## Chapter 1

# Monotonic Reasoning from a Proof-Theoretic Perspective

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Abstract. The article presents the first results we have obtained studying natural reasoning from a proof-theoretic perspective. In particular, we focus our attention on monotonicity reasoning: Inferences are made using structurally parsed sentences on which monotonic positions are displayed. The monotonicity markers are propagated through the proofs via the combined structural and logical rules for the unary operators of Multimodal Categorial Grammar (MMCG). We have chosen to work with such an expressive 'grammar logic' in order to avoid both the use of extra-logical marking devices as made in [SV91] and a too complex lexicon [Dow94]. With MMCG as the parser, the system is able to make the derivations fully within the logic.

## 1.1 Introduction: 'Natural Logic'

The task of accounting for the role of language in drawing inferences has been commonly considered to belong to the domain of formal semantics. Most of the literature in natural reasoning assumes a model-theoretic perspective and uses a formal language as an intermediate step into which natural language expressions are translated.

However, within the generalized type-logical framework [Moo97] the challenge is to assume a direct proof-theoretic perspective; this entails that instead of employing logical forms as vehicles of inference, natural language expressions are used directly, and instead of taking models into account, the validity of an inference is read off the derivation. A system satisfying these criteria will be called a Natural Logic.

We restrict our attention to monotonicity inference in natural language, i.e., inferences that involve replacing an expression with an expression the denotation of which is a subset or a superset of the denotation of the original expression. Our goal is to have a system which automatically accounts for this kind of inference using (1) A formal grammar which calculates and marks the monotonicity positions in a sentence while it is (syntactically) parsing it; and (2) a logic which derives the inferences using the marked output.



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A similar approach has been assumed by Sanchez in [SV91], where linguistic expressions are analyzed by a non-directional Categorial Grammar – i.e., Lambek calculus with Permutation (LP) – enriched with an extra-logical marking algorithm. LP, in fact, cannot account for the propagation of the markers on its own. Moreover, the system based on it, is not direct: it calculates the polarity markers in three independent steps, without automatically giving the marked output.

Finally, because of the simplicity of the grammar the system cannot express any logical control on the sentence structure and the word order. Natural language is sensible to these two aspects, which also have effect on (monotonicity) inferences. For these reasons we have chosen to work with a more expressive version of categorial grammar.

As we will see in the next section, Multimodal Categorial Grammar (MMCG) is able to mark the polarity position directly without using any extra-logical algorithm. Starting from the lexical entries it propagates the monotonicity markers and calculates the polarity simply via its logical and structural rules, yielding an output which can serve as input of the inference to be derived.

## 1.2 A Fragment of Natural Logic

### 1.2.1 Monotonic Reasoning

Natural language inference turns out to cover many forms of reasoning of quite different complexities (see [Ben87]). In this paper we concentrate on monotonicity inferences, which are a widespread phenomenon in natural reasoning. The following examples illustrate the inferences we want to account for.

1. Mary reads an *interesting newspaper* $\Rightarrow$ Mary reads a *newspaper*.
2. No boy bought a *newspaper* $\Rightarrow$ No boy bought an *interesting newspaper*.
3. Mary *carefully listens* to the news $\Rightarrow$ Mary *listens* to the news.
4. No boy *listens* to the news $\Rightarrow$ No boy *carefully listens* to the news.
5. Mary doesn't *carefully listen* to the news $\Rightarrow$ Mary doesn't *eagerly and carefully listen* to the news.

The inferences all involve substituting an expression by an expression the denotation of which is a superset (see (1) and (3)) or a subset (see (2), (4) and (5)) of the denotation of the original expression. We will denote this inclusion relation by $[\![\mathrm{P}]\!] \leq [\![\mathrm{Q}]\!]$, i.e., P denotes a subset of the set denoted by Q, and we summarize the described behavior with the following inference schema:

If $[\![P]\!] \leq [\![Q]\!]$ and N is a monotonic context, then

$$\frac{N[Q]}{N[P]}\ (a) \qquad \text{or} \qquad \frac{N[P]}{N[Q]}\ (b)$$

The examples (1) and (3) instantiate ($b$), while (2), (4) and (5) exemplify ($a$).

The monotonicity calculus can predict the above inferences. This calculus is based on the individuation of the monotonicity property of the single word, information needed to know which kind of replacements can be done. Establishing the monotonicity of all positions within a sentence is based on the following definition.

**Definition 1.2.1** *Let $f : A \rightarrow B$ be a function and let $\leq_A$, $\leq_B$ be partial orders on $A$ and $B$, respectively. Then*

a. $f$ is an *increasing monotonic function* (not. $\uparrow$Mon) iff $\forall x, y \in A$, if $x \leq_A y$ then $f(x) \leq_B f(y)$;

b. $f$ is a *decreasing monotonic function* (not. $\downarrow$Mon) iff $\forall x, y \in A$, if $x \leq_A y$ then $f(y) \leq_B f(x)$; and

c. $f$ is a *non-monotonic function* (not. $\not\uparrow$Mon) iff it is neither $\uparrow$Mon nor $\downarrow$Mon.

