigma^2_{l,a}}$ for $l \in \{1, 2\}$. Incorporating (3.8) into the Philips curve derived above delivers imperfect-common-knowledge Philips curves:

$$p_1 = (1-\theta_1)\int_i \mathbb{E}[p+\tilde{y}|\mathbb{I}_{1,i}]di + (1-\theta_1)n_2(K_2a_2 - K_1a_1), \tag{3.9}$$

$$p_2 = (1-\theta_2)\int_i \mathbb{E}[p+\tilde{y}|\mathbb{I}_{2,i}]di + (1-\theta_2)n_1(K_1a_1 - K_2a_2). \tag{3.10}$$

The imperfect-common-knowledge Philips curve nests the full-information Philips curve as a special case in which $\sigma_{l,e} = 0$. In this special case, $K_1 = K_2 = 1$. Comparing the imperfect-common-knowledge Philips curve (when $K_l < 1$) with the full-information Philips curves, it is evident that information frictions dampen the effects of shocks on prices. By setting $\theta_1 = \theta_2 = 0$ and $K_1 = K_2 = 1$, a direct implication from the imperfect-common-knowledge Philips curves is that the relative price under flexible price and perfect information is given by $p_1^e - p_2^e = a_2 - a_1$.

The central bank chooses its degree of transparency $\sigma_{l,e}^2$ for $l \in \{1,2\}$. Throughout this study, the term central bank communication, or the degree of transparency, refers to the value of $\sigma_{l,e}^2$ relative to the volatilities of fundamental shocks $\sigma_{l,a}^2$.

3.2 The Welfare Loss Function and Monetary Policy

The Welfare Loss Function We now discuss the welfare loss function used to evaluate alternative policies.

Formally, the welfare loss function is obtained by taking the second-order approximation of the household's utility function:[12]

$$\mathbb{E}\mathbb{L} = \mathbb{E}\Bigg\{\epsilon \sum_{k=1}^{2} n_k \underbrace{\Big[\underbrace{(1-\theta_k)\theta_k(p_k^*)^2}_{\text{sticky-price}} + \underbrace{(1-\theta_k)\int_i (p_{k,i}^* - p_k^*)^2 di}_{\text{dispersed-belief}}\Big]}_{\text{price-dispersion}}$$

$$+ \underbrace{\underbrace{\tilde{y}^2}_{\text{output-gap}} + \eta n_1 n_2 \underbrace{\Big[(p_1 - p_2) - (p_1^e - p_2^e)\Big]^2}_{\text{relative-price-gap}}}_{\text{inefficient-decision}}\Bigg\}. \tag{3.11}$$

Welfare loss increases in the volatilities of the output gap and relative price gaps, which

[12]Note that we assume the presence of optimal wage subsidies that offset the distortions arising from monopolistic competitions in the steady state.

are defined as the corresponding variables in deviations from their efficiency levels. Both information frictions and nominal rigidity lead to inefficient coordination in firms' production and pricing decisions, ultimately resulting in inefficient gaps. We label them as the *inefficient-decision component*.

More importantly, the welfare loss function includes the *price-dispersion component*. Two features of the CES aggregators are worth highlighting to understand the welfare loss function component. First, C_k is concave in $C_{k,i}$. Second, an individual good $C_{k,i}$ matters for the aggregate C_k symmetrically. These two features imply the socially optimal allocation features $C^*_{k,i} = C^*_{kj}, \forall i, j$. In other words, dispersions in quantities of goods produced reduce social welfare. Together with the demand functions (3.1), they imply that dispersions in prices reduce social welfare.

Both information frictions and nominal rigidity contribute to the price-dispersion component. The importance of the price stickiness is captured by $(1-\theta_k)\theta_k(p^*_k)^2$. This subcomponent is called the *sticky-price component*. This term measures the dispersion in prices originating from nominal rigidity, the dispersion between price-resetting firms (p^*_k), and these firms whose prices are staggered at their initial level. In the extreme case when prices in sector k are flexible; that is, $\theta_k = 0$, this term vanishes, and the price-dispersion component consists solely of the dispersed-belief component that we now discuss.

The importance of information frictions is captured by $(1-\theta_k)\int_i (p^*_{k,i} - p^*_k)^2 di$, which is called the *dispersed-belief component*. This component captures the fact that price dispersions exist among price-resetting firms because of information friction and dispersed beliefs. Firms set different prices because they have different assessments of the unobserved aggregate economic conditions.

Monetary Policy In addition to selecting the degree of central bank communication, the central bank can influence economic activities by conducting monetary policy. Throughout the study, we consider the following (alternative) monetary policy rules: (i) price index stabilization policy and (ii) Ramsey optimal monetary policy.

With the price index stabilization policy, the central bank *chooses* an aggregate price index (ω) and commits to fully stabilizing it. Formally:

$$\omega p_1 + (1-\omega)p_2 = 0. \tag{3.12}$$

Within the price index stabilization policy, we consider two alternative policies. The first is called the CPI stabilization policy, formally, $\omega = n_1$. The second is the optimal price index stabilization (OII) policy: ω is optimally chosen to minimize the expected welfare loss. Note that such a price index stabilization policy is widely used in multi-sector New Keynesian models (see, e.g., Aoki 2001, Mankiw and Reis 2003, Benigno 2004, Woodford 2003, and Eusepi et al. 2011).

This paper also considers the optimal communication strategy under the Ramsey optimal monetary policy. The latter refers to the case where the central bank is free to choose the allocation of aggregate output (or aggregate price) as a function of state variables, respecting the constraints of the economy. Formally, the central bank optimally chooses p_k,p and $\tilde{y}$ such that given the information structure (3.7), equation (3.5) is satisfied and the welfare loss (3.11) is minimized.

A detailed definition of the equilibrium is presented in section (B.1).

The Timing of the Model The model consists of four stages. At stage one, the central bank chooses the degree of transparency ($\sigma_{1,e}^2, \sigma_{2,e}^2$), and the inflation index (ω) that it commits to stabilize (if the central bank conducts a price stabilization policy). At stage two, the nature draws fundamental shocks a_l, and each firm k, i receives a signal $s_{ki,l}$ about a_l for $l \in \{1, 2\}$. Each firm decides on its supply curve in the goods market at stage three. At stage four, the representative household observes the state of the economy and makes consumption and labor decisions. At the same stage, the goods and labor markets clear, and if the central bank conducts the optimal monetary policy, it chooses the optimal (constraint) allocations.

3.3 Results

The description of the model is complete, and it can be solved analytically. This study aims to contribute to the debate on the optimal level of central bank transparency (σ_e^2/σ_a^2). To this end, we investigate how welfare loss depends on σ_e^2/σ_a^2, and how this relationship depends on the degree of nominal rigidity. For the analysis conducted in the current section, we abstract from sectoral heterogeneities in the volatilities of shocks, that is, $\sigma_{1,a} = \sigma_{2,a} = \sigma_a$, and we consider economy-wide central bank communication, that is, $\sigma_{1,e}^2 = \sigma_{2,e}^2 = \sigma_e^2$. Central bank communication that only affects the noise-signal ratio of one sector delivers results similar to those presented below.

3.3.1 Central Bank Communication when Prices are Flexible

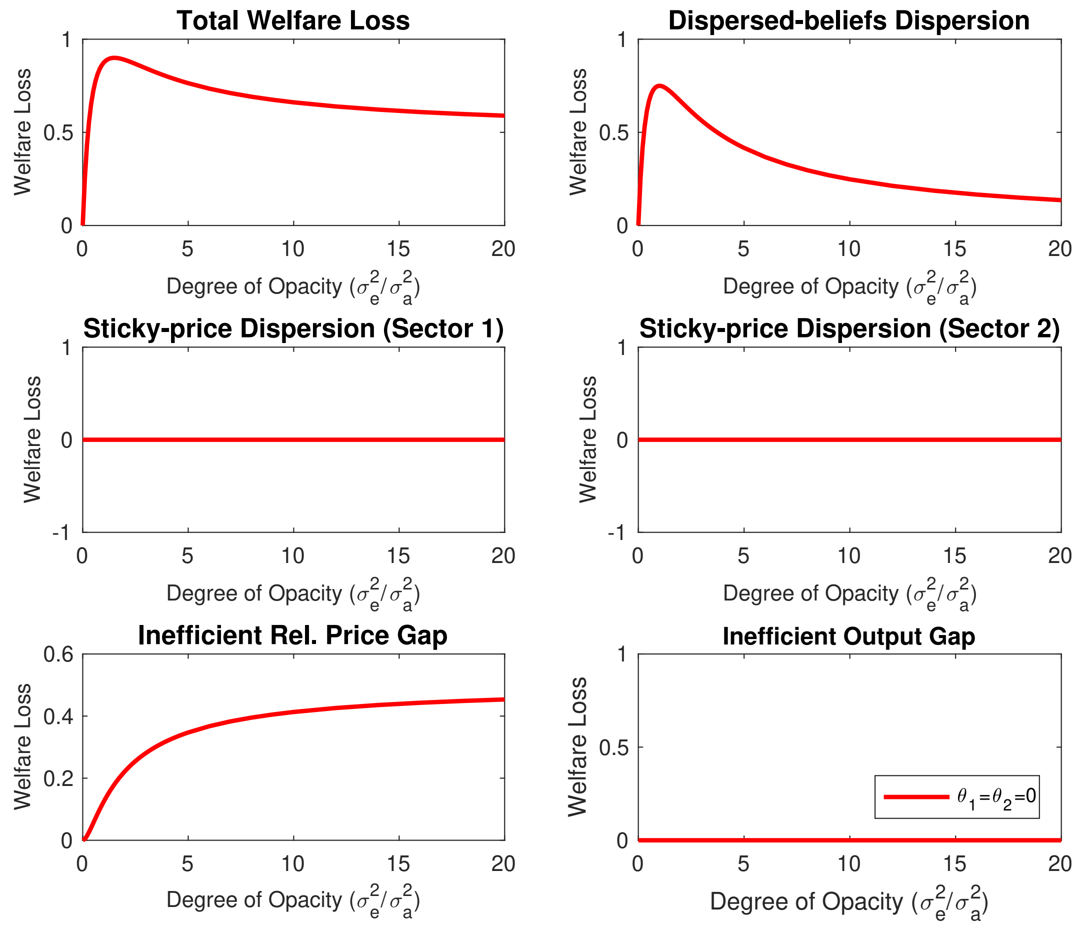

Figure 1: This figure plots the welfare consequence of central bank communication when prices are flexible. The top left panel plots the total welfare loss, and the remaining panels plot the subcomponents of welfare loss as functions of the degree of opacity σ_e/σ_a. The model calibration is as follows: $\theta_1 = \theta_2 = 0$, $\epsilon = 6$, $\eta = 1$, and $\sigma_a = 1$.

The first policy experiment considers an economy with symmetric characteristics and flexible prices ($\theta_1 = \theta_2 = 0$), Figure 1 plots the welfare loss function and its subcomponents as functions of σ_e/σ_a. In this numerical exercise, we normalize the standard deviations of productivity shocks to one. The elasticities of substitutions are set to be $\epsilon = 6$ and $\eta = 1$, which correspond to values that are widely used in the literature. The policy experiment is conducted under the CPI stabilization policy. In this baseline setting, the CPI stabilization policy is equivalent to the OII, and more importantly, both are very close to the optimal monetary policy. This is shown in Aoki (2001) and Woodford (2003), in a model with full information. The same result applies to the current setting—a model with information frictions.

Figure 1 illustrates the overall social welfare loss is a hump-shaped function of central bank communication. Note that σ_e/σ_a (the x-axis) equals zero indicates full central bank transparency and $\sigma_e/\sigma_a = \infty$ corresponds to full opacity. This hump-shaped feature is entirely driven by the dispersed-belief component. Intuitively, the disagreement between firms about the aggregate economic condition vanishes in the two extremes. On one extreme, if the central bank is fully transparent, firms have full information, and they all agree on the actual realization of shocks. On the other extreme, with full opacity, firms do not value their idiosyncratic signals and form their beliefs solely by relying on common priors—the unconditional distribution of shocks. The disagreement emerges in the intermediate case when central bank communication is valuable but not perfect. The sticky-price component equals zero due to flexible prices, and the inefficient output gap component is equal to zero. The latter is a result of the CPI stabilization policy.[13]

The inefficient-decision component arising from the relative price gap is decreasing with the degree of central bank transparency. This result confirms that more information allows firms to make more efficient decisions. This component and the dispersed-belief price dispersion component imply that overall welfare loss is hump-shaped and full transparency is optimal. In fact, with full transparency, the model is frictionless. Therefore, there is no welfare loss.

The location of the hump, that is, the degree of central bank communication associated with the maximum welfare loss, can be important for policymakers if full central bank communication is not achievable in real life. In this case, the current analysis suggests that central bank communication improves welfare if and only if the status quo is on the left-hand side of the hump. Intuitively, the location of the hump depends on the relative importance between the price-dispersion and the inefficient-decision components for social welfare. This relative importance is characterized by parameters ϵ and η, as evident from the welfare loss function. Proposition 2 summarizes the findings and provides an analytical expression for the location of the hump.

