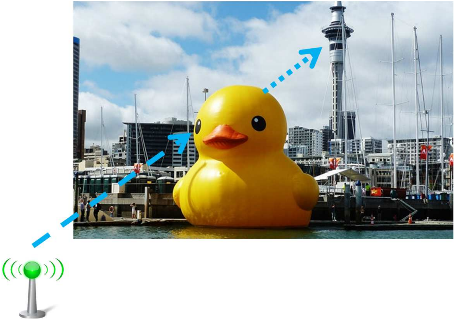
Chapter 02
Characterization of Wireless Channels

Kuang-Hao Liu

Institute of Communications Engineering
National Tsing Hua University

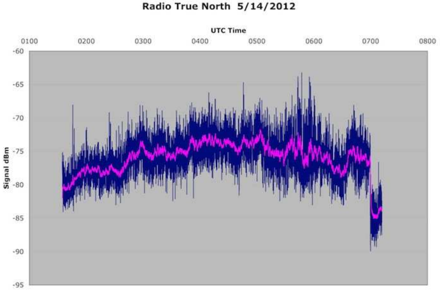
Wireless Channel Impairment



It is common to model the received signal strength in the form

$$P_r[\text{dB}] = \underbrace{\overline{P_r(d)}}_{\text{pathloss}} + \underbrace{X}_{\text{shadowing}} + \underbrace{Y(t)}_{\text{multipath fading}}$$

Wireless Channel Impairment (cont'd)



- The average received signal (magenta line) gradually decreases as a function of the distance $\Rightarrow$ can be predicted
- When Tx/Rx moves, the received signal experiences fluctuations, causing the received signal power varying severely. In deep fades, connection may be lost $\Rightarrow$ need effective fading-mitigation techniques.

Wireless Channel Impairment (cont'd)

$$P_r[\text{dB}] = \overline{P_r(d)} + X + Y(t)$$

- $\overline{P_r(d)}$: Average received power level as a function of distance d,

  $$\overline{P_r(d)} = P_t - L_p(d)$$

  where $L_p(d)$ represents the pathloss
- X (random variable):
  Large-scale variations in the average received power level due to shadowing
- $Y(t)$ (random process):
  Small-scale variations in the received power level due to multipath propagation

Commonly Used Models

- Pathloss
  - Free space model
  - Ground reflection model
  - 3GPP model
  - Simplified model

Free Space Propagation Model

The *free space propagation model* is used to predict received signal strength when the transmitter and receiver have a clear, unobstructed line-of-sight (LOS) path between them.

$$P_r = P_t G_t G_r L_p(d), \quad L_p(d) \triangleq \left(\frac{\lambda}{4\pi d}\right)^2$$

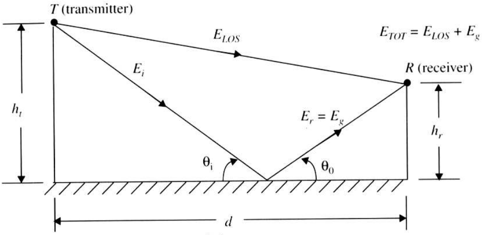
where

- P_t: transmitter power
- λ: wavelength of the carrier signal
- G_t/G_r: transmitter/receiver antenna gain

In decibel,

$$L_p(d) = 10\log_{10}\left(\frac{\lambda}{4\pi d}\right)^2 \xlongequal{\lambda=c/f_c} 147.56 - 20\log_{10} f_c - 20\log_{10} d \text{ (dB)}$$

Ground Reflection Model

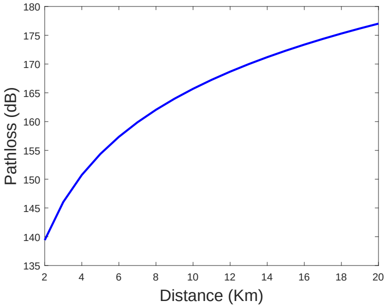


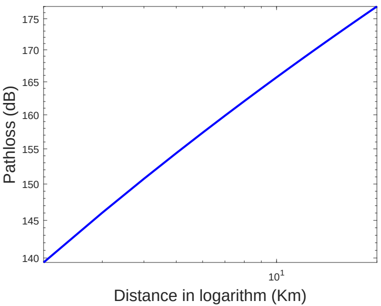
Read Sec. 2.4.1 in Goldsmith's book.

