Chapter 6 Cooperative Communications with Multiple Sources

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Previous chapters focus on the cooperative network with one source node.

Multi-source scenario arises in many practical systems, such as uplink cellular and ad hoc networks.

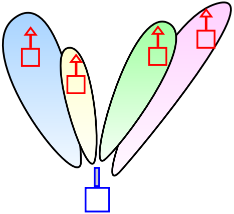
Key issues

- How to separate signals from multiple sources?
- How to perform relay cooperation?

Multiple Access of Source Nodes

How to separate signals from multiple sources?

- time-division multiple access (TDMA)
- frequency-division multiple access (FDMA)
- code-division multiple access (CDMA)
- space-division multiple access (SDMA)



TDMA/FDMA: easy to achieve interference-free access, but need proper resource allocation.

CDMA/SDMA: interference-free access is more complex and difficult to fully eliminate multiple access interference (MAI).

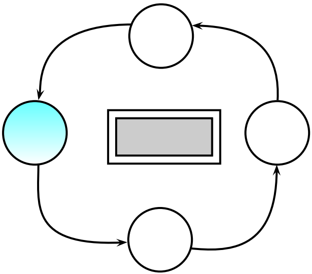
TDMA/FDMA

Each sender is assigned with a dedicated time or frequency channel.

Since available orthogonal channels are limited, sources may compete for resources. Need efficient resource allocation policies.

Two representative resource allocation schemes

- Round-Robin: sources take turn to transmit
- Opportunistic scheduling: relays dynamically serve the source with the best effective channel



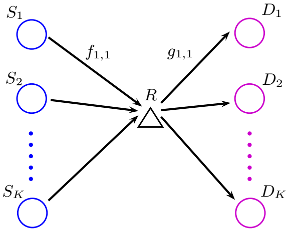
Round-Robin Scheduling

When accessing the network, each source is assigned with a unique ID.

Time is divided into equal-length time slots and each source takes turns in accessing the relay.

Very simple to implement. What about the performance?

Consider the scenario in the following figure with only one relay. The case of multiple relays can be treated using the strategies introduced in Ch. 5.



Power Allocation

With K s-d pairs, each pair is allocated $1/K$ share of time to transmit.

For DF relaying, the capacity of the k-th s-d pair is

$$C_k = \min\left\{\frac{1}{2K}\log_2\left(1+\frac{P_{S_k}|f_{S_k,D_k}|^2}{\sigma_d^2}+\frac{P_R^{(k)}|g_{R,D_k}|^2}{\sigma_d^2}\right), \frac{1}{2K}\log_2\left(1+\frac{P_{S_k}|h_{S_k,R}|^2}{\sigma_r^2}\right)\right\}$$
$$= \frac{1}{2K}\min\left\{\log_2\left(1+P_{S_k}\gamma_{S_k,D_k}+P_R^{(k)}\gamma_{R,D_k}\right), \log_2\left(1+P_{S_k}\gamma_{S_k,R}\right)\right\} \quad (1)$$

Adjust each relay's transmit power used to maximize the total capacity

$$\max_{P_R^{(k)}} \quad \sum_{k=1}^{K} C_k$$
$$\text{subject to} \quad \sum_{k=1}^{K} P_R^{(k)} \le P,\ P_R^{(k)} \ge 0,\ k=1,2,\cdots,K.$$

Power Allocation (cont'd)

Let first neglect the second term inside the minimum in (1) by assuming that R always decodes successfully, resulting in the reduced problem

$$\max_{P_r^{(k)}} \quad \sum_{k=1}^{K} \log\left(1 + P_{S_k}\gamma_{S_k,D_k} + P_R^{(k)}\gamma_{R,D_k}\right)$$

$$\text{subject to} \quad \sum_{k=1}^{K} P_R^{(k)} \le P,\ \ P_R^{(k)} \ge 0,\ \ k = 1, 2, \cdots, K.$$

