Advanced Econometrics HW2

Paul Braverman

October, 31, 2018

1 Classification vs Regression

In class on October 16 (lecture 4), I went through a proof that "classification is easier than regression". The math I sketched was
Let $\hat{f}$ be any estimate of f. Let $\hat{g}(X) = \mathbb{1}(\hat{f}(X) > 1/2)$. Then, we want to show that:

$$\mathbb{P}(Y \neq \hat{g}(X)|X) - \mathbb{P}(Y \neq g_*(X)|X) = (2f_*(X) - 1)(\mathbb{1}(g_*(X) = 1) - \mathbb{1}(\hat{g}(X) = 1))$$

first we note several things, the fist of which are definitions

$$\begin{aligned} f_*(X) &= \mathbb{P}(Y = 1|X) \\ g_*(X) &= \mathbb{1}(f_*(X) > 1/2) \\ \hat{g}(X) &= \mathbb{1}(\hat{f}(X) > 1/2) \end{aligned}$$

We note that $\mathbb{1}(Y \neq \hat{g}(X)|X) = 1$ can occur two different ways:

$$\mathbb{1}(Y \neq \hat{g}(X)|X) = \begin{cases} 1, & \text{if} \quad Y = 1 \,\&\, \hat{g}(X) \neq 1 \\ 1, & \text{if} \quad Y \neq 1 \,\&\, \hat{g}(X) = 1 \\ 0, & \text{if} \quad \text{otherwise} \end{cases}$$

Similarly, $\mathbb{1}(Y \neq g_*(X)|X) = 1$ can occur two different ways:

$$\mathbb{1}(Y \neq g_*(X)|X) = \begin{cases} 1, & \text{if} \quad Y = 1 \,\&\, g_*(X) \neq 1 \\ 1, & \text{if} \quad Y \neq 1 \,\&\, g_*(X) = 1 \\ 0, & \text{if} \quad \text{otherwise} \end{cases}$$

with this in mind we can do the following manipulations

$$
\begin{aligned}
&\mathbb{P}(Y \neq \hat{g}(X)|X) - \mathbb{P}(Y \neq g_*(X)|X) \\
=&\mathbb{P}(Y = 1 \,\&\, \hat{g}(X) \neq 1|X) + \mathbb{P}(Y \neq 1 \,\&\, \hat{g}(X) = 1|X) - \mathbb{P}(Y = 1 \,\&\, g_*(X) \neq 1|X) - \mathbb{P}(Y \neq 1 \,\&\, g_*(X) = 1|X) \\
=&\Big[\mathbb{P}(Y = 1 \,\&\, \hat{g}(X) \neq 1|X) - \mathbb{P}(Y = 1 \,\&\, g_*(X) \neq 1|X)\Big] + \Big[\mathbb{P}(Y \neq 1 \,\&\, \hat{g}(X) = 1|X) - \mathbb{P}(Y \neq 1 \,\&\, g_*(X) = 1|X)\Big] \\
=&\Big[\mathbb{P}(Y = 1 \,\&\, \hat{g}(X) = 0|X) - \mathbb{P}(Y = 1 \,\&\, g_*(X) = 0|X)\Big] + \Big[\mathbb{P}(Y = 0 \,\&\, \hat{g}(X) = 1|X) - \mathbb{P}(Y = 0 \,\&\, g_*(X) = 1|X)\Big] \\
=&\mathbb{E}\Big[\mathbb{1}(Y = 1 \,\&\, \hat{g}(X) = 0) - \mathbb{1}(Y = 1 \,\&\, g_*(X) = 0)\Big|X\Big] + \mathbb{E}\Big[\mathbb{1}(Y = 0 \,\&\, \hat{g}(X) = 1) - \mathbb{1}(Y = 0 \,\&\, g_*(X) = 1)\Big|X\Big] \\
=&\mathbb{E}\Big[\mathbb{1}(Y = 1)(\mathbb{1}(\hat{g}(X) = 0) - \mathbb{1}(g_*(X) = 0))\Big|X\Big] + \mathbb{E}\Big[\mathbb{1}(Y = 0)(\mathbb{1}(\hat{g}(X) = 1) - \mathbb{1}(g_*(X) = 1))\Big|X\Big] \\
=&\mathbb{E}\Big[\mathbb{1}(Y = 1)(\mathbb{1}(\hat{g}(X) = 0) - \mathbb{1}(g_*(X) = 0)) + \mathbb{1}(Y = 0)(\mathbb{1}(\hat{g}(X) = 1) - \mathbb{1}(g_*(X) = 1))\Big|X\Big] \\
=&\mathbb{E}\Big[\mathbb{1}(Y = 1)(\mathbb{1}(\hat{g}(X) = 0) - \mathbb{1}(g_*(X) = 0)) + [1 - \mathbb{1}(Y = 1)]([1 - \mathbb{1}(\hat{g}(X) = 0)] - [1 - \mathbb{1}(g_*(X) = 0)]\Big|X\Big] \\
=&f_*(X)(\mathbb{1}(\hat{g}(X) = 0) - \mathbb{1}(g_*(X) = 0)) + [1 - f_*(X)]([1 - \mathbb{1}(\hat{g}(X) = 0)] - [1 - \mathbb{1}(g_*(X) = 0)] \\
=&(2f_*(X) - 1)(\mathbb{1}(g_*(X) = 1) - \mathbb{1}(\hat{g}(X) = 1)) \quad \blacksquare
\end{aligned}
$$