Proposition 2. *The full transparency is optimal in a two-sector economy with symmetric characteristics and flexible prices ($\theta_1 = \theta_2 = 0$). Moreover, welfare loss reaches its maximum at $\frac{\sigma_e^2}{\sigma_a^2} = \frac{\epsilon}{\epsilon - 2\eta}$ if $\epsilon > 2\eta$. That is, a marginal increase in central bank transparency improves welfare*

[13]This can be derived using equations (3.9) and (3.10) together with CPI stabilization $p = n_1 p_1 + n_2 p_2 = 0$.

if and only if the status quo is on the left-hand side of $\frac{\epsilon}{\epsilon-2\eta}$.

Proof. See Appendix B. □

3.3.2 Central Bank Communication when Prices are Rigid

We now move to the main focus of the current study: a model with nominal rigidity. We begin by presenting the results that are important for understanding this mechanism.

Proposition 3. *The presence of information frictions dampens the effects of shocks on* p^*. *Therefore, the sticky-price component of welfare loss increases with the degree of central bank communication.*

Proof. See the Appendix B. □

The intuition behind this result is discussed in Section 3.1 by comparing the imperfect-common-knowledge Philips curve with the full-information Philips curves. The following result crucially depends on the fact that a more transparent central bank increases the sticky-price component of welfare loss.

The following policy experiment is similar to that conducted in the previous subsection 3.3.1, but with a crucial difference: now, prices are sticky. In particular, we introduce price rigidity to sector 2 ($\theta_2 > 0$) whereas prices in sector 1 are flexible ($\theta_1 = 0$).

Figure 2 plots these results. The dashed blue lines plot the case in which the degree of nominal rigidity in sector 2 is small ($\theta_2 = 0.2$). The solid red lines depict the results with more rigid prices in sector 2 ($\theta_2 = 0.75$), corresponding to the median duration of the prices uncovered by Nakamura and Steinsson (2008). Similar to the previous case with flexible prices, the dispersed-belief component is a humped function of the degree of the central bank communication, and the inefficient-decision component decreases if the central bank is more transparent.

The novelty of the current analysis is that the sticky-price component emerges, particularly in the sticky-price sector (sector 2). There are two types of firms in the sticky-price sector: those that can reset prices and those that are staggered with steady-state prices. Differences in the prices of these two groups of firms give rise to the sticky-price component of welfare loss.

Proposition 3 shows that the sticky-price component increases with the degree of central bank transparency. Consider the extreme case of full opacity. No firms believe in the realization of unobserved shocks; therefore, there are no price dispersions among the two types of firms. This describes the benefit of opacity. Thus, in the presence of nominal rigidity, optimal central bank communication should balance the dispersed belief with the inefficient-decision component *and*, taking into account the sticky-price component.

In addition, a more rigid price *reduces the gain* from more efficient decisions. The presence of nominal rigidity prohibits firms from re-optimizing their actions and, therefore, providing those firms with more information does not improve the efficiency of their decisions. These results can be viewed in the bottom-left panel in Figure 2, which corresponds to the inefficient-decision component of the welfare loss function. When information friction is reduced, the efficiency gain is reduced if prices are more rigid.

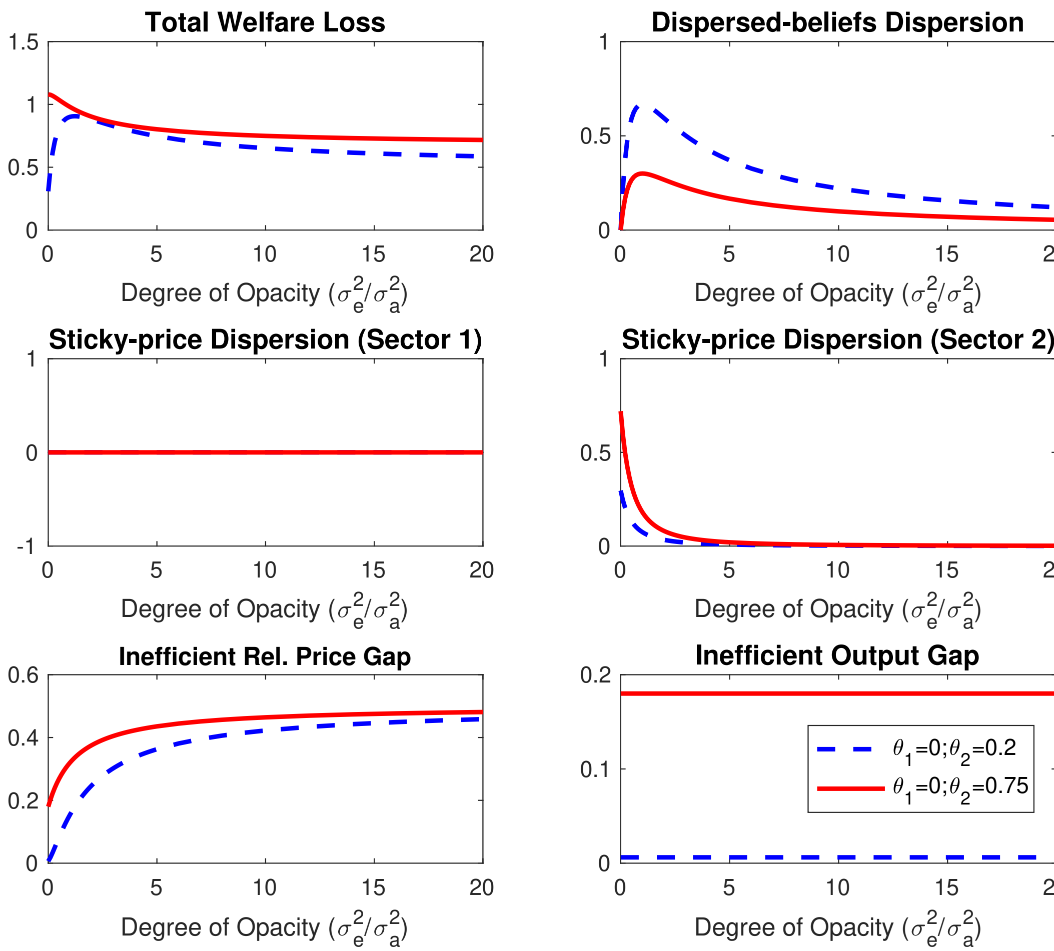

Figure 2: This figure plots the welfare consequence of central bank communication in an economy that consists of one flexible-price sector ($\theta_1 = 0$) and one sticky-price sector. The solid red (dashed blue) lines correspond to the case in which $\theta_2 = 0.75$ ($\theta_2 = 0.2$). The top left panel plots the total welfare loss, and the remaining panels plot the subcomponents of welfare loss as functions of the degree of opacity σ_e/σ_a. The remaining calibration of the model is as follows: $\epsilon = 6$, $\eta = 1$, and $\sigma_a = 1$.

Combining all components of the welfare loss function, the top-left panel in Figure 2 shows that the previous result with flexible prices is, qualitatively, unchanged when nominal rigidity is small. However, when prices are reasonably rigid, the sticky-price component overturns the previous result: Full opacity is optimal, and central bank communication reduces welfare independent of the current degree of information frictions.

Proposition 4. *Consider a two-sector economy that consists of one flexible-price sector ($\theta_1 = 0$) and one sticky-price sector ($\theta_2 = \theta$), and the central bank targets the consumer price index. A threshold level of nominal rigidity $\overline{\theta} = 2\eta/\epsilon$ exists such that*

1. *the full transparency is optimal if $\theta < \overline{\theta}$;*

2. *the full opacity is optimal if $\theta > \overline{\theta}$.*

Moreover, the threshold level of nominal rigidity $\overline{\theta}$ increases with the importance of the inefficient-decision component relative to the importance of the price-dispersion component (η/ϵ) in the welfare loss function.

Proof. See Appendix B □

Proposition 4 analytically characterizes Figure 2. In particular, we derive the threshold level of nominal rigidity ($\overline{\theta}$), such that the central bank communication reduces social welfare if and only if the degree of nominal rigidity (in sector 2) is greater than $2\eta/\epsilon$.[14] Note that the two parameters determining the threshold values ϵ and η characterize the relative importance of price-dispersions and inefficient decisions for social welfare. The more important the price-dispersions component is (a larger ϵ), the smaller the parameter space for θ such that a more transparent central bank is socially desirable. η affects the gain from increased efficiency, and ϵ determines the cost of higher price dispersions owing to nominal rigidity. When $\epsilon \leqslant 2\eta$, which corresponds to the case in which the efficiency gain from central bank communication is extremely important, $\overline{\theta}$ is larger than 1, and therefore, full transparency would be optimally independent of the degree of nominal rigidity. However, such a calibration ($\epsilon \leqslant 2\eta$) is far from the estimates provided in the empirical literature; for example, Rotemberg and Woodford (1995) and Basu and Fernald (1997) suggest that ϵ is equal to six and Hobijn and Nechio (2019) estimate that η is equal to one. These estimates are widely used to calibrate macroeconomic models in the literature, see, for example, Golosov and Lucas (2007) and Galí (2015).

3.3.3 The Role of Monetary Policy

The previous analysis is conducted under the CPI stabilization policy. While the CPI stabilization policy is optimal when sectors have symmetric characteristics, it is no longer the case if sectors have different degrees of nominal rigidity. The heterogeneous effects of central bank communication on the sticky-price component in the two sectors hint at the important role that monetary policy *can* play in altering the previous findings.

[14] Note that we exclude the discussions of the following two limiting cases. First, if $\theta = \overline{\theta}$, the economy attains the highest level of social welfare under either full transparency or full opacity. Second, if prices are fully rigid ($\theta = 1$), social welfare does not depend on the precision of signals.

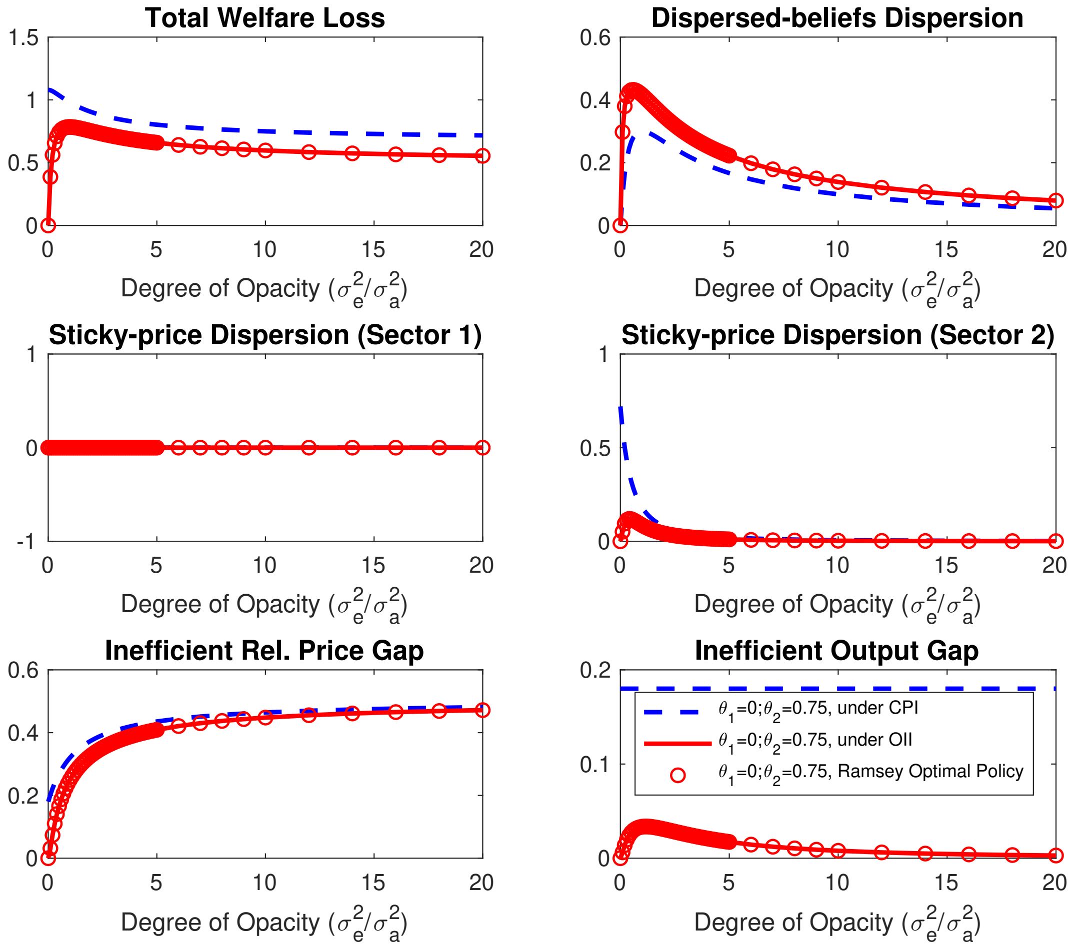

Figure 3: This figure plots the welfare consequence of central bank communication in an economy that consists of one flexible-price sector ($\theta_1 = 0$) and one sticky-price sector ($\theta_2 = 0.75$) under alternative monetary policy rules. The dashed blue lines, solid red lines, and red circles correspond to the cases under CPI stabilization, OII stabilization, and Ramsey optimal monetary policy. The top left panel plots the total welfare loss, and the remaining panels plot the subcomponents of welfare loss as functions of the degree of opacity σ_e/σ_a. The model calibration is as follows: $\epsilon = 6$, $\eta = 1$, and $\sigma_a = 1$.

