Chapter 4 Cooperative Communications with Single Relay

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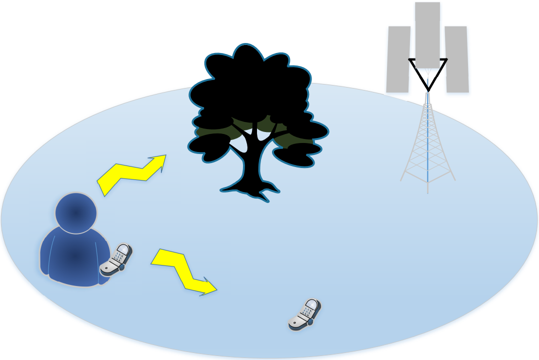
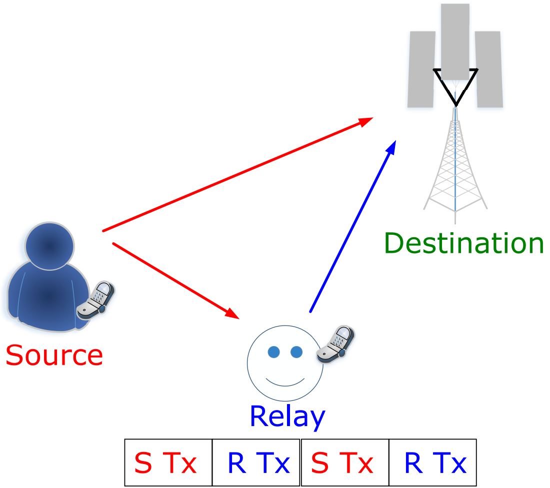


How to enable the cooperation between two users?



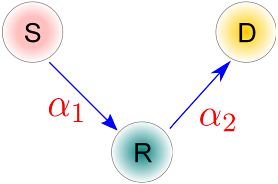
Protocol is simple: two phases.

But what the relay should do to the received signal before forwarding?

Main themes of this chapter

- How the relay deal with the received signal
- Evaluation of relay's gain
- Power consumption minimization

Amplify-and-Forward (AF)



The received signal at R is

$$r_R(t) = a_1 s(t) + n_1(t)$$

where $a_1 = |\alpha|$ is the fading amplitude of the S-R channel, $s(t)$ is the transmitting signal with power P_1, and $n_1(t)$ is the AWGN signal with double-sided PSD N_0.

R amplifies the received signal with a certain gain G and forwards.

System Model (cont'd)

The received signal at D is

$$
\begin{aligned}
r_D(t) &= a_2 G r_R(t) + n_2(t) \\
&= a_2 G\big(a_1 s(t) + n_1(t)\big) + n_2(t)
\end{aligned}
$$

The overall received SNR at D is

$$
\gamma_{\text{eq}} = \frac{P_1[a_2 G a_1]^2}{\big[(a_2{}^2 G^2 + 1)\big] N_0} = \frac{\frac{P_1 a_1{}^2}{N_0}\frac{a_2{}^2}{N_0}}{\frac{a_2{}^2}{N_0} + \frac{1}{G^2 N_0}} \tag{1}
$$

Three variants for choosing G:

- Variable-gain: relay has the CSI of the first hop.
- Fixed-gain: relay is not aware of any CSI [1].
- Semi-blind: relay knows the statistical CSI of the first hop.

Variable-Gain AF

To satisfy the maximum power constraint of relay, say P_2, choose the relay gain as

$$P_2 = G^2 a_1^2 P_1 + G^2 N_0$$
$$\Rightarrow G^2 = \frac{P_2}{N_0 + P_1 {a_1}^2}, \tag{2}$$

For a small ${a_1}^2$, the relay amplifier should increase its gain. when ${a_1}^2 \to 0$. Since N_0 is never zero, the output power of the relay is still limited if α_1 is low.

Fixed-Gain AF

However, CSI-assisted AF requires continuously estimating the channel fading amplitude, rendering it less attractive in practice.

Alternatively, the relay may use a constant gain G regardless of the fading amplitude on the first hop.

As a result, (1) can be written as

$$\gamma_{\text{eq}}^{\text{FAF}} = \frac{\gamma_1 \gamma_2}{C + \gamma_2} \tag{3}$$

where $\gamma_i = P_i {a_i}^2 / N_0,\ i = 1, 2$ and $C = P_2 / (G^2 N_0)$.

Semi-blind AF

Instead of continuous monitoring the first hop as in variable-gain AF, relay may acquire the statistical CSI, e.g., the average channel gain power $\overline{a_1^2}$. This significantly reduces the overhead for frequent channel estimation and is suitable when channel fading changes slowly.

To maintain the same average power as (2), the relay gain is chosen:

$$G^2 = \mathbb{E}\left[\frac{P_2}{P_1 a_1^2 + N_0}\right] \tag{4}$$

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For Rayleigh fading channel, what is G^2 [1]?