We note the second to last step follows from the tower property.

2 Another replication-like exercise

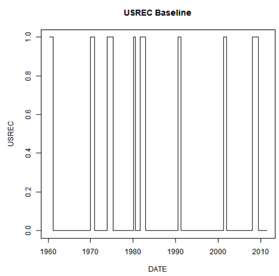
The repo includes an article from Journal of Money, Credit, and Banking as well as data nearly the same as theirs (it is from an earlier version). Load the data and produce out of sample forecasts for $h = 0, 1, 2, 3$. Consider the following methods:

1. Logistic regression
2. Lasso-logistic regression
3. Elnet-logistic regression ($\alpha = 0.5$)

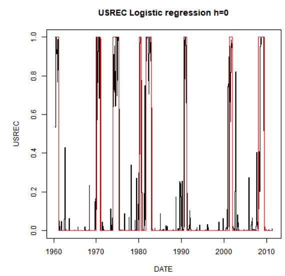
For the regularized methods, try also adding all the squared predictors, and all the squared and cubic predictors. This means a total of 7 models. For the regularized models, use CV to choose λ in each case. Estimate the models using only data up to the year 2000. Produce a table with out-of-sample error rates for each model and each forecast horizon. Use the QPS score on page 855. Do any of these predict the 2008-09 recession? Feel free to try any other classifiers you like.

The following code loads the data and does some processing. The **alldat** object contains the data for h = 0. It is easily adaptable for the other values of h.

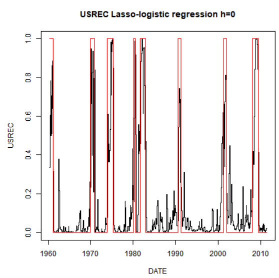
For all seven models and the h = 0 horizon, produce a figure that plots (for the whole time period), the true recessions and the forcasts. Think about how you might make a nice looking graph. the true recession



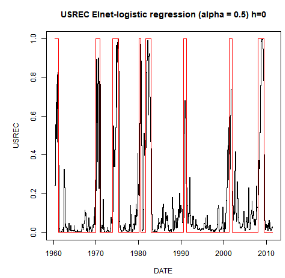
(a) Baseline



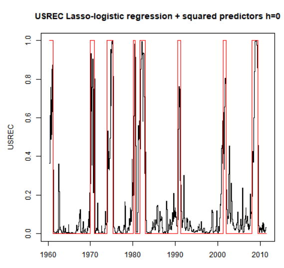
(b) Logistic regression



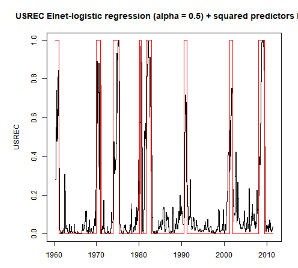
(c) Elnet-logistic regression ($\alpha = 0.5$)



