

Building temperature regulation using a distributed model predictive control

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ABSTRACT

This paper presents a predictive control structure for thermal regulation in buildings. The proposed method exploits the intermittently operating mode of almost all types of buildings. Usually the occupation profile can be known in advance and this fact will be used to reduce the energy consumption without decreasing the thermal comfort during the occupation. For that purpose, the predictive control strategy is first presented for a single zone building then extended to a multizone building example. Two opposite control strategies commonly exists: the decentralized control structure, which does not offer good performances especially when the thermal coupling among adjacent rooms is not negligible, and on the other hand, the centralized control for which the computational demand grows exponentially with the size of the system, being very expensive for large scale buildings. Our solution is based on a distributed approach which takes the advantages of both methods mentioned above. A distributed MPC algorithm with one information exchange per time step is proposed with good control performances and low computational requirements. Simulations and a comparison performance table end the article.

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1. Introduction

The scientific and the political communities have been aware for several years of the global warming problem. By consequence, a European target is the reduction of greenhouse gases by 20% until 2020 while allowing economic and demographic growths. This can be reached only if the energy consumption is optimized. According to [1], in 2007 the services and households sectors use 40% of the total final European (EU-27) energy. Within the buildings, the heating systems consume more than 50% which means about 23% of total energy consumption [2]. Even if the trends are to construct new energy-efficient buildings, an overall energy consumption reduction cannot be achieved without an optimization in the existent buildings. As renovations and isolations have high costs and are time demanding, in this context, an advanced control law is the optimal solution. The challenges of indoor heating system control are to find a compromise between the user thermal comfort and the energy consumption.

Even if many studies were performed in order to optimize the energy efficiency of heating systems, the controllers that are used today remain basic on/off type or PID. To ensure proper regulation auto-tuning methods of PID parameters have been proposed [3,4]. The major problem of thermal systems is their slow dynamic, usu-

ally with time delays [5]. Therefore, other approaches have been proposed in the literature like fuzzy logic [6], neural networks [7,8] or genetic algorithms [9].

During the last two decades a growing interest has been granted to model predictive control (MPC). In MPC, the control input is calculated by solving an optimal problem (minimization of a cost function) over a given horizon. Only the first element of the open-loop command sequence is applied to the system. At the next instant, a new optimization is performed based on current measurements. The predictive control has been successfully used in many and varied applications [10,11]. In particular, for heating and cooling systems, different formulations of cost functions and constraints have been analyzed in [12] to minimize the consumption or to guarantee a desired comfort level.

In this paper, a predictive control law is proposed in order to regulate the indoor temperature. The idea is to use the future occupation profile of the rooms (zones) and to obtain a certain degree of thermal comfort while the room is occupied. In order to reduce the energy consumption, no particular temperature setpoint is imposed when the rooms are empty (without occupants). In the second part of this work, we intend to generalize our approach to a multi-zone building considering the thermal coupling between the zones.

The paper is organized as follows. Section 2 introduces the control problem in a single zone example defining the minimization criterion which includes the future occupation as an error weighting factor. In Section 3, we generalize the proposed predictive approach to a multi-zone building (comparing the

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decentralized, centralized and distributed approaches). A tractable dMPC algorithm is proposed, which offers high performances with low computation cost. The efficiency of the proposed control strategy is illustrated by a comparison between different control structures performances. Conclusions and future directions are proposed in Section 4.

2. Single zone approach

2.1. Presentation

The control problem of a room heating system is to minimize the energy consumption maintaining a certain thermal comfort for the occupants. Assuming that the comfort in this case is defined by a reference temperature, then why do we need a complicated method as predictive control while a simple PI could be sufficient? The reason comes from the fact that the comfort is defined only while the room is occupied and most of the buildings are intermittently occupied. The current room temperature controllers have an inoccupation setpoint [13] which is usually motivated only by transient time constraints and to facilitate the building thermal load calculation. This means that the controller maintains a certain indoor temperature only to avoid long transient periods between inoccupation and occupation setpoints. The existence of an inoccupation minimal temperature setpoint is not efficient (from the energy consumption point of view) especially if the building is equipped with an electric heating system. In this case, using a simple reactive control law as PI and due to the slow dynamics of the thermal system, the steady state can be reached after few minutes or several hours depending on heater characteristics, isolation, internal and external perturbations. The anticipative effect of the MPC can be used to overcome this issue. Modifying the minimization criterion of the MPC according to future occupation profile allows us to handle the absence of the setpoint during inoccupation periods without reducing the comfort of the occupation phases. The only assumption made is that the future occupation profile is known in advance at least over a finite prediction horizon window.

