Chapter 3 Receiver Techniques for Fading Channels

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Diversity Techniques for Fading Channels

Diversity: Primary technique used to improve performance on an fading channel

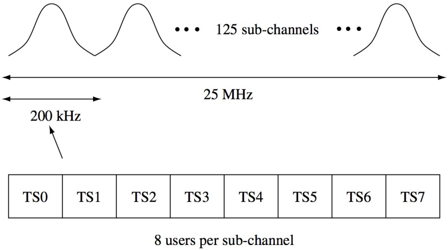
Main idea: provide the receiver with multiple versions of the same tx signal over independent channels.Intuitively, the probability of *all* signals being faded will be less than the probability that *just one* is faded!

Diversity Techniques for Fading Channels (Cont'd)

How to obtain independent channels needed for diversity?

- ***Frequency Diversity***: Use different frequency carriers separated by a distance larger than the coherence bandwidth of the channel. Not bandwidth-efficient!
- ***Time Diversity***: Use different time slots separated by an interval longer than the coherence time of the channel.
- ***Space Diversity***: Use multiple antennas separated wide enough w.r.t. carrier wavelength

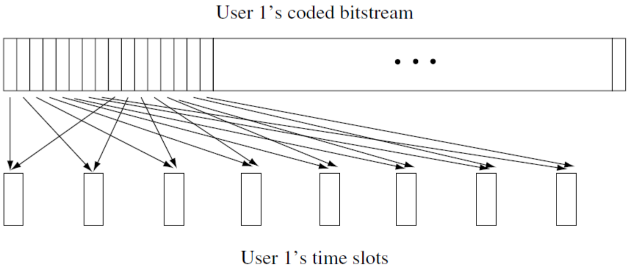
Time Diversity in GSM



GSM (2G): a digital cellular standard developed in Europe in the 1980s

- Two 25-MHz bands are used for uplink and downlink based on frequency division duplex (FDD).
- Each band is further divided into 200-kHz sub-channels and each sub-channel is shared by eight users in a time-division fashion (time-division multiple access (TDMA)). Eight users together form a frame of length 4.615 ms.

Time Diversity in GSM (cont'd)



- GSM targets on voice application coded by a speech encoder into speech frames each of length 20 ms.
- The bits in each speech frame are encoded by a convolutional code of rate 1/2, with two different generator polynomials.
- To achieve time diversity, these coded bits are interleaved across eight consecutive time slots assigned to that specific user.

Time Diversity in GSM (cont'd)

The maximum possible time diversity gain is 8, but the actual gain that can be obtained depends on how fast the channel varies, and that depends primarily on the mobile speed.

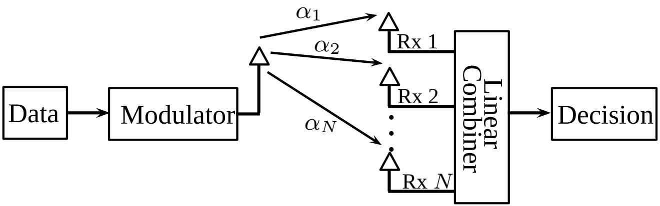
If the mobile speed is v, then the largest possible Doppler spread is $f_d = 2f_c v/c$, where f_c is the carrier frequency and c is the speed of light. The coherence time is roughly $T_d = 1/(4f_d)$. For the channel to fade more or less independently across the different time slots for a user, the coherence time should be less than 5 ms. For $f_c = 900$ MHz, this translates into a mobile speed of at least 30 km/h.

Time Diversity in GSM (cont'd)

For a walking speed of say 3 km/h, there may be too little time diversity. In this case, GSM can go into a frequency hopping mode, where consecutive frames (each composed of the time slots of the eight users) can hop from one 200-kHz sub-channel to another.

With a typical delay spread of about 1 μs, the coherence bandwidth is 500 kHz. The total bandwidth equal to 25MHz is thus much larger than the typical coherence bandwidth of the channel and the consecutive frames can be expected to fade independently. This provides the same effect as having time diversity.

Maximum Ratio combining (MRC)



$r_n(t) = \alpha_n s(t) + z_n(t)$

- N: Number of RX antennas
- $\alpha_n = a_n e^{j\theta_n}$: Complex fading coefficient of the nth channel

MRC (cont'd)

The decision is based on the output of linear combiner

$$r = \sum_{n=1}^{N} c_n \cdot r_n$$

where c_n are the weighting coefficient of the nth branch

How to choose the weighting coefficients?