For Rayleigh fading and let $\overline{a_1^2} = \Omega_1$,

$$\begin{aligned}
G^2 &= \mathbb{E}\left[\frac{P_2/N_0}{P_1 a_1^2/N_0 + 1}\right] \\
&= \int_0^\infty \frac{P_2/N_0}{\gamma_1 + 1} f_{\gamma_1}(\gamma) d\gamma \overset{\gamma_1 \sim \exp(1/\bar{\gamma}_1)}{=} \frac{P_2}{N_0 \bar{\gamma}_1} \int_0^\infty \frac{1}{\gamma + 1} e^{-\frac{\gamma}{\bar{\gamma}_1}} d\gamma \\
&\overset{(\star)}{=} \frac{P_2}{P_1 \Omega_1} e^{1/\bar{\gamma}_1} E_1\left(\frac{1}{\bar{\gamma}_1}\right)
\end{aligned}$$

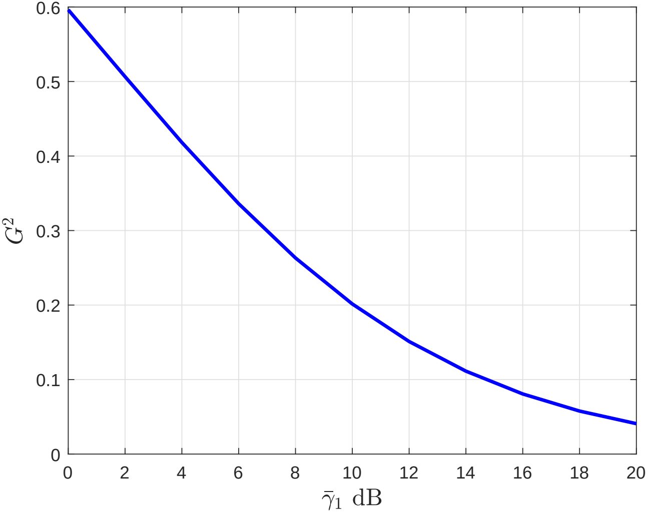
where $(\star)$ uses change of variable $t := \gamma + 1$ so that

$$\begin{aligned}
&\int_0^\infty \frac{1}{\gamma + 1} e^{-\frac{\gamma}{\bar{\gamma}_1}} d\gamma = \int_1^\infty \frac{1}{t} e^{-\frac{t-1}{\bar{\gamma}}} dt = e^{\frac{1}{\bar{\gamma}}} \int_1^\infty \frac{1}{t} e^{-\frac{1}{\bar{\gamma}} t} dt \\
&= e^{\frac{1}{\bar{\gamma}}} E_1\left(\frac{1}{\bar{\gamma}}\right)
\end{aligned}$$

where $E_1(\cdot)$ is the exponential integral function [2].

Semi-blind AF (cont'd)

Given $P_2/N_0 = 1$



Outage Probability

A popular performance measure for communications techniques in fading channels.

In information-theoretical sense, a message cannot be reliably decoded at the receiver if the channel capacity is less than the transmission rate. The outage probability is

$$\begin{aligned}\mathbb{P}_{\text{out}} &= \mathbb{P}[\log_2(1+\gamma) < \mathtt{R}] \\ &= \mathbb{P}\left[a^2 < \frac{2^{\mathtt{R}}-1}{\mathtt{SNR}}\right]\end{aligned}$$

where $\mathtt{SNR} = P/N_0$, $a = |\alpha|$ and $\mathtt{R}$ is the transmission rate.

Outage Probability (Cont'd)

Given the underlying channel characteristics (e.g., Rayleigh fading), we can find the CDF of a^2 and thus the outage probability

$$\mathbb{P}_{\text{out}} = \int_0^{\mathtt{T}} f_{a^2}(x)dx = F_{a^2}(\mathtt{T})$$

where $\mathtt{T} = \frac{2^{\mathtt{R}}-1}{\mathtt{SNR}}$ is a constant, and $F(\cdot)$ is the CDF of a RV.

Q

- Derive the outage probability of a point-to-point communication over the Rayleigh fading channel.

P_{out} in Case I: No Direct Link

Consider VAF. The maximum achievable end-to-end rate is

$$C^{\mathsf{VAF}} = \frac{1}{2}\log\left(1+\gamma_{\mathsf{eq}}^{\mathsf{VAF}}\right)$$

Thus the outage probability is

$$\mathbb{P}_{\mathsf{out}} = \mathbb{P}[C^{\mathsf{VAF}} < \mathtt{R}] = \mathbb{P}[\gamma_{\mathsf{eq}}^{\mathsf{VAF}} < 2^{2\mathtt{R}} - 1], \tag{5}$$

where $\gamma_{\mathsf{eq}}^{\mathsf{VAF}}$ can be obtained by substituting (2) into (1)

$$\gamma_{\mathsf{eq}}^{\mathsf{VAF}} = \frac{\gamma_1 \cdot \gamma_2}{\gamma_1 + \gamma_2 + 1}, \tag{6}$$

This is not tractable due to the complexity in finding the statistics of the equivalent SNR in (6).