As is well-known, in Categorial Grammar (CG) each word is assigned either a primitive or a complex category. The latter are functions $\alpha\backslash\beta$ and $\beta/\alpha$ which take arguments of category $\alpha$ to yield values of category $\beta$.

In this way CG can assign a functor-argument structure to arbitrary sentences. Consequently, the formalism allows the application of Definition 1.2.1 to natural language expressions. Words of complex categories can display different monotonicity behavior. In [Zwa86] detailed tests are given for establishing these properties of linguistic expressions.

Sanchez [SV91] proposes a three-step algorithm which assigns the right monotonicity properties to the positions in a syntactically parsed sentence. The first step is called *Lexical Monotonicity Marking* and makes the monotonicity properties of each word visible on the type level of the formal grammar. The algorithm takes the syntactic derivation given by LP and labels the leaves with the monotonicity markers. It assigns '+' and '–' to the argument of an increasing and decreasing monotonic function, respectively, and leaves the argument of a non-monotonic function unmarked. Let $(a, b)$ denote a function with argument $a$ and value $b$,

(i) if $(a, b)$ is $\uparrow$Mon, then $(a^+, b)$;
(ii) if $(a, b)$ is $\downarrow$Mon, then $(a^-, b)$;
(iii) if $(a, b)$ is $\not\uparrow$Mon, then $(a, b)$.

We will say that $a$ is in a positive position in (i) and in a negative position in (ii). By way of an example we give some entries.

**Lexical Entries**

$$\begin{array}{ll} \textbf{walks} \in (e^+, t) & \textbf{a} \in ((e,t)^+, ((e,t)^+, t)) \\ \textbf{reads} \in (e^+, (e^+, t)) & \textbf{no} \in ((e,t)^-, ((e,t)^-, t)) \\ \textbf{doesn't} \in ((e,t)^-, (e,t)) & \textbf{exactly2} \in ((e,t), ((e,t), t)) \end{array}$$

Thus the determiner 'a' is an increasing monotonic function with respect to both of its arguments, which are therefore marked with '+'.

To automatically derive monotonicity inferences we still need (1) to propagate the markers from the lexical entries to the parsed output; and (2) to assume a partial order among the linguistic expressions of the one and same type, e.g., $[\![\text{interesting newspaper}]\!] \leq [\![\text{newspaper}]\!]$, $[\![\text{every woman}]\!] \leq [\![\text{mary}]\!] \leq [\![\text{a woman}]\!]$. The latter is established by means of semantic tests, the results of which are stored in the system.

In order to account for (1), Sanchez enriches the logical rules of LP with monotonicity markers following the idea that (i) functors are always in a positive monotonic position [Ben87]; and (ii) arguments inherit their marker from their functor. These two aspects are embodied in the Functional Application rule. The second step of the algorithm, *External Monotonicity Marking*, propagates the markers (cf. (i)) and marks each functor with a '+' (cf. (ii)). Let $* \in \{+, -\}$,

$$\frac{(a^*, b) \quad a}{b} \quad \leadsto \quad \frac{[(a^*, b)]^+ \quad a^*}{b}$$

However, the monotonicity markers on their own do not suffice for marking each word within the sentence with the right polarity. A comparison of the examples (3) "Mary carefully listens to the news" and (5) "Mary doesn't carefully listen to the news", reveals the important role that (monotonic) function composition plays here: the presence of "doesn't" in (5) changes the polarity of the verb phrase, and this entails that different substitutions are allowed.

In order to account for this combinatorial aspect, Sanchez' algorithm [SV91] contains a third step, called *Polarity Determination*, which assigns a polarity to each node of a derivation in which monotonicity markers have been assigned by Monotonicity Marking.

**Definition 1.2.2** *Let $D$ be a derivation with root $\alpha$ that has undergone Monotonicity Marking. Then*

(*i*) A node $\gamma$ *has polarity* in D iff all the nodes in the path from $\gamma$ to $\alpha$ are marked and

(*ii*) A node $\gamma$ *is positive* iff $\gamma$ has polarity, and the number of nodes marked by '$-$' in the path from $\gamma$ to $\alpha$ is even.

(*iii*) A node $\gamma$ *is negative* iff $\gamma$ has polarity, and the number of nodes marked by '–' in the path from $\gamma$ to $\alpha$ is odd.