2.2. Defining a dynamic cost function

The anticipative effect of MPC consists in using a model of the process in order to predict its behavior during a finite horizon. A linear discrete time representation of the system for a single room building can be the following ARX form:

$$A(q^{-1})y(k) = B(q^{-1})u(k-1) + \xi(k), \quad (1)$$

where $u(k)$ and $y(k)$ are the input, indicating the heating power, respectively the output (the indoor air temperature) of the system, $\xi(k)$ is the perturbation acting as a zero mean white noise, q^{-1} the one step delay operator and $A(q^{-1})$ and $B(q^{-1})$ are polynomials defined by:

$$\begin{cases} A(q^{-1}) = 1 + a_1 q^{-1} + \dots + a_{n_a} q^{-n_a} \\ B(q^{-1}) = b_0 + b_1 q^{-1} + \dots + b_{n_b} q^{-n_b} \end{cases}$$

The controller computes the command sequence minimizing a cost function. This optimization criterion has usually two terms, one that includes the error and the other that contains the control effort. One of the common cost functions in predictive control is:

$$J = \sum_{j=N_1}^{N_2} \delta(j) [\hat{y}(k+j|k) - w(k+j)]^2 + \lambda \sum_{j=0}^{N_u-1} \Delta u^2(k+j), \quad (2)$$

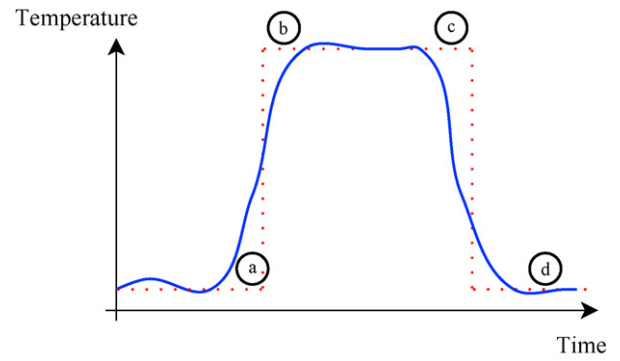


Fig. 1. MPC with the classic cost function.

where N_1 and N_2 are the minimum and the maximum bounds of the prediction horizon, $\hat{y}(k+j|k)$ is the predicted output, $w(k+j)$ the future reference, δ and λ are the weighting coefficients for the error and for the command respectively, N_u is the control horizon and Δu the command increment. The sequence of predicted outputs (3) are computed as follows:

$$\begin{aligned} \hat{y}(k+j|k) = & F_j(q^{-1})y(k) + H_j(q^{-1})u(k-1) \\ & + G_j(q^{-1})u(k+j-1) \\ & + J_j(q^{-1})\xi(k+j), \end{aligned} \quad (3)$$

where the polynomials F_j , H_j , G_j and J_j are obtained by solving (recursively) two Diophantine equations, where the model polynomials, $A(q^{-1})$ and $B(q^{-1})$, are included (see [11] for details). The optimal prediction equation is obtained considering the mean (zero here) as the best prediction for the white noise ξ .

To understand our approach it is better to analyze the output behavior of the MPC related to (2), shown in Fig. 1.

It can be seen that the anticipative effect is present (a) and (c), but the necessity of a temperature setpoint (d) during inoccupation causes a decreasing in the comfort level at the beginning (b) and at the end (c) of the occupation period. In order to remove this drawback another cost function is proposed, as the first main contribution of the paper, that incorporates the future occupation profile as the error weighting term:

$$\begin{aligned} J(k) = & \sum_{j=N_1}^{N_2} \delta^k(j) |\hat{y}(k+j|k) - w(k+j)| \\ & + \lambda \sum_{j=0}^{N_2-N_1} u(k+j), \end{aligned} \quad (4)$$

subject to

$$0 \leq u(k+j) \leq P_{max}, \quad \forall j = 0 \dots N_2 - N_1,$$

$$u(k+j) = u(k+N_u-1), \quad \forall j = N_u \dots N_2 - N_1,$$

where $\delta^k(j)$ is defined as:

$$\delta^k(j) = \begin{cases} 1, & \text{if } k+j \in \text{Occupation} \\ 0, & \text{if } k+j \in \text{Inoccupation} \end{cases}. \quad (5)$$

The vector δ^k represents the future occupation profile and enables to manage the absence of a setpoint during the inoccupation periods when the criterion is minimizing only the consumption. A graphical representation of how δ^k changes is shown in Fig. 2. If a person enters the zone and this occupation has not been foreseen then δ^k will be forced to have all the elements equal to 1.

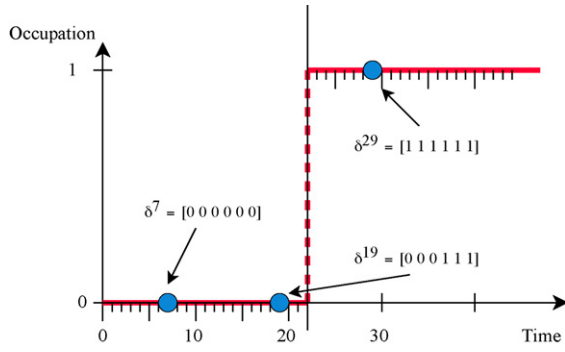


Fig. 2. Construction of δ^k ($N_1 = 1$, $N_2 = 6$).

Note that the second term of the cost function (2) was changed in (4) because the objective is to minimize the energy consumption u and not the increment Δu . The quadratic form in (2) was definitely abandoned considering classical control performance indices evaluating energy consumption (6) and thermal comfort (7), which are not in a quadratic form.

