Spatial Heterogeneity and Inflation

Paul Bousquet*

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Abstract

This project seeks to better understand inflation disparities through the use of non-public BLS data on expenditures, prices, and wages. The standard measurement for inflation is tracking changes in a price index tied to a given bundle of goods. Yet we would expect there to be substantive differences across geography in this bundle's price, which implies different welfare and general equilibrium effects from changes in prices across regions that a simple averaging would not include. Further, what constitutes a "typical" basket of goods also varies by location. Related economic literature details how differing beliefs and preferences can affect the results our models give and the implementation of policy. But there is currently no analysis on how such aspects are spatially distributed, or a consideration of the consequences of location-based dispersion. I posit that it is unlikely questions policymakers, economists, and citizens alike are asking can be fully answered without accounting for these disparities. My research will add to the wealth of heterogeneous agent model analysis of the macroeconomy and inflation by including a spatial component. Calibrating such a model requires access to information not in the BLS' public microdata. In particular, while there is insufficient data to model each county autonomously, a model where locations are grouped more precisely than "rural or urban" will better isolate current trends. In terms of output, because the focus of this project is on the macroeconomy, there will be no discussion or references tied to any individual or specific location, only about (aggregated) location characteristics in general (e.g., output on the relationship between population size and inflation).

Table of Contents

*Department of Economics, University of Virginia, ptb8zf@virginia.edu.

Introduction

The law of one price is almost always assumed in economic modeling, despite significant and widely known deviations. For instance, Handbury and Weinstein (2015) show that, all else equal, cities with higher population tend to have a lower price level, while the Regional Price Parity measure from the BEA suggests states that are less densely populated have a lower price level. There is also an extensive literature on *inflation inequality* which recognizes that consumers themselves face a different price level because the composition of consumption can be fundamentally different along demographic lines, like income or location. In other words, because groups, like wealthy and impoverished people, have unequal expenditure shares across goods, price changes lead to differing rates of inflation. Relevant to this proposal, Zélity (2023) finds persistent differences in inflation between rural and urban areas, showing that from 2001-2021 the average inflation differential was 1.9x (whichever area had more inflation in a given year on average had about twice as much inflation as the other).

Despite these well-documented differences, it's likely they are still underreported. To my knowledge, all research on inflation inequality, following the precedent of Hobijn and Lagakos (2005), assumes that while urban rural areas can have a different price level, prices themselves (in a given product category) grow at the same rates. However, my own analysis of consumer expenditure data from the BLS' public CEX series suggests this assumption creates large downward bias in estimating inflation differences. Zeroing in on food expenditures as an example, notable features of the data include that people in rural areas spend less on food and correlations of rural and urban food expenditures are weak, despite the tendency to comove. These results still hold even when looking at rural and urban consumers in the same income groups, meaning that unless one is willing to say that people in rural and urban areas have intrinsically different, time-variant preferences over food, the price of food must not be changing at the same rate. Similar trends are seen in other product categories as well.

Clearly, rural and urban inflation are distinct objects, but the important question for both policymakers and researchers is do these differences matter. A modal Economist might suggest that rural areas make up a low percentage of both population and GDP, so in the aggregate, the persistent divergence is not noteworthy. While this might be a compelling point for someone interested in measuring economic activity, this proposal is concerned with the hypothesis that these differences have a meaningful equilibrium effect. The intuition behind this hypothesis is that the relationship between consumers and firms, both employee-employer and buyer-seller, ingrains linkages that create a feedback effect that can ripple through supply and demand chains, so these differences in inflation (and inflation expectations) do not occur in a vacuum. As a simple illustration, a firm responding to a supply shock may change their price, which alters consumer (and worker) behavior, leading firms to respond to this response, and so on. We know these sorts of domino dynamics are pervasive in modern economies, and given inflation's importance to the population, we should expect that varied experience with inflation can create ripple effects that don't happen in isolation.

With respect to the need for restricted data access from the BLS, the public Consumer Expenditure series does give an indication of whether a consumer is from a rural or urban area, but more specific location data is not available. In order to isolate the effects at a more granular level, I plan to use the restricted data to put locations into groups more specific than rural and urban, and aggregate the series based on these location groups. This will be helpful for both empirical and model-based exercises. To help calibrate my approach, I am also requesting access to the Consumer Prince Index and Occupational Employment and Wage Statistics series. For example, the latter will be useful for a model in which wage growth contributes to inflation. The rest of the proposal will give more specific details on the specific need and use of these data.

The next section give a broader overview on the adjacent literature, namely work that sets precedent for the models and methods that will be used. The final section, informed by the proposed methodology and empirical exercises, will detail the data usage and institutional relevance of the project, as well as offering a brief conclusion on future direction. For simplicity and to respect length requirements, there is an online appendix[1] that has some additional information, including more model details and description of preliminary data analysis.

Approach: Past Literature and Proposed Methods

The quantitative aim of this project is to **(i)** extensively document the inflation disparities that exist along spatial lines and then **(ii)** see if a model in which these differences meaningfully affect the US economy can replicate these differences well. This is a common playbook in the Macroeconomics literature in general, but not in research about differences in inflation experiences, and there is to my knowledge no past research that looks at rural/urban disparities in a general equilibrium context. But to better understand the proposed approach, the proximate research and empirical facts of note will be established. One notable strand of literature is connected on a purely topical basis. Other papers outside of this focus will be presented in the body of describing planned methodology to contextualize modeling approaches.