Case I: No Direct Link (Cont'd)

Can we find a reasonable approximation?

- Accurate, at least at high SNR
- Analytically tractable with useful guideline for system designers

Let's ignore the constant 1 in the denominator (valid at high SNR), leading to the approximated equivalent SNR

$$\gamma_{\mathsf{eq}}^{\mathsf{VAF}} \approx \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} = \frac{1}{\frac{1}{\gamma_1} + \frac{1}{\gamma_2}}, \tag{7}$$

which is related the harmonic mean of γ_1 and γ_2.

Case I: No Direct Link (Cont'd)

Definition 1

Given two numbers X_1 and X_2, their harmonic mean is defined as the reciprocal of the arithmetic mean of the reciprocals of X_1 and X_2, that is

$$\mu_H(X_1, X_2) = \frac{2}{\frac{1}{X_1} + \frac{1}{X_2}} = \frac{2X_1X_2}{X_1 + X_2}.$$

Theorem 1 in Appendix gives the PDF and CDF of $\mu_H(X_1, X_2)$.

Using notations $X := \mu_H(\gamma_1, \gamma_2)$ and $\Gamma := \gamma_{\text{eq}}^{\text{VAF}}$, the relation $\Gamma = X/2$ implies we can find the PDF of Γ by transformation of RV,

$$f_\Gamma(\gamma) = f_X(x = 2\gamma)\frac{d}{d\gamma}(x = 2\gamma) = 2f_X(2\gamma).$$

Case I: No Direct Link (Cont'd)

According to (5), the outage probability is [3, Eq. (27)]

$$\mathbb{P}_{\text{out}}^{\text{VAF}} = \mathbb{P}[\Gamma < \mathtt{T}] = \int_0^{\mathtt{T}} f_\Gamma(\gamma) d\gamma$$
$$= 1 - \frac{2\mathtt{T}}{\sqrt{\bar{\gamma}_1 \bar{\gamma}_2}} K_1\left(\frac{2\mathtt{T}}{\sqrt{\bar{\gamma}_1 \bar{\gamma}_2}}\right) e^{-\mathtt{T}(1/\bar{\gamma}_1 + 1/\bar{\gamma}_2)} \quad (8)$$

where $\mathtt{T} \triangleq 2^{2\mathtt{R}} - 1$, $\bar{\gamma}_i = 1/\lambda_i, i = 1, 2$ in (λ_i defined in 17)).

$\boxed{\text{Diversity gain: how (8) scales with } \mathtt{SNR}?}$

Asymptotic analysis

- When $\mathtt{SNR} \to \infty$, the argument of $K_1(\cdot)$ approaches to zero.
- $K_1(x) \approx 1/x$ when $x \to 0$.

$$\mathbb{P}_{\text{out}}^{\text{VAF}} \approx 1 - e^{-\mathtt{T}(1/\bar{\gamma}_1 + 1/\bar{\gamma}_2)} \tag{9}$$

- Expanding the exponential term ($e^{-x} \approx 1 - x + \cdots$)

$$\begin{aligned} \mathbb{P}_{\text{out}}^{\text{VAF}} &\approx \mathtt{T}\left(\frac{1}{\bar{\gamma}_1} + \frac{1}{\bar{\gamma}_2}\right) \\ &\overset{(\star)}{=} \mathtt{T}\left(\frac{1}{\mathbb{E}[a_1^2]} + \frac{1}{\mathbb{E}[a_2^2]}\right)\frac{1}{\mathtt{SNR}} \end{aligned} \tag{10}$$

where $(\star)$: $P_1 = P_2 = P$ and $\mathtt{SNR} = P/N_0$.

Conclusion

With one relay, VAF achieves diversity order of 1.

Case II: With Direct Link

In case the direct link between S and D is not blocked. D can combine the signals from S and R.

By optimally choosing the combining weight and assuming VAF, the output SNR using MRC is

$$\gamma_c = \gamma_0 + \gamma_{\text{eq}}^{\text{VAF}}$$

where γ_0 is the instantaneous SNR of the S-D link and $\gamma_{\text{eq}}^{\text{VAF}}$ is given by (6).The maximum achievable end-to-end transmission rate is

$$C = \frac{1}{2}\log_2\left(1 + \gamma_0 + \gamma_{\text{eq}}^{\text{VAF}}\right).$$

Hence the outage probability can be given as

$$\mathbb{P}_{\text{out}} = \mathbb{P}[C \leq \mathtt{R}] = \mathbb{P}\left[\gamma_0 + \gamma_{\text{eq}}^{\text{VAF}} < \mathtt{T}\right] \tag{11}$$

Case II: With Direct Link (cont'd)

With the assumption of Rayleigh fading, γ_i's are exponentially distributed with mean $\bar{\gamma}_i = P_i\sigma_i^2/N_0$, where $\mathbb{E}[a_i{}^2] = \sigma_i^2$.