Indeed, the consumption index (I_W), in kWh, is the integral of heating power required over the simulation period:

$$I_W = \int_{t_0}^{t_f} u(t) dt. \quad (6)$$

The comfort index (I_C), in $^\circ\text{C h}$, acts as a penalty when the room temperature does not meet the comfort objective. As the comfort is defined only within the occupation periods, it can be written as:

$$I_C = \int_{\text{Occupation}} |w(t) - y(t)| dt. \quad (7)$$

The input inequality constraint is necessary to guarantee the positivity of the criterion and to define the maximum heat power of the actuators. The role of the equality constraint is to reduce the optimization argument dimensions from $N_2 - N_1 + 1$ to N_u . This decreases the computational demand of the optimization but with a loss in optimality. If it is strongly necessary, a minimal temperature constraint can be added to avoid low temperatures while the inoccupation periods:

$$\hat{y}(k+j|k) \geq T_{\min}, \quad \forall j = N_1 \dots N_2.$$

2.3. First results and discussions

Fig. 3 shows a simulation (using SIMBAD Toolbox) of the proposed control law for a 12 m² room heated by a 1200 W elec-

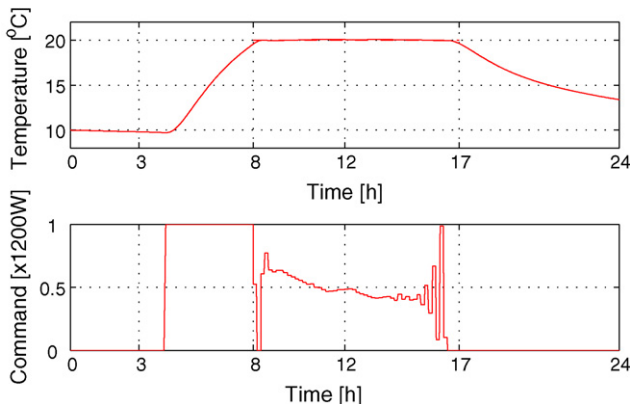


Fig. 3. Temperature and command signals using the proposed MPC.

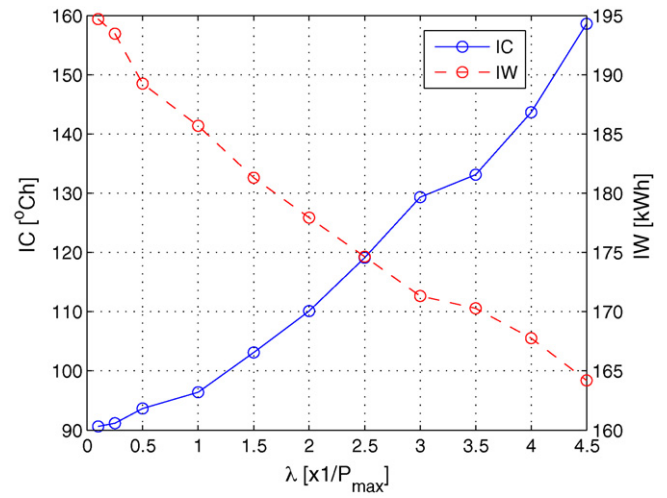


Fig. 4. Comfort and consumption indices achieved for different values of λ .

tric convector, under meteorological conditions of 01/01/1998 in Rennes, France. The occupation period is a priori known being between 8:00 and 17:00, during which temperature setpoint equals 20 $^\circ\text{C}$.

The prediction horizon is chosen to offer enough time for the control system to increase the indoor temperature up to the desired setpoint in the worse situation (low indoor and low external temperatures). In our example $N_2 = 30$, $N_1 = 1$ because no dead time was considered in the model and using a time step $T_s = 10$ min, we obtain a prediction window of 5 h. As it can be seen in Fig. 3, even if the controller 'sees' the first occupation setpoint at 3:00 the heating starts later, at the optimal time. A similar effect appears at the end of the occupation period when the heater is turned off before the end of the occupation, using in an optimal way the thermal inertia of the building. The command prediction horizon N_u can be chosen between 1 and $N_2 - N_1 + 1$ knowing that a smaller value means less computational demands but in the same time a loss in optimality. An analysis of the command horizon influence over the control performances can be found in [14]. For the simulations presented in this paper, we used $N_u = 10$.

The command weighting factor λ influences the steady state error. A big value of λ means that the energy is very expensive and by consequence the comfort quality will be decreased. Analyzing the Fig. 4 we obtain a mean of 2 $^\circ\text{C h}$ for 1 kWh. However, we can see that the average slope of the two curves are modified for values of λ below $1/P_{\max}$ and a small gain in comfort will be reached with a relatively big amount of energy. Even if values between $1/P_{\max}$ and $2/P_{\max}$ are a good choice, in the simulations presented in this paper, we are using $\lambda = 1/P_{\max}$. Note that for $\lambda > 2/P_{\max}$, the thermal comfort will be decreased at the beginning and at the end of the occupation periods which will diminish the advantage of using a dynamic cost function.