The term inflation inequality refers to idea that a single headline number for inflation is not representative of the inflation faced by different groups of people. In a research context, the objective of studying inflation inequality is to identify the exact nature of where divergences occur and how they may be relevant for understanding the economy and optimal policy. Hobijn and Lagakos (2005) can be considered the seminal paper for solidifying the current path of the inflation inequality literature, which found non-negligible inflation differences between

[1] The most recent version of the proposal can be found here, with the appendix at the very bottom of the document https://pbousquet.com/assets/Bousquet_RDA.pdf

rural and urban areas, among a host of other individual characteristics. As detailed in the extensive review of Jaravel (2021), most research following this paper has focused on inflation inequality along an income dimension. Specifically, there is compelling evidence that preferences over goods are not homothetic (types of goods are not consumed in equal proportions across income types). This implication is if we were to form a price level for income groups in a similar fashion to the default of the Consumer Price Index (CPI), these price levels would be different and grow at different rates. Virtually all of the current work on inflation inequality has treated the rural/urban divide as a tangential issue, meaning the careful, preliminary empirical work needed to see out this project on its own would be a contribution to the literature.

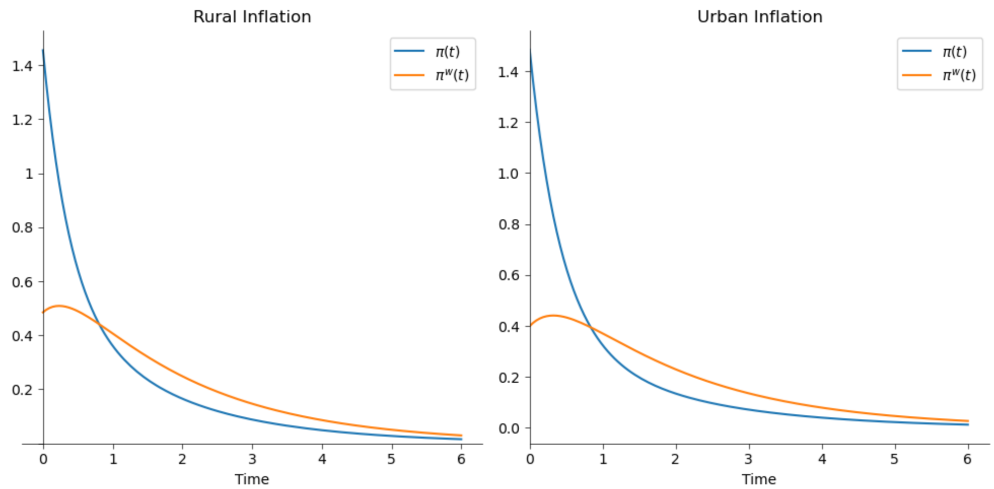
As put forward in Hobijn and Lagakos (2005), the biggest stumbling block from studying the issue of inflation inequality between rural and urban households is a measurement issue. Noteworthy inflation measures like the CPI intend to be representative of almost all of the US' population but explicitly specify that it's a metric for urban areas. In particular, while some rural households are surveyed to calculate the proper weighting of prices (into an aggregate price level), the prices themselves come directly from data in metropolitan areas. Consequently, all research analyzing rural/urban inflation make a simplifying assumption that prices in rural and urban areas change at the same rates across all product categories or use even more indirect measures of rural inflation. Chakrabarti et al. (2023) use a GDP deflator alongside a state's proportion of rural citizens as one indirect method but are transparent about the fact that such approaches are not suitable for peer-reviewed scholarly research. Similar caveats are given in Swagel (2022), a CBO report to Congress on rural inflation, which relies on an even noisier approach of using the proportion of rural citizens in the 9 Census geographic areas. This report also notes the importance of tracking the growth of nominal wages alongside the growth in prices, but again its analysis is hindered by the fact that Census Bureau does not publicly report an individual's location if they live in a county of less than 100,000 people. The BEA posts a Regional Price Parity (RPP) series that is informative for this question, but can only account for state-level data or urban areas.

The quantitative tasks for this project can be put into two categories: data analysis and model+calibration exercises. As alluded to previously, while the former is less sophisticated at face value, because of the measurement hurdles, many basic empirical questions have not been answered. As done in Zélity (2023), one can calculate rural and urban inflation rates by assuming price changes are equal. But it would be useful to perform these calculations at a more granular level and also is critical to relax the assumption of uniform price changes. One way to relax this assumption would be to attempt to infer the price level directly using the Consumption Expenditure series (CEX) from the BLS, in conjunction with publicly available data used to calculate GDP. This approach would be similar to the Geary Index used to create the BEA's RPP series. Using expenditure and population weights from the CEX, the quantity of a given product category purchased by urban consumers can be estimated, then subtracting the resulting total expenditure from national expenditure and dividing by the implied rural quantity (the leftover amount of domestic consumption) results in a prospective rural price level. These steps would need to be mindful of aggregation bias discussed at length in Jaravel (2021). The practicality of the data analysis procedures discussed in this paragraph in the context of restricted data access guidelines will be discussed in the next section