Further assume independent channel statistics, (11) can be derived as

$$\begin{aligned}
\mathbb{P}_{\text{out}} &= \int_0^{\mathtt{T}} \mathbb{P}\left[\gamma_{\text{eq}}^{\text{VAF}} < \mathtt{T} - x \middle| \gamma_0 = x\right] f_{\gamma_0}(x)dx \\
&\overset{(8)}{=} \int_0^{\mathtt{T}} \left(1 - \frac{2(\mathtt{T}-x)}{\sqrt{\bar{\gamma}_1\bar{\gamma}_2}} K_1\left(\frac{2(\mathtt{T}-x)}{\sqrt{\bar{\gamma}_1\bar{\gamma}_2}}\right) e^{-(\mathtt{T}-x)(1/\bar{\gamma}_1+1/\bar{\gamma}_2)}\right) f_{\gamma_0}(x)dx \\
&\overset{(9)}{\approx} \int_0^{\mathtt{T}} \left(1 - e^{-\frac{\mathtt{T}-x}{\bar{\gamma}_1} - \frac{\mathtt{T}-x}{\bar{\gamma}_2}}\right) f_{\gamma_0}(x)dx \\
&= \int_0^{\mathtt{T}} \left(1 - e^{-\frac{\mathtt{T}-x}{\bar{\gamma}_1} - \frac{\mathtt{T}-x}{\bar{\gamma}_2}}\right) \frac{1}{\bar{\gamma}_0} e^{-\frac{x}{\bar{\gamma}_0}} dx \\
&= 1 - e^{-\frac{\mathtt{T}}{\bar{\gamma}_0}} - \frac{\bar{\gamma}_1\bar{\gamma}_2}{\bar{\gamma}_1\bar{\gamma}_2 - \bar{\gamma}_0\bar{\gamma}_2 - \bar{\gamma}_0\bar{\gamma}_1}\left[e^{-\mathtt{T}(\frac{1}{\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_2})} - e^{-\frac{\mathtt{T}}{\bar{\gamma}_0}}\right]
\end{aligned}$$

Case II: With Direct Link (cont'd)

High SNR Approximation

Second-order Taylor approximation: $e^{-x} \approx 1 - x + \frac{x^2}{2}$ when $x \to 0$

$$\begin{aligned}\mathbb{P}_{\text{out}} \approx& \frac{1}{2}\frac{\mathtt{T}}{\bar{\gamma}_0}\left[\frac{\mathtt{T}}{\bar{\gamma}_1} + \frac{\mathtt{T}}{\bar{\gamma}_2}\right] \\ =& \frac{1}{2}\left(\frac{\mathtt{T}}{\mathtt{SNR}}\right)^2 \frac{1}{\sigma_0^2}\left[\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right],\end{aligned}$$

assuming $P_0 = P_1 = P_2 = P$ and $\mathtt{SNR} = P/N_0$.

Conclusion

VAF with direct link achieve diversity order of 2.

Optimal Power Allocation: Case I

The simplified expression for outage probability in (10) allows us to formulate the optimal power allocation problem as follows.

Suppose $\alpha_i \sim \mathcal{CN}(0, \sigma_i^2)$ for $i = 1, 2$. Then (10) can written as

$$P_{out}^{\text{VAF}} \approx \mathtt{T}\left(\frac{1}{\mathbb{E}[\frac{P_1}{N_0}a_1^2]} + \frac{1}{\mathbb{E}[\frac{P_2}{N_0}a_2^2]}\right)$$
$$= \mathtt{T}\left(\frac{1}{\frac{P_1}{N_0}\sigma_1^2} + \frac{1}{\frac{P_2}{N_0}\sigma_2^2}\right)$$

Optimal Power Allocation: Case I (cont'd)

Given the total power constraint $P_1 + P_2 \leq 2P$, the optimal power allocation between P_1 and P_2 can be found by minimizing the outage probability, i.e.

$$\min_{P_1, P_2} \quad \frac{1}{\frac{P_1}{N_0}\sigma_1^2} + \frac{1}{\frac{P_2}{N_0}\sigma_2^2} \tag{12}$$

$$\text{subject to} \quad P_1 + P_2 \leq 2P \text{ and } P_1, P_2 \geq 0 \tag{13}$$

Optimal Power Allocation: Case I (cont'd)

Back to our problem, define

$$L = -\frac{1}{P_1}\frac{N_0}{\sigma_1^2} - \frac{1}{P_2}\frac{N_0}{\sigma_2^2} - \lambda(P_1 + P_2 - 2P)$$