The following policy experiment uses the previous model setup with $\theta_1 = 0$ and $\theta_2 = 0.75$. It evaluates the welfare consequences of the central bank communication under three alternative monetary policy rules: (i) CPI stabilization, (ii) OII stabilization, and (iii) optimal monetary policy.

Figure 3 presents these findings. The dashed blue lines depict the results under the CPI stabilization policy: full opacity is optimal. The findings under the CPI stabilization policy are already discussed above (red lines in Figure 2) but are repeated in Figure 3 for comparison. Interesting results emerge when the central bank conducts the OII stabilization policy (solid red lines) or optimal monetary policy (red circles). Note that the overlap between the two optimal monetary policies is a well-known result, see, for example, Woodford (2003).

The optimal policy can overturn the results under the CPI stabilization policy regarding the optimality of central bank communication. When the central bank conducts monetary policy optimally, it internalizes the consequences of central bank communications. Particularly, the central bank is aware of the increased sticky-price dispersion in the sticky-price sector accompanied by increased transparency. In response, an optimizing central bank chooses to stabilize prices in the sticky-price sector more aggressively than in the flexible-price sector. Such a policy substantially reduces the sticky-price component, and as a result, full transparency is optimal. Proposition 5 shows that full transparency is optimal regardless of the degree of nominal rigidity in the sticky-price sector if the central bank stabilizes the OII in a two-sector economy consisting of one flexible price sector.

Proposition 5. *In a two-sector economy consisting of one flexible-price sector and one sticky-price sector with equal sizes and the central bank stabilizes the optimal price index, the full transparency is optimal regardless of the degree of nominal rigidity in the sticky-price sector.*

Proof. See the Appendix B. □

3.3.4 Monetary Policy and the Conditions for the Return of Greenspan

Before moving to the quantitative analysis, the paper proceeds to the case in which all sectors are subject to nominal rigidity. Proposition 6 provides the analytical results, allowing for an interaction between price stabilization and communication policies.

Proposition 6. *Consider a two-sector economy with symmetric characteristics, and the central bank conducts a price index stabilization policy. Let $\Omega \equiv (1-2\omega)^2$ denote the deviation of the monetary policy rule from the optimal inflation stabilization policy. The social welfare loss increases with Ω.*

If $\eta < \frac{\epsilon}{2} + (\frac{\epsilon}{2} - 1)\Omega$, then there exists a threshold level of nominal rigidity $\overline{\theta} \equiv \frac{\eta+\Omega}{(\epsilon-1)\Omega+(\epsilon-\eta)}$, such that

1. *full transparency is optimal if $\theta < \overline{\theta}$;*
2. *full opacity is optimal if $\theta > \overline{\theta}$.*

With $\frac{\partial\overline{\theta}}{\partial\Omega} \leq 0$, $\frac{\partial\overline{\theta}}{\partial\epsilon} < 0$, and $\frac{\partial\overline{\theta}}{\partial\eta} > 0$. That is, $\overline{\theta}$ decreases if monetary policy is less optimal and decreases (increases) with the importance of the price dispersion (inefficient-decision) component in the welfare loss function.

Moreover, if $\eta \geqslant \frac{\epsilon}{2} + (\frac{\epsilon}{2} - 1)\Omega$, full transparency is optimal.

Proof. See Appendix B. □

Corollary 1. *When the degree of nominal rigidity in the entire economy is sufficiently high, $\theta > \frac{\eta}{\epsilon-\eta}$, the full opacity is optimal, even if the central bank conducts the OII stabilization policy ($\Omega = 0$).*

Proof. It follows from Proposition 6 by setting $\Omega = 0$ □

Under the price index stabilization policy, a symmetric two-sector economy permits us to derive the analytical results. The OII stabilization policy coincides with the CPI stabilization policy ($\omega = 0.5$) in such an economy. Hence, intuitively, $\Omega \equiv (1 - 2\omega)^2$ denotes the deviation of the monetary policy rule from the optimal inflation stabilization policy. Ω reaches its minimum value of 0 if $\omega = 0.5$, and in the worst policy case (if $\omega = 0$ or 1), Ω reaches its maximum value of 1.

Proposition 6 adds two new insights to the discussion above. First, when the degree of nominal rigidity in the entire economy is sufficiently high, full opacity is optimal *even if* the monetary policy is conducted optimally. This is shown in Corollary 1. Figure 4 shows this result. It plots the welfare consequences of central bank communication under optimal policy for three cases: flexible prices (black lines), low nominal rigidity (blue lines), and high nominal rigidity (red lines). In contrast to other cases, when nominal rigidity is high, a more transparent central bank hurts social welfare.

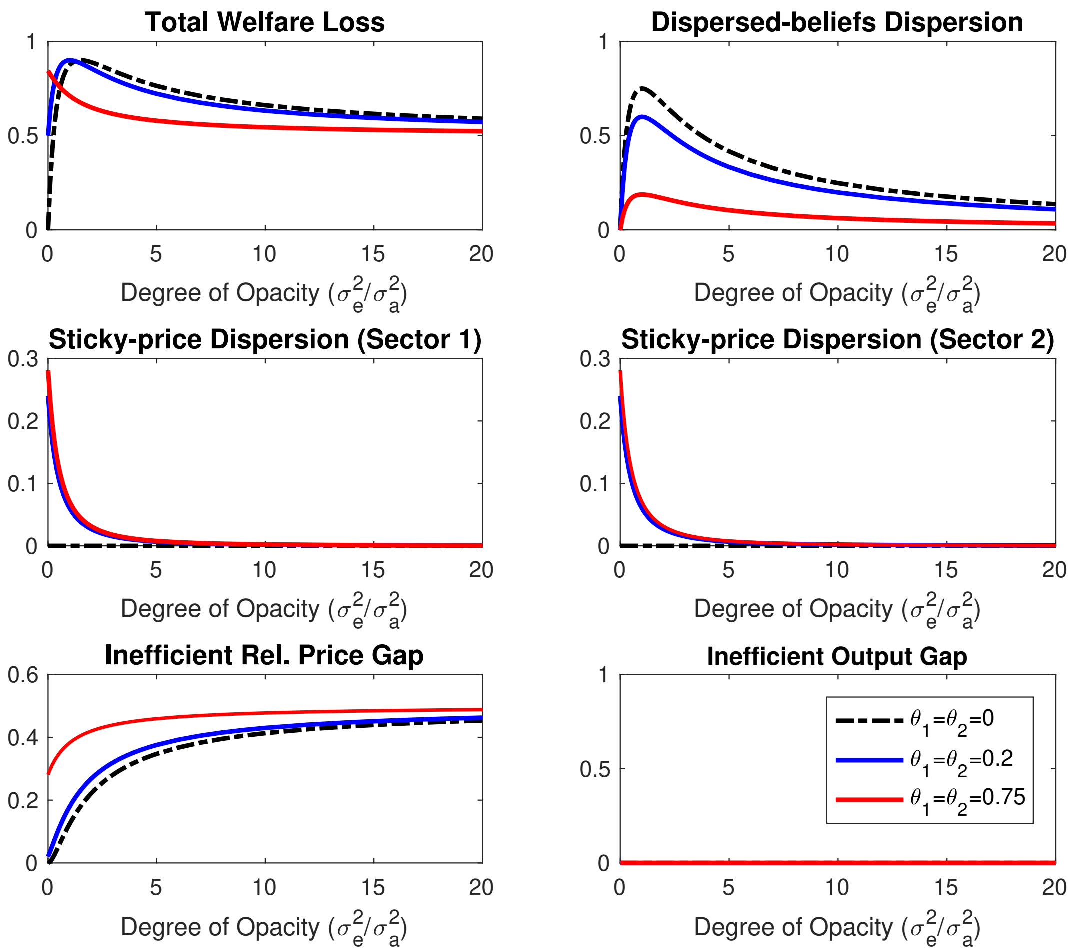

Figure 4: This figure plots the welfare consequence of central bank communication in a symmetric multi-sector economy under the optimal monetary policy. The black, blue, and red lines correspond to cases where $\theta_1 = \theta_2 = 0, 0.2$, and 0.75, respectively. The top left panel plots the total welfare loss, and the remaining panels plot the subcomponents of welfare loss as functions of the degree of opacity σ_e/σ_a. The remaining calibrations of the model are as follows: $\epsilon = 6$, $\eta = 1$, and $\sigma_a = 1$.

Second, the conduct of monetary policy interacts with communication policy. The threshold value of nominal rigidity ($\overline{\theta}$) now depends on Ω, which is defined as the gap between the monetary policy rule and OII stabilization policy. In particular, $\overline{\theta}$ increases slightly as the monetary policy rule moves toward optimal inflation stabilization policy; that is, Ω decreases. In other words, a worse monetary policy is associated with a larger parameter space for θ such that full opacity is optimal.

These two results rely on the condition $\eta < \frac{\epsilon}{2} + (\frac{\epsilon}{2} - 1)\Omega$, which is related to, but weaker than, the $\epsilon > 2\eta$ condition discussed in Section 3.3.2. In summary, based on a two-sector model, we have demonstrated that the optimality of the central bank communication depends on the degree of nominal rigidity and the conduct of monetary policy.

Discussions Proposition 4 to 6 holds true regardless of whether the communication pertains to productivity in a specific sector or all sectors simultaneously. This is because both sector shocks and the signals are independent. Therefore, communication about a particular sector does not exert any effect on the impact of other sectoral shocks, and the validity of the proposition remains unaffected.

3.4 Generalization: the Signal and Shock Structure of the Model

We have presented a micro-founded NK model that incorporates the main mechanism highlighted in Section 2. The baseline model is stylized in its signal structure; specifically, firms are restricted to observing only one signal about one shock in an ad-hoc fashion. In Section 3.4.1, we generalize the signal structure of the model and demonstrate that the ad-hoc assumption made in the baseline model has a micro-foundation. Simultaneously, the baseline model is rich and contains many features. Specifically, it is a general equilibrium model where monetary policy plays a crucial role, and it features asymmetric shocks. Section 3.4.2 clarifies the roles of these two key features.

3.4.1 The Generalization of Signal Structure

This section micro-founds the stylized signal structure adopted in the baseline model. Specifically, it shows that rational inattentive firms, who have access to a continuum of signals and face an attention constraint, will endogenously choose to pay attention to one signal. A similar result is first shown by Mondria (2010) and Li and Wu (2016). Details of the setting and proofs can be found in a companion paper Ou et al. (2023).

Let $\boldsymbol{a} = (a_1, a_2)'$ and $\boldsymbol{s}_{ki} = (s_{ki,1}, s_{ki,2})'$ denote the vectors of sectoral shocks and signals, respectively. Firms have the access to a continuum of signals of the following structure:

$$\boldsymbol{s}_{ki} = \widehat{\boldsymbol{M}}_k \boldsymbol{a} + \boldsymbol{e}_{ki}. \tag{3.13}$$

where $\widehat{\boldsymbol{M}}_k$ is a 2×2 matrix. The observational errors are encapsulated in the 2-dimensional Gaussian vector $\boldsymbol{e}_{ki} = (e_{1,ki}, e_{2,ki})'$ with zero mean and a variance-covariance matrix denoted as $\widehat{\Sigma}_{e,k}$. Firms have access to a continuum of signals, each of which is associated with one matrix $\widehat{\boldsymbol{M}}_k$ and one matrix $\widehat{\Sigma}_{e,k}$. To easy the notation burden, we omit $\widehat{\boldsymbol{M}}_k$ and $\widehat{\Sigma}_{e,k}$ from $\boldsymbol{s}_{ki}$.[15]

Rationally inattentive firms face an attention constraint and, as a result, do not consider the entire population of signals. Firms endogenously choose which signal(s) to pay attention to, i.e., the choice of $\widehat{\boldsymbol{M}}_k$, and how much attention to be paid to that signal(s), i.e., $\widehat{\Sigma}_{e,k}$.