3. Multizone approach

This section will analyze the generalization of the predictive control law proposed above for multi-zone (large scale) buildings. Even if the controllers working in almost all buildings are zone-independent, the thermal coupling factor can be important (the internal walls isolation is weak). The thermal influences between rooms of the same building occur through internal walls and/or door openings. In this study, the coupling is assumed to be caused only between two adjacent rooms through walls. For simplicity purposes, a three-zone building (Fig. 5) equipped with three independent convector heaters was used in the simulations and for

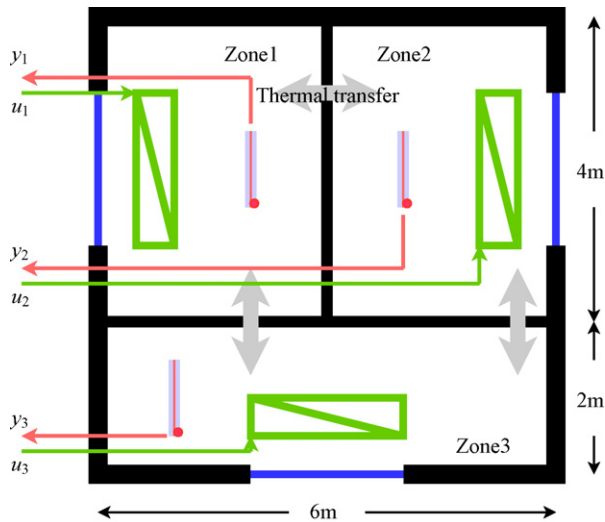


Fig. 5. Three-zone building configuration.

theory description. However, generalization for several zones can be easily achieved.

As we already mentioned, the experimental results were obtained using SIMBAD Toolbox. The simulated building is a three zones ($3 \times 42 \text{ m}^3$) with three independent electrical convectors of 1200 W maximal power. It has a double glazed window of 2 m^2 surface on the larger external wall of each zone. The external wall sandwich consists of 1 cm of gypsum, 8 cm of extruded polystyrene and 20 cm of concrete. The internal wall is 7.2 cm thick of gypsum board. The simulator supposes a well mixed indoor air. Concerning the building orientation, the common external wall for zones 1 and 2 faces to the NW.

The simulation results presented in the following sections were obtained using a daily occupation profile summarized in Table 1.

3.1. Decentralized MPC

As mentioned above, the most used building thermal control structure is a decentralized one. In this case each room air temperature is regulated by an independent controller (Fig. 6). The thermal influences among the subsystems (rooms) are considered as external unknown perturbations. For the indoor heating control system, the positive perturbations have a significant action because the control input is bounded (between 0 and P_{max}) and positive, which means that the controller can only heat up the zone. The decrease of the indoor temperature is caused mainly by losses through walls and infiltrations.

The decentralized MPC approach for the three-zone building is the simplest generalization of the MPC presented in Section 2 for a multi-zone strategy. This implies that each room temperature is regulated by its own controller independently of all others. Intuitively, as the thermal coupling between the rooms is ignored by the prediction models when these influences are important (and positive) they will not be quickly rejected and certain output overshoots will appear (Fig. 7). The slowness of the MPC controller in the

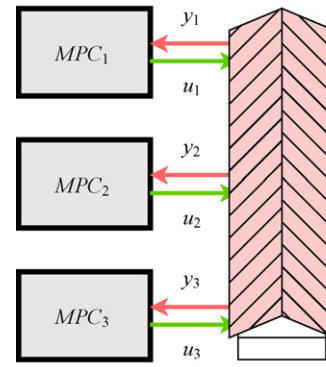


Fig. 6. Decentralized MPC configuration.

perturbation rejection is due to the relatively large prediction window. We can expect that considering in the control law the entire coupled system will diminish or even eliminate these overshoots and as a result the overall energy consumption will decrease ($I_W \searrow$) and the thermal comfort will be improved ($I_C \searrow$).

3.2. Centralized MPC

In the centralized control structure case, the entire multi-zone system is controlled by one MPC law (Fig. 8). The model used for prediction includes the coupling elements (non diagonal elements of A_g in (9)).

Knowing that in particular for multi-zone heating systems the coupling element is the output of each subsystem (the measured temperatures) then a model of one zone including this influence with the adjacent rooms can be expressed in a state space formulation as:

$$\begin{cases} \mathbf{x}_i(k+1) = \mathbf{A}_i \mathbf{x}_i(k) + \mathbf{B}_i \begin{bmatrix} u_i(k) \\ \mathbf{y}_{h_i}(k) \end{bmatrix}, & i = 1 \dots 3, \\ \mathbf{y}_i(k) = \mathbf{C}_i \mathbf{x}_i(k) \end{cases} \quad (8)$$

where \mathbf{y}_{h_i} includes the outputs of all adjacent rooms (h_i) of i . Using the local models and the building structure, we can derive the global