The above definition incorporates a fundamental aspect of monotonicity calculus– viz., the composition of monotonic functions – into the system. Let Mon range over $\nmid$Mon, $\uparrow$Mon or $\downarrow$Mon,

**Monotonic Function Composition**

| | | | | | |
|---|---|---|---|---|---|
| (i) | $\nmid\text{Mon} \circ \text{Mon}$ | $=$ | $\text{Mon} \circ \nmid\text{Mon}$ | $=$ | $\nmid\text{Mon}$; |
| (ii) | $\uparrow\text{Mon} \circ \uparrow\text{Mon}$ | $=$ | $\downarrow\text{Mon} \circ \downarrow\text{Mon}$ | $=$ | $\uparrow\text{Mon}$; |
| (iii) | $\downarrow\text{Mon} \circ \uparrow\text{Mon}$ | $=$ | $\uparrow\text{Mon} \circ \downarrow\text{Mon}$ | $=$ | $\downarrow\text{Mon}$. |

Having the distinct steps of Monotonicity Marking and Polarity Determination makes it possible for a word to be, for example, in a negative polarity position while it is marked by a positive monotonicity marker. We will come back to this in section 1.3.

For reason of space we cannot go into the details of Sanchez' algorithm. See [SV91] for further details. For our present purposes, it suffices to conclude that although Sanchez' algorithm can predict the right monotonicity inferences, it obtains this result using extra-logical marking devices.

As announced in the introduction, our main goal is to present a system able to properly account for the output marking within its logic, and to avoid the independent steps of monotonicity marking and polarity determination. The same goal has been pursued in [Dow94]. We will first describe our proposal, and then discuss the advantages of using MMCG rather than a simpler categorial grammar.

### 1.2.2 Monotonic Reasoning within MMCG

MMCG is a development of Categorial Grammar that overcomes certain expressive limitations of "Classical" Categorial Systems. It consists of two components: (i) a base logic in which no structural rules are available; and (ii) packages of structural rules that can be lexically controlled via $\Diamond$, $\Box^{\downarrow}$ – a pair of unary operators added to $/$, $\bullet$, $\backslash$, the familiar binary ones.

The base logic, with its Introduction, Elimination rules for the logical constants, and their Curry-Howard interpretation captures invariants of grammatical composition. The structural packages make it possible for the form/meaning correspondence to be realized in different ways across languages. See [Moo97] for more details.

We will use the $\Diamond$, $\Box^{\downarrow}$, and the structural rules for these operators to propagate the monotonicity markers and to calculate the polarity of the single nodes step by step in the process of dynamically building a proof, so that the output sentence will have its polarity already assigned.

Assuming that the reader is familiar with the Introduction/Elimination rules for $/$ and $\backslash$, we present the logical rules for the new unary operators. In what follows $\Gamma \vdash A$ denotes that the structure $\Gamma$ is of category A. Structures are built up from formulas with the binary operation $(.. \circ ..)$ and the unary operation $< .. >$, structural counterparts of $\bullet$ and $\diamond$, respectively. Logical (and structural) operators can be indexed, so that structural inferences can be relativized to these indices.

**Logical rules for the unary operators**

$$\frac{\Delta \vdash \diamond_i A \quad \Gamma[\langle A \rangle^i] \vdash B}{\Gamma[\Delta] \vdash B}\,[\diamond_i E] \qquad \frac{\Gamma \vdash A}{\langle \Gamma \rangle^i \vdash \diamond_i A}\,[\diamond_i I]$$

$$\frac{\Gamma \vdash \Box_i^{\downarrow} A}{\langle \Gamma \rangle^i \vdash A}\,[\Box_i^{\downarrow} E] \qquad \frac{\langle \Gamma \rangle^i \vdash A}{\Gamma \vdash \Box_i^{\downarrow} A}\,[\Box_i^{\downarrow} I]$$

In what follows the index $i$ will stand for the monotonicity markers. We use '+' and '$-$' to mark positive and negative positions, respectively and '0' to mark the non-monotonic ones.

Reflecting the fact that words in functor position are always in an increasing monotonic position, functors are decorated with $\Box_+^{\downarrow}$. This will keep track, via the $[\Box^{\downarrow}$ E$]$ rule, of the lexical entries which figure as functors in the proof. The propagation of monotonicity properties from a functor to the words that function as its arguments is given by the logical rules for the $\diamond$ operator.

In order to understand how the logical rules of the unary operators can be employed we look at some examples. One of the advantages of assuming a direct proof-theoretic perspective in the study of natural reasoning is that the Natural Logic thus obtained can be implemented more easily. The system described in this paper has been implemented in Grail, an automated theorem prover for CG Logics developed by Moot [Moo98]. We will use its proof format in the presentation of the examples below.

When attempting to prove (the grammaticality of) a sentence, Grail starts from the lexical entries stored in a database and applies the logical and structural rules of the system.

Let us take *John walks* as our first example. The category assigned to 'walks' in the lexicon incorporates the semantic fact that intransitive verbs are always monotonic increasing functions.