Formally, firms choose $\boldsymbol{M}_k$ and the variance-covariance matrix of the noise $\Sigma_{e,k}$ to minimize the profit loss arising from the information frictions:[16]

$$\min_{\{\boldsymbol{M}_k,\Sigma_{e,k}\}} \frac{\epsilon-1}{2} E[(p^*_{ki} - p^\diamond_{ki})^2 \, | s^*_{ki}] \tag{3.14}$$

Subject to the constraint on information flow:

$$\frac{det\left(\boldsymbol{M}_k \boldsymbol{\Sigma}_{aa} \boldsymbol{M}'_k + \Sigma_{e,k}\right)}{det\left(\Sigma_{e,k}\right)} \quad \leq \quad 2^{2\kappa_k}, \tag{3.15}$$

Where $p^\diamond_{ki}$ denotes the profit-maximizing price for firm ki had this firm fully observe the state of the economy, defined as $p^\diamond_{ki} = p + (y - y^e) + u_k$. p^*_{ki} is the optimal price conditional on the information set of the firm $\{ki\}$, i.e., $p^*_{ki} = E(p^\diamond_{ki} | s^*_{ki})$. Equation (3.15) characterizes the constraint faced by firms when choosing the amount of information contained in the signal s^*_{ki}. κ_k measures the degree of information frictions. The term *det* denotes the determinant of a matrix.

Proposition 7. *Consider the economy described above with a general signal structure where firms have access to a continuum of signals of the functional form:* $\boldsymbol{s}_{ki} = \widehat{\boldsymbol{M}}_k \boldsymbol{a} + \boldsymbol{e}_{ki}$ *for any* $\widehat{\boldsymbol{M}}_k$ *and variance-covariance matrix* $\widehat{\Sigma}_{e,k}$*. A firm ki optimally chooses to narrow its attention to one signal:* $s_{ki} = u + e_{ki}, e_{ki} \sim N(0, \sigma^2_{e,k})$ *and* $u = a_2 - a_1$.

Proof. See Appendix B. □

The intuition is that u and the belief about u are sufficient states for both firms' profit optimization and for solving the model. In the baseline model, we allowed firms to observe one signal about each shock separately. However, the results of central bank communication presented in Propositions 2 to 6 are not affected, or are observationally equiv-

[15] A more rigorous notation is $\boldsymbol{s}_{ki}(\widehat{\boldsymbol{M}}_k, \widehat{\Sigma}_{e,k})$.

[16] $\boldsymbol{M}_k$ and $\Sigma_{e,k}$ are linear transformations of $\widehat{\boldsymbol{M}}_k$ and $\widehat{\Sigma}_{e,k}$, respectively; see Appendix B.

alent, if firms only observe $s_{ki} = u + e_{ki}$ and central bank communication aims to reduce $\sigma^2_{e,k}$.

In the setting with rational inattention, the degree of information frictions ($\sigma^2_{e,k}$) is determined by κ_k, which measures the inattention constraint. Recent developments in AI technology reduce information processing costs and can be viewed as an increase in κ_k.

Note that the noise variance $\sigma^2_{e,k}$ decreases monotonically as the information processing capacity κ_k expands. This is equivalent to the previous case where central bank communication increases the precision of information. If κ converges to infinity, then the information in the economy is perfect, identical to the full transparency case in our previous discussion.

Proposition 8 illustrates that our baseline findings, where the social value of information depends on the degree of nominal rigidities, carry over to the scenario with rational inattention.

Proposition 8. *Consider a two-sector economy in which the central bank conducts a price index stabilization policy. The welfare implications of the progress in information processing capacity (κ_k) are identical to the optimal central bank communication strategy characterized by Propositions 2 to 6.*

Proof. See Appendix B. □

3.4.2 The Generalization of Shock Structure

Next, we discuss the role of asymmetric shocks and generalize the shock structure.

Without Asymmetric Shocks We begin by considering a special case of the model that involves only aggregate shocks, specifically, $a_1 = a_2 = a$. The following result can be derived.

Proposition 9. *In a two-sector economy where the central bank implements a price index stabilization policy, the economy experiences only aggregate shocks, i.e., $a_1 = a_2 = a$. The first-best allocation is achieved independently of central bank communication.*

Proof. See Appendix B. □

At the core of this finding is that in the Phillips curve, the trade-off between price stabilization and output stabilization arises only if there are asymmetrical shocks that affect

a_1 and a_2, meaning $a_1 - a_2 \neq 0$. In the absence of asymmetry ($a_1 - a_2 = 0$), price stabilization leads to the closure of the output gap, and the first-best allocation is achieved. Consequently, central bank communication becomes irrelevant.

In the previous example, the irrelevance of information or central bank communication stems from the absence of trade-off shocks combined with an effective monetary policy. However, the baseline results presented in Section 3.3 deviate from this special case by introducing asymmetric shocks, preventing the attainment of the first-best allocation. In the next example, we illustrate that a similar result can be obtained without asymmetric shocks, but with an inefficient monetary policy–specifically, a nominal demand stabilization policy

Proposition 10. *Consider a two-sector economy in which the central bank conducts a nominal demand stabilization policy: $p + y = 0$. The economy features aggregate shocks only, i.e., $a_1 = a_2 = a$. A threshold level of nominal rigidity, denoted as $\overline{\theta} = 1/\epsilon$, exists such that*

1. *Full transparency is optimal if $\theta < \overline{\theta}$.*
2. *Full opacity is optimal if $\theta > \overline{\theta}$.*

Proof. See Appendix B. □

Proposition 10 illustrates that central bank communication plays a role when the economy is not at its first-best allocation. In such cases, aggregate shocks lead to fluctuations in prices, generating welfare loss and resulting in a non-negligible sticky-price dispersion component of the welfare loss. Consequently, the social value of information depends on the degree of nominal rigidity, similar to the baseline model.

Heterogeneous Exposure to Common Aggregate Shocks We have emphasized the importance of asymmetric shocks by assuming exogenous sector-specific shocks. In reality, these shocks can materialize as, for instance, oil shocks affecting the oil sector or supply chain disruptions impacting specific sectors.

However, it is crucial to note that the mechanism we highlight can remain applicable even if the underlying shocks are aggregate. Proposition 11 specifically demonstrates that the baseline findings remain unaffected in an economy with aggregate shocks only, but with heterogeneous exposures to these common shocks across sectors.

Proposition 11. *Consider a two-sector economy in which the central bank conducts a price index stabilization policy. The economy features aggregate shocks only, but sectors have heterogeneous* *exposures to a common shock. That is,* $a_1 = \beta_1 a$ *and* $a_2 = \beta_2 a$*. The condition characterizing the optimal central bank communication about* a *is identical to Propositions 2 to 6.*

Proof. See Appendix B. □

For central bank communication to be relevant, it requires the economy to deviate from the first-best allocation. This is evident in the economy described in Proposition 11 because, as observed in the Phillips curve, heterogeneous exposure to aggregate shocks gives rise to an endogenous trade-off between stabilizing the output gap and price: $u = a_2 - a_1 = (\beta_2 - \beta_1)a \neq 0$. In such an economy, business cycle fluctuations due to aggregate shocks a adversely impact social welfare, and the social benefits and costs of information highlighted in the baseline carry over to this case.

A General Shock Structure One stylized feature of the baseline model is that shocks are sector-specific and uncorrelated across sectors. This section demonstrates that relaxing this assumption to allow for aggregate shocks and an arbitrary positive correlation between a_1 and a_2 does not affect the baseline findings.

Specifically, we consider the following productivity process:

$$a_1 = a + \xi_1$$
$$a_2 = a + \xi_2$$

where ξ_1 and ξ_2 are independent and identically distributed (i.i.d.) sector-specific shocks, and a is the aggregate shock. The shocks are drawn from normal distributions with mean zero and $\text{Var}(a) = \sigma_a^2$, $\text{Var}(\xi_1) = \text{Var}(\xi_2) = \sigma_\xi^2$. In this setting, the correlation between a_1 and a_2,

$$\text{corr}(a_1, a_2) = \frac{\sigma_a^2}{\sigma_a^2 + \sigma_\xi^2}$$

is an arbitrary positive number between zero and one depending on the relative variances.

This general productivity process encompasses the baseline setting as a special case, where $\sigma_a^2 = 0$ and $\text{corr}(a_1, a_2) = 0$. In the other extreme case, if $\sigma_\xi^2 = 0$, then $\text{corr}(a_1, a_2) = 1$, and the economy is the same as the one described in Proposition 9. Henceforth, we exclude this special case and consider $\sigma_\xi^2 > 0$.

The signal structure is the same as in the baseline model, see Equation 3.7. Similar to the baseline, we assume economy-wide central bank communication, i.e., $\sigma_{1,e}^2 = \sigma_{2,e}^2 = \sigma_e^2$, and central bank communication refers to changes in σ_e^2.

Proposition 12. *Consider a two-sector economy in which the central bank conducts a price index stabilization policy. The economy features both aggregate and sectoral shocks, i.e., $a_1 = a + \xi_1$ and $a_2 = a + \xi_2$. The condition characterizing the optimal central bank communication strategy is identical to Propositions 2 to 6.*

Proof. See Appendix B. □

Proposition 12 demonstrates that the baseline results remain applicable in an economy with a general shock structure, incorporating both aggregate shocks and an arbitrary positive correlation between sectoral productivities.

The intuition is as follows: In an economy with both aggregate shocks a and sector-specific shocks ξ_1 and ξ_2, given aggregate shocks a, Proposition 9 applies, and the price stabilization policy manages to achieve the first-best allocation. This is attributed to effective monetary policy: the price stabilization policy fully stabilizes aggregate shocks. Consequently, in terms of implications for the social value of information, the model behaves as if there were no aggregate shocks.

Communication Through Aggregate Endogenous Variables The baseline model explores the social value of information by manipulating the precision of exogenous signals about sector-specific shocks. In reality, it might be more efficient for the central bank or any other information provision institute to convey more information through an aggregate variable, such as the average price or the aggregate output gap ($\tilde{y}$) in the economy. We will now discuss the generalization of our findings to such a policy. Without the loss of generality, let's consider the situation where the central bank communicates via the aggregate output gap $\tilde{y}$.

Note that the aggregate output gap is a linear function of $(a_2 - a_1)$, denoted as $\tilde{y} = \phi_x(a_2 - a_1)$. As discussed earlier, the relevant state variable in the economy is the asymmetric component of shocks (u), with $u = a_2 - a_1$. Suppose firms observe the output gap with noise. That is, firms observe $\tilde{y}^s = \tilde{y} + e$ with $e \sim N(0, \sigma_e^2)$. Information provision, or central bank communication, refers to the reduction of noise contained in the aggregate output gap signal, denoted as σ_e. Intuitively, the mechanisms we highlight are present, and the results are not qualitatively affected. However, there is a complication in such a setting.

The main results are not unaffected qualitatively. To see this, relabel $\tilde{y}^s$ as s. The setting described above–communication via an aggregate variable–is similar to a setting with exogenous signals $s = \tilde{y} + e = \phi_x u + e$. If s, a signal about the aggregate economy, is exogenous, then the baseline findings of Propositions 2 to 6 are unaffected.

However, the complication arises because the output gap is endogenous, i.e., ϕ_x depends on central bank communication. As a result, the output gap signal $\tilde{y}^s$ becomes an endogenous signal. Due to this complication, analytically characterizing the conditions for the optimal communication strategy in our setting is not possible.

We show the social value of information in this scenario quantitatively. Note that the aggregate variable, $\tilde{y}^s$, is only a signal if it responds to the relevant state of the economy. Under the CPI stabilization policy, the output gap is closed, i.e., $\tilde{y} = 0$ in an economy with symmetric characteristics. For $\tilde{y}^s$ to fluctuate, we need to consider an economy with asymmetric characteristics, such as the one underlying Proposition 4. Figure 5 shows that a similar result holds if communication works through an endogenous price signal: more information hurts (improves) social welfare if nominal rigidity is high (low); see the red (blue) lines.