- What is the fundamental idea of ground reflection model?
- Why free space propagation model is over simplified?

3GPP Model

3GPP adopts distance-dependent pathloss model. For example, the pathloss between Macro BS to UE in outdoor is [TS 36.872] $L_p(d) = 128.1 + 37.6\log_{10}(d)$ for d in km.





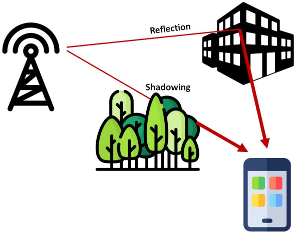
Simplified model

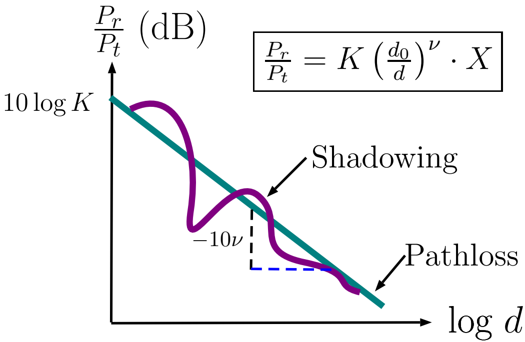
In general,

$$L_p(d) \propto \frac{d}{d_0}^{\nu}$$

Environment	Path loss exponent, ν
Free space	2
Urban area cellular radio	2.7 to 3.5
Shadowed urban cellular radio	3 to 5
In building line-of-sight	1.6 to 1.8
Obstructed in building	4 to 6
Obstructed in factories	2 to 3

Shadowing





- Shadowing refers to variations in received signal strength due to specific geometries of the paths between the transmitter and receiver and is effected by such objects as trees, hills, etc.
- Shadowing causes random fluctuations from average values which are typically modeled as a log-normal random variables (RVs).

Lognormal Shadowing

X is log-normal distributed if $10\log_{10} X \triangleq Z$ is a Gaussian (normal) distributed random variable with mean of zero and standard deviation of σ

$$f_Z(z) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{z^2}{2\sigma^2}\right), \quad \sigma = \begin{cases} 5 \text{ dB}, & \text{outdoor cellular} \\ 8 \text{ dB}, & \text{indoor environment} \end{cases}$$

Lognormal Shadowing (cont'd)

Let's find the distribution for "log-normal" RV.

Z [dB] $= 10\log_{10}(X) \longrightarrow X = 10^{Z/10}$

$$f_X(x) = \sum_i \frac{f_Z(z_i)}{|g'(z_i)|}, \quad \begin{cases} z_i : & \text{Roots of } x = g(z) \\ g'(z) : & \text{Derivative of } g(z) \end{cases}$$

Solving for z yields one root, i.e., $z_1 = 10\log_{10}(x)$

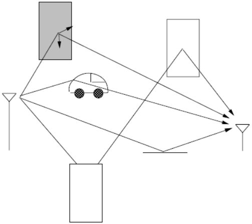
$$\left.\frac{\partial 10^{z/10}}{\partial z}\right|_{z=z_1} = \left.\frac{\ln 10}{10} 10^{\frac{z}{10}}\right|_{z_1=10\log_{10} x} = \frac{\ln 10}{10} x.$$

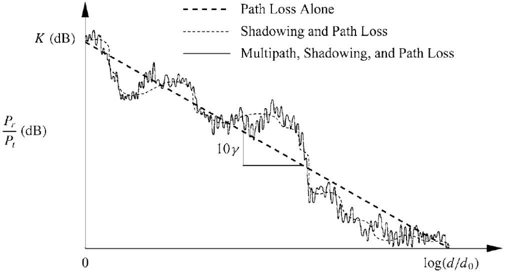
Replacing these into the above formula gives the following pdf for X

$$f_X(x) = \frac{10/\ln 10}{\sqrt{2\pi} x\sigma} \exp\left[-\frac{(10\log x)^2}{2\sigma^2}\right].$$

Small-Scale (Short-Term) Fading

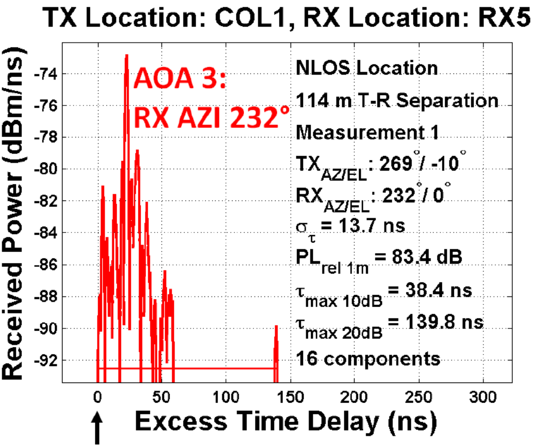
Short-term fading is due to "multipath propagation" and is independent of the distance between the transmitter and receiver.