Then solve

$$\begin{aligned}
\frac{\partial L}{\partial P_1} &= \frac{1}{{P_1}^2}\frac{N_0}{\sigma_1^2} - \lambda = 0 \\
\frac{\partial L}{\partial P_2} &= \frac{1}{{P_2}^2}\frac{N_0}{\sigma_2^2} - \lambda = 0 \\
P_1 + P_2 &= 2P
\end{aligned}$$

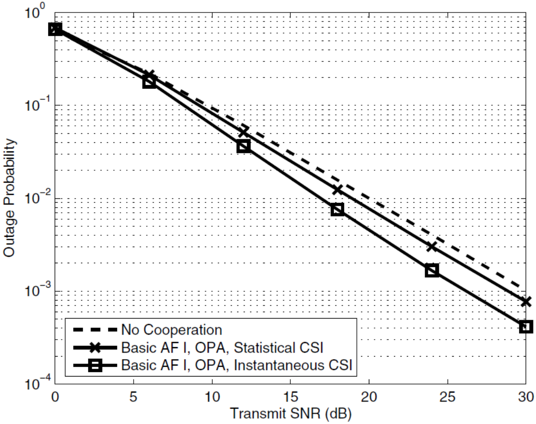
The optimal allocation is

$$\begin{aligned}
P_1 &= 2P\frac{\sigma_2}{\sigma_1 + \sigma_2} \\
P_2 &= 2P\frac{\sigma_1}{\sigma_1 + \sigma_2}
\end{aligned}$$

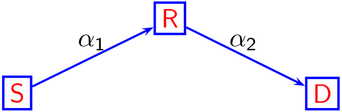
Optimal Power Allocation: Case I (cont'd)

The above power allocation only requires the statistics of the channel.

If instantaneous CSI is available (i.e. α_1 and α_2) at S and R, the optimal power allocation can be found in a similar manner.



Decode-and-Forward (DF)



The received signal at R is

$$r_R(t) = \alpha_1 s(t) + n_1(t)$$

where $\alpha_1 \sim \mathcal{CN}(0, \sigma_1^2)$ is the fading coefficient of the S-R channel, $s(t)$ is the transmitting signal with power P_1, and $n_1(t)$ is the AWGN signal with one sided PSD N_0.

DF (cont'd)

R decodes the received signal and performs error detection. If R can correctly decode, R re-encodes the message into a new one $\hat{s}$ such that $\hat{s} = s$ The received signal at D is

$$r_D(t) = \alpha_2 s(t) + n_2(t)$$

where $\alpha_2 \sim \mathcal{CN}(0, \sigma_2^2)$ is the fading coefficient of the channel between R and D, and $n_2(t)$ is the AWGN signal with one sided PSD N_0.

P_{out} in Case I: No Direct Link

The maximum rate is bounded by the capacity of both links, i.e., $\mathtt{R} \leq \frac{1}{2}\min\{\log_2(1+\gamma_1), \log_2(1+\gamma_2)\}$. Hence, the end-to-end achievable rate is

$$C = \frac{1}{2}\min\{\log_2(1+\gamma_1), \log_2(1+\gamma_2)\}$$

Thus outage occurs with probability

$$\begin{aligned}
\mathbb{P}_{\text{out}} &= \mathbb{P}\left[C < \mathtt{R}\right] = \mathbb{P}\left[\frac{1}{2}\min\{\log_2(1+\gamma_1), \log_2(1+\gamma_2)\} < \mathtt{R}\right] \\
&= 1 - \mathbb{P}\Big[\min\{\log_2(1+\gamma_1), \log_2(1+\gamma_2)\} \geq 2\mathtt{R}\Big] \\
&= 1 - \mathbb{P}\Big[\log_2(1+\gamma_1) \geq 2\mathtt{R}, \log_2(1+\gamma_2) \geq 2\mathtt{R}\Big] \\
&= 1 - \mathbb{P}\Big[\gamma_1 \geq 2^{2\mathtt{R}} - 1, \gamma_2 \geq 2^{2\mathtt{R}} - 1\Big]
\end{aligned}$$

Case I: No Direct Link (cont'd)

For Rayleigh fading, γ_1 and γ_2 are exponentially distributed RVs. Assuming *S*-*R* and *R*-*D* channels are independent,

$$\mathbb{P}_{\text{out}} = 1 - \mathbb{P}\left[\gamma_1 \geq \mathtt{T}\right] \cdot \mathbb{P}\left[\gamma_2 \geq \mathtt{T}\right] = 1 - e^{-(\frac{1}{\bar{\gamma}_1} + \frac{1}{\bar{\gamma}_2})\mathtt{T}}$$

where $\mathtt{T} \triangleq 2^{2\mathtt{R}} - 1$.

High SNR approximation

Assume $P_1 = P_2 = P$,

$$\mathbb{P}_{\text{out}} \approx (\tfrac{1}{\bar{\gamma}_1} + \tfrac{1}{\bar{\gamma}_2})\mathtt{T} = \frac{\mathtt{T}}{\mathtt{SNR}}\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)$$

With one relay, DF achieves diversity order of 1.

