Cooperative Communications and Networking

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Midterm Exam
10:10AM~12:00PM, April 13, 2022.

1. This exam contains 4 questions carrying 100 marks in total of **3** pages.
2. The exam is open book. You may use any of the course materials prepared by your own but can not exchange them during the exam. Using electronic devices is prohibited too.
3. Write your answer neatly and show intermediate steps. Be sure to answer the questions in the order in which they appear in the exam paper. Answers without supporting work or do not follow the question numbering will not be given credit.

Question 1 (20 points)

Consider a cellular system where the received signal power is distributed according to a log-normal distribution with mean μ dBm and standard deviation σ dBm. Assume the received signal power must be above 10 dBm for acceptable performance.

(a) (10 points) If X is log-normal distributed with mean μ_{dB} and standard deviation σ_{dB}. Show that $10\log_{10}(X) \triangleq Y$ is Gaussian distributed with the density function

$$f_Y(y) = \frac{1}{\sqrt{2\pi}\sigma_{\text{dB}}} \exp\left(\frac{(y-\mu_{\text{dB}})^2}{2\sigma_{\text{dB}}^2}\right)$$

(b) (10 points) What is the outage probability when the log-normal distribution has $\mu = 15$ dBm and $\sigma = 8$ dBm? Show your answer using Q function, which is defined as $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp(-\frac{t^2}{2})dt$.

Question 2 (30 points)

This question is about the effect of the unequal SNRs on the diversity reception. Recall in Assignment 2, we study the performance of selection combining (SC) assuming an equal SNR of the branches.

Consider SC for N receiving antennas over independent Rayleigh fading channels. Let γ_n be the received SNR of the nth branch, $n = 1, \cdots, N$.

(a) (5 points) Let Γ be the output of the SC receiver (i.e., $\Gamma = \max\{\gamma_1, \gamma_2, \cdots, \gamma_N\}$). What is the cumulative distribution function (CDF) of γ?

(b) (10 points) Suppose the first $N-1$ branches have the same average received SNR (i.e., $\bar{\gamma}_1 = \bar{\gamma}_2 = \cdots = \bar{\gamma}_{N-1} = \bar{\gamma}$). The Nth branch has a value of mean SNR much smaller than that of the other branches (i.e., $\bar{\gamma}_N \ll \bar{\gamma}$) . Given the SNR threshold γ_0 for successful reception. Find the approximated outage probability of SC as a function c when $\gamma_0 \ll \bar{\gamma}_N$.

(c) (10 points) Suppose $\bar{\gamma}_N < \gamma_0 \ll \bar{\gamma}$. Find the approximated outage probability of SC as a function c.

(d) (5 points) Suppose $\bar{\gamma}_N \ll \gamma_0 \ll \bar{\gamma}$. In this case, the Nth branch has a very low value of the average SNR. Derive the resultant diversity gain.

Question 3 (20 points) Answer the following questions.

(a) (10 points) Choose the correct statement about "diversity gain" in a communication system. (1) It is the negative slope of the outage probability curve in the logarithmic scale. (2) It specifies how many errors can be reduced by increasing the signal-to-noise ratio. (3) If a transmitter sends the same signal using two antennas, the diversity gain is two. (4) Selection combining achieves the same diversity gain as maximum ratio combining.

(b) (10 points) Which of the following channel impairment can be overcome by diversity technique? (1) Distance-dependent path loss (2) Log-normal shadowing (3) Multipath fading.

Question 4 (30%)

Consider a three-node network consisting of a source, a destination, and a relay. All nodes are single-antenna device. The relay performs the fixed-gain amplify-and-forward (AF) where the amplification gain is chosen to meet the maximum power constraint of relay. The direct link is not available due to severe blockage. All the channels suffer independent Rayleigh fading. Answer the following question using the notations: G as the constant gain, T as the SNR threshold, $\gamma_i = P_i a_i^2 / N_0$ as the received SNR of the i-th hop, where $i = 1$ corresponds to the source-relay channel and $i = 2$ the relay-destination channel.

(a) (10 points) Derive the outage probability of the single-relay fixed-gain AF.

Hint: use the integral $\int_0^\infty \exp\left(-\frac{\beta}{4x} - \gamma x\right) dx = \sqrt{\frac{\beta}{\gamma}} K_1\left(\sqrt{\beta\gamma}\right)$.

(b) (10 points) Find the approximated outage probability of fixed-gain AF when P_i/N_0 approaches infinity,

(c) (10 points) Justify why the outage probability of fixed-gain AF at high SNR is not dependent on the second-hop channel gain.

Hint: consider the end-to-end-SNR of fixed-gain AF.