**Lexical entries**

$\texttt{John} \vdash np$

$\texttt{walks} \vdash \Box_+^{\downarrow}(\diamond_+ np \backslash s)$

**Derivation**

$$\frac{\dfrac{\texttt{John} \vdash np}{\langle \texttt{John} \rangle^+ \vdash \diamond_+ np}\,[\diamond_+ I] \quad \dfrac{\texttt{walks} \vdash \Box_+^{\downarrow}(\diamond_+ np \backslash s)}{\langle \texttt{walks} \rangle^+ \vdash \diamond_+ np \backslash s}\,[\Box_+^{\downarrow} E]}{\langle \texttt{John} \rangle^+ \circ \langle \texttt{walks} \rangle^+ \vdash s}\,[\backslash E]$$

As we can see from the proof, rule $[\Diamond_+ I]$ is applied because of the $\Diamond_+$ which heads the argument *np* of the functor 'walks' and it marks the argument, 'John', with the required polarity. As sketched above, rule $[\Box_+ E]$, instead, keeps track of the functional application, marking the lexical entry 'walks', which is the functor, with the structural unary operator $< .. >^+$.

Based on this marked and structurally analyzed sentence which is the output of our "grammar logic" and making use of the fact that $[\![\text{walks}]\!] \leq [\![\text{moves}]\!]$, the Natural Logic can draw the following inference:

$$\frac{\langle \texttt{John}\rangle^+ \circ \langle \texttt{walks}\rangle^+ \vdash s}{\langle \texttt{John}\rangle^+ \circ \langle \texttt{moves}\rangle^+ \vdash s}$$

For dealing with more complex sentences, we now introduce structural rules. For the binary operators, we will simply assume a *global* associative rule (space limitations make it impossible to discuss appropriate forms of control here).

**Structural Rules**

$$\frac{\Gamma[\Delta_1 \circ (\Delta_2 \circ \Delta_3)] \vdash C}{\Gamma[(\Delta_1 \circ \Delta_2) \circ \Delta_3] \vdash C}\,[P1] \qquad \frac{\Gamma[(\Delta_1 \circ \Delta_2) \circ \Delta_3] \vdash C}{\Gamma[\Delta_1 \circ (\Delta_2 \circ \Delta_3)] \vdash C}\,[P2]$$

For the propagation of monotonicity markers, we need information about the composition of monotonicity markers within more complex structures, i.e., about the determination of polarity on the basis of monotonicity markers. We encode the above table of monotonic function composition by means of structural rules which derives $\Gamma[\Delta'] \vdash C$ from $\Gamma[\Delta] \vdash C$, where $\Delta$ and $\Delta'$ are as specified in the table below:

| | $\Delta$ | $\Delta'$ | | $\Delta$ | $\Delta'$ |
|---|---|---|---|---|---|
| $[Mon0i]$ | $\langle\langle\Sigma\rangle^0\rangle^i$ | $\langle\Sigma\rangle^0$ | $[Moni0]$ | $\langle\langle\Sigma\rangle^i\rangle^0$ | $\langle\Sigma\rangle^0$ |
| $[Mon++]$ | $\langle\langle\Sigma\rangle^+\rangle^+$ | $\langle\Sigma\rangle^+$ | $[Mon--]$ | $\langle\langle\Sigma\rangle^-\rangle^-$ | $\langle\Sigma\rangle^+$ |
| $[Mon-+]$ | $\langle\langle\Sigma\rangle^-\rangle^+$ | $\langle\Sigma\rangle^-$ | $[Mon+-]$ | $\langle\langle\Sigma\rangle^+\rangle^-$ | $\langle\Sigma\rangle^-$ |

The monotonicity distribution of a marker over all the elements within its scope is accounted for by the following rule:

$$\frac{\Gamma[\langle\Delta_1 \circ \Delta_2\rangle^i] \vdash C}{\Gamma[\langle\Delta_1\rangle^i \circ \langle\Delta_2\rangle^i] \vdash C}\,[Moni]$$

In order to see how these postulates together with the logical rules succeed in determining the polarity markers in the process of a proof, we consider the example of the object wide scope reading of the sentence *No boy reads a book.*

**Lexical entries**

$$\begin{array}{ll} \texttt{no} \vdash \Box^{\downarrow}_{+}((s/\Diamond_{-}(\Diamond_{i}np\backslash s))/\Diamond_{-}n) & \texttt{boy} \vdash n \\ \texttt{a} \vdash \Box^{\downarrow}_{+}((\Diamond_{+}(s/\Diamond_{i}np)\backslash s)/\Diamond_{+}n) & \texttt{book} \vdash n \\ \texttt{reads} \vdash \Box^{\downarrow}_{+}((\Diamond_{+}np\backslash s)/\Diamond_{+}np) & \texttt{exactly2} \vdash \Box^{\downarrow}_{+}((s/\Diamond_{0}(\Diamond_{i}np\backslash s))/\Diamond_{0}n) \end{array}$$