An aspiration of this research is to change the academic understanding of inflation by developing a model that understands these pervasive inflation differences well. Following the recent inflationary episode, many have echoed the prescient words of Tarullo (2017) in highlighting that there is no consensus on very basic questions about inflation, and accounting for the rural/urban gap could provide an important piece of this puzzle. In part because no past research has attempted to answer this question, the standard check of seeing how other "benchmark" models perform is not relevant (these models essentially assume away the question). So a useful comparison exercise will be evaluating whether the models developed for this project are able to account for other important features of economic data, ones that its not specifically designed to answer. A calibrated model that is able to account for some niche feature of the data well but also outputs that the US economy shrinks every quarter, for example, casts doubt on the story that model is telling. These sorts of considerations will be important going forward to make sure the results are robust to specifications outside of the target area of inflation inequality.

In line with the stated objective at the beginning of this section, there are several ways to create models of the economy where differences in urban and rural inflation matter. One such model is the wage-price spiral model of Lorenzoni and Werning (2023). The foundation of this model is that firm and consumer are linked through wages. Firm's set prices based on what they expect marginal costs (and therefore wages) to be, and consumers set their wage expectations based on what they expect prices to be. A wage-price spiral is when these expectations disagree, and then one tries to catch up with the other, which in turn provokes a reaction making it harder to catch up, creating a "spiral". Such a spiral can happen if there is a negative supply shock, resulting in a price increase, that simultaneously does not affect a worker's utility maximizing preferences for leisure, provoking workers to demand a higher nominal wage because their desired real wage just fell. Because this model in particular provides a transparent motivation for requesting wage data, we can construct a simple illustration[2]. Consider an economy with price and wage rigidities with trade between two regions, rural and urban, where both have a tradeable and non-tradeable sector, labor is immobile, and there is one wage rate per region set by a monopolistic union. If there is a negative supply shock in the rural tradeable good, it directly affects urban inflation and indirectly wages because the union will try to make nominal income adjust to prices. The graphs on the next page show the impulse responses to such a shock under analogous assumptions used in Lorenzoni and Werning (2023). We see a spiral here because wage inflation (π^w) prolongs and eventually overtakes price inflation (π) in both areas. Workers in both areas face similar amounts of inflation if expenditure shares are similar, meaning both make essentially the same wage demand in response, enhancing the feedback effect through the traded goods

[2]To respect length constraints, an extensive discussion on assumptions, derivations, and computation can be found in the online appendix



Price and Wage Inflation following a Supply Shock to Rural Tradeables

Another possible avenue is using a production network model advanced by David Baqaee and Emmanuel Farhi, including in Baqaee and Farhi (2022). These models allow for goods to be used as inputs, which creates linkages betwen rural and urban areas, and have recently been expanded to include small open economies (Silva, 2023), particularly noteworthy since rural areas are inherently small open economies (namely, they take interest rate as given). Paradigmatically, any modeling approach will bear resemblance to a combination of research that has employed heterogeneous agents, combined multi-sector models with trade and inflation (Comin et al., 2023), treated intra-country trade similar to usual environments inter-country trade, like trading between states (Caliendo et al., 2018), and showed how differences in inflation experiences matter for aggregate shock and monetary policy transmission (Lee, 2023).

Finally, we conclude with empirical trends that provide important context for this work. Previously referenced work has established the link between location and inflation. Performing basic regression analysis with the BEA's RPP shows a highly significant positive relationship between a city's population and its price level relative to the US average. In the years during and following the COVID pandemic, there is a statistically significant negative relationship between population *growth* and change in the relative price level, more in line with the findings of Handbury and Weinstein (2015). In terms of internal area structure, it's a common image that most people in rural areas work on farms, but in reality most people work in the service or public sectors and only 7% work in agriculture, with much of that employment being part time (Davis et al., 2022). Though there is some truth to the conception – agriculture does often serve as the economic backbone, even if not the biggest supplier of jobs, and undoubtedly is consistently the most identifiable way the rest of the economy is dependent on rural areas, despite the smaller population. Yet there are intricacies of geography and industry that make the nature of these internal and macro dependencies varied across the country. Rasool and Abler (2023) captured the nuances of many of these trends by identifying groupings of US counties that have distinct characteristics in agricultural development (type,scale, and sophistication), again suggesting an important spatial amplification of the described effects.

Rural areas had their own experiences with COVID; in some ways they were already social distancing by construction so the immediate impact was generally not as severe. The increase in the unemployment rate was only about a third of the national average and the labor market recovery only took about a quarter (Sanders, 2023). This trend is compounded by the fact that rural areas have a generally more elastic labor market, which increases their bargaining power even more during a pandemic where the firm-side is experiencing never before seen emergencies. Rural areas also have a unique relationship with monetary policy because of agriculture. Government stimulus accounted for half of farm income in 2020 (Penson, 2022), and when that stopped, farms supplemented that income with low-interest debt. Firms also began taking out bigger loans shortly before the time their effective interest rate double because of the Federal Reserve's tightening (Kauffman and Kreitman, 2023). Farms also had a harder time adapting to the tightening – according to the Census Bureau, in 2022 interest payments went up almost 30% while plateauing for most other major industries (Huffstutter and Flowers, 2022). This is a stark change from the beginning of the pandemic, as farms were one of the few profitable sectors of the economy in 2020 (Zwilling, 2022). Since farms frontload costs, they were likely less affected by an input price shock. Once they had to buy new supplies, it was bearable from stimulus and low-interest loans. Now, this correction has become problematic, and definitely something models with internal economic linkages should be considering.