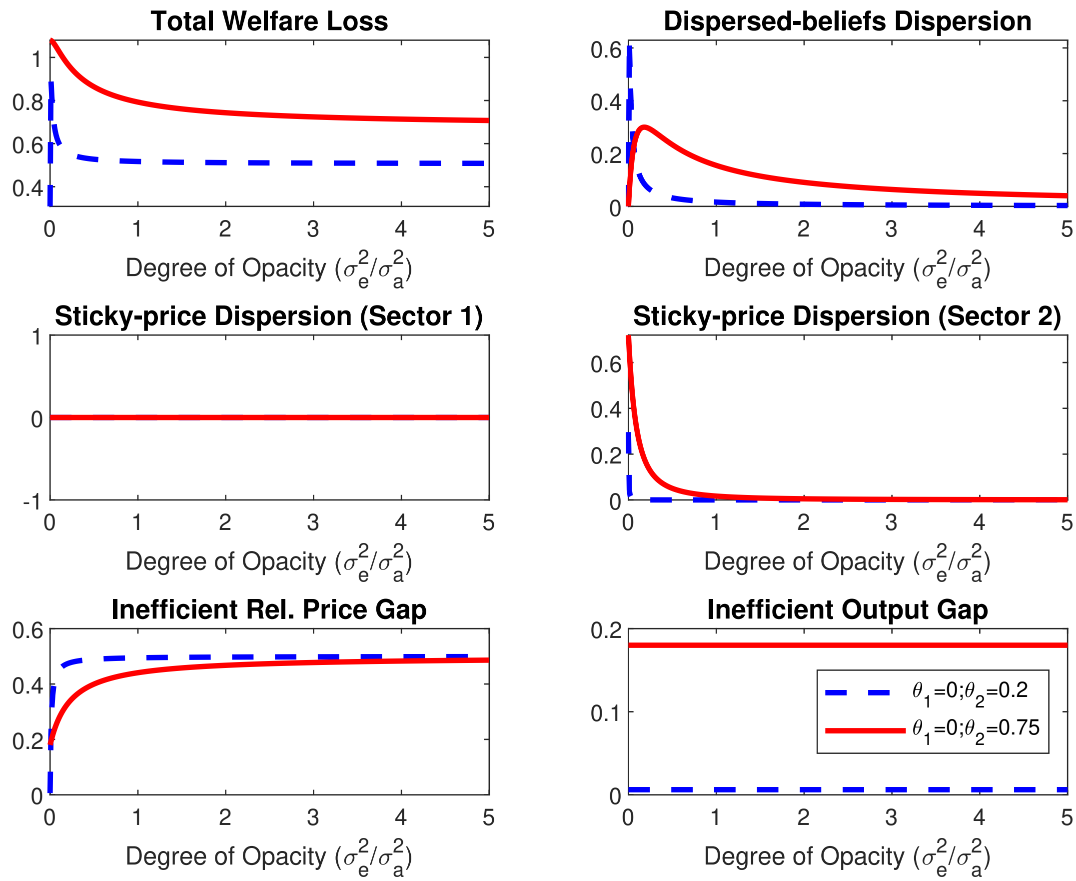

Figure 5: This figure plots the welfare consequence of central bank communication, via an endogenous signal $\tilde{y}^s = \tilde{y} + e$, in an economy that consists of one flexible-price sector ($\theta_1 = 0$) and one sticky-price sector. The solid red (dashed blue) lines correspond to the case in which $\theta_2 = 0.75$ ($\theta_2 = 0.2$). The top left panel plots the total welfare loss, and the remaining panels plot the subcomponents of welfare loss as functions of the degree of opacity σ_e/σ_a. The remaining calibration of the model is as follows: $\epsilon = 6$, $\eta = 1$, and $\sigma_a = 1$.

3.5 Relationship with Literature

This section discusses our paper's relationship to the extant literature, namely, Angeletos and Pavan (2007) (see also Baeriswyl and Cornand 2010, and Angeletos et al. 2016), and Fujiwara and Waki (2022).

Relationship with Angeletos and Pavan (2007) We demonstrate how to relate our findings to the theories shown by Angeletos and Pavan (2007).

Angeletos and Pavan (2007) define that a shock in an economy is efficient (inefficient) if the equilibrium outcomes coincide with (deviate from) the first-best allocation under full information. In a game-theoretical model, Angeletos and Pavan (2007) then prove that it is optimal to be fully transparent about efficient shocks; however, the optimal communication strategy concerning inefficient shocks is *ambiguous*. Baeriswyl and Cornand (2010) and Angeletos et al. (2016) demonstrate similar findings in macroeconomic models. These previous studies made convincing claims that communicating efficient shocks – aggregate supply shocks – is optimal.

Our paper contributes to this discussion by presenting a micro-founded model in which asymmetric supply shocks are inefficient. That is, asymmetric supply shocks generate welfare losses even without information frictions. We emphasize that the *degree of inefficiency* of the economy depends on the degree of nominal rigidity — the central message in Proposition 6. Another way to understand Proposition 6 is: the full opacity is optimal if the degree of inefficiency surpasses a certain threshold. The following exercise illustrates this interpretation.

Building upon Angeletos and Pavan (2007), we define the degree of inefficiency of a shock in the economy as the welfare loss (deviation from the first-best allocation) that the shock generates under full information. The solid red line in Figure 6 plots the degree of inefficiency of asymmetric shocks as a function of nominal rigidities in the economy. The economy we considered is a two-sector model with symmetric characteristics (e.g., $\theta_1 = \theta_2 = \theta$), but sectors are subject to sector-specific supply shocks. The central bank conducts an OII policy. As one can see, the degree of inefficiency is a hump-shaped function of price rigidity. It is hump-shaped because the sticky-price dispersion component of the welfare loss function is maximized in an interior point where there is a roughly equal mass of price resetting and price-staggered firms.[17] The result is virtually unaffected if we allow the central bank to conduct a Ramsey optimal monetary policy: i.e., allowing the central bank to react optimally conditional on shocks.

The dashed black line in Figure 6 plots the welfare loss in the same economy but with information frictions and a fully opaque central bank.[18] The intersection of the two curves (blue circle) determines the threshold level of nominal rigidity $\bar{\theta}$ in proposition 6 when full transparency and opacity are indifferent. When the inefficiency of the economy surpasses the threshold level, full opacity is superior to full transparency. Note that Figure 6 is helpful for the comparison of the full opacity with the full transparency allocations.

[17] The sticky-price dispersion component is minimized at the two corners where either all or no firms can reset prices.

[18] Note that under full opacity, the welfare loss of the economy is the same as when the price is fully rigid under perfect information because the price will not adjust under either case.

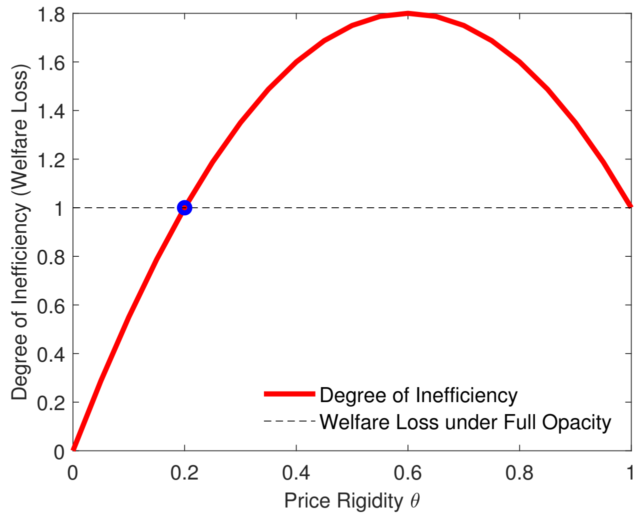

Figure 6: This figure plots the degree of inefficiency (welfare loss) against the degree of nominal rigidity in the economy under perfect information in a symmetric multi-sector economy under the OII targeting policy. The solid red line plots the degree of inefficiency. The dashed black line corresponds to the value of the total welfare loss in the same economy but with information frictions and a fully opaque central bank.

The readers are referred to the detailed analysis conducted above, for example, Figure 4, to fully understand the welfare losses in an intermediate case and the parameters that determine the cutoff point (blue circle).

Moreover, the degree of inefficiency of the economy also depends on the conduct of monetary policy. This result is captured by Ω in Proposition 6. As discussed above, the Ramsey optimal monetary policy cannot overturn our findings due to the asymmetric nature of supply shocks, which is confirmed in a quantitative analysis conducted in Section 4. Our findings can be generalized to an economy whose fluctuations are generated by *symmetric* supply shocks with a suboptimal monetary policy.

Another related paper is Angeletos et al. 2016. Our paper differs from Angeletos et al. (2016) in two important dimensions. First, Angeletos et al. (2016) defined nominal rigidity as suboptimal pricing decisions resulting from information frictions. We consider price stickiness supported by infrequent price adjustments observed in the data. Second, in Angeletos et al. (2016)'s setting, an optimal monetary policy can un-do nominal frictions. They investigate optimal communication results conditional on an optimal monetary policy replicating flexible price allocations. In our setting, if a policy that fully addresses nominal frictions existed, together with a fully transparent central bank, one would replicate the first best allocation. However, in a multi-sector setting, monetary policy cannot replicate flexible price allocations (see, e.g., Woodford 2003). Our discussions of the central bank communication rest on the fundamental result that there is welfare loss arising from nominal frictions.

Relationship with Fujiwara and Waki (2022) In their study, Fujiwara and Waki (2022) demonstrate that, within the standard New Keynesian framework, advanced information, or news, regarding future shocks can result in a reduction in welfare. Their analysis is grounded in the observation that news about any type of aggregate shock effectively functions as additional noise in the Phillips curve, akin to mark-up shocks. In other words, forward information generates inefficient shocks, amplifying inflation volatility and output gaps. They further illustrate that the optimal communication strategy depends on the specific model, the type of shocks involved, and the frictions incorporated in the model.

The most significant contribution of our paper, in contrast to Fujiwara and Waki (2022), is our focused emphasis on the role of the 'two-agent' feature in the model, which influences the social value of information. Our model, specifically tailored to analytically illustrate both the social costs and benefits of information, provides the analytical conditions that characterize the optimal communication strategy depending on the degree of nominal rigidities.

Furthermore, our paper distinguishes itself from Fujiwara and Waki (2022) in several aspects. First, we emphasize communication about *current* shocks within a model featuring *incomplete* information and dispersed beliefs. Second, our model diverges from the first-best economy by incorporating asymmetric shocks in a multi-sector setting. Our approach introduces two distinctive features: (i) optimal policy cannot achieve the first best, and (ii) asymmetric shocks exhibit both efficient and inefficient characteristics. The second feature ultimately underpins the propositions outlined in our paper: the social value of information depends on the degree of nominal rigidities.

4 Fifteen-sector Model

In this section, we extend the two-sector model to a fifteen-sector economy in order to provide a quantitative evaluation. The key parameters, nominal rigidities and sectoral sizes, are taken from the U.S. data.

4.1 Static model: fifteen-sector

A general multisector model consists of the following optimal price setting rules:

$$p^*_{k,i} = E[p + \tilde{y} + \sum_{k=1}^{N_k} n_k a_k - a_k | \mathcal{I}_{1i}] \ \forall\ k = 1, 2, ..., N_k \tag{4.1}$$

where $\tilde{y} \equiv y - y^e$ denotes the output gap and $y^e = \sum_{k=1}^{N_k} n_k a_k$. Moreover, $p_k = (1-\theta_k) p^*_{k,i}$ $\forall$ k due to the presence of nominal rigidity.

Firms set prices subject to information frictions: productivities are not observable. Firm i in sector k receives signal $s_{ki,l}$ about sectoral shock a_l $\forall l \in 1, 2, ..., N_k$:

$$s_{ki,l} = a_l + e_{ki,l}$$

with $e_{ki,l} \sim N(0, \sigma^2_{l,e})$ and $a_l \sim N(0, \sigma^2_{l,a})$. The precisions of signals are common across firms in the entire economy. Firms update their beliefs according to Bayes' rule in (3.8).

The micro-founded welfare loss function in a general multisector model is:

$$E\mathbb{L} = E\left\{ \epsilon \sum_{k=1}^{N_k} n_k \left[(1-\theta_k)\theta_k p^{*2}_k + (1-\theta_k) \int_i (p^*_{ki} - p^*_k)^2 di \right] + \sigma x^2 + \eta \sum_{k=1}^{N_k} n_k \widetilde{p}^2_{R,k} \right\},$$

where $\widetilde{p}_{R,k} = (p_k - p) - (p^e_k - p^e)$ and p^e is the prevailing price in the absence of nominal and information frictions.

A monetary policy rule is required to close the model. As it is done above, we consider three types of monetary policy rule: (i) CPI stabilization, (ii) OII stabilization, and (iii) optimal monetary policy.