The Lagrange function is

$$L(P_R^{(k)}) = \sum_{k=1}^{K} \log\left(1 + P_{D_k}\gamma_{S_k,D_k} + P_R^{(k)}\gamma_{R,D_k}\right) + \lambda\left(P - \sum_{k=1}^{K} P_R^{(k)}\right)$$

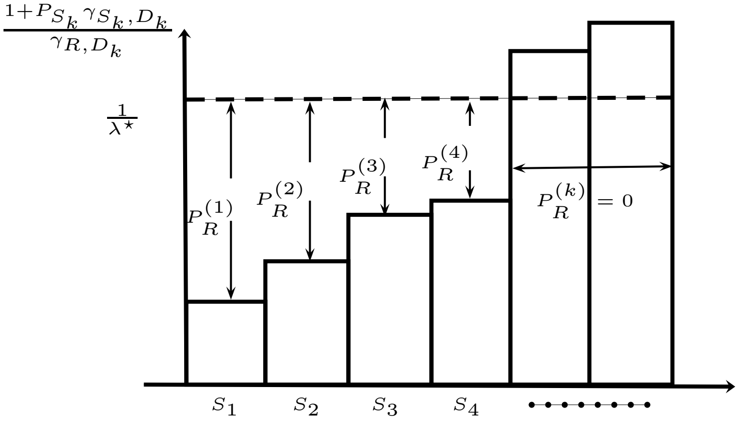
From the KKT condition (see appendix), the optimal power allocation is

$$P_R^{(k)} = \left(\frac{1}{\lambda^\star} - \frac{1 + P_{S_k}\gamma_{S_k,D_k}}{\gamma_{R,D_k}}\right)^+ \tag{2}$$

Power Allocation (cont'd)

In (2), $\lambda^\star$ is chosen to satisfy the total power constraint $\sum_{i=1}^{K} P_R^{(k)} \leq P$.

This is in the form of a **water-filling** solution, where power is distributed among the various bins corresponding to different users in a way that is analogous to the distribution of water when poured into a vessel.



Power Allocation (cont'd)

When considering the second term inside the minimum in (1), the capacity of each S-D pair is upper bounded by that of the S-R link (i.e., $\frac{1}{2K}\log_2(1 + P_{S_k}\gamma_{S_k,R})$.

$$\begin{aligned} & 1 + P_{S_k}\gamma_{S_k,D_k} + P_R^{(k)}\gamma_{R,D_k} \leq 1 + P_{S_k}\gamma_{S_k,R} \\ \Rightarrow & P_R^{(k)} \leq P_{S_k}\frac{\gamma_{S_k,R} - \gamma_{S_k,D_k}}{\gamma_{R,D_k}}. \end{aligned}$$

Any $P_R^{(k)}$ exceeding $P_{S_k}\frac{\gamma_{S_k,R} - \gamma_{S_k,D_k}}{\gamma_{R,D_k}}$ does not increase the system sum rate.

The optimal power allocation:

$$P_R^{(k)} = \min\left(\underbrace{\left(\frac{1}{\lambda^\star} - \frac{1 + P_{S_k}\gamma_{S_k,D_k}}{\gamma_{R,D_k}}\right)^+}_{T_1}, \underbrace{P_{S_k}\frac{\gamma_{S_k,R} - \gamma_{S_k,D_k}}{\gamma_{R,D_k}}}_{T_2}\right) \quad (3)$$

Power Allocation (cont'd)

When $T_2 > T_1$,

$$\frac{1}{\lambda^\star} < P_{S_k}\frac{\gamma_{S_k,R}-\gamma_{S_k,D_k}}{\gamma_{R,D_k}} + \frac{1+P_{S_k}\gamma_{S_k,D_k}}{\gamma_{R,D_k}}$$
$$\Rightarrow \frac{1}{\lambda^\star} < \frac{1+P_{S_k}\gamma_{S_k,R}}{\gamma_{R,D_k}}.$$