Data, Disclosure, and Relevance

The primary BLS data of interest is the Consumer Expenditure Survey (CEX) series. Currently, the public use microdata (PUMD) available for the CEX series does offer whether a given household resides in a rural or urban area. But as described in the preceding section, rural areas themselves are not structurally homogeneous, in particular with respect to labor markets and industry composition. In a perfect world, the centerpiece of this project would be a model where each US county was treated as an independent region, but not enough data is collected on each county for this to be feasible. The best path forward is to then do some sort of aggregation by clustering counties. One possible partitioning that aligns with the themes of this research would be to use clustering by the makeup (sophistication, scale, and development) of the agricultural industry, which would be able to account for some of the stylized facts we observe. Such a clustering has already been performed and made available by the aforementioned Rasool and Abler (2023). Other, more qualitative, clustering can also be considered, for instance grouping by a simple combination of geography and population.

The first step of data processing will be aggregating individual-level data to its corresponding county cluster using its FIPS code. Ideally, this process would yield enough observations for each group of counties such that the data frequency can remain at a monthly basis. This step itself would allow for the ability to conduct further preliminary analysis, such as gauging whether the persistent inflation differences described earlier still remain along these differently aggregated lines. If this is the case for a given clustering, it would speak to the merit of separating counties along these lines, especially in a modeling context.

Referring back to the stated aim of this project at the start of the preceding section, we will use this data to grade the performance of the model. Given the measurement issues at hand, it will be important to test the performance along several lines. To aid in the evaluation portion, as well as document other important facts, data requests have also been made for the BLS' Consumer Price Index (CPI) and Occupational Employment and Wage Statistics (OEWS). The former could be useful for trying to assess how confident we should be in the approach on a measurement basis. However, the CPI series is largely requested for completeness; because much thought has gone into its tabulation, it's less likely anything new can be gleaner, but there are still some robustness exercises that could be performed with its inclusion. For example, breaking down inflation based only on the metropolitan areas for which price data is explicitly gathered to create a state index similar to the BEA's Regional Price Parity, a comparison model could be developed along the lines of Caliendo et al. (2018) to think about state-based inflation and trade. It's also useful to have this granular data as a guide to develop an index for these county clusters, since they would be totally novel measures, in addition to having a benchmark for the inferred rural price level discussed in the previous section. The OEWS would be used to develop a more accurate accounting of real wages, which is a useful empirical exercise and inform modeling structures resembling Lorenzoni and Werning (2023). The OEWS series is also important given the established issues with the income data in the CEX series outlined in Blundell et al. (2006). Ideally[3], we would use data with a monthly frequency, but can perform analysis at a monthly level using a combination of interpolation and sensitivity analysis (creating a bound on possible month by month paths to get from one yearly data point to the next). This won't be sufficient to claim that the data has been matched by the model, but it is a worthwhile robustness exercise, considering the smoothness of aggregate wages and well-established notions about wage frictions in the literature about wage frictions, also anecdotally support this being worthwhile, though certainly not complete.

In terms of data needs, the entire timespan of the CEX, OEWS, and CPI data would be used. Granularity would occur at either the county level (or city if unavailable). Similarly, the only variable needed that is not in the public files would be related to location (otherwise the collection of publicly available variables are sufficient).

With respect to privacy concerns: while the process of aggregation requires direct contact with individual level data, none of this analysis takes place at the household level. To be explicit, no steps to complete this project require even looking directly at individual observations; if filtration is necessary it will happen based on a blind threshold. Relevant variables are expenditure and location data from the CEX, effectively all variables (related to

[3]The Census Bureau's Current Population survey would be used in a perfect world but only annual data is available to non-Census employees

price, product, and location) in the CPI, and income and occupation data from the OEWS. The outputs will not be tied to any particular city or county, and will include broad statistics such as correlation (and perhaps some motivating regression tables), newly calculated inflation measures tied to county clusters, and graphs to depict the model's performance. These exercises can only be performed with non-publicly available data for reasons previously described, though in the meantime of waiting for approval, more preliminary analysis can be conducted using publicly available data to make the process more efficient once data access is granted. Logistically, I will commute to the onsite BLS location in DC from University of Virginia when needed.

The relevance to the BLS appears on many fronts. This proposal will produce a novel measurement (of disaggregated) inflation, which provides more context on how generalizable the current approach is. These results would then shed light on proper use of current use of BLS metrics for policymakers. Even if it's found that these rural/urban divides are not noteworthy, that on its own will lend confidence to the use of the headline figures in decision making. But if these results prove to be significant, it may inform the tracking of new types of statistics. There is also a connection in the need to keep up with the way the American landscape is changing. Rural and urban households are simultaneously becoming more linked through online infrastructure but drifting further apart in terms of differences in physical infrastructure, so accounting for inflation, one of the most important economic aspects for many households, will help make sense of these paradoxical trends. In summary, this research has relevance both from the administrative measurement duties of the BLS as well as its role of informing policymakers. In the time awaiting the decision, I will continue to update this proposal at the online link and do what I can with the publicly available data. One useful exercise would be to extend current models to include inflation inequality with respect to heterogeneity in income levels, as there is much more precedent for such models in general (compared to this type of spatial heterogeneity), in addition to work in the context of inflation inequality (Clayton et al., 2018). I will also continue to develop stylized facts to better motivate the utility and usefulness of this vision. Overall, the work that has already been done makes clear the gulf between rural and urban households with respect to the most discussed economic metric in the post-pandemic era, and I suspect continued work will only reveal the gulf to be bigger and the necessity to develop this area more immediate.