One can perform power allocation to achieve further gain.

$\mathbb{P}_{\text{out}}$ in Case II: With Direct Link

When the direct link is available, D will receive two copies of the signal. The received signal at D in vector form

$$\mathbf{r}_D = \begin{bmatrix} r_0(t) \\ r_D(t) \end{bmatrix} = \begin{bmatrix} \sqrt{P_1}\alpha_0 \\ \sqrt{P_2}\alpha_2 \end{bmatrix} s(t) + \begin{bmatrix} n_0(t) \\ n_2(t) \end{bmatrix},$$

which is similar to conventional single-input multiple-output (SIMO) systems. The received SNR can be maximized by employing MRC at the destination by choosing the weight coefficients as

$$\mathbf{w} = \begin{bmatrix} \sqrt{P_1}\alpha_0{}^* \\ \sqrt{P_2}\alpha_2{}^* \end{bmatrix}.$$

P_{out} in Case II: With Direct Link (cont'd)

The combined signal is

$$\tilde{r}_D(t) = \mathbf{w}^T \mathbf{r}_D = ({P_1 \alpha_0}^2 + {P_2 \alpha_2}^2) s(t) + \tilde{n}(t)$$

where $\tilde{n}(t) = \sqrt{P_1}{\alpha_0}^* n_1(t) + \sqrt{P_2}{\alpha_2}^* n_2(t)$, $\tilde{n}(t) \sim \mathcal{CN}\left(0, (P_1{\alpha_0}^2 + P_2{\alpha_2}^2)\sigma^2\right)$.

The SNR at the output of the MRC is

$$\gamma_c^{\mathrm{DF}} = \frac{P_1{\alpha_0}^2}{\sigma^2} + \frac{P_2{\alpha_2}^2}{\sigma^2} = \gamma_0 + \gamma_2.$$

With diversity combining, the achievable rate in Phase II is $\frac{1}{2}\log_2(1+\gamma_0+\gamma_2)$, given that R successfully decodes in Phase I.This requires the rate transmitted by S is less than the capacity of the S-R link, i.e, $\frac{1}{2}\log_2(1+\gamma_1)$. Hence, the maximum achievable end-to-end rate is

$$C^{\mathrm{DF}} = \frac{1}{2}\min\{\log_2(1+\gamma_1), \log_2(1+\gamma_0+\gamma_2)\}$$

Case II: With Direct Link (cont'd)

$$\begin{aligned}\mathbb{P}_{\text{out}} &= \mathbb{P}\left[C^{\text{DF}} < \mathtt{R}\right] \\ &= \mathbb{P}\left[\tfrac{1}{2}\log_2(1+\gamma_1) < \mathtt{R}\right] \\ &\quad + \mathbb{P}\left[\tfrac{1}{2}\log_2(1+\gamma_1) \geq \mathtt{R}\right] \times \mathbb{P}\left[\tfrac{1}{2}\log_2(1+\gamma_0+\gamma_2) < \mathtt{R}\right] \\ &= \mathbb{P}\left[\gamma_1 < \mathtt{T}\right] + \mathbb{P}\left[\gamma_1 \geq \mathtt{T}\right]\mathbb{P}\left[\gamma_0+\gamma_2 < \mathtt{T}\right] \\ &= 1 - e^{-\frac{\mathtt{T}}{\bar{\gamma}_1}} + e^{-\frac{\mathtt{T}}{\bar{\gamma}_1}} \cdot \mathbb{P}\left[\gamma_0+\gamma_2 < \mathtt{T}\right]\end{aligned}$$

Here we need the distribution of $\gamma_0 + \gamma_2$, the sum of two independent exponential RVs. It is a well-known result and can be found in Appendix.

Case II: With Direct Link (cont'd)

Using (18),

$$\mathbb{P}_{\text{out}} = \begin{cases} 1 - e^{-\frac{\mathtt{T}}{\bar{\gamma}_1}} \left(\frac{\bar{\gamma}_0}{\bar{\gamma}_0 - \bar{\gamma}_2} e^{-\frac{\mathtt{T}}{\bar{\gamma}_0}} + \frac{\bar{\gamma}_2}{\bar{\gamma}_2 - \bar{\gamma}_0} e^{-\frac{\mathtt{T}}{\bar{\gamma}_2}} \right), & \text{for } \bar{\gamma}_0 \neq \bar{\gamma}_2 \\ 1 - \left(1 + \frac{\mathtt{T}}{\bar{\gamma}_0}\right) e^{-\frac{\mathtt{T}}{\bar{\gamma}_1} - \frac{\mathtt{T}}{\bar{\gamma}_0}}, & \text{for } \bar{\gamma}_0 = \bar{\gamma}_2 \end{cases}$$