For example, the category assigned to 'no' expresses that it is a decreasing monotonic function in both its arguments $\Diamond_{-}n$ and $\Diamond_{-}(\Diamond_{i}np\backslash s)$. This is captured by the fact that both argument are headed by $\Diamond_{-}$, while the second argument of 'no' can be an increasing, a decreasing or a non-monotonic function – since its $np$ argument is headed by $\Diamond_{i}$, which ranges over $\Diamond_{+}, \Diamond_{-}, \Diamond_{0}$. Applying these entries and applying the logical and the structural rules to the above entries, the following proof is derived:

$$\dfrac{[\mathrm{r}_0 \vdash \Diamond_{+}np]^1 \quad \begin{array}{c}\vdots\\ \langle \mathrm{p}_1\rangle^{+} \circ (\langle\texttt{reads}\rangle^{+} \circ \langle \mathrm{r}_2\rangle^{+}) \vdash s\end{array}}{\mathrm{r}_0 \circ (\langle\texttt{reads}\rangle^{+} \circ \langle \mathrm{r}_2\rangle^{+}) \vdash s}\,[\Diamond E]$$

$$\dfrac{\mathrm{r}_0 \circ (\langle\texttt{reads}\rangle^{+} \circ \langle \mathrm{r}_2\rangle^{+}) \vdash s}{\langle\texttt{reads}\rangle^{+} \circ \langle \mathrm{r}_2\rangle^{+} \vdash \Diamond_{+}np\backslash s}\,[\backslash I]^1$$

$$\dfrac{\langle\texttt{reads}\rangle^{+} \circ \langle \mathrm{r}_2\rangle^{+} \vdash \Diamond_{+}np\backslash s}{\langle\langle\texttt{reads}\rangle^{+} \circ \langle \mathrm{r}_2\rangle^{+}\rangle^{-} \vdash \Diamond_{-}(\Diamond_{+}np\backslash s)}\,[\Diamond_{-}I]$$

$$\dfrac{\begin{array}{c}\vdots\\ \langle\texttt{no}\rangle^{+} \circ \langle\texttt{boy}\rangle^{-} \vdash s/\Diamond_{-}(\Diamond_{+}np\backslash s)\end{array} \quad \langle\langle\texttt{reads}\rangle^{+} \circ \langle \mathrm{r}_2\rangle^{+}\rangle^{-} \vdash \Diamond_{-}(\Diamond_{+}np\backslash s)}{(\langle\texttt{no}\rangle^{+} \circ \langle\texttt{boy}\rangle^{-}) \circ \langle\langle\texttt{reads}\rangle^{+} \circ \langle \mathrm{r}_2\rangle^{+}\rangle^{-} \vdash s}\,[/E]$$

$$\dfrac{(\langle\texttt{no}\rangle^{+} \circ \langle\texttt{boy}\rangle^{-}) \circ \langle\langle\texttt{reads}\rangle^{+} \circ \langle \mathrm{r}_2\rangle^{+}\rangle^{-} \vdash s}{(\langle\texttt{no}\rangle^{+} \circ \langle\langle\texttt{boy}\rangle^{+}\rangle^{-}) \circ (\langle\langle\texttt{reads}\rangle^{+}\rangle^{-} \circ \langle\langle \mathrm{r}_2\rangle^{+}\rangle^{-}) \vdash s}\,[Mon-]$$

$$\dfrac{(\langle\texttt{no}\rangle^{+} \circ \langle\langle\texttt{boy}\rangle^{+}\rangle^{-}) \circ (\langle\langle\texttt{reads}\rangle^{+}\rangle^{-} \circ \langle\langle \mathrm{r}_2\rangle^{+}\rangle^{-}) \vdash s}{(\langle\texttt{no}\rangle^{+} \circ \langle\texttt{boy}\rangle^{-}) \circ (\langle\texttt{reads}\rangle^{-} \circ \langle \mathrm{r}_2\rangle^{-}) \vdash s}\,[Mon+-]$$

$$\dfrac{(\langle\texttt{no}\rangle^{+} \circ \langle\texttt{boy}\rangle^{-}) \circ (\langle\texttt{reads}\rangle^{-} \circ \langle \mathrm{r}_2\rangle^{-}) \vdash s}{((\langle\texttt{no}\rangle^{+} \circ \langle\texttt{boy}\rangle^{-}) \circ \langle\texttt{reads}\rangle^{-}) \circ \langle \mathrm{r}_2\rangle^{-} \vdash s}\,[P1]$$

$$\dfrac{[\mathrm{p}_2 \vdash \Diamond_{-}np]^4 \quad ((\langle\texttt{no}\rangle^{+} \circ \langle\texttt{boy}\rangle^{-}) \circ \langle\texttt{reads}\rangle^{-}) \circ \langle \mathrm{r}_2\rangle^{-} \vdash s}{((\langle\texttt{no}\rangle^{+} \circ \langle\texttt{boy}\rangle^{-}) \circ \langle\texttt{reads}\rangle^{-}) \circ \mathrm{p}_2 \vdash s}\,[\Diamond_{+}E]^3$$