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0 Appendix

The first three subsections deal with details on one of the proposed models. The final subsection is more detail on the data analysis that informed the research proposal, including the treatment of the CEX data.

0.1 Wage-Price Spirals and Inflation Inequality

This is an extension of the wage-price spiral model of Lorenzoni and Werning (2023) to a partial open economy setting (partial due to the ad hoc restrictions on production) with multiple sectors. These results are meant to match the case where there are a continuum of labor supply and good varieties (and corresponding monopolistic unions and firms). Because of the form of the production function, firms within industry behave identically to a first order approximation, even in a dynamic setting. So for notational and conceptual clarity, we use a representation in which there is a single monopolist in each industry that behaves like a firm in the scenario with variety (also a single union in each country). In summary, the following writeup is more symbolic than literal, but for transparency, the fully derived extension with variety can be found in the next subsection.

There are two regions, $n \in \{\text{Urban,Rural}\}$, each populated by a representative agent with the same intrinsic preferences over 3 types of consumables: $i \in I = \{\text{food, goods, services}\}$. We assume there is only source of each for a given region: only Rural produces food, only Urban produces goods, and services are non-tradeable. The utility function, common to both countries, is

$$\frac{C_n^{1-\eta}}{1-\eta} - \frac{L_n^{1+\gamma}}{1+\gamma}$$

where households are subject to a budget constraint (p_{ni} is price in region n of the industry i consumable),

$$\sum_{i \in I} p_{ni} c_{ni} \leq W_n L_n$$

we use a CES aggregator,

$$C_n = \left(\sum_{i \in I} c_{ni}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

and assume the economy is at its (deterministic) steady state. This implies a price index of

$$P_n = \left(\sum_{i \in I} p_{ni}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

Consumption is equal to real income by $C_n = \frac{W_n}{P_n} L_n$

The labor demand for a service sector firm is N_{ns}, while the demand for the food and good sectors are just N_f and N_g. We assume labor is immobile, so labor market clearing is

$$N_{Rs} + N_f = L_R \ \underline{\textbf{and}} \ N_{Us} + N_g = L_U$$

The production processes are

$$Y_{ns} = Z_{ns} N_{ns} \qquad Y_f = Z_f N_f^{1-\alpha_f} \qquad Y_g = Z_g N_g^{1-\alpha_g}$$

Log-linearized nominal marginal products of labor are therefore

$$2z_{ns} \qquad 2z_f - \alpha_f n_f \qquad 2z_g - \alpha_g n_g$$

Now that the basic model is established, we now shift into a dynamic setting in order to consider a shock that pushes the economy away from the steady state.

We assume a Calvo pricing setting and use a continuous time representation with rate of discounting ρ. Firms are randomly selected to reset their price with Poisson arrival λ_p, and unions are selected with arrival λ_w. Firms when behaving optimally set prices as if they won't be able to change again, equal to average marginal costs We will denote these "flexible" (log-linear) prices by p^*_{nit}, which is given by

$$p^*_{nit} = (\rho + \lambda_p) \int_t^\infty e^{-(\rho+\lambda_p)(\tau - t)} (w_{n\tau} - mpl_{ni\tau}) \mathrm{d}\tau$$

Under a similar philosophy, unions set nominal wage (in case it wasn't clear up until this point – firms are price takers in the labor market) in relation to the marginal rate of substitution between consumption and leisure, which is given by $\eta y_{nt} + \gamma l_{nt}$. We call w^*_{nt} the flexible wage and write it as

$$w^*_{nt} = (\rho + \lambda_w) \int_t^\infty e^{-(\rho+\lambda_w)(\tau - t)} (p_\tau + mrs_{n\tau}) \mathrm{d}\tau.$$

where p_τ is the log-linear representation of the earlier derived price index.

The tension in this model is garnered from the fact that the wage w_τ is in the firm's reset problem, and similarly p_τ is in the union's reset problem. This is precisely the wage-price spiral mechanism: nominal prices are set to catch up with future wages, and symmetrically wages are set to match future prices. Shocks that create a diversion in aspirations can yield a "ping-pong" effect that is smoothed out by price-stickiness. Related, this environment does not follow a traditional Bertrand paradigm because actors cannot freely adjust price, and a reasonable calibration of this model will set wages are more rigid than prices to match overwhelming empirical evidence to that effect.