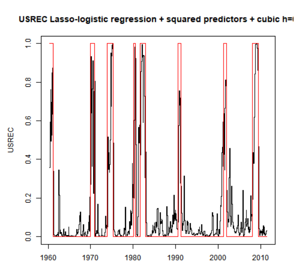
(d) Lasso-logistic regression



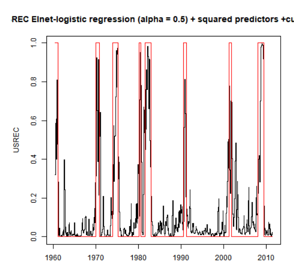
(e) Elnet-logistic regression ($\alpha = 0.5$) [square]



(f) Lasso-logistic regression [square]

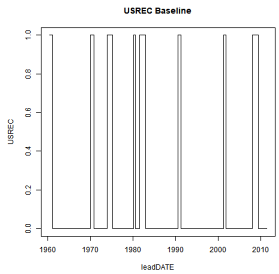


(g) Elnet-logistic regression ($\alpha = 0.5$) [cubed]

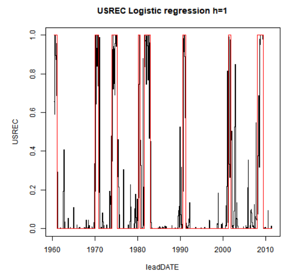


(h) Lasso-logistic regression [cubed]

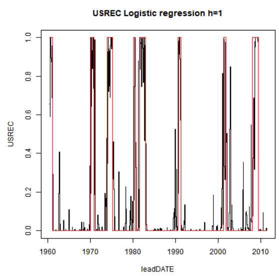
Figure 1: nowcasts for $h = 0$



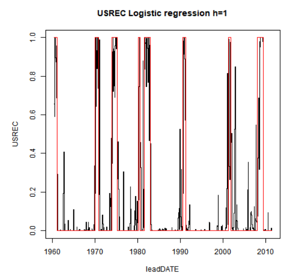
(a) Baseline



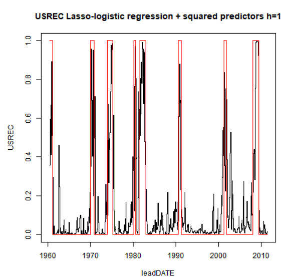
(b) Logistic regression



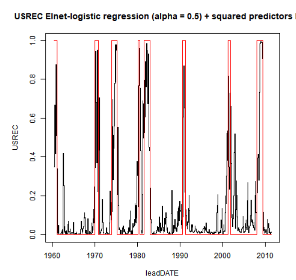
(c) Elnet-logistic regression ($\alpha = 0.5$)



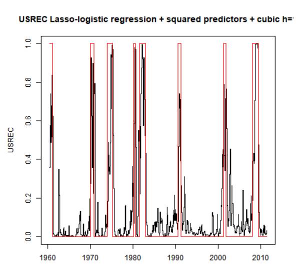
(d) Lasso-logistic regression



(e) Elnet-logistic regression ($\alpha = 0.5$) [square]



(f) Lasso-logistic regression [square]

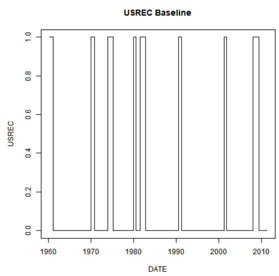


(g) Elnet-logistic regression ($\alpha = 0.5$) [cubed]

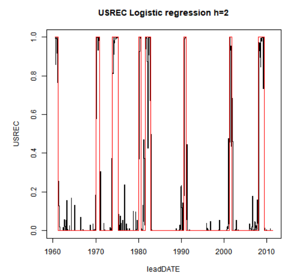
`myplot_7_h1.png`

(h) Lasso-logistic regression [cubed]