$$\dfrac{((\langle\texttt{no}\rangle^{+} \circ \langle\texttt{boy}\rangle^{-}) \circ \langle\texttt{reads}\rangle^{-}) \circ \mathrm{p}_2 \vdash s}{(\langle\texttt{no}\rangle^{+} \circ \langle\texttt{boy}\rangle^{-}) \circ \langle\texttt{reads}\rangle^{-} \vdash s/\Diamond_{-}np}\,[/I]^4$$

$$\dfrac{(\langle\texttt{no}\rangle^{+} \circ \langle\texttt{boy}\rangle^{-}) \circ \langle\texttt{reads}\rangle^{-} \vdash s/\Diamond_{-}np}{\langle(\langle\texttt{no}\rangle^{+} \circ \langle\texttt{boy}\rangle^{-}) \circ \langle\texttt{reads}\rangle^{-}\rangle^{+} \vdash \Diamond_{+}(s/\Diamond_{-}np)}\,[\Diamond_{+}I]$$

$$\dfrac{\langle(\langle\texttt{no}\rangle^{+} \circ \langle\texttt{boy}\rangle^{-}) \circ \langle\texttt{reads}\rangle^{-}\rangle^{+} \vdash \Diamond_{+}(s/\Diamond_{-}np) \quad \langle\texttt{a}\rangle^{+} \circ \langle\texttt{book}\rangle^{+} \vdash \Diamond_{+}(s/\Diamond_{-}np)\backslash s}{\langle(\langle\texttt{no}\rangle^{+} \circ \langle\texttt{boy}\rangle^{-}) \circ \langle\texttt{reads}\rangle^{-}\rangle^{+} \circ (\langle\texttt{a}\rangle^{+} \circ \langle\texttt{book}\rangle^{+}) \vdash s}\,[\backslash I]$$

$$\dfrac{\langle(\langle\texttt{no}\rangle^{+} \circ \langle\texttt{boy}\rangle^{-}) \circ \langle\texttt{reads}\rangle^{-}\rangle^{+} \circ (\langle\texttt{a}\rangle^{+} \circ \langle\texttt{book}\rangle^{+}) \vdash s}{((\langle\texttt{no}\rangle^{+} \circ \langle\texttt{boy}\rangle^{-}) \circ \langle\texttt{reads}\rangle^{-}) \circ (\langle\texttt{a}\rangle^{+} \circ \langle\texttt{book}\rangle^{+}) \vdash s}\,[Mon+]$$

On the basis of this proof, the following inference can be made:

If $[\![\text{carefully reads}]\!] \leq [\![\text{reads}]\!]$, then

$$\dfrac{((\texttt{no} \circ \texttt{boy}) \circ \langle\texttt{reads}\rangle^{-}) \circ (\texttt{a} \circ \texttt{book}) \vdash s}{((\texttt{no} \circ \texttt{boy}) \circ \langle\texttt{carefully} \circ \texttt{reads}\rangle^{-}) \circ (\texttt{a} \circ \texttt{book}) \vdash s}$$

Adopting a top to bottom perspective, we see that the proof is given by hypothetical reasoning, assuming the arguments taken by 'reads' – $\mathrm{r}_0$ and $\mathrm{r}_2$ – and substituting them with 'no boy' and 'a book', respectively. In particular, the second substitution which proceeds via $[/I]^4$, is made possible by applying the structural rules $[Mon-]$ $[Mon+-]$, which distribute the monotonicity markers in accordance with the monotonic function composition. Thus in the application of $[/I]^4$ the fact that the $np$ is abstracted from a negative context is marked by the $\Diamond_{-}$. After that, the proof goes on as usual. A clear explanation of the use of hypothetical reasoning in MMCG can be found in [Moo97].

The narrow scope object reading of *Nobody reads a book*, gives rise to a marked output which justifies the following monotonicity inference:

$$\frac{(\langle \texttt{no}\rangle^{+} \circ \langle \texttt{boy}\rangle^{-}) \circ (\langle \texttt{reads}\rangle^{-} \circ (\langle \texttt{a}\rangle^{-} \circ \langle \texttt{book}\rangle^{-})) \vdash s}{(\langle \texttt{no}\rangle^{+} \circ \langle \texttt{boy}\rangle^{-}) \circ (\langle \texttt{reads}\rangle^{-} \circ (\langle \texttt{a}\rangle^{-} \circ \langle \texttt{nice} \circ \texttt{book}\rangle^{-})) \vdash s}$$

Scope ambiguity phenomena interact with monotonicity inference in an interesting way. Note that the Natural Logic described so far derives inferences which are *logically* correct but *intuitively* wrong.[1] This reveals something about the inadequacy of the interpretations that are commonly assigned to the relevant examples. Considers, e.g., "Mary reads no book", for which we can derive the following inferences. They are both logically correct, although only the second could intuitively be accepted.