A counterfactual exercise to illustrate the interesting transmission at play: suppose that there is a negative shock to the productivity of the Rural service sector. This leads to a fall in the industry's marginal product of labor. Accordingly, prices will in this sector will rise. This leads to a rise in the nominal wage in Rural, which pushes up the food price level, thus creating inflation in Urban and necessitating a rise in its wages. More specifically, this is where having a continuum of unions/firms makes a difference, but by the law of large numbers these movements will happen eventually, its just which a continuum of firms the movements are smooth. Also, an important factor at play here is that the wage must be equal across industries within countries. This is not merely an imposition of the union structure, but more broadly an equilibrium outcome in a sectoral model in which labor is substitutable across production processes.

The next subsection will go into detail of the aggregation that's behind the abstraction we just presented. The subsection after will go into detail about the modeling exercise presenting in the main body of the proposal.

0.2 Aggregation Detail

Now, all industries consist of a continuum of monopolist firms that produce a unique good variety which are aggregated, yielding 3 aggregate consumables. The utility function, common to both countries, is

$$\frac{C_n^{1-\eta}}{1-\eta} - \frac{L_n^{1+\gamma}}{1+\gamma}$$

where households are subject to a budget constraint ($p_{ni}(\theta)$ is price in region n of the variety θ for industry i),

$$\sum_{i\in I}\left\{\int_{\theta\in\Theta_{ni}} p_{ni}(\theta)c_{ni}(\theta)\mathrm{d}\theta\right\} \leq w_n L_n$$

we use a CES aggregator,

$$C_n = \left(\sum_{i\in I}\int_{\theta\in\Theta_{ni}} \left[c_{ni}(\theta)\right]^{\frac{\sigma-1}{\sigma}} \mathrm{d}\theta\right)^{\frac{\sigma}{\sigma-1}}$$

and assume the economy is at its steady state. This implies a price index of

$$P_n = \left(\sum_{i\in I}\int_{\theta\in\Theta_{ni}} p_{ni}(\theta)^{1-\sigma}\mathrm{d}\theta\right)^{\frac{1}{1-\sigma}}$$

Consumption is equal to real income by $C_n = \frac{w_n}{P_n} L_n$

The labor demand for each firm $N_{ni}(\theta)$ is an aggregate of a continuum of labor varieties $N_{ni}(\theta) = \left(\int_0^1 \left[N_{nij}(\theta) \right]^{1-\varepsilon} \mathrm{d}j \right)^{\frac{1}{1-\varepsilon}}$. Each labor variety j is supplied by a monopolistic union that employs labor from households and turns it, one for one, into specialized labor services of type j. Assuming labor is immobile across countries and imposing market clearing, we integrate over firms and unions for a total labor supply of

$$L_n = \int_0^1 \left\{ \sum_{i \in I} \int_{\theta \in \tilde{\Theta}_{ni}} N_{nij}(\theta) \mathrm{d}\theta \right\} \mathrm{d}j$$

where $\tilde{\Theta}_{ni} \subseteq \Theta_{ni}$ respects the fact that some industries don't exist in a given n.

The production process for the service sector is

$$Y_{ns}(\theta) = Z_{ns} N_{ns}(\theta)$$

The production process for the food sector is

$$Y_f(\theta) = Z_f N_{nf}(\theta)^{1-\alpha_f}$$

The production process for the goods sector is

$$Y_g(\theta) = Z_g N_{ng}(\theta)^{1-\alpha_g}$$

0.3 Proposal Example

To keep this subsection relatively standalone, first we will do some review of notation (some small changes) and structure. We will use continuous time with price and wage rigidities, where prices and wages can be reset per a realization of a Poisson arrival (parameters λ_p and λ_w, respectively). In log-linear terms, price of a good from sector s in region i is $p_{t,is}$, the (nominal) wage in region i is $\dot{w}_{t,i}$, and "flexible" objects are denoted with a *. Because we are using continuous time, we use the following notation for price and wage inflation: $\pi_{t,is} = \dot{p}_{t,is}$ and $\pi^w_{t,i} = \dot{w}_{t,i}$. We will also let $\omega_{t,i}$ denote real wages. As far as simplifying assumptions, we initially assume labor is immobile across countries, wages do not vary by sector, and will make further assumptions later as needed for simplicity. To begin the derivations, note that the form of differential equations for variables of our type follows

$$\dot{p}_{t,is} = \lambda_p (p^*_{t,is} - p_{t,is}) \ \underline{\textbf{and}} \ \dot{w}_{t,i} = \lambda_w (w^*_{t,i} - w_{t,i})$$

As shown in Lorenzoni and Werning (2023) Appendix A1, for a given sector s and region i, for our general multi-sector, multi-region setup this implies

$$\rho\pi_{t,is} = \Lambda_p(\omega_t - mpl_{t,is}) + \dot{\pi}_{t,is} \ \underline{\textbf{and}}\ \rho\pi^w_{t,i} = \Lambda_\omega(mrs_{t,is} - \omega_{t,i}) + \dot{\pi}^w_t$$

where $\Lambda_p = \lambda_p(\rho + \lambda_p)$ and $\Lambda_w = \lambda_w(\rho + \lambda_w)$. Because we are in a multi-sector economy, to define an aggregate inflation level, we stick to the usual formula of using a weighted average of product inflation rates based on expenditure shares. For our counterfactual exercise looking at a shock, we note that in the immediate aftermath of a shock, expenditure shares do not change, and it is simple to extend the following to a case in which it's allowed to change after a fixed interval of time. For our exercise, assume there is an isolated, singular shock in rural non-tradeable sector (to $mpl_{0,rn}$). If goods in rural are consumed equally, [4] rural inflation is