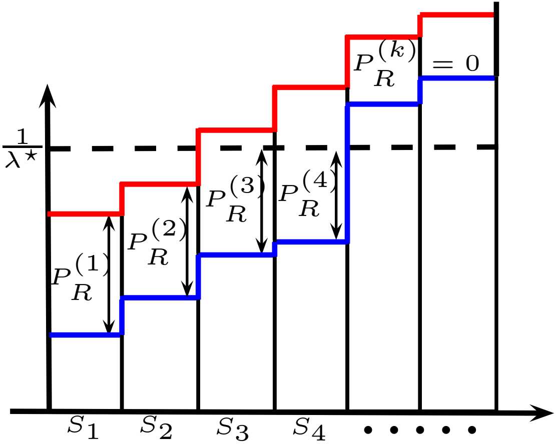
The allocated power $P_R^{(k)} = \frac{1}{\lambda^\star} - \frac{1+P_{S_k}\gamma_{S_k,D_k}}{\gamma_{R,D_k}}$.

When $T_2 < T_1 \Rightarrow \frac{1}{\lambda^\star} > \frac{1+P_{S_k}\gamma_{S_k,R}}{\gamma_{R,D_k}}$,
The allocated power

$$P_R^{(k)} = P_{S_k}\frac{\gamma_{S_k,R}-\gamma_{S_k,D_k}}{\gamma_{R,D_k}}$$
$$= \frac{1+P_{S_k}\gamma_{S_k,R}}{\gamma_{R,D_k}} - \frac{1+P_{S_k}\gamma_{S_k,D_k}}{\gamma_{R,D_k}}.$$

Power Allocation (cont'd)

Cavefilling algorithm



Power Allocation (cont'd)

Additional notes

- The above power allocation problem can be applied to AF relays
- In the case of multi-relay networks, individual power constraints at the relays make the problem more difficult
- Although round-robin is easy to implement in practice, it does not fully exploit the spatial diversity in cooperative relay networks

Opportunistic Scheduling

Idea: leverage *multiuser diversity* (MUD) by always selecting the *best* source to transmit in phase I.

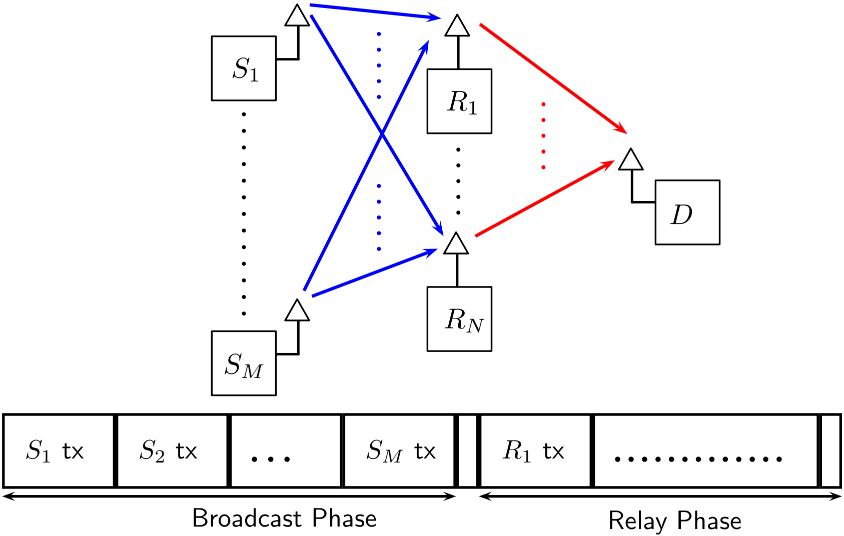
The best source is then assisted by one or multiple relays in phase II.

Can jointly exploit MUD and cooperation diversity (CD), but may not be fair after all.

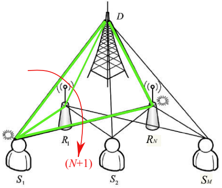
The selected relay needs to share its transmit power among multiple sources, leading to **power splitting** that deteriorate cooperative benefits.

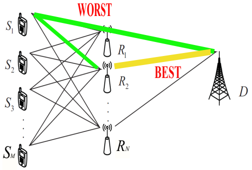
Optimal partner selection strategy must jointly consider the selection among all source users.

