# Cooperative Communication and Networks Assignment 2

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– Numerical Questions –

Q1) Outage probability

Derive the outage probability of selection combining (SC) for N receive antennas over Rayleigh fading channels.

1. $P_{outage} = P[\gamma_{max} < T]$, given highest SNR $\gamma_{max}$, threshold $T$

$= P[\max_{n=1,\dots,N} \gamma_n < T]$, $\gamma_n \triangleq |\alpha_n|^2 \cdot \frac{E}{N_0}$, $\gamma_n \sim E(\frac{1}{\overline{\gamma_n}})$

$= P[\gamma_1, \gamma_2, \dots, \gamma_n < T]$

$= P[\gamma_1 < T] \cdot P[\gamma_2 < T] \cdots P[\gamma_n < T]$ $\because$ iid

$= \prod_{n=1}^{N} P[\gamma_n < T]$

$= \prod_{n=1}^{N} F_X(x)$ $\qquad F_X(x;\beta) = 1 - e^{-\frac{1}{\beta}x}$, $x \geq 0$

$= \prod_{n=1}^{N} (1 - e^{-\frac{1}{\overline{\gamma_n}}T})$, $T \geq 0$, given average SNR $\overline{\gamma_n}$

$= (1 - e^{-\frac{1}{\overline{\gamma_n}}T})^N$, $T \geq 0$, $\overline{\gamma_n} = E(|\alpha_n|^2) \cdot \frac{E}{N_0}$ ※

Q2) High-SNR approximation

The outage probability derived in Q2 is a function of the average SNR of each branch. If the average SNR of the branches are the same and approaching to infinity, show that the outage probability is proportional to the average SNR to the power of −N.

2.

$$\lim_{\bar{\gamma}_n \to \infty} P_{outage} = \lim_{\bar{\gamma}_n \to \infty} \prod_{n=1}^{N} (1 - e^{-\frac{1}{\bar{\gamma}_n} T})$$

$$= \prod_{n=1}^{N} \cdot \lim_{\bar{\gamma}_n \to \infty} \left[ 1 - (1 - \frac{T}{\bar{\gamma}_n} + \frac{T^2}{2! \bar{\gamma}_n^2} - \cdots) \right]$$

$$\approx \prod_{n=1}^{N} \left[ 1 - (1 - \frac{T}{\bar{\gamma}_n}) \right] = \prod_{n=1}^{N} \frac{T}{\bar{\gamma}_n}$$

$$= (\frac{T}{\bar{\gamma}_n})^N = T^N \cdot (\bar{\gamma}_n)^{-N}$$

$\therefore$ when average SNR $\bar{\gamma}_n \to \infty$

$P_{outage} \propto (\bar{\gamma}_n)^{-N}$ #

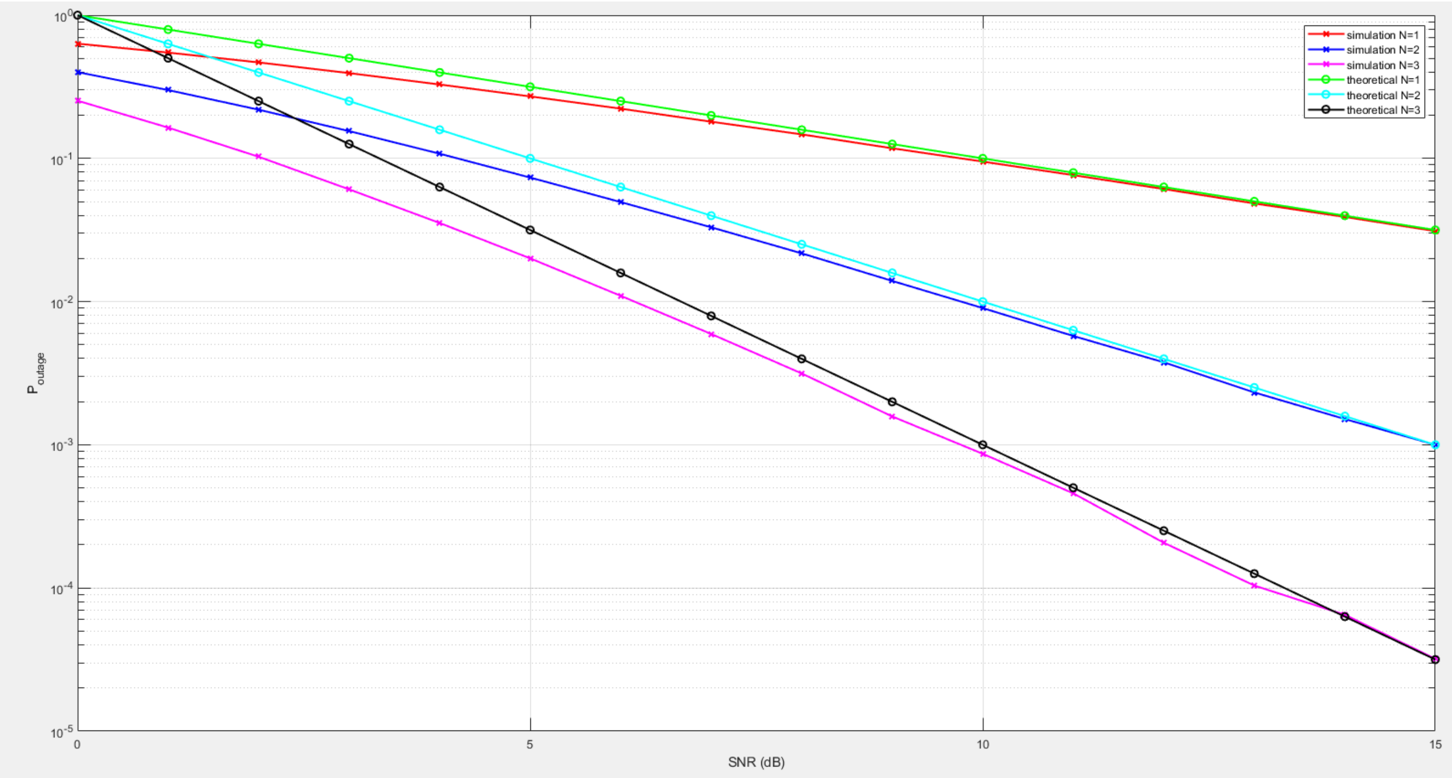
– Simulation –

Q3) Selection Combining

Perform Monte Carlo simulation for the outage probability of Selection Combining (SC).

- ◆ Consider the average SNR from 0 dB to 15 dB, the number of receive antennas N = 1, 2, 3, the decoding threshold T = 1.
- ◆ Plot both the theoretical outage probability derived from Q2 and the simulated ones versus SNR in the same figure. Full credits are given when the following requirements are all satisfied.
  - – Clearly indicate the value of N for each curve and whether it is a theoretical curve or a simulated one.
  - – Set y-axis as logarithm scale.
  - – Colors are not differentiable when printed in black and white. Use different line styles and symbols to represent different curves.

- ◆ Explain your simulation program by plotting a flow chart.
- ◆ Explain how you determine the diversity gain of each curve.




- flow chart

初始化參數

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for (跑 avg. SNR = 0~15 dB 迴圈)

直接算理論 outage prob.

for (跑 $10^6$ 次 monte carlo simulation)

模擬 N=1~3 diversity branch

end for

end for

↓

畫圖輸出

- diversity gain is the slope when the curve converged. First, we just arbitrarily take two points (x1, y1) and (x2, y2), then calculate the slope by using the formula, slope = (y2 – y1) / (x2 – x1). In this way, we could have the diversity gain of every branch.