$$\rho\pi_{t,r} = \widetilde{\Lambda}_p\left(\omega_{t,r} + \frac{1}{2}(\omega_{t,u} - mpl_{t,rn})\right) + \dot{\pi}_{t,r}$$

where we exploit the additivity of the aggregate inflation equation ($\dot{\pi}_{t,r}$ can be easily defined as weighted sum) and redefine $\tilde{\Lambda}_p = \frac{2}{3}\Lambda_p$. Similarly, urban inflation is

$$\rho\pi_{t,u} = \widetilde{\widetilde{\Lambda}}_p\left(\omega_{t,u} + \frac{s_r}{1-s_r}\omega_{t,r}\right) + \dot{\pi}_{t,u}$$

where s_r denotes the expenditure share of the rural tradeable good and $\widetilde{\widetilde{\Lambda}}_p = (1-s_r)\Lambda_p$. Let $\gamma = \frac{-s_r}{1-s_r}$. Using the results from Lorenzoni and Werning (2023) Appendix A2, we get the following expression for urban wages

$$\omega_{t,u} = \gamma \int_0^\infty H^u_{s,t}\,\omega_{s,r}\,\mathrm{d}s$$

where $H^u_{s,t} = \frac{\widetilde{\widetilde{\Lambda}}}{r_4 - r_3}\left(e^{\min\{r_3(t-s),-r_4(s-t)\}} - e^{-r_3 t - r_4 s}\right)$, $\widetilde{\widetilde{\Lambda}} = \widetilde{\widetilde{\Lambda}}_p + \Lambda_w$, and r_3, r_4 solve $x(x-\rho) = \widetilde{\widetilde{\Lambda}}$ with $r_3 < r_4$. Where this gets (even more) complicated is that the term for rural wages depends on urban wages in a similar way. So substituting in urban wages into the analogous expression for rural wages, we get rural wages on the RH and LHS of the expression. We can reduce the complexity slightly by assuming the shock decays exponentially at rate δ

$$\omega_{t,r} = \frac{\gamma}{2}\int_0^\infty H^r_{s,t}\left\{\int_0^\infty \left(H^u_{s,t}\,\omega_{s,u} - mpl_{0,rn}e^{-\delta s}\right)\mathrm{d}s\right\}\mathrm{d}s \tag{1}$$

where $H^r_{s,t} = \frac{\widetilde{\Lambda}}{r_2 - r_1}\left(e^{\min\{r_1(t-s),-r_2(s-t)\}} - e^{-r_1 t - r_2 s}\right)$, $\widetilde{\Lambda} = \widetilde{\Lambda}_p + \Lambda_w$, and r_1, r_2 solve $x(x-\rho) = \widetilde{\Lambda}$ with $r_1 < r_2$.

[4]For $S > 3$ sectors and unequal expenditure share $r_s \neq \frac{1}{S}$, it's $\rho\pi_{t,r} = \widetilde{\Lambda}_p\left(\omega_{t,r} + \frac{r_s}{1-r_s}(\omega_{t,u} - mpl_{t,rn})\right) + \dot{\pi}_{t,r}$ with $\tilde{\Lambda}_p = (1-r_s)\Lambda_p$

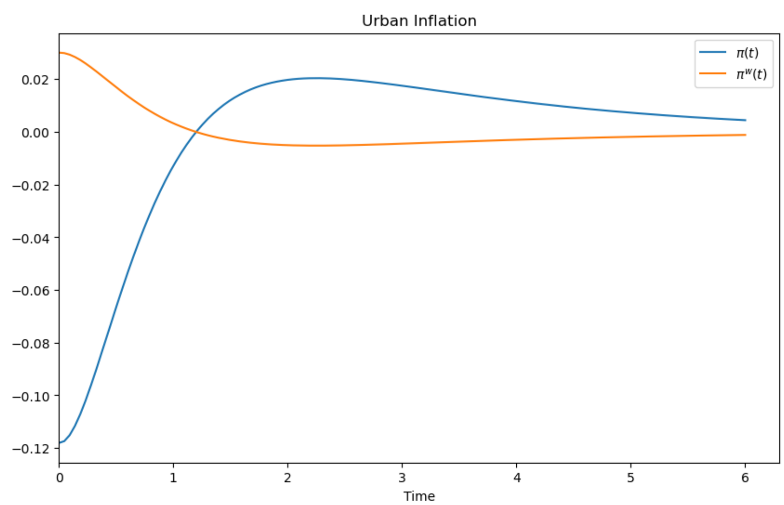
(1) is an example of a non-linear Fredholm equation of the second kind, meaning there is no hope for getting an analytical expression for wages. We instead turn to numerical methods to be able to solve this system, specifically the algorithm from Li and Huang (2016). The algorithm is essentially an application of Newton iteration, and at least for this application, is remarkably powerful: using a grid of $n = 100$ was imperceptibly different from using a grid 100 times a large. This procedure gives us an approximation for $\omega_{t,r}$ allowing us to solve