High SNR Approximation

At high SNR, $e^{-x} \approx 1 - x$ for $x \to \infty$ and thus

$$\mathbb{P}_{\text{out}} \approx \begin{cases} \frac{\mathtt{T}}{\bar{\gamma}_1}, & \text{for } \bar{\gamma}_0 \neq \bar{\gamma}_2 \\ \frac{\mathtt{T}}{\bar{\gamma}_1} + \frac{\mathtt{T}^2}{\bar{\gamma}_0} \left(\frac{1}{\bar{\gamma}_1} + \frac{1}{\bar{\gamma}_0} \right), & \text{for } \bar{\gamma}_0 = \bar{\gamma}_2 \end{cases}$$

Let $P_1 = P_2 = P$, we can express $\mathbb{P}_{\text{out}}$ in terms of $\mathtt{SNR}$ as

$$\mathbb{P}_{\text{out}} \approx \begin{cases} \frac{\mathtt{T}}{\sigma_1^2} \frac{1}{\mathtt{SNR}}, & \text{for } \bar{\gamma}_0 \neq \bar{\gamma}_2 \\ \frac{\mathtt{T}}{\sigma_1^2} \frac{1}{\mathtt{SNR}} + \mathcal{O}\left(\frac{1}{\mathtt{SNR}^2} \right), & \text{for } \bar{\gamma}_0 = \bar{\gamma}_2 \end{cases} \tag{14}$$

$\mathbb{P}_{\text{out}}$ in Case II: With Direct Link (cont'd)

This shows that the diversity order of the basic DF relaying scheme is still equal to 1 even when diversity combining is employed at the destination. This is due to the fact that the relay must first be able to successfully decode the message before the cooperative transmission can take place. Thus, the achievable rate is limited by the capacity of the S-R link. However, this can be improved upon if we allow the source to repeat the information if the relay can not decode successfully, which leads to the so called selection DF relaying scheme discussed in the following.

Selection Decode-and-Forward

The performance of DF is limited by the quality of the S-R link, because the rely is required to successfully decode the source signal.

Selection DF (SDF): It works similarly to DF, except that S retransmits in phase II if R can not decode the source signal.

- If the same codeword is used by S in Phase II, it is like the traditional automatic repeat-request (ARQ) protocol that exploits repetition coding gain.
- More powerful codes can be used, but the repetition code is sufficient to achieve full diversity order of 2 (See Appendix for the end-to-end capacity of SDF).

Demodulate-and-Forward (DmF)

In DF, R needs to decode every received packet. This introduces a large amount of overhead and power consumption for relaying. For energy-constraint applications, DmF is an appealing choice to exploit cooperative diversity.

In DmF, R demodulates the received signals r_R on a symbol by symbol basis. Suppose that BPSK modulation is employed at S such that the transmit signal $x_s \in \{-1, +1\}$.

The ML detector at R is

$$\hat{x}_s = \mathrm{Re}(r_R \cdot {\alpha_{s,r}}^*) \gtrless_1^0 0.$$

In Phase II, R forwards the remodulated symbol $\hat{x}_r$ to D.

DmF (cont'd)

The received signal at D is

$$r_D^{(2)} = \alpha_{r,d}\hat{x}_r + n_2.$$

Given the received signal at both phases, i.e., $r_D^{(1)}$ and $r_D^{(2)}$, D attempts to decode the source signal.

- Without decoding the received signal, $\hat{x}_r$ may be different from x_s, leading to error propagation.
- Typical ML detector can not combat error propagation.
- A specialized MRC called *C-MRC* is proposed in [4] by using the SNR information of the *S*-*R* and the *R*-*D* channels.

Idea of C-MRC

1) if R detects correctly, implying that the S-R link is reliable, the optimal weight is $w_2 = \alpha^*_{r,d}$.
2) if the S-R link is not reliable such that R detects wrongly, then the R-D link should be given a smaller weight by a factor $\gamma_{\text{eq}}/\gamma_{r,d} \leq 1$, i.e., [4]

$$w_2 = \frac{\gamma_{\text{eq}}}{\gamma_{r,d}}\alpha^*_{r,d} \tag{15}$$

where γ_{eq} is the SNR of a one-hop AWGN channel that has the same error rate as that of the $S - R - D$ channel using DmF.

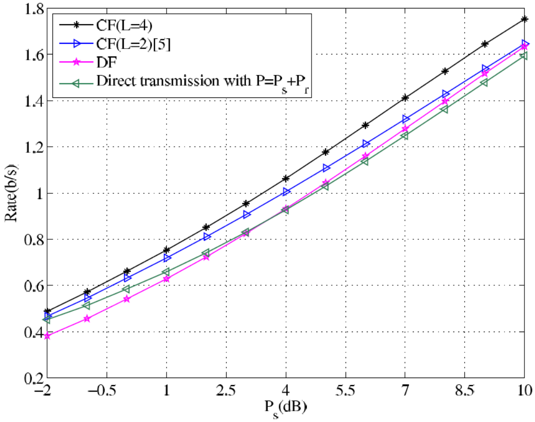
It is shown in [4] that C-MRC achieves diversity order of 2.