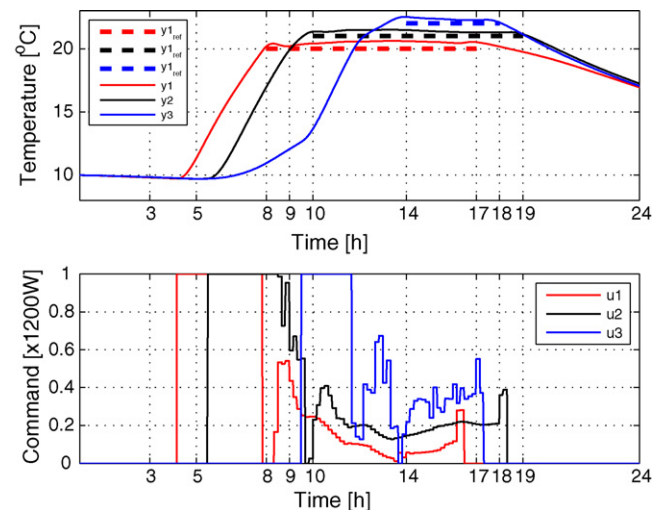


Fig. 7. Decentralized MPC behavior.

Table 1
Zone occupation profile.

Zone	Daily ^a occupation	Occupation setpoint
1	8:00–17:00	20 °C
2	10:00–19:00	21 °C
3	14:00–18:00	22 °C

^a Monday to Friday.

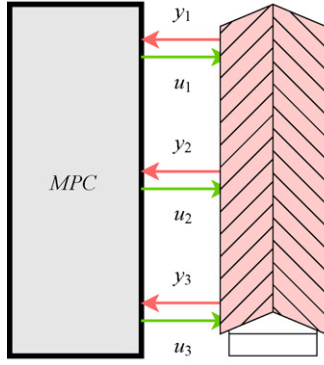


Fig. 8. Centralized MPC configuration.

model (10) matrices as:

$$\mathbf{A}_g = \begin{bmatrix} \mathbf{A}_1 & \mathbf{B}_{12}\mathbf{C}_2 & \mathbf{B}_{13}\mathbf{C}_3 \\ \mathbf{B}_{22}\mathbf{C}_1 & \mathbf{A}_2 & \mathbf{B}_{23}\mathbf{C}_3 \\ \mathbf{B}_{32}\mathbf{C}_1 & \mathbf{B}_{33}\mathbf{C}_2 & \mathbf{A}_3 \end{bmatrix}, \quad (9)$$

$$\mathbf{B}_g = \begin{bmatrix} \mathbf{B}_{11} & 0 & 0 \\ 0 & \mathbf{B}_{21} & 0 \\ 0 & 0 & \mathbf{B}_{31} \end{bmatrix},$$

$$\mathbf{C}_g = \begin{bmatrix} \mathbf{C}_1 & 0 & 0 \\ 0 & \mathbf{C}_2 & 0 \\ 0 & 0 & \mathbf{C}_3 \end{bmatrix},$$

where \mathbf{B}_{ij} represents the column j of \mathbf{B}_i .

The global state space representation of the entire (centralized) system can be written as:

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{A}_g\mathbf{x}(k) + \mathbf{B}_g\mathbf{u}(k) \\ \mathbf{y}(k) = \mathbf{C}_g\mathbf{x}(k), \end{cases} \quad (10)$$

where

$$\begin{aligned} \mathbf{x}(k) &= [\mathbf{x}_1^T(k) \quad \mathbf{x}_2^T(k) \quad \mathbf{x}_3^T(k)]^T, \\ \mathbf{u}(k) &= [\mathbf{u}_1^T(k) \quad \mathbf{u}_2^T(k) \quad \mathbf{u}_3^T(k)]^T, \\ \mathbf{y}(k) &= [\mathbf{y}_1^T(k) \quad \mathbf{y}_2^T(k) \quad \mathbf{y}_3^T(k)]^T, \end{aligned}$$

are respectively the state, the control signal and the output of the centralized model.

Considering the positivity and the additivity properties of the cost function used, the global criterion for the 3×3 system can be written as:

$$J(k) = \sum_{i=1}^3 J_i(k) \quad (11)$$

where

$$J_i(k) = \sum_{j=N_1}^{N_2} \delta_i^k(j) |\hat{y}_i(k+j|k) - w_i(k+j)| + \lambda_i \sum_{j=0}^{N_2-N_1} u_i(k+j). \quad (12)$$

Each output prediction \hat{y}_i will be computed including the modeled coupling factors. In the simulation results (Fig. 9) using the same occupation profiles and external conditions as in the

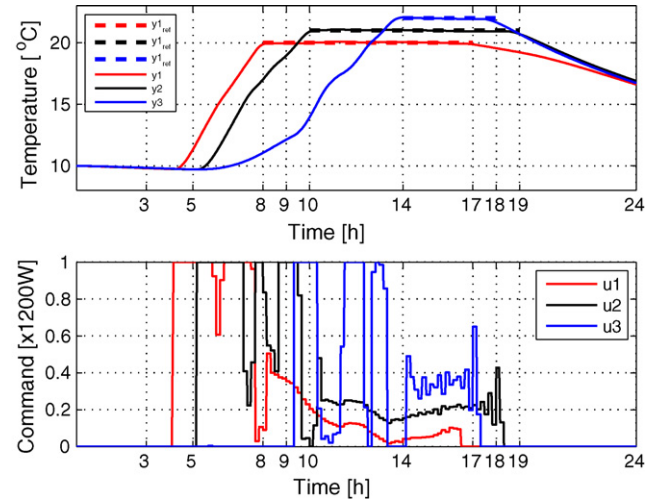


Fig. 9. Centralized MPC behavior.

decentralized example (Fig. 7), the zone temperatures present no overshoots.