MRC (cont'd)

$$r = \sum_{n=1}^{N} c_n r_n = \overbrace{\sum_{n=1}^{N} c_n \alpha_n s}^{\text{Signal term}} + \overbrace{\sum_{n=1}^{N} c_n z_n}^{\text{Noise term}}$$

where $|s|^2 \triangleq E$, and $\sum_{n=1}^{N} c_n z_n \sim \mathcal{N}\left(0, N_0 \sum_{n=1}^{N} |c_n|^2\right)$.

The SNR at the output of linear combiner is given as

$$\frac{E}{N_0} \frac{\left|\sum_{n=1}^{N} c_n \alpha_n\right|^2}{\sum_{n=1}^{N} |c_n|^2} \overset{(*)}{\leq} \frac{E}{N_0} \frac{\sum_{n=1}^{N} |c_n|^2 \sum_{n=1}^{N} |\alpha_n|^2}{\sum_{n=1}^{N} |c_n|^2}$$

$$= \frac{E}{N_0} \sum_{n=1}^{N} |\alpha_n|^2$$

MRC (cont'd)

($*$) "Schwarz Inequality" $\left|\sum_{n=1}^{N} c_n\alpha_n\right|^2 \leq \sum_{n=1}^{N} |c_n|^2 \cdot \sum_{n=1}^{N} |\alpha_n|^2$, which holds with equality for $c_n = \alpha_n^*$

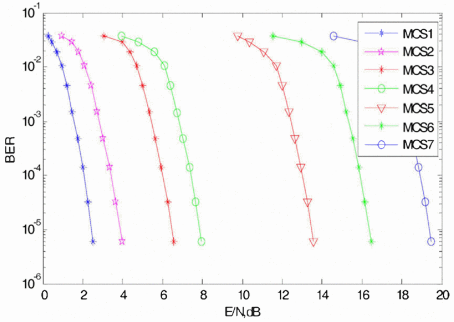
The choice of weighting coefficients $c_n = \alpha_n^*$ maximizes SNR.

Example: Outage Probability for MRC

A receiver can not decode a message if the received SNR is less than the threshold.

$$\mathbb{P}_{\text{out}} = \mathbb{P}[\text{SNR} < T].$$

Modulation and Coding Scheme in LTE



MCS index	modulation	code rate x 1024	Threshold dB	efficiency
1	QPSK	308	2.90	0.6016
2	QPSK	602	5.15	1.1758
3	16QAM	378	7.12	1.4766
4	16QAM	490	12.10	1.9141
5	64QAM	466	15.85	2.7305
6	64QAM	666	19.25	3.9023
7	64QAM	873	>19.25	5.1152

Example: Outage Probability for MRC

For MRC,

$$\mathrm{SNR} = \left(\frac{E}{N_0}\right)\sum_{n=1}^{N}|\alpha_n|^2 \triangleq \sum_{n=1}^{N}\gamma_n.$$

Considering Rayleigh fading, $|\alpha_n|^2$'s are independent and identically distributed (i.i.d.) exponential random variable (RV).

- So is γ_n: $\gamma_n \sim \mathrm{Exp}(1/\bar{\gamma})$.

What is the sum of N i.i.d. exponential RVs?

- It is gamma distributed: the waiting time until the N-th customer arrives.
- $\mathbb{P}_{\text{out}}$ is equivalent to the CDF of SNR.

Example: Outage Probability for MRC

The CDF of a gamma RV $X = \sum_{n=1}^{N} |\alpha_n^2|$ is

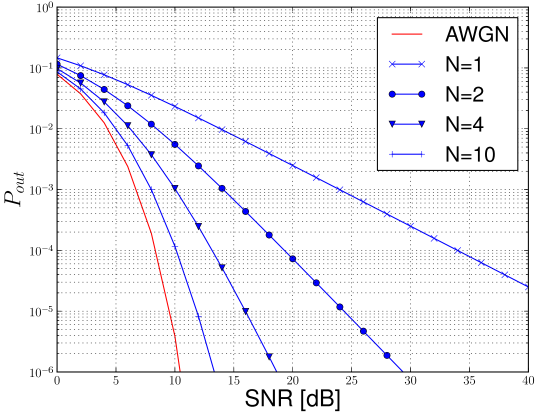
$$F_X(x) = \frac{\gamma(N, x/\bar{\gamma})}{\Gamma(N)}$$

where $\gamma(c, x) = \int_0^x e^{-t} t^{c-1} dt$ is the incomplete gamma function.