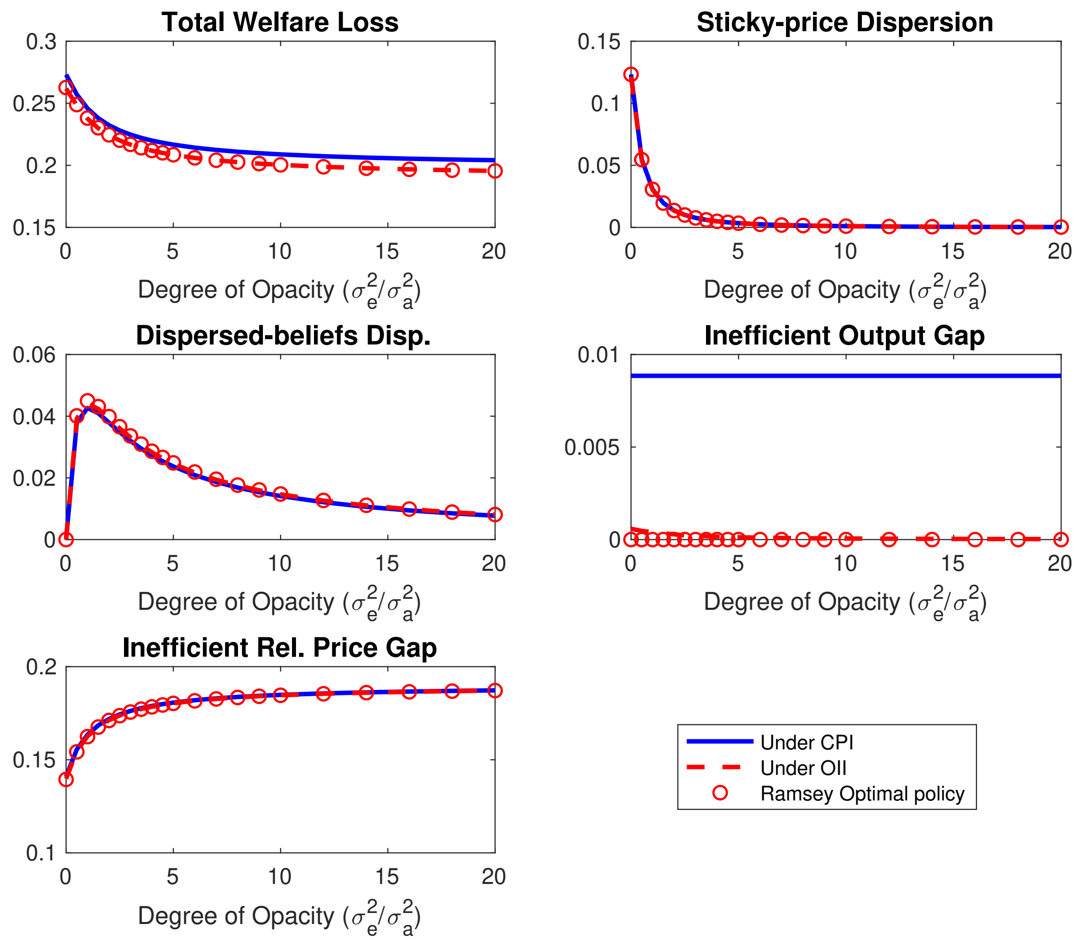

Figure 7: This figure plots the welfare consequence of central bank communication in a fifteen-sector static model under alternative monetary policy rules. The blue lines, dashed red lines, and red circles correspond to the cases under CPI stabilization, OII stabilization, and Ramsey optimal monetary policy. The top left panel plots the total welfare loss, and the remaining panels plot the subcomponents of welfare loss as functions of the degree of opacity σ_e/σ_a.

The calibration of the model is the following. We calibrate the model to a fifteen-sector economy ($N_k = 15$). Sectors share the same standard deviation for productivity shocks ($\sigma = 0.02$) and the same elasticity of substitution within each sector ($\epsilon = 6$). The signed value of ϵ is consistent with the empirical estimates provided by Rotemberg and Woodford (1995) and Basu and Fernald (1997). The elasticity of substitution across sectors (η=1) is taken from Hobijn and Nechio (2019). The degree of nominal rigidity (θ_k) and sector sizes (n_k) are taken from Eusepi et al. (2011) (see Table 1 of the current study), who extended Nakamura and Steinsson (2008)'s estimates of nominal rigidity to 15 sectors. According to this calibration, the median duration of prices across sectors is approximately nine months, and the average price duration, weighted by sector size, is approximately seven months.

Figure 7 shows the results based on the calibration of the static model with 15 sectors). In contrast to the current practices of central banks, the calibrated model predicts that full opacity is optimal. As discussed above, the result is driven by the sticky-price component. Moreover, compared with the CPI stabilization policy, the Ramsey optimal monetary policy reduces social welfare loss. However, optimal monetary policy does not alter the prediction that central bank communication reduces social welfare. What drives these results is the significant nominal rigidity revealed by Nakamura and Steinsson (2008) and Eusepi et al. (2011). These quantitative results are consistent with the analytical results shown in Proposition 6.

4.2 Counterfactual Analysis

Alternative calibrations of ϵ Propositions 4 and 6 demonstrate the relevance of the value of ϵ, relative to the value of η, for the central bank's optimal communication strategy. The following robustness check evaluates the welfare consequence of central bank communication in a fifteen-sector static model using $\epsilon = 3$, a value corresponding to the steady-state markup of 50%. Figure 8 shows that the results presented above are robust to using a small value of ϵ.

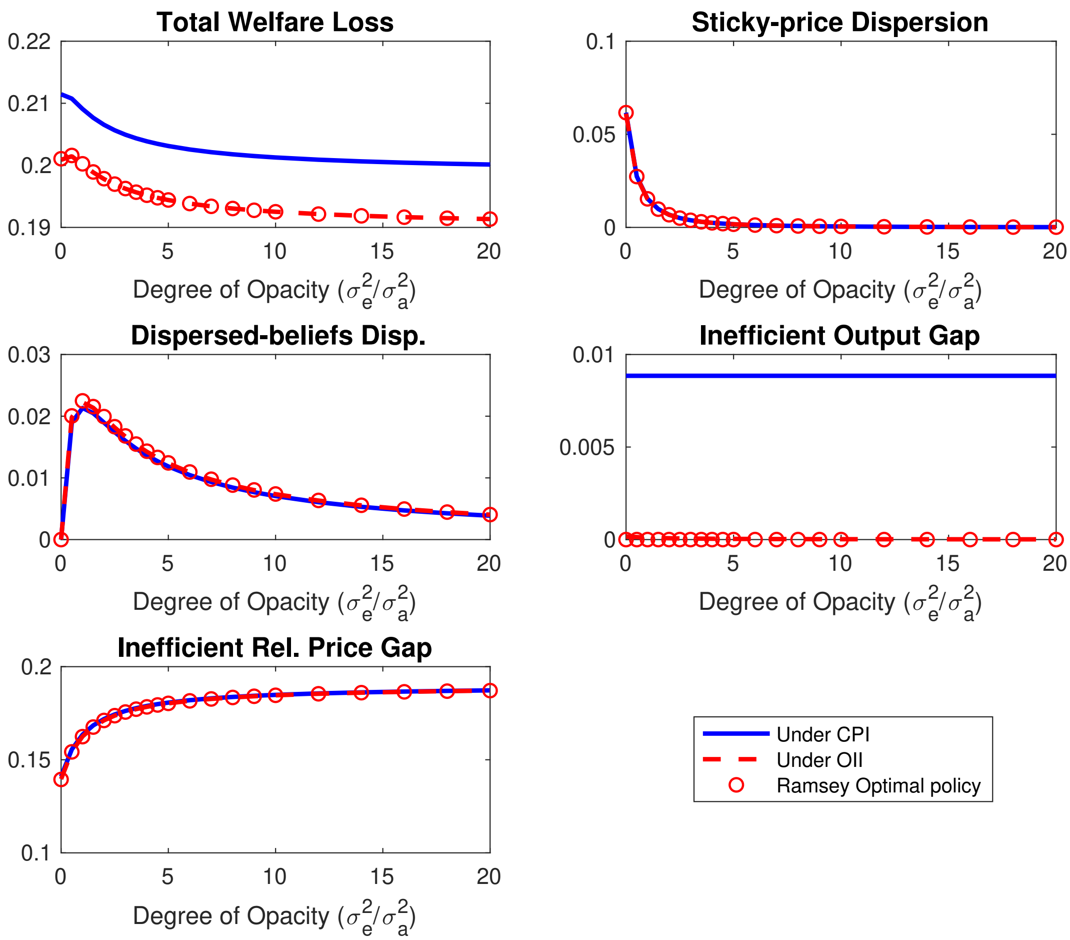

Figure 8: This figure plots the quantitative result using $\epsilon = 3$. The figure plots the welfare consequence of central bank communication in a fifteen-sector static model under alternative monetary policy rules. The blue lines, dashed red lines, and red circles correspond to the cases under CPI stabilization, OII stabilization, and Ramsey optimal monetary policy. The top left panel plots the total welfare loss, and the remaining panels plot the subcomponents of welfare loss as functions of the degree of opacity σ_e/σ_a.

Alternative calibrations of θ_k The degree of nominal rigidities observed in the data determines the quantitative results discussed. This result is formally shown in Propositions 4 and 6: the threshold level of nominal rigidity determines the optimality of central bank communication. How close are the calibrated degrees of nominal rigidity to their threshold values? The following exercise addresses this question.

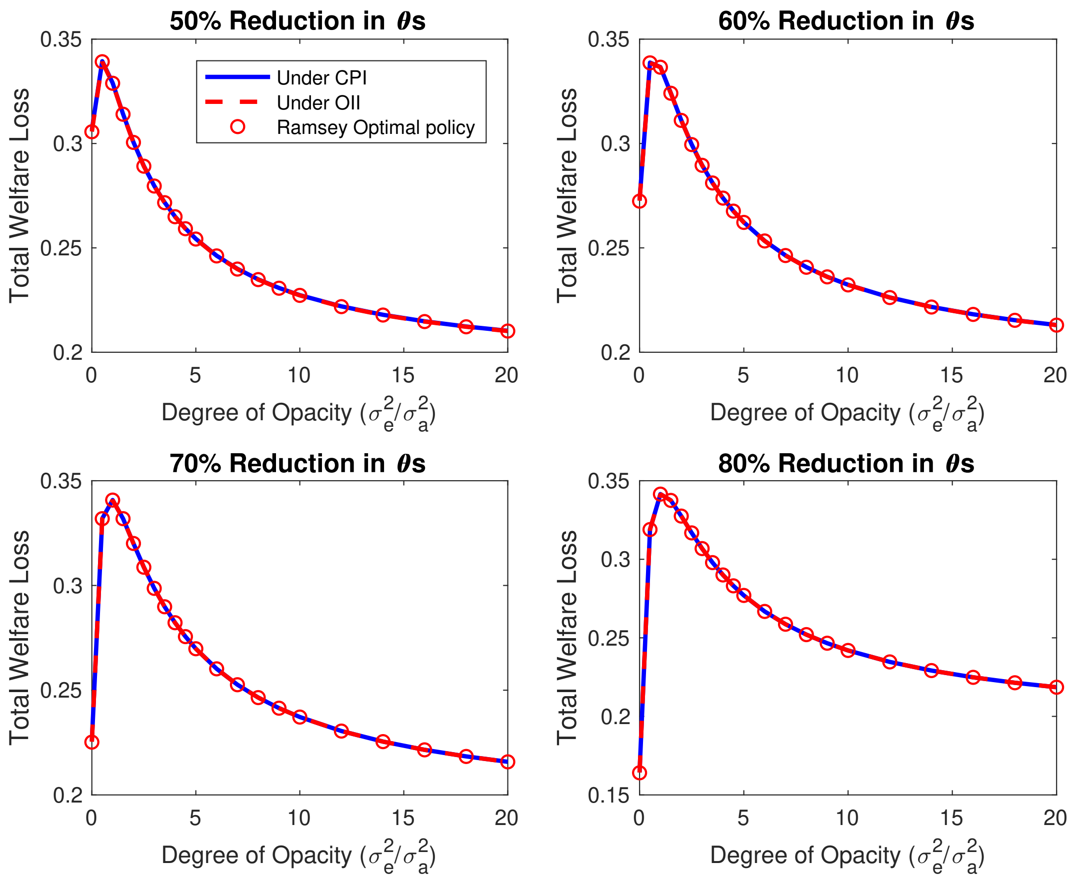

Figure 9: This figure plots the total welfare loss as functions of the degree of opacity σ_e/σ_a under alternative calibrations of nominal rigidities across sector θ_k—reducing θ_k by x% compared with its value in the data. The blue lines, dashed red lines, and red circles correspond to the cases under CPI stabilization, OII stabilization, and optimal monetary policy.

We recalibrate the degree of nominal rigidity across sectors in the model. In particular, we consider four alternative scenarios corresponding to a 50%, 60%, 70%, and 80% reduction in θ_k $\forall k$ compared with its value in the data reported in Table 1. Figure 9 plots the total welfare loss as a function of the degree of opacity σ_e/σ_a for these four alternative scenarios. As one can see, cutting the degree of nominal rigidity by half would not be sufficient for a transparent central bank to be optimal. For increased transparency to be a sensible policy, a reduction is required in nominal rigidity across all 15 sectors by roughly 80%. Such a reduction implies reducing the median duration of prices from 9 months to 1.3 months—a magnitude of decline that is unlikely to occur in the foreseeable future.

5 Extensions

This section provides two extensions of the baseline model. Section 5.1 presents results in a dynamic model. Section 5.2 shows that the findings carry over to a setting in which the central bank communicates via a public signal with a common noise.

5.1 The Fifteen-sector Dynamic Model

Firms and households live in a dynamic economy in the real world with persistent shocks. The following analysis shows that this dynamic feature does not alter the quantitative results described above.

We extend the household and firms' problems to their dynamic versions. The productivity in sector k, denoted as $a_{k,t}$, consists of two components: the common component a_t and the sector-specific components $\xi_{k,t}$. Particularly, it follows the process: $a_{k,t} = a_t + \xi_{k,t}$, $a_t = \rho a_{t-1} + \eta_t$, and $\xi_{k,t} = \rho \xi_{k,t-1} + v_{k,t}$. Here, η_t and $v_{k,t}$ are stochastic processes with the distributions $N(0, \sigma_\eta^2)$ and $N(0, \sigma_{v,k}^2)$, respectively. The firm ki receives a noisy signal about the sectoral productivity $a_{l,t}$, i.e., $s_{ki,l,t} = a_{l,t} + \epsilon_{l,t}$, where $\epsilon_{l,t} \sim N(0, \sigma_{e,l}^2)$.