Even if the control performances are good, the computational demand of a centralized MPC grows exponentially with the system size. The implementation of this control law for large scale buildings is time-consuming because of the high necessary computational power of the controller. Moreover, a damage of the central controller will cause the failure of the entire building heating system.

3.3. Distributed MPC

Because of the computational complexity of the centralized MPC, the application area of this type of control is restricted to only relatively small-scale MIMO systems. A distributed approach (dMPC) seems to be the only solution for large-scale dynamically coupled systems. The dMPC is structured as a decentralized law, with a local controller for each subsystem (Fig. 10). In order to converge to the global optimal solution [15,16] or to a Nash equilibrium point [17,18], the local MPCs exchange informations related to their future behavior. A communication network and an algorithm, that allow the collaboration among the local control laws, permit the improvement of global system performance compared to decentralized structure. On the other hand, the computational demand should be significantly reduced compared to the centralized case.

The multi-zone heating system dMPC idea is to use for each local controller the *future output prediction of the neighbor rooms*. Based on the model developed in Section 3.2, the output prediction

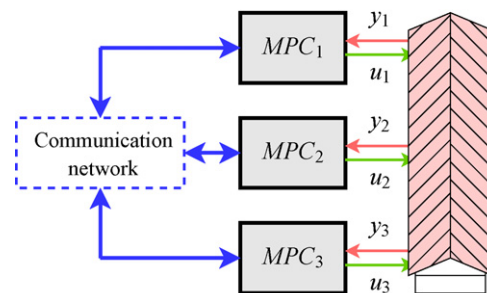


Fig. 10. Distributed MPC configuration.

equation of subsystem i can be written as:

$$\hat{\mathbf{y}}_i(k) = \Psi_i \mathbf{x}_i(k) + \Phi_{i1} \mathbf{u}_i(k) + \sum_{s \in \bar{h}_i} \Phi_{is} \mathbf{y}_s(k) \quad (13)$$

with the following notations:

$$\hat{\mathbf{y}}_i(k) = [\hat{y}_i(k + N_1|k) \quad \dots \quad \hat{y}_i(k + N_2|k)]^T,$$

$$\mathbf{u}_i(k) = [u_i(k|k) \quad \dots \quad u_i(k + N_u - 1|k)]^T,$$

$$\Psi_i = [\mathbf{C}_i \mathbf{A}_i^{N_1} \quad \dots \quad \mathbf{C}_i \mathbf{A}_i^{N_2}]^T,$$

$$\Phi_{i1} = \begin{bmatrix} \phi_{i1}^{N_1-1} & \dots & \phi_{i1}^0 & 0 & \dots & \dots \\ \vdots & & \vdots & \ddots & & \\ \phi_{i1}^{N_2-1} & \dots & \dots & \phi_{i1}^{N_2-N_u+1} & \sum_{k=0}^{N_2-N_u} \phi_{i1}^k & \end{bmatrix},$$

$$\Phi_{is} = \begin{bmatrix} \phi_{is}^{N_1-1} & \dots & \phi_{is}^0 & 0 & \dots & 0 \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ \phi_{is}^{N_2-1} & \dots & \phi_{is}^{N_2-N_1} & \phi_{is}^{N_2-N_1-1} & \dots & \phi_{is}^0 \end{bmatrix},$$

$$\phi_{ij}^k = \mathbf{C}_i \mathbf{A}_i^k \mathbf{B}_{ij},$$

$$\mathbf{y}_s(k) = \begin{bmatrix} y_s(k) \\ \hat{y}_s(k + N_1|k - 1) \\ \vdots \\ \hat{y}_s(k + N_2 - 1|k - 1) \end{bmatrix}.$$

Replacing (13) in (12) and writing the local cost function in a matrix form we have:

$$J_i(k) = \delta_i^k \left| \Psi_i \mathbf{x}_i(k) + \Phi_{i1} \mathbf{u}_i(k) + \sum_{s \in \bar{h}_i} \Phi_{is} \mathbf{y}_s(k) - \mathbf{w}_i(k) \right| + \lambda_i \mathbf{e} \mathbf{u}_i(k),$$

where

$$\delta_i^k = [\delta_i^k(1) \quad \dots \quad \delta_i^k(N_2 - N_1 + 1)],$$

$$\mathbf{e} = [1 \quad \dots \quad 1] \in \mathbb{R}^{N_2 - N_1 + 1}.$$

If the command prediction horizon is shorter than the output prediction window ($N_2 - N_1 + 1 > N_u$) then the line vector \mathbf{e} will have the following form:

$$\mathbf{e} = [1 \quad \dots \quad 1 \quad N_2 - N_1 + 2 - N_u] \in \mathbb{R}^{N_u}.$$

Now we are able to describe the algorithm for the i th controller at time step k .