When $\bar{\gamma} \approx \infty$, $\gamma(N, x/\bar{\gamma}) \to \frac{1}{N}(x/\bar{\gamma})^N$, and hence

$$\mathbb{P}_{\text{out}} \approx \frac{1}{N!} \left(\frac{T}{\bar{\gamma}} \right)^N$$

Example: Outage Probability for MRC



- MRC dramatically improves the performance.
- For the limiting case of $N \to \infty$ the performance converges to that of AWGN.

Selection Combining (SC)

$r_n(t) = \alpha_n s(t) + z_n(t)$

The combiner chooses the branch with the highest SNR

$$\gamma_{\max} = \max_n \left(|\alpha_n|^2 \frac{E}{N_0} \right)$$

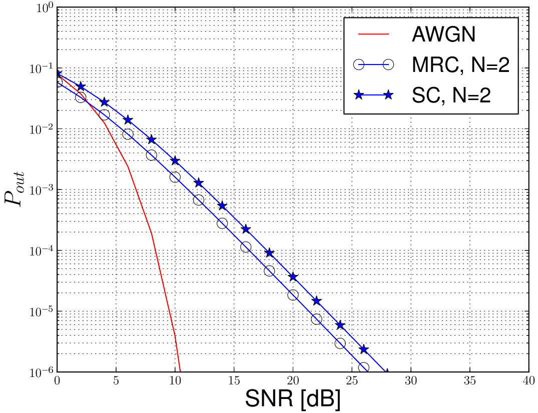
Outage probability can be given as

$$\mathbb{P}_{\text{out}} = \mathbb{P}[\gamma_{\max} < T].$$

Q

- What is the distribution of $\gamma_{\max}$?
- $\mathbb{P}_{\text{out}} \approx ?$ when $\bar{\gamma} \to \infty$?

Example: Outage Probability for SC (cont'd)



- The same diversity order (i.e. the same slope) with that of MRC is preserved.
- There is a performance degradation compared to that of MRC (i.e. a horizontal shift)

Equal Gain Combining (EGC)

Each diversity branch is assigned with the same weight of $c_n = e^{-\theta_n}$

The SNR using EGC (c.f. (2))

$$\mathrm{SNR}_{EGC} = \frac{E}{N_0} \frac{|\sum_{n=1}^{N} c_n \alpha_n|^2}{\sum_{n=1}^{N} |c_n|^2} = \frac{E}{N_0} \frac{\sum_{n=1}^{N} |\alpha_n|^2}{N} \tag{1}$$

For $N > 2$, there is no closed form for the distribution of SNR_{EGC}.

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- What is the advantage of EGC compared with MRC?

Transmit Diversity

So far, we have studied "space diversity" assuming multiple receive antennas. Another powerful form (though more difficult to implement) to exploit spatial diversity is to use multiple transmit antennas. First, we need to clarify why we need transmit diversity.

	Transmitter	**Receiver**	**Diversity type**
Uplink	Mobile station	Base station	RX-Diversity
Downlink	Base station	Mobile station	TX-Diversity

- In downlink, receive diversity is difficult to implement as it requires multiple antennas and additional processing at the mobile station. This is not suitable due to size and battery power limitation at mobile.
- Put additional processing and complexity at the base station $\Rightarrow$ TX-Diversity.

Transmit Diversity (cont'd)

Assume the same signal is transmitted from two different antennas <u>without</u> any processing. $r = \sqrt{\frac{1}{2}}\alpha_1 s + \sqrt{\frac{1}{2}}\alpha_2 s + z$

Now, rewrite the above received signal as

$$r = \left(\frac{\alpha_1}{\sqrt{2}} + \frac{\alpha_2}{\sqrt{2}}\right) s + z$$

Define $\alpha = \frac{\alpha_1}{\sqrt{2}} + \frac{\alpha_2}{\sqrt{2}}$, we have (equivalent single-antenna model)

$$r = \alpha s + z$$

No diversity provided!

Strategy: manipulate the transmit signal by pre-processing or pre-coding prior to transmission

- Open-loop precoding
- Closed-loop precoding

Precoding in Transmit Diversity

Denote c_m the precoding coefficient for the mth tx antenna

$$r = \sqrt{\frac{1}{M}} \sum_{m=1}^{M} \alpha_m c_m s + z.$$

where α_m is the fading coefficient from mth transmit antenna to the receive antenna (scaling by transmit antenna number M is necessary to keep the transmit power fixed).