In this framework, firms' profits depend on the actions of other firms. Therefore, price-setters need to consider the actions of the other firms. In the presence of information frictions and idiosyncratic noises, agents need to form beliefs about other agents' actions. Moreover, firms need to form expectations about other firms' beliefs about their actions. In other words, higher-order expectations matter in firms' decisions. As a result, the sectoral imperfect-common-knowledge Philips curve depends on the higher-order expectations.

Let $x_{t|kt}^{(1)}(i) \equiv E(x_t | \mathbb{I}_{k,t}(i))$ denote an agent i's first-order expectation about the unobserved state x_t. Then, the average first-order expectation is $x_{t|kt}^{(1)} \equiv \int E(x_t | \mathbb{I}_{k,t}(i)) di$. Similarly, the average jth order expectation is $x_{t|kt}^{(j)} \equiv \int E(x_{t|t}^{(j-1)} | \mathbb{I}_{k,t}(i)) di$. Using this notation, the imperfect-common-knowledge Philips curve for each sector k is

$$\pi_{k,t} = (1-\theta_k)(1-\beta\theta_k) \sum_{j=1}^{\infty} (1-\theta_k)^{j-1} (\widehat{mc}_{kt|kt}^{(j)} - \widehat{p}_{R,kt|kt}^{(j)}) + \beta\theta_k \sum_{j=1}^{\infty} (1-\theta_k)^{j-1} \pi_{kt+1|kt}^{(j)},$$

where $\widehat{mc}$ represents the marginal cost. The information structure is the same as that discussed above. Note that, for tractability, we assume a_{t-1} and $\xi_{k,t-1}$ is observed at time t. A detailed description of the equilibrium conditions can be found in Appendix B.1.

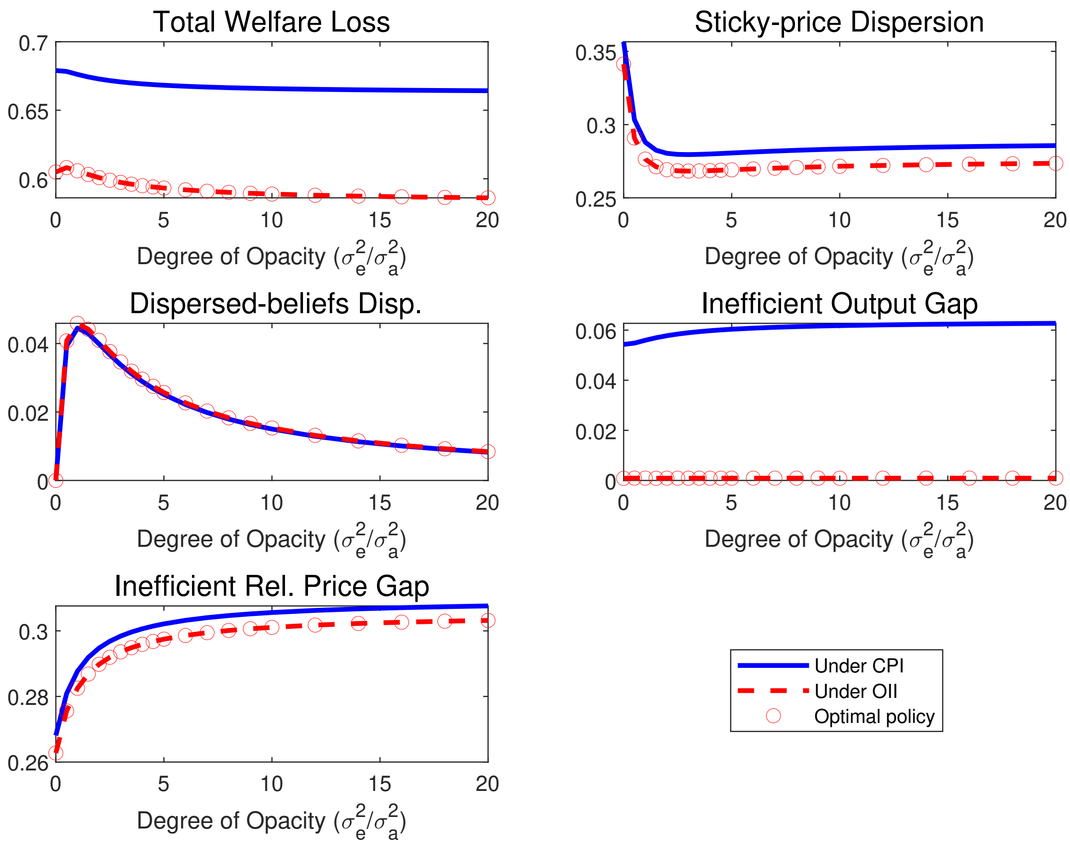

Figure 10: This figure plots the welfare consequence of central bank communication in a fifteen-sectors dynamic model under alternative monetary policy rules. The blue lines, red circles, and dashed red lines correspond to the cases under CPI stabilization, OII stabilization, and time-consistent optimal monetary policy. The top left panel plots the total welfare loss, and the remaining panels plot the subcomponents of welfare loss as functions of the degree of opacity σ_e/σ_a.

We calibrate the standard deviations of the sector-specific component of productivity to match the standard deviations of sectoral inflation in the data. The noise level of signals is calibrated such that the sum of weights assigned to each signal used for estimating each sectoral shock is 0.5, consistent with the estimates of an aggregate Kalman gain of 0.5 by Coibion and Gorodnichenko (2015). The moments in the calibrated model and the data are shown in Table 2. Table 1 reports calibrated parameters.

Figure 10 shows the predictions of the calibrated dynamic model, with 15 sectors. Qualitatively, the results are unchanged compared with the fifteen-sector static model, and the main message remains unchanged: the full opacity is optimal.

5.2 Public Signal

We now consider a setting in which the central bank communicates via a public signal with a common noise.

Two-sector Model with Analytical Results We begin by providing the analytical results based on a two-sector model. The model is identical to that presented in Section 3 with one exception: the noise contained in the public signal released by the central bank is common across firms. Formally, for each sector k, the central bank releases a public signal z_k:

$$z_k = a_k + \epsilon_k^z \tag{5.1}$$

with $\epsilon_k^z \sim N(0, \sigma_{k,z}^2)$. A more transparent central bank is modeled as a public signal with a smaller $\sigma_{k,z}$

Proposition 13. *Consider a two-sector economy with symmetric characteristics, under a price index stabilization policy, and the central bank communication affects the precision of public signals. Let $\Omega \equiv (1-2\omega)^2$ denote the deviation of the monetary policy rule from the optimal inflation stabilization policy. The social welfare loss increases with Ω.*

A threshold level of nominal rigidity $\bar{\theta} \equiv \frac{\eta+\Omega}{(\epsilon-1)\Omega+(\epsilon-\eta)}$ exists such that

1. *full transparency is optimal if $\theta < \bar{\theta}$*
2. *full opacity is optimal if $\theta > \bar{\theta}$;*

Moreover, $\bar{\theta}$ decreases if monetary policy is less optimal: $\frac{\partial\bar{\theta}}{\partial\Omega} \leq 0$

Proof. See the Appendix B. □

Proposition 13 summarizes the analytical results obtained in a two-sector economy with symmetric characteristics and a price index stabilization policy, and the central bank communication affects the precision of public signals. As can be observed, the results presented in Proposition 6 carry over to the current model, in which central bank communication affects the common noise of public signals.[19]

[19]The social welfare is independent of central bank communication if $\theta = \bar{\theta}$.

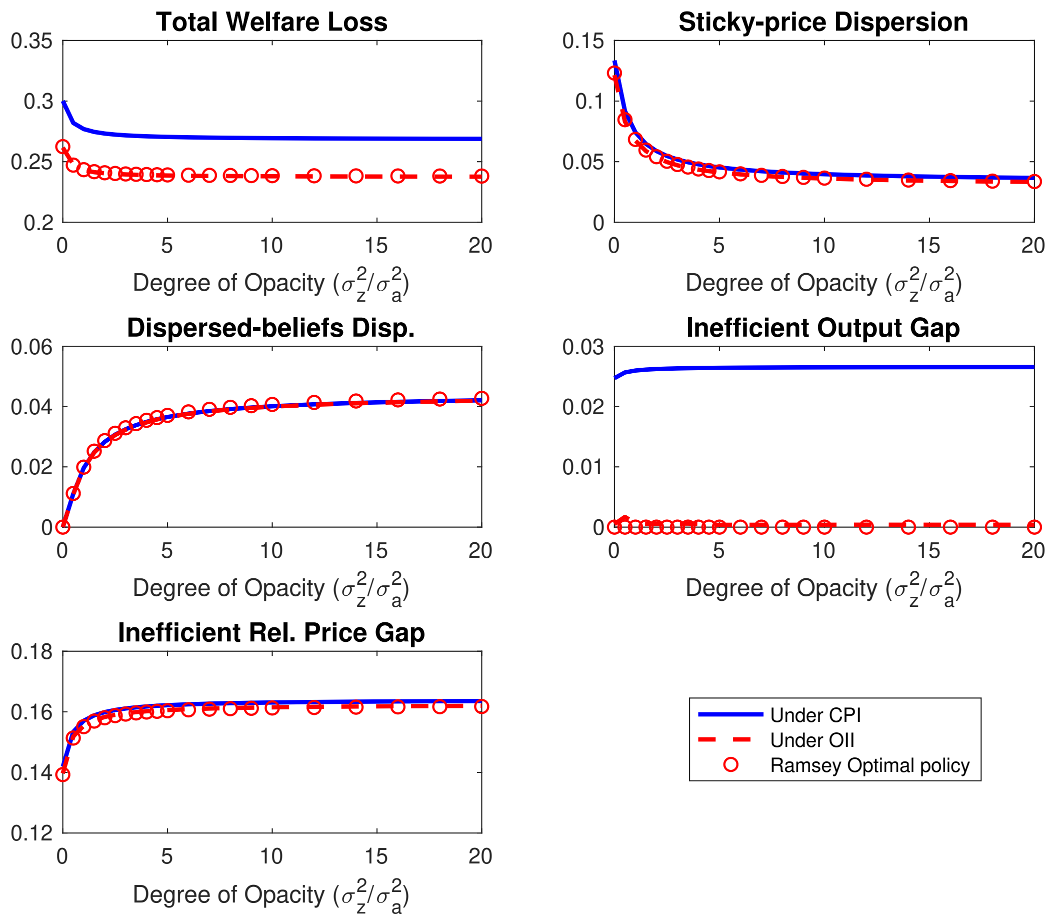

Figure 11: This figure plots the results in a fifteen-sector static model when central bank communication affects the precisions of public signals. The blue lines, dashed red lines, and red circles correspond to the cases under CPI stabilization, OII stabilization, and Ramsey optimal monetary policy. The top left panel plots the total welfare loss, and the remaining panels plot the subcomponents of welfare loss as functions of the degree of opacity σ_z/σ_a.

Fifteen-sector Results Similar to the baseline model, the optimality of the central bank communication is a quantitative question that relies on the degree of nominal rigidity. We extend the two-sector model to a fifteen-sector version. The calibration of the model is the same as that of the baseline fifteen-sector model with one additional feature. For the quantitative analysis, we allow firms to observe a private signal $s_{ki,l}$ in addition to the public signal released by the central bank. The precisions of these private signals are calibrated such that the noise-signal ratios are equal to one—a value that is comparable to the weight that firms assign to new signals, that is, the Kalman gain, estimated in the data, see, for example, Coibion and Gorodnichenko (2012, 2015).The introduction of public signals brings additional state variables. Therefore, under the optimal monetary policy, where$\tau_u = \frac{1}{\sigma_u^2}, \tau_z = \frac{1}{\sigma_z^2}$.

The welfare effects of public information:

$$\frac{\partial W}{\partial \tau_z} = \left\{ \left[2\Delta\varepsilon\theta + (1-\theta)\left(\eta + (1-2\omega)^2\right) - 2\left(\eta + (1-2\omega)^2\right) \right] \tau_z + \left[2\Delta\varepsilon\theta + (1-\theta)\left(\eta + (1-2\omega)^2\right) - 2\left(\eta + (1-2\omega)^2\right) \right] \tau_u \right\} \times \frac{1-\theta}{(\tau_u + \tau_z)^3}$$

where $\Delta = (1-\omega)^2 + \omega^2$.

We focus on the cases that $\theta \in [0,1)$. For the special scenario $\theta = 1$,the social welfare is independent of the precision of information.

Denote $\frac{\partial W}{\partial \tau_z} = f'(\tau_z) = (a\tau_z + b\tau_u) \times \frac{1-\theta}{(\tau_u+\tau_z)^3}, a = 2\Delta\varepsilon\theta + (1-\theta)\left(\eta + (1-2\omega)^2\right) - 2\left(\eta + (1-2\omega)^2\right), b = \left[2\Delta\varepsilon\theta + [(1-\theta) - 2]\left(\eta + (1-2\omega)^2\right)\right]\tau_u, \Omega = (1-2\omega)^2$.