Algorithm 1. dMPC with one communication step and output coupled model

- 1: Send $\hat{\mathbf{y}}_i(k-1)$ and $y_i(k)$ to all $j \in \bar{h}_i$
- 2: Receive $\hat{\mathbf{y}}_j(k-1)$ and $y_j(k)$ from all $j \in \bar{h}_i$
- 3: Replace $\hat{y}_j(k + N_1 - 1|k - 1)$ in $\hat{\mathbf{y}}_j(k-1)$ with $y_j(k)$ for all $j \in \bar{h}_i$
- 4: Solve the local optimization problem $\min_{\mathbf{u}_i(k)} J_i(k)$ and compute $\hat{\mathbf{y}}_i(k)$
- 5: Apply the first element of $\mathbf{u}_i(k)$ to the local subsystem
- 6: $k = k + 1$ and go to step 1

The presented algorithm is close to the idea of [18] with few modifications. Using the output coupled model (8) the information exchanged by the controllers is the predicted output sequence and not the future control input. An innovative aspect is that we

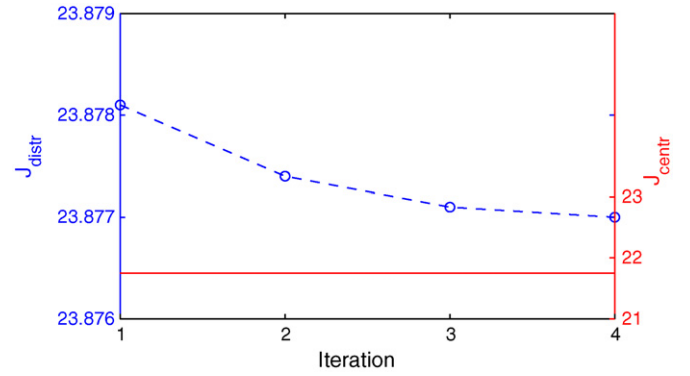


Fig. 11. Convergence of the multiple iteration distributed algorithm in one time step.

included the current measures of neighbors' outputs in the first element of the prediction sequence, adding a robustness degree of the command. The convergence and the stability conditions for an unconstrained distributed MPC can be easily formulated using the explicit solution as in [18]. In the constrained case these conditions are an open problem. This paper focuses only on the control performances.

A multiple iteration version of the Algorithm 1 has been tested, using a stop condition of the following form:

$$\|\mathbf{u}_i^{(l+1)}(k) - \mathbf{u}_i^{(l)}(k)\| \leq \epsilon_i, \quad i = 1 \dots 3$$

and for $\epsilon_i = 10^{-3}$, $i = 1 \dots 3$, the maximum number of iterations was 3. The fast convergence of the algorithm (Fig. 11) is due to the output coupling of the model. Knowing the slowness of the thermal systems, the coupling element has small variations between two consecutive iterations. Then, the iterative algorithm will converge to a Nash equilibrium.

Using the same control parameters as in the previous two cases (decentralized and centralized), we observe in Fig. 12 that the overshoots are eliminated.

From a computational point of view, the proposed distributed MPC (Algorithm 1) has the same complexity as the decentralized approach, considering that the calculation of $\hat{\mathbf{y}}_i(k)$ requires less time than the optimization routine. In the distributed approach, we should also consider the communication efficiency, which can be very important in the overall efficiency of the algorithm.

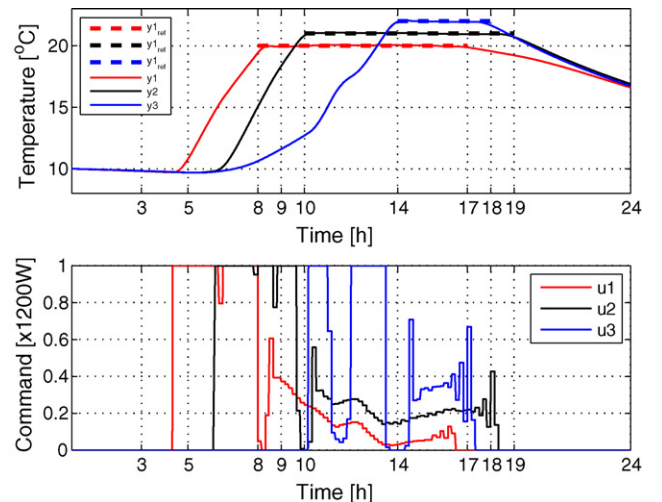


Fig. 12. dMPC behavior using Algorithm 1.

Table 2

Comparison between different control structures.