$$\dot{\omega}_{t,r} = r_1 \omega_{t,r} + \frac{\widetilde{\Lambda}_p}{2} \int_t^\infty e^{-r_2(\tau - t)} \left(mpl_0 e^{-\delta t} - \omega_{\tau,r} \right) d\tau$$

which comes from subtracting rural price inflation from rural wage inflation, which yields a second order ODE using $\dot{\omega}_{t,r} = \pi^w_{t,r} - \pi_{t,r}$. In case there is any confusion about how this has solved our problems: we have essentially found a function $f(t) = \omega_{t,r}$, which means we can compute this integral. Finally, we can now use the closed form expressions from Lorenzoni and Werning (2023)

$$\pi_{t,r} = \Pi_{t,r} - \alpha \dot{\omega}_{t,r} \ \underline{\textbf{and}} \ \pi^w_t = \Pi_{t,r} + (1-\alpha)\dot{\omega}_{t,r}.$$

where[5] $\Pi_{t,r} = e^{-(\rho+\delta)t} \frac{\Lambda_p \Lambda_w}{3\Lambda} \frac{-mpl_0}{\rho+\delta}$, $\Lambda = \Lambda_p + \Lambda_w$, and $\alpha = \frac{\Lambda_p}{\Lambda}$ is relative price stickiness. Similar expressions can be derived for urban, though there is no Π term because there is no aspirational conflict. The actual graphs in the proposal were from a slightly different premise, a shock to rural tradeables, since it produced slightly more intuitive graphs. However, the case of a shock to rural non-tradeables is a bit more conceptually interesting because there is no direct effect on inflation, yet inflation still occurs. In fact, there is initially wage inflation because real wages rise in urban since they have fallen in rural (graph there is same as proposal)



[5] Recall: can replace $\frac{1}{3}$ in $\Pi_{t,r}$ with generic expenditure share

It's important to note that the results in this subsection assume there is no general equilibrium effect resulting in labor reallocations that further distort marginal products. There are several production technologies where that would be the case, but in general would not hold. Future updates to this proposal will hopefully consider the full case outlined in the first appendix subsection. Some complementing code and more detail is available online, full code available upon request.

0.4 Extra

$$\pi_t = \Pi_t - \alpha\dot{\omega}_t \ \underline{\textbf{and}}\ \pi_t^w = \Pi_t + (1-\alpha)\dot{\omega}_t.$$

$$\pi_t = e^{-(\rho+\delta)t}\frac{\Lambda_p\Lambda_w}{\Lambda}\frac{-mpl_0}{\rho+\delta} - \alpha\dot{\omega}_t \ \underline{\textbf{and}}\ \pi_t^w = e^{-(\rho+\delta)t}\frac{\Lambda_p\Lambda_w}{\Lambda}\frac{-mpl_0}{\rho+\delta} + (1-\alpha)\dot{\omega}_t.$$

$$\dot{\omega}_t = r_1\int_0^\infty \frac{\Lambda}{r_2-r_1}\left(e^{\min\{r_1(t-s),-r_2(s-t)\}} - e^{-r_1 t - r_2 s}\right)\alpha\left(mpl_0 e^{-\delta t}\right)ds + \Lambda\int_t^\infty e^{-r_2(\tau-t)}\alpha\left(mpl_0 e^{-\delta t}\right)d\tau,$$

$$mpl_0 = -2, \omega_0 = mrs_t = 0, \Lambda_p = 4.08, \Lambda_w = 1.04, \Lambda = 5.12, \alpha = 0.796875, r_1 = -2.2428, r_2 = 2.2828, \delta = .5$$

where r_1 and r_2 are the roots of the quadratic equation $r(r-\rho) = \Lambda$,

and satisfy $r_1 < 0 < \rho < r_2$. The solution of (24) is

$$\omega_t = e^{r_1 t}\omega_0 + \int_0^\infty H_{s,t}\tilde{\omega}_s\, ds,$$

where $H_{s,t}$ is defined as

$$H_{s,t} = \frac{\Lambda}{r_2-r_1}\left(e^{\min\{r_1(t-s),-r_2(s-t)\}} - e^{-r_1 t - r_2 s}\right).$$

$$\dot{p}_t = \lambda_p(p_t^* - p_t)\ \underline{\textbf{and}}\ \dot{\omega}_t = \lambda_\omega(\omega_t^* - \omega_t).$$

$$\rho\pi_t = \Lambda_p(\omega_t - mpl_t) + \dot{\pi}_t\ \underline{\textbf{and}}\ \rho\pi_t^w = \Lambda_\omega(mrs_t - \omega_t) + \dot{\pi}_t^w$$

$$\dot{p}_t^* = -(\rho+\lambda_p)(\omega_t - mpl_t) + (\rho+\lambda_p)p_t^*,$$

$$x_t = 1.09682\int_0^\infty \left\{\left(e^{\min\{r_1(t-s),-r_2(s-t)\}} - e^{-r_1 t - r_2 s}\right)\left(\int_0^\infty \left(e^{\min\{r_3(t-s),-r_4(s-t)\}} - e^{-r_3 t - r_4 s}\right)x_s \mathrm{d}s\right)\right\}\mathrm{d}s$$

$$\alpha_r = \frac{.5\widetilde{\Lambda}_p}{\widetilde{\Lambda}}$$