Define $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \cdots, \alpha_M]$, $\mathbf{c} = [c_1, c_2, \cdots, c_M]$.

$$r = \sqrt{\frac{1}{M}} \boldsymbol{\alpha} \mathbf{c}^T s + z$$

Precoding in Transmit Diversity (cont'd)

To maximize SNR, weight coefficients are chosen as

$$\mathbf{c}^T = \sqrt{M}\frac{\boldsymbol{\alpha}^*}{\|\boldsymbol{\alpha}\|^2} \Rightarrow |\boldsymbol{\alpha}\mathbf{c}^T|^2 = M\|\boldsymbol{\alpha}\|^2 = M\sum_{m=1}^{M}|\alpha_m|^2$$

The received SNR is

$$\mathrm{SNR} = \frac{\frac{E}{M}|\boldsymbol{\alpha}\mathbf{c}^T|^2}{N_0} = \frac{E}{N_0}\sum_{m=1}^{M}|\alpha_m|^2 \tag{2}$$

which is the same result as using MRC RX diversity!

Close-Loop Transmit Diversity - TDD

In Time Division Duplex (TDD), a single radio channel is shared in time so that a portion of the time is used to transmit from BS to the MS and the remaining time is used to transmit from the MS and BS.

Channel reciprocity: the downlink and uplink channels are approximately the same in TDD mode. In this case, the BS can get the necessary feedback information of the computation of weight vector from the received signal through the uplink channel.

Close-Loop Transmit Diversity - FDD

In Frequency Division Duplex (FDD), we have simultaneous radio transmission between the MS and BS. At the BS, separate transmit and receive antennas are used to accommodate the two separate channels. At the MS, a single antenna with duplexer is used. Transmit and receive frequencies are separated by 5% of the nominal frequency for enabling the use of single antenna for simultaneous transmission/reception.

Since uplink and downlink channels are significantly different, MSs must provide feedback to the BS. For this purpose, in general, a dedicated digital uplink channel is used.

Open-Loop Transmit Diversity

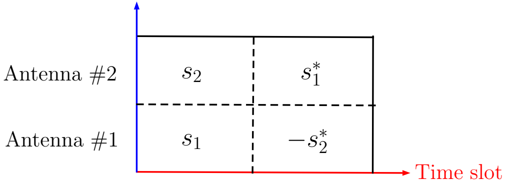
Space-Time Coding (STC) was introduced in 1998 and provides a revolutionary solution for the long-standing problem of designing open-loop transmit diversity schemes.

Dr. S. Alamouti

- included in 3G and WiFi standard.
- achieves full $M \times N$ diversity without channel knowledge at TX.

A recent interview: Link

Alamouti STC: 2×1 case



$$\text{Received symbol in slot 1}: \; y_1 = h_1 s_1 + h_2 s_2 + n_1$$
$$\text{Received symbol in slot 2}: \; y_2 = -h_1 s_2^* + h_2 s_1^* + n_2$$

$$\text{In matrix form: } \mathbf{y} = \underbrace{\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} s_1 \\ s_2 \end{bmatrix}}_{\mathbf{s}} + \underbrace{\begin{bmatrix} n_1 \\ n_2 \end{bmatrix}}_{\mathbf{n}} \Rightarrow \mathbf{H}^H\mathbf{H} = (|h_1|^2 + |h_2|^2)\mathbf{I}_2$$

Alamouti STC: 2×1 case (cont'd)

The diagonal structure of $\mathbf{H}^H\mathbf{H}$ suggests the receiver filter as $\mathbf{H}^H$ such that

$$\mathbf{H}^H\mathbf{y} \triangleq \mathbf{z} = (|h_1|^2 + |h_2|^2)\mathbf{I}_2\mathbf{s} + \tilde{\mathbf{n}}$$

where $\mathbf{z} = [z_1 \ z_2]^T$ and $\tilde{\mathbf{n}} = \mathbf{H}^H\mathbf{n}$ is complex Gaussian with mean 0 and covariance matrix $\mathbb{E}[\tilde{\mathbf{n}}\tilde{\mathbf{n}}^*] = (|h_1|^2 + |h_2|^2)N_0\mathbf{I}_2$.

Now the receiver can detect two symbols separately using

$$z_i = (|h_1|^2 + |h_2|^2)s_i + \tilde{n}_i, \ \ i = 1, 2$$

and the received SNR $\boxed{\dfrac{(|h_1|^2 + |h_2|^2)E}{2N_0}}$.

- Alamouti scheme achieves diversity order of 2
- Despite the array gain is half of that of MRC, Alamouti precoding requires no CSI at the transmitter!

Summary

- Diversity techniques have been proven useful to overcome fading channel impairments
- Time diversity through interleaving is commonly used in practical systems
- Frequency diversity remain widely adopted in current OFDM systems
- Spatial diversity is the main stream of 4G & 5G systems

Topics to be covered next week

Cooperative Communications with Single Relay

The END