Centralized Selection



Design Dimensions





Joint utilize MUD and CD [Sun'10]

- Choose the source-relay pair that has the best end-to-end SNR.
- Unfair to source nodes.

Take fairness into account [Zhang'11]

- Each source node is granted with a transmission opportunity in each scheduling cycle, followed by a relay phase in which a best relay node is chosen to assist the source node with the worst direct link quality.

Below we show a formulation for minimizing the number of relays used for cooperative relaying with improved per-source throughput and fairness guarantee.

Relay Capacity

Define $f_j(P)$ the power allocation function of relay R_j.

Feasible condition (for AF relays)

Assigning relay R_j to assist source S_i is feasible if $\gamma_{i,j} \geq \mathsf{T}$.

$$\gamma_{i,j} = \frac{\frac{Pa^2_{S_i,R_j}}{N_0}\frac{f_j(P)a^2_{R_j,D}}{N_0}}{\frac{Pa^2_{S_i,R_j}}{N_0} + \frac{f_j(P)a^2_{R_j,D}}{N_0} + 1} \geq \mathsf{T}.$$

R_j can help at most m_j number of sources without outage. Under equal power allocation $f_j(P) = P/m_j$,

$$m_j \leq \min_{S_i \in \mathcal{S}'} \left(\frac{\frac{\gamma_{S_i,R_j}\gamma_{R_j,D}}{\mathsf{T}} - \gamma_{R_j,D}}{\gamma_{S_i,R_j} + 1} \right)^+$$

where $\mathcal{S}' = \{S_i | \gamma_{S_i,D} < \mathtt{T}\}$ is the set of sources that need help.

- m_j is dominated by the worst source-relay link ($\gamma_{R_j,D}$ is fixed for R_j).
- m_j can be thought of as the **relay capacity**.

Relay Assignment

$$\boxed{\min \sum_{R_j \in \mathcal{R}} y_j}$$

s.t.

$$\boxed{\sum_{R_j \in \mathcal{R}} x_{ij} = 1, \qquad \forall\ S_i \in \mathcal{S}',}$$

$$\boxed{\sum_{i=1}^{|\mathcal{S}'|} x_{ij} \le m_j y_j, \quad \forall\ R_j \in \mathcal{R},}$$

$$\boxed{x_{ij} \in \{0,1\}, y_j \in \{0,1\}, \qquad \forall\ S_i \in \mathcal{S}', \forall\ R_j \in \mathcal{R},}$$

- Minimize the number of relays used
- Each source is served by exactly one relay
- Feasibility constraint

Equivalent to *bin-packing* problem: source$\leftrightarrow$ items, relay$\leftrightarrow$bin, $m_j \leftrightarrow$ bin size.

Heuristic

Variable-sized bin-packing is *NP-hard*.

Well-known heuristic: Best First Decreasing

- For each item (source), load it to the "best" already-selected bin (relay).
- If a bin was full, select a new bin.
- Best bin: the bin maximizing the merit function.
- Merit function: free space.

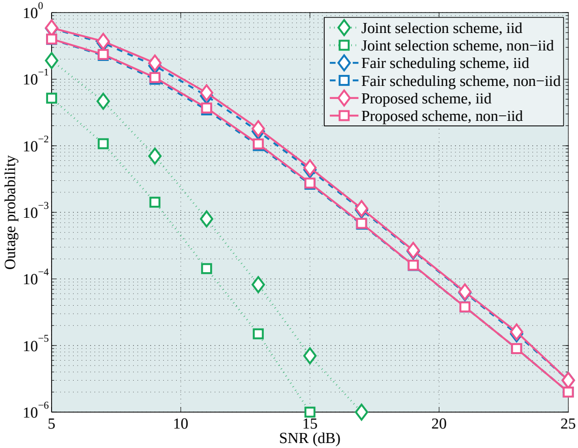
Algorithm 1 Relay Assignment

Sort relays according to their capacity m_j
$j = 1$
while $\mathcal{S}' \neq \emptyset$ or $j > N$ **do**
    Assign R_j to assist the first m_j sources in $\mathcal{S}'$
    Remove the first m_i elements in $\mathcal{S}'$
    $j = j + 1$
end while