If $[\![(\text{every} \circ \text{woman})]\!] \leq [\![\text{mary}]\!] \leq [\![(\text{a} \circ \text{woman})]\!]$, then

the wide-scope object reading entails:

$$\frac{(\langle \texttt{mary}\rangle^{-} \circ \langle \texttt{reads}\rangle^{-}) \circ (\langle \texttt{no}\rangle^{-} \circ \langle \texttt{book}\rangle^{-}) \vdash s}{(\langle \texttt{every} \circ \texttt{woman}\rangle^{-} \circ \langle \texttt{reads}\rangle^{-}) \circ (\langle \texttt{no}\rangle^{-} \circ \langle \texttt{book}\rangle^{-}) \vdash s}$$

and the narrow-scope object reading entails:

$$\frac{\langle \texttt{mary}\rangle^{+} \circ (\langle \texttt{reads}\rangle^{-} \circ (\langle \texttt{no}\rangle^{+} \circ \langle \texttt{book}\rangle^{-})) \vdash s}{\langle \texttt{a} \circ \texttt{woman}\rangle^{+} \circ (\langle \texttt{reads}\rangle^{-} \circ (\langle \texttt{no}\rangle^{+} \circ \langle \texttt{book}\rangle^{-})) \vdash s}$$

Sanchez [SV91] accounts for these phenomena by assuming a *Polarity convention* according to which "[I]n the derivation corresponding to the string x Y, x must be positive" (pag. 150). This convention will block the first inference. Discussing this issue he observes that it has the philosophical implication that proper names are scope-less *semantically*, but they are not so *inferentially*.

Another important linguistic phenomenon which can be explored using Natural Logic involves *polarity items*, i.e., expression which can occur only in negative or positive contexts. Such expressions are called *negative polarity items* and *positive polarity items*, respectively. In [Dow94], Dowty presents an alternative formulation of Sanchez' algorithm which is able to account for these phenomena. He builds his Natural Logic on a simpler formalism than MMCG. The reader maybe wonder whether the full power of a unary multi-modal logic is really needed in order to obtain our goal. In the next section, we will address this question, comparing our proposal with Dowty's and showing the advantages offered by using a stronger logic.

[1]If, for example, we translate the sentence *Mary reads no book* into Predicate Logic we clearly see that the inference is logically correct: if $\forall x(B(x) \rightarrow \neg R(m,x))$ and $W(m)$ holds, then $\forall x(B(x) \rightarrow \neg\forall y(W(y) \rightarrow R(y,x)))$ also holds, although nobody will consider the corresponding natural language inference valid.

## 1.3 Comparison: Dowty's Polarity Marking

[Dow94] presents a direct Natural Logic, in which the independent steps of Monotonicity Marking and Polarity Determination collapse into a syntactic derivation. In this approach the symbols '+' and '–' are used unambiguously only to indicate the (final) logical polarity.

The main characteristics of Dowty's system are:

a. Since one and the same word can appear with positive polarity in one derivation and with negative polarity in another, most lexical items – with the important exception of polarity items – will have both a '+' and a '–' marked category, with the same interpretation.

b. ↑Mon functors appear assigned categories of the forms $\mathrm{A}^+/\mathrm{B}^+$ and $\mathrm{A}^-/\mathrm{B}^-$, i.e., they *preserve* the polarity markers. We will mark them as $\mathrm{A}^x/\mathrm{B}^x$.

c. ↓Mon functors are assigned categories of the forms $\mathrm{A}^+/\mathrm{B}^-$ and $\mathrm{A}^-/\mathrm{B}^+$, i.e., they *reverse* the polarity markers, we will mark them as $\mathrm{A}^x/\mathrm{B}^y$.

Functional Application respects the polarity markers in the following way. Let $\alpha, \beta \in \{+, -\}$,

$$\frac{A^\alpha/B^\beta \quad B^\beta}{A^\alpha}$$

where '$\alpha$' and '$\beta$' coincide when the main premise $A^\alpha/B^\beta$ is a ↑Mon function, and they differ when is a ↓Mon function. Moreover, when a non-monotonic function is applied, the argument and the minor premise have no marker '$\beta$'.

The grammar thus defined generates sentences of category $\mathrm{S}^+$ or $\mathrm{S}^-$. The former is the category of independent sentences, the latter of sentences embedded inside a ↓Mon function.

Using this grammar for monotonicity inference, we will assume the below lexical entries. Let $x \in \{+, -\}$, let $y$ be the "negation" of $x$, and let $VP = NP\backslash S$,

**Lexical Entries**

| | |
|---|---|
| **walks** $= VP^x$ | **a** $= (S^x/VP^x)/CN^x$ |
| **reads** $= VP^x/NP^x$ | **no** $= (S^x/VP^y)/CN^y$ |
| **John** $= S^x/VP^x$ | **any** $= (S^-/VP^-)/CN^-$ |
| **does not** $= VP^x/VP^y$ | **several** $= (S^+/VP^+)/CN^+$ |

For decoding the given lexical entries we need the following polarity marking definition for complex categories: $(A/B)^x =_{def} (A^x/B)^x =_{def} (A^x/B)$. This definition is based on the fact proven by Sanchez that in a function-argument combination the function always has the same polarity as the combination as a whole. Of course an analogous definition can be given for $(A\backslash B)^x$. We present some examples of polarity marking.