Compress-and-Forward

In Compress-and-Forward (CF), the relay quantizes the received signal and encodes the quantization indices. Then the relay transmits the encoded quantization indices to the destination.

- CF is similar to AF: the relay does not attempt to decode.
- The destination D first tries to recover the quantization indices and then decode the source message. To this end, the quantization indices are encoded with D's received signal.
- CF can approach the capacity region because the second slot can be made arbitrarily small (compression gain).

Compress-and-Forward (cont'd)



- CF(L=4): variable tx rate with 4 uniform quantization levels [5].
- CF(L=2): fixed tx rate with 2 uniform quantization levels [6].
- At rate of 1.0 b/s, CF has a coding gain of 1.15 dB over DF.

Conclusion

- Cooperative relaying is promising to achieve diversity gain using a single relay.
- Some practical challenges
  - Need reliable channel estimations $\rightarrow$ costly in bandwidth and time
  - Accuracy degrades in fast-fading environment
- When multiple relays are available, the major issue is how to use them efficiently.

P_{out} in Case I: No Direct Link (cont'd)

Theorem 1 (*PDF and CDF of the Harmonic Mean of Two Exponential RVs*)

Let X_1 and X_2 be two independent exponential RVs with parameters λ_1 and λ_2, respectively, then the CDF of $X = \mu_H(X_1, X_2)$ is given by [3, Eq. (2)]

$$f_X(x) = \frac{1}{2}\lambda_1\lambda_2 x e^{-x/2(\lambda_1\lambda_1)}\left[\frac{\lambda_1+\lambda_2}{\sqrt{\lambda_1\lambda_2}}K_1(x\sqrt{\lambda_1\lambda_2}) + 2K_0(x\sqrt{\lambda_1\lambda_2})\right] \quad (16)$$

$$F_X(x) = 1 - x\sqrt{\lambda_1\lambda_2}e^{-x/2(\lambda_1+\lambda_2)}K_1\left(x\sqrt{\lambda_1\lambda_2}\right) \quad (17)$$

where $K_0(\cdot)$ and $K_1(\cdot)$ are the zeroth and first order modified Bessel function of the second kind [2].

Lagrangian method

$$\begin{aligned} \max \quad & f(x, y) \\ \text{subject to} \quad & g(x, y) < 0 \end{aligned}$$

Basic idea: augment the objective function with a weighted sum of constraints, resulting in an unconstraint optimization problem.

Define a Lagrangian $L(x, y, \lambda) = f(x, y) - \lambda g(x, y)$ where λ is called the Lagrange multiplier. If changing the constraint will affect the solution (i.e. binding constraint), $\lambda \neq 0$. Otherwise, $\lambda = 0$.

Then we solve the following equations

$$\frac{\partial L}{\partial x} = 0, \quad \frac{\partial L}{\partial y} = 0$$
$$g(x, y) = 0$$

Sum of Two Exponential RVs

Denote $W = U + V$ with U and V being independent exponential RVs. The CDF of W is [7]

$$F_W(w) = \begin{cases} 1 - \left(\frac{\bar{U}}{\bar{U}-\bar{V}} e^{-\frac{w}{\bar{U}}} + \frac{\bar{U}}{\bar{V}-\bar{U}} e^{-\frac{w}{\bar{V}}} \right), & \text{for } \bar{U} \neq \bar{V} \\ 1 - \left(1 + \frac{w}{\bar{U}} \right) e^{-\frac{w}{\bar{U}}}, & \text{for } \bar{U} = \bar{V} \end{cases} \tag{18}$$

P_{out} in SDF

Suppose MRC is used and the channel coefficient remains constant during a slot time but differs from one to the other. The effective SNR at the output of MRC is

$$\gamma_c^{\mathsf{SDF}} = \begin{cases} \gamma_0^{(1)} + \gamma_2, & \text{if } \gamma_1 \geq \mathtt{T}, \\ \gamma_0^{(1)} + \gamma_0^{(2)}, & \text{if } \gamma_1 < \mathtt{T}, \end{cases} \qquad \mathtt{T} \triangleq 2^{2\mathtt{R}} - 1,$$

where $\gamma_0^{(i)}, i = 1, 2$ are the SNRs on the S-D link in Phase I and II, respectively.

The achievable capacity of SDF is

$$C^{\mathsf{SDF}} = \begin{cases} \frac{1}{2}\log_2\left(1 + \gamma_0^{(1)} + \gamma_2\right), & \text{if } \gamma_1 \geq \mathtt{T}, \\ \frac{1}{2}\log_2\left(1 + \gamma_0^{(1)} + \gamma_0^{(2)}\right), & \text{if } \gamma_1 < \mathtt{T}. \end{cases}$$

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