Simulation Results

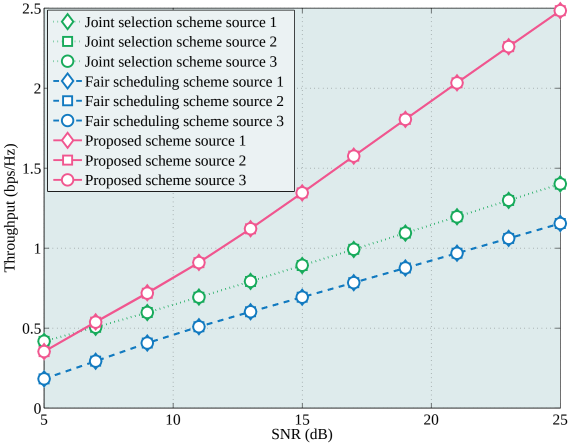
- $M = 3$ source nodes, $N = 2$ relay nodes
- I.I.D. configuration: $\sigma^2 = 1$
- I.N.D. configuration: $[\sigma^2_{S_1,D}, \sigma^2_{S_2,D}, \sigma^2_{S_3,D}] = [0.5, 1, 1.5]$
- SNR threshold $\mathtt{T} = 5$ dB

Outage Probability



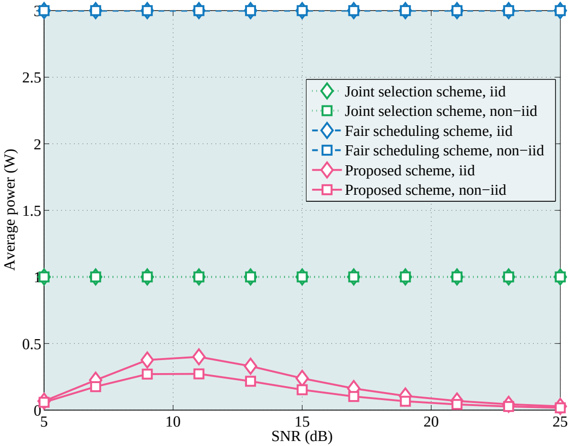
- Fair scheduling and proposed scheme achieve diversity order of two ⇒ cooperative diversity gain
- Joint selection scheme achieves diversity order of five ⇒ combine MUD and CD

Average Throughput (I.I.D.)



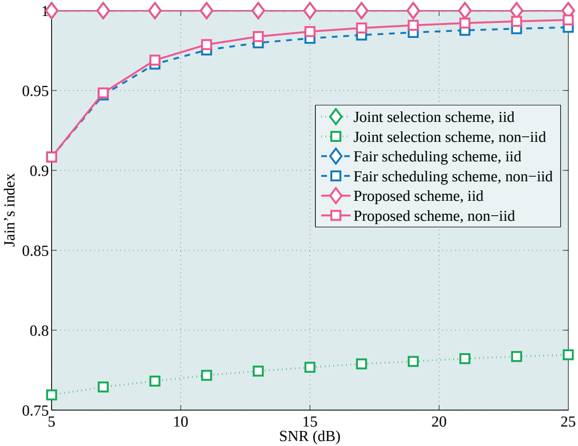
- All sources have the same throughput due to i.i.d. channel
- Relays in the proposed scheme are used only when needed $\Rightarrow$ shorter relay phase and in turn better bandwidth efficiency.
- Fair scheme has the longest broadcast phase (equal to the number of source nodes).

Average Relay Power Consumption



- Joint selection scheme: only one relay is used for a selected source.
- Fair scheduling scheme: a relay may be used by multiple sources.
- Proposed scheme: relays are used according to channel condition.