Control law	I_C [°Ch]	I_W [kWh]
On/off (± 0.1 , $T_s = 60$ s)	306	312
P ($k = 0.5$)	328	295
PI	306	308
Decentralized MPC	319	288
Centralized MPC	191	279
Distributed MPC	195	273

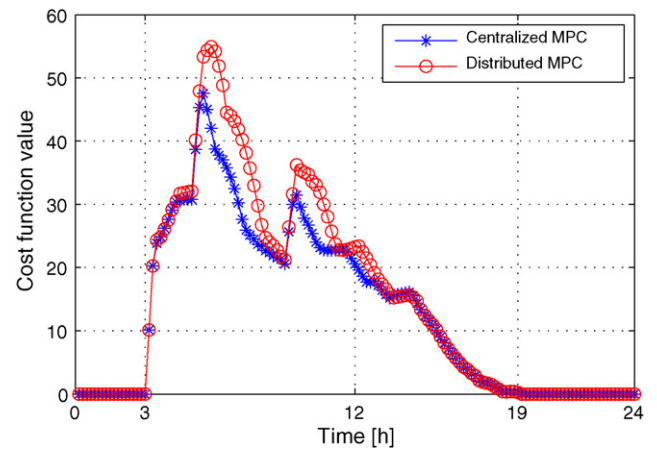
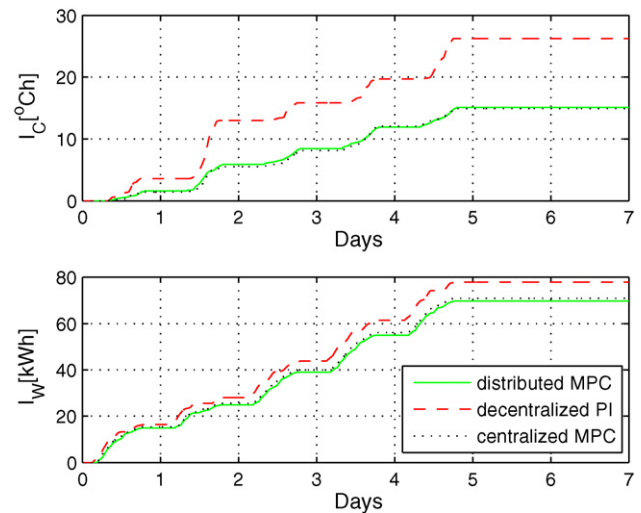
3.4. Results analysis

To have a better comparison of the control methods, we imposed for the classic on/off and P/PI controllers, a very low inoccupation reference temperature and we also increased the occupation periods with the MPC prediction horizon, so that the comfort temperature appears 5 h earlier than the real occupation. The comfort temperature was set at 20 °C for all zones while the occupation schedule is the same as in Table 1.

Table 2 makes a comparison of the proposed MPC algorithms with other common room temperature controllers. The control performances of the conventional regulators (On/Off, P and PI) are quite similar. By their nature, these controllers act in a decentralized manner, each of them controls the temperature of its own zone, without any knowledge of the others' behavior. These strategies offer low comfort performances. This is due to the fact that the thermal coupling between adjacent zones is not considered by the control law. The decentralized MPC scheme improves slightly the performances (reduces the consumption with 5.5%¹), only optimizing the transitions between occupation and inoccupation periods. The thermal coupling is not included in the prediction model in this case. The centralized and the distributed MPC strategies can improve the thermal comfort with 36.7%¹ and in the mean time reducing the energy consumption with 13.4%.¹ These results show the importance of considering the thermal transfer through the interior wall of a building in the control model. Of course, the energy saving and the comfort improvements vary with the thermal insulation of these walls. Fig. 14 compares the evolution of the two performances indices, defined in Section 2.2, over a week. The simulation results show that for the case of a multizone heating system control problem, the centralized and the distributed MPC offer very close performances. This means that the centralized solution is relatively close to the Nash equilibrium of the distributed strategy (see Fig. 13), due to the fact that the thermal coupling between adjacent rooms is stable and relatively less important comparing to direct influences (the effect of the local control input over the local output).

The centralized and the distributed MPC give close performances, with the mention that dMPC is less computational demanding. For example, using a Dual CPU at 3.00 GHz and Matlab routines, we obtained a mean of 0.618 s for the centralized optimization time versus 0.19 s, the time spent by each distributed controller to minimize its local criterion. As the number of subsystems (rooms) increases, the gap between the computational time of the centralized and distributed MPC strategies becomes more important. Even if in the dMPC case we should add the communication between local controllers, it should not be very important for a large-scale system case. The performance gain of the dynamic MPC algorithm over a classic control law with anticipation can be even greater if the occupation periods have a higher frequency.

¹ Comparing to the mean performance indices of the conventional controllers (On/Off, P, PI).

**Fig. 13.** Cost function evolution for centralized and distributed structures.**Fig. 14.** Evolution of the performance indices over one week.

4. Conclusion

A model predictive control strategy has been proposed for building temperature regulation using electrical convectors. Significant consumption reductions can be achieved optimizing the transitions between inoccupation and occupation phases. The proposed control design is based on the optimization of a dynamic cost function that includes the future occupation profile, which is the first main contribution of the paper. For large-scale buildings, especially when the internal walls have a low thermal isolation, a one-step distributed algorithm was proposed, which gives good results with low computational demand. The distributed method exploits the output coupling between the subsystems, improving the convergence. By consequence, a one-step communication algorithm (second main contribution) offers a high degree of performance with a low computational demand.

Future work will focus on the analysis of the performance improvements of the distributed MPC architecture (comparing to decentralized strategies) over different internal walls sandwiches, as well as the impact of the thermal coupling between zones through open doors. Another research topic is the control problem with multiple heat sources, with different dynamics and energy costs.

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