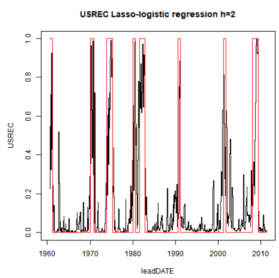
Figure 2: forcasts for $h = 1$



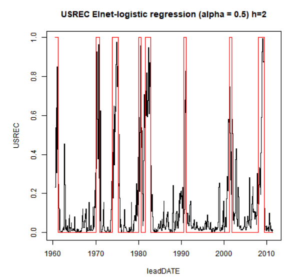
(a) Baseline



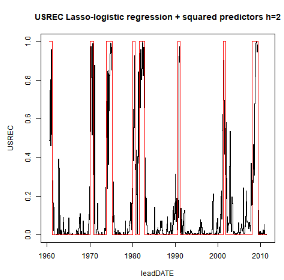
(b) Logistic regression



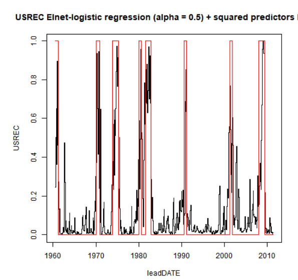
(c) Elnet-logistic regression ($\alpha = 0.5$)



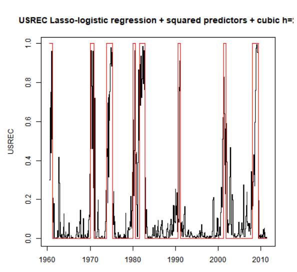
(d) Lasso-logistic regression



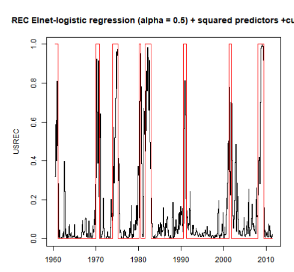
(e) Elnet-logistic regression ($\alpha = 0.5$) [square]



(f) Lasso-logistic regression [square]

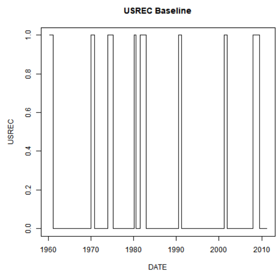


(g) Elnet-logistic regression ($\alpha = 0.5$) [cubed]

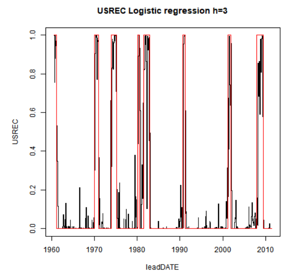


(h) Lasso-logistic regression [cubed]

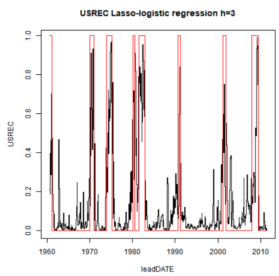
Figure 3: forcasts for $h = 2$



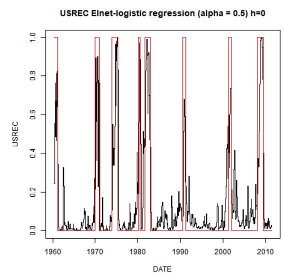
(a) Baseline



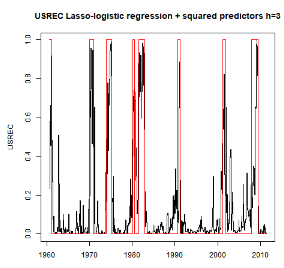
(b) Logistic regression



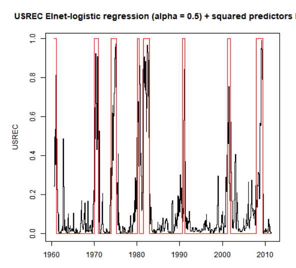
(c) Elnet-logistic regression ($\alpha = 0.5$)



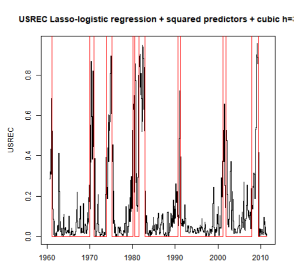
(d) Lasso-logistic regression



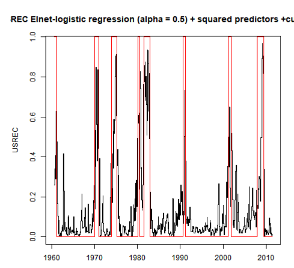
(e) Elnet-logistic regression ($\alpha = 0.5$) [square]



(f) Lasso-logistic regression [square]



(g) Elnet-logistic regression ($\alpha = 0.5$) [cubed]



(h) Lasso-logistic regression [cubed]

Figure 4: forcasts for $h = 3$