Throughput Fairness



$$\boxed{J_i = \frac{\left(\sum_{i=1}^{M} \bar{\eta}_i\right)^2}{M\sum_{i=1}^{M}(\bar{\eta}_i)^2}}$$

- I.I.D. case: all the schemes are strictly fair
- I.N.D. case: joint scheme favors a single source and thus is not fair

Decentralized Scheduling

Above discussion assumes centralized relay selection: global (instantaneous or statistical) CSI is available either at each user or at the central unit.

In distributed partner selection, each relay is selected based on local information only.

Simplest decentralized strategy: random selection. Does it achieve full diversity?

Assume K users in the network, and each user can serve as the relay of other users. If each user randomly chooses n users to serve, the probability that a user is served by less than n users is

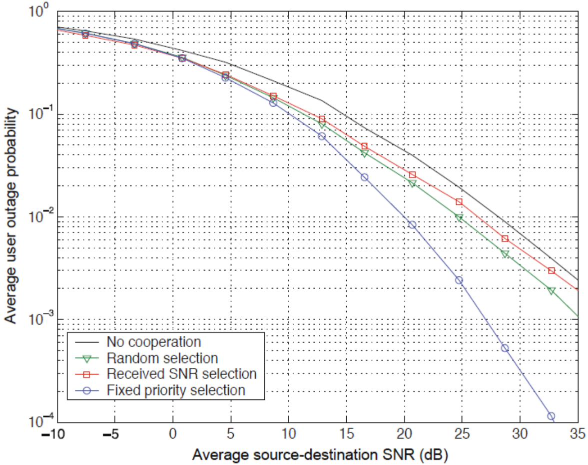
$$\Pr[|\mathcal{R}| < n] = \sum_{i=0}^{n-1} \binom{K-1}{i} \left(\frac{n}{K-1}\right)^{i} \left(1 - \frac{n}{K-1}\right)^{K-i-1}$$

Since this probability does not vanish as the SNR increases, full diversity order of n will not be attainable.

Decentralized Selection (cont'd)

How to guarantee that each source is served by n relays?

Fixed priority protocol: order all users in a ring and have each user help the n closest neighbors in the clockwise direction [1] ($K = 10$ users considered in the figure).



Lagrangian

standard form problem

$$\begin{aligned} \min \quad & f_0(x) \\ \text{subject to} \quad & f_i(x) \le 0,\ i = 1, \cdots, m \\ & h_i(x) = 0,\ i = 1, \cdots, p \end{aligned}$$

variables $x \in \mathbf{R}^n$, optimal value $p^\star$

Lagrangian: $: \mathbf{R}^n \times \mathbf{R}^m \times \mathbf{R}^p \to \mathbf{R}$

$$L(x, \lambda, \nu) = f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x) + \sum_{i=1}^{p} \nu_i h_i(x)$$

- weighted sum of objective and constraint functions
- λ_i is Lagrange multiplier associated with $f_i(x) \le 0$
- ν_i is Lagrange multiplier associated with $h_i(x) = 0$

S. Boyd, *Convex Optimization.*

Karush-Kuhn-Tucker (KKT) conditions

If $x^\star$ minimizes $L(x, \lambda^\star, \nu^\star)$ over x, it follows that its gradient must vanish at $x^\star$, i.e,

(i) primal constraints: $f_i(x^\star) \leq 0, i = 1, \cdots, m$; $h_i(x^\star) = 0, i = 1, \cdots, p$

(ii) dual constraints: $\lambda_i^\star \geq 0, i = 1, \cdots, m$

(iii) complementary slackness: $\lambda_i^\star f_i(x^\star) = 0, i = 1, \cdots, m$

(iv) gradient of Lagrangian w.r.t. $x^\star$ vanishes:

$$\nabla f_0(x^\star) + \sum_{i=1}^{m} \lambda_i^\star \nabla f_i(x^\star) + \sum_{i=1}^{p} \nu_i^\star \nabla h_i(x^\star) = 0$$

References

[1] A. Nosratinia and T. E. Hunter, "Grouping and partner selection in cooperative wireless networks," *IEEE J. Select. Areas Commun.*, vol. 25, no. 2, pp. 369–378, Feb. 2007.