Taken from Goldsmith, Fig. 3.8

Measurement results of mmWave multipath



M. K. Samimi and et al., IEEE Trans. Microwave Theory and Techniques, 2016.

Small-Scale (Short-Term) Fading

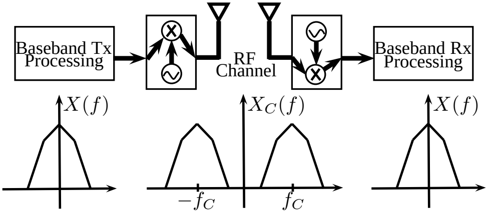
- Multiple delayed reception of the transmitted signals due to the reflections off of buildings, hills, cars and other obstacles, etc.
- A LOS may or may not not exist.
- Each path has a different attenuation, time delay, and phase shift.
- Due to the relative phase shifts, the signals from different paths add constructively sometimes or cancel each other resulting in a weak signal other times. This phenomenon is known as fading.

Mathematical Model for Multipath Fading

The band-pass transmit signal:

$$s_b(t) = \Re\{s(t)e^{j2\pi f_c t}\}$$

$s(t)$: baseband signal
f_c: carrier frequency



$$x(t)e^{j2\pi f_c t} \Leftrightarrow X(f - f_c)$$

Multipath Fading Model (Cont'd)

Ignore the additive noise and assume that the channel consists of N paths.

The received band-pass signal:

$$r_b(t) = \Re\{r(t)e^{j2\pi f_c t}\}$$

where

$$r(t) = \sum_{k=1}^{N} \alpha_k(t) e^{-j2\pi f_c \tau_k(t)} s\big(t - \tau_k(t)\big)$$

$\alpha_k(t)$: Attenuation of the kth path

$\tau_k(t)$: Delay of the kth path

$\theta_k(t) = 2\pi f_c \tau_k(t)$: Phase shift of the kth path

Statistical Multipath Fading Model

For narrowband channel, $s\big(t-\tau_k(t)\big) \approx s(t)$ and thus

$$r(t) = s(t)\underbrace{\boxed{\sum_{k=1}^{N(t)} \alpha_k(t) e^{j\theta_k(t)}}}_{\triangleq r_I(t)+jr_Q(t)} \tag{1}$$

where back

$$r_I(t) = \sum_{k=1}^{N(t)} \alpha_k(t)\cos\big(\theta_k(t)\big) \quad \text{and} \quad r_Q(t) = \sum_{k=1}^{N(t)} \alpha_k(t)\sin\big(\theta_k(t)\big)$$

When the following conditions holds,

- the number of paths N is large
- no dominating term (i.e., no LOS component)

m the Central Limit Theorem can be applied to $x(t)$ and $y(t)$.

Statistical Multipath Fading Model (cont'd)

According to CLT, $r_I(t)$ and $r_Q(t)$ are approximately independently Gaussian random process.

Let X and Y denote the samples taken from $r_I(t)$ and $r_Q(t)$

$$X \sim N(0, \sigma^2) \quad \text{and} \quad Y \sim N(0, \sigma^2)$$

Then $X + jY$ is a zero-mean complex Gaussian random variable.

$$f_{X,Y}(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

$$\left.\begin{array}{ll} \text{Fading amplitude (envelope)} & a = \sqrt{X^2 + Y^2} \\ \text{Fading phase} & \phi = \arctan(Y/X) \end{array}\right\} f_A(a) =?\ f_\Phi(\phi) =?$$

Statistical Multipath Fading Model (cont'd)

From Euler formula, $X = a\cos\phi$ and $Y = a\sin\phi$.

Using transformation formula between random variable pairs (X, Y) and (A, Φ)

$$f_{A,\Phi}(a,\phi) = |J(a,\phi)| f_{X,Y}(x,y)\Big|_{\substack{x=a\cos\phi \\ y=a\sin\phi}},$$ J: Jacobian of the transform

$$|J(a,\