After some algebras:

$a = [2\Delta\epsilon - (\eta + \Omega)]\theta - (\eta + \Omega), b = a$. Then $f'(\tau_z) = a(\tau_z + \tau_u) \times \frac{1-\theta}{(\tau_e+\tau_u+\tau_z)^3}$

We focus on the parameterization $2\Delta\epsilon - (\eta + \Omega) > 0$.

- Case 1: $a > 0 \iff \theta > \frac{\eta+\Omega}{2\Delta\epsilon-(\eta+\Omega)}$,The welfare loss achieves the minimum at $\tau_z = 0$.
- Case 2: $a = 0 \iff \theta = \frac{\eta+\Omega}{2\Delta\epsilon-(\eta+\Omega)}$,The welfare loss is independent of central bank communication;
- Case 3: $a < 0 \iff \theta < \frac{\eta+\Omega}{2\Delta\epsilon-(\eta+\Omega)}$The welfare loss achieves the minimum at $\tau_z = \infty$.

Note that $\Delta = \frac{\Omega+1}{2}, 2\Delta\epsilon - \Omega = \epsilon + (\epsilon - 1)\Omega, \frac{\eta+\Omega}{2\Delta\epsilon-(\eta+\Omega)} = \frac{\eta+\Omega}{(\epsilon-1)\Omega+\epsilon-\eta}$, the proposition is easily verified and it is trivial to show that W is increasing in Ω. □

B.1 Equilibrium Conditions

In this section, we collect the equilibrium conditions for the models in the paper.

B.1.1 The model in section 3.1

The system of equilibrium conditions are listed below:

The firm ki's information set $\mathbb{I}_{k,i} = \{s_{ki,1}, s_{ki,2}\}$.

Information structure

$$s_{ki,l} = a_l + e_{ki,l}, \tag{B.15}$$

where $e_{ki,l} \sim N(0, \sigma^2_{l,e})$, $l \in \{1, 2\}$.

Non-policy block

$$p^*_{k,i} = \mathbb{E}[p + \tilde{y} + u_k | \mathbb{I}_{k,i}], \tag{B.16}$$

$$p^*_k = \int_0^1 p^*_{k,i} di \tag{B.17}$$

$$p_k = (1 - \theta_k) p^*_k \tag{B.18}$$

$$p = \sum_{n=1}^{K} n_k p_k \tag{B.19}$$

Policy block under price stabilization policy

$$\sum_{n=1}^{K} \omega_k p_k = 0 \tag{B.20}$$

The equilibrium under price stabilization policy consists of variables $p^*_{k,i}, p^*_k, p_k, p$, and $\tilde{y}$ such that given the information structure (B.15), equations from(B.16) to (B.19), and the monetary policy (B.20) are satisfied.

The equilibrium under Ramsey problem consists of variables $p^*_{k,i}, p^*_k, p_k, p$, and $\tilde{y}$ such that given the information structure (B.15), equations from (B.16) to (B.20) are satisfied, and the welfare loss given by (3.11) is minimized.

B.1.2 The model in section 5.1

We first present the dynamic model in detail and then show the system of equilibrium conditions for the model in section 5.1 below.

Households Households have complete information. The representative household chooses consumption, bond holding, and labor supply to maximize its lifetime utility:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \Big(\frac{C_t^{1-\sigma}}{1-\sigma} - \sum_{k=1}^{N_k} \frac{L_{k,t}^{1+\varphi}}{1+\varphi} \Big),$$

where β is the discount factor, σ is the relative risk aversion, φ governs the elasticity of labor supply, and N_K is the total number of sectors. The budget constraint of the household in period t is

$$P_t C_t + Q_t B_{t+1} = B_t + \sum_{k=1}^{N_k} W_{k,t} L_{k,t} + \sum_{k=1}^{N_k} \Pi_{k,t} + T_t, \tag{B.21}$$

where P_t is the price level of the composite good, Q_t is the nominal risk-free bond price, $W_{k,t}$ is the sectoral nominal wage, $\Pi_{k,t}$ represents sectoral profit, and T_t represents lump-sum taxes/transfers. The aggregate consumption C_t has a Dixit-Stiglitz aggregator of the following form:

$$C_t = \left[\sum_{k=1}^{N_k} n_k^{1/\eta} C_{k,t}^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)},$$

where n_k is the size of sector k with the weights n_k summing up to 1 and $C_{k,t}$ is sectoral output, which is a Dixit-Stiglitz aggregator of the following form:

$$C_{k,t} = \left[n_k^{-1/\varepsilon} \int_0^{n_k} C_{ki,t}^{(\varepsilon-1)/\varepsilon} di \right]^{\varepsilon/(\varepsilon-1)}.$$

Solving the consumer's problem yields the following demand functions:

$$C_{k,t} = \left(\frac{P_{k,t}}{P_t} \right)^{-\eta} n_k C_t \qquad C_{ki,t} = \left(\frac{P_{ki,t}}{P_{k,t}} \right)^{-\varepsilon} \frac{1}{n_k} C_{k,t} \tag{B.22}$$

Firms There is a continuum of monopolistic competitive firms in each sector. Each firm i in sector k is endowed with a linear production function:

$$Y_{ki,t} = e^{a_{k,t}} L_{ki,t}.$$

Firms are subject to nominal rigidity à la Calvo (1983b): each firm in sector k may reset its price with probability $(1-\theta_k)$. The price-resetting firms set price $p^*_{ki,t}$ with following problem:

$$\max_{p^*_{ki,t}} \sum_{h=0}^{\infty} \theta_k^h E_{ki,t} \left\{ Q_{t,t+h} (P^*_{ki,t} Y_{ki,t+h,t} - \Psi_{ki,t+h}(Y_{ki,t+h,t})) \right\}$$

subject to the demand schedule specified in (B.22), where $Q_{t,t+h} \equiv \beta^h (C_{t+h}/C_t)^{-\sigma} (P_t/P_{t+h})$

denotes the stochastic discount factor, $\Psi_{ki,t+h}$ is the nominal cost function, $Y_{ki,t+h,t}$ is the output for firm i in sector k that last reset its price in period t, and $E_{ki,t}$ denotes the firm's expectation conditional on its information at time t, which we specify later.

The optimality condition implied by the firm's problem is

$$\sum_{h=0}^{\infty} \theta_k^h E_{ki,t} \left\{ Q_{t,t+h} Y_{ki,t+h,t} (P_{ki,t}^* - \frac{\varepsilon}{\varepsilon - 1} \Psi'_{ki,t+h}(Y_{ki,t+h,t})) \right\} = 0.$$

The Central Bank We will discuss two different scenarios related to the central bank's monetary policy. In the first case, the central bank aims to stabilize the inflation index, which is the weighted average of sectoral inflation,

$$\sum_{k=1}^{N_k} \omega_k \pi_{k,t} = 0.$$

In the second case, the central bank follows an optimal monetary policy. We will provide detailed explanations of this policy later.

Information structure Both firms and households are rational and understand the structure of the economy. However, firms cannot directly observe sectoral productivity shocks; instead, they receive signals regarding these shocks. Specifically, firm ki receives a signal $s_{ki,l,t}$ about sectoral shock $a_{l,t}$ for all sector l in period t:

$$s_{ki,l,t} = a_{l,t} + e_{ki,l,t} \quad with \quad e_{ki,l,t} \sim N(0, \sigma_{e,k}^2) \tag{B.23}$$

where $e_{ki,l,t}$ is a white noise. $a_{k,t} = a_t + \xi_{k,t}$, $a_t = \rho_a a_{t-1} + \eta_t$, and $\xi_{k,t} = \rho_\xi \xi_{k,t-1} + v_{k,t}$. η_t and $v_{k,t}$ are stochastic process with the distribution $N(0, \sigma_\eta^2)$ and $N(0, \sigma_{v,k}^2)$. The signal precisions are common among all firms across the entire economy. The information set of firm i in sector k is

$$\mathcal{I}_{ki,t} \equiv \{ s_{ki,l,\tau}, a_{t-1}, \xi_{l,t-1}, y_{R,lt-1}, P_{k,\tau}(i) : \forall l \in \{1, 2, ...N_k\}, \forall \tau \leq t \}.$$

For tractability, we assume all state variables are revealed after one period. Moreover, firms are allowed to observe the lagged relative output $y_{R,lt-1} \equiv y_{l,t-1} - y_{t-1}$.

B.2 Linearized Equilibrium Conditions

We log-linearize the household's and firm's optimality conditions around the deterministic steady-state with perfect information to the equilibrium conditions. A variable with a tilde denotes that this variable is deviating from its natural level, and a variable with a hat indicates that this variable is deviating from its steady state. Let $z_{t,k}(j) \equiv p^*_{t,k}(j) - p_{t-1}$, we obtain the following imperfect-common-knowledge sectoral NKPC:

$$
\begin{aligned}
z_{k,t}(j) \quad = \quad & E_{k,t}(j)\left(\sum_{k=1}^{N_k} n_k \pi_{k,t}\right) \\
& +(1-\beta\theta_k)E_{k,t}(j)\left((\sigma+\varphi)y_t + \varphi y_{R,kt} - (1+\varphi)a_{k,t}\right) \\
& +\beta\theta_k E_{k,t}(j) z_{k,t+1}(j)
\end{aligned}
\tag{B.24}
$$

where $y_{R,kt} \equiv y_{k,t} - y_t$ is the relative output.

The law of motion of $y_{R,kt}$ is

$$
y_{R,kt} = -\eta(\pi_{k,t} - \sum_{k=1}^{N_k} n_k \pi_{k,t}) + y_{R,kt-1}, \tag{B.25}
$$

where we use the definitions of $\pi_{k,t}$, π_t, and $p_{k,t} - p_t = -\eta^{-1}\left(y_{k,t} - y_t\right)$ which arises from log-linearizing (B.22).

The relation that links sectoral inflation to $z_{k,t}(j)$ is

$$
\pi_{k,t} \quad = \quad (1-\theta_k)\int z_{k,t}(j)dj + \frac{1-\theta_k}{\eta}(y_{k,t-1} - y_{t-1}), \tag{B.26}
$$

which makes use the definition of $\pi_{k,t}$ and $z_{t,k}(j)$.

The household's Euler equation is

$$
\tilde{y}_t \quad = \quad E\tilde{y}_{t+1} + \frac{1}{\sigma}(i_t - E\sum_{k=1}^{N_k} n_k \pi_{k,t+1} - r_t^N) \tag{B.27}
$$

The output gap is defined as

$$
\tilde{y}_t = y_t - y_t^N. \tag{B.28}
$$

The relative price gap is

$$\tilde{y}_{R,kt} \;=\; y_{k,t} - \sum_{k=1}^{N_k} n_k y_{k,t} - \Phi(a_{k,t} - \sum_{k=1}^{N_k} n_{N_k} a_{k,t}) \tag{B.29}$$

The monetary policy under inflation stabilization policy:

$$\sum_{k=1}^{N_k} \omega_k \pi_{k,t} = 0. \tag{B.30}$$

where $r_t^N = \rho + \sigma \Psi^a \sum_{k=1}^{K} n_k E \triangle a_{k,t+1}$, $y_t^N = \Psi^a \sum_{k=1}^{K} n_k a_{k.t}$, $\Phi = \frac{1+\varphi}{\eta^{-1}(1-\alpha)+\varphi+\alpha}$, $\Psi^a = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$.

Equilibrium under inflation stabilization policy The equilibrium under inflation stabilization policy consist of variables $z_{k,t}(j)$,$y_{R,kt}$, $\pi_{k,t}$, $\tilde{y}$,i_t,$\tilde{y}$,$\tilde{y}_{R,kt}$ such that given the information structure (B.25), equations from (B.25) to (B.29), and the monetary policy (B.30) is satisfied.

Equilibrium under optimal policy In the dynamic model, we consider the optimal time consistent monetary policy. The equilibrium under optimal time consistent monetary policy is that the central bank optimally chooses variables $z_{k,t}(j)$,$y_{R,kt}$,$\pi_{k,t}$,i_t,$\tilde{y}_t$,$\tilde{y}_{R,kt}$ as a function of state variables $[a_{t-1}, \xi_{l,t-1}, y_{R,lt-1}, s_{ki,l,t}, \forall l \in \{1, 2, ...N_k\}]$ such that given the information structure (B.25), equations from (B.25) to (B.29) are satisfied, and the welfare loss below the is minimized:

$$E_0 \sum_{t=0}^{\infty} \beta^t \Big[\sum_{k=1}^{N_k} \Big(\frac{\epsilon n_k}{1-\beta\theta_k} [(1-\theta_k) \int_i (p^*_{ki,t} - p^*_{k,t})^2 di + \frac{\theta_k}{1-\theta_k} \pi^2_{k,t}] \Big) + (\sigma+\varphi)\tilde{y}_t^2 + (\eta^{-1}+\varphi) \sum_{k=1}^{K} n_k \tilde{y}^2_{R,kt} \Big],$$

which is derived as the second-order approximation of the representative consumer's period welfare loss expressed in consumption equivalent variation.