1. *John walks*

$$\dfrac{\dfrac{\text{John}}{S^+/VP^+} \qquad \dfrac{\text{walks}}{VP^+}}{S^+}$$

2. *John doesn't walk*

$$\dfrac{\dfrac{\text{John}}{S^+/VP^+} \qquad \dfrac{\dfrac{\text{does not}}{VP^+/VP^-} \qquad \dfrac{\text{walk}}{VP^-}}{VP^+}}{S^+}$$

Comparing these two derivations, note how the polarity of the $VP$ is changed by the presence of the ↓Mon 'does not'. Using a bottom-up reading the inference says that: if the string "John does not walk" is a well-formed independent sentence $S^+$, then 'John', last function applied, has to have $S^+$ as a value. Since 'John' is a ↑Mon function, this means its argument must be marked with a '+' as well. This requires 'does not walk' to be of category $VP^+$. Therefore, the value of the ↓Mon function 'does not' must be marked with the '+' as well, and consequently its argument is marked negatively.

To deal with more complex sentences in which a generalized quantifier occurs in the object position, Dowty includes in the lexicon for each determiner $(S^\alpha/VP^\beta)/CN^\gamma$ an object counterpart, of category $(TV^\beta\backslash VP^\alpha)/CN^\gamma$, where $TV^\beta = (NP^\beta\backslash S^\beta)/NP^\beta$. The object-counterparts of the above given lexical entries of 'a', 'no' 'any' and 'several' are as follows:

$$\mathbf{a} = \left\{ \begin{array}{l} (TV^+\backslash VP^+)/CN^+ \\ (TV^-\backslash VP^-)/CN^- \end{array} \right\} \qquad \mathbf{no} = \left\{ \begin{array}{l} (TV^-\backslash VP^+)/CN^- \\ (TV^+\backslash VP^-)/CN^- \end{array} \right\}$$

$$\mathbf{any} = (TV^-\backslash VP^-)/CN^- \qquad \mathbf{several} = (TV^+\backslash VP^+)/CN^+$$

We can now give the derivation of the wide-scope reading of *No boy reads a book.*

$$\dfrac{\dfrac{\dfrac{\text{no}}{(S^+/VP^-)/CN^-} \quad \dfrac{\text{boy}}{CN^-}}{S^+/VP^-} \qquad \dfrac{\dfrac{\text{reads}}{TV^-} \quad \dfrac{\dfrac{\text{a}}{(TV^-\backslash VP^-)/CN^-} \quad \dfrac{\text{book}}{CN^-}}{TV^-\backslash VP^-}}{VP^-}}{S^+}$$

The formalism used by Dowty is able to directly account for the same monotonic inferences as the ones we propose. But in order to avoid the independent steps of Sanchez' algorithm, Dowty's formalism needs to duplicate the category assignment for each lexical entry. Moreover, the system so described needs four different categories for the determiner in order to be able to account for scope ambiguity phenomena. Finally, the lack of a distinction between the structural and the logical part of the system makes the parser unable to give an output marked in such a way that the inference can be derived automatically.

However, Dowty's system does shed light on important phenomena that involve monotonicity, namely polarity items. The lexical entries assigned to 'any' and 'several' show that an account of these expressions requires the presence of information on both monotonicity and polarity marking already in the syntactical parsing. Observe that MMCG can account for these double properties via its unary operators, viz., in the following way:

- **any** $\vdash \Box^{\downarrow}_{+}((\Diamond_{+}tv\backslash\Box^{\downarrow}_{-}\Diamond_{-}iv)/\Diamond_{+}n)$
- **several** $\vdash \Box^{\downarrow}_{+}((\Diamond_{+}tv\backslash\Box^{\downarrow}_{+}\Diamond_{+}iv)/\Diamond_{+}n)$

where $tv = (\Diamond_{+}np\backslash s)/\Diamond_{+}np$, and $iv = (\Diamond_{+}np\backslash s)$. As the reader can find out computing the derivation of 'John does not read any book' and 'John does not read several books', the $\Diamond$ of the $iv$ forces the polarity item to be taken as argument of the required monotonic function, (decreasing in the case of the negative polarity item *any*, and increasing in case of positive polarity item *several*). The $\Box^{\downarrow}$ heading the $\Diamond$iv, instead, accounts for the right polarity of the whole expression which function as argument.

## 1.4 Conclusion and further research

The examples discussed in the paper make us think that polarity determination incorporate semantic explanations, as well as syntactic properties. We would like to explore these phenomena further, focusing on scope-ambiguity phenomena particularly within negative contexts. Besides, we plan to investigate negative polarity items, developing the idea here presented and studying their behavior from a cross-linguistic perspective. If our insights are correct, we will have semantic properties involved in the monotonic reasoning, and the system will fully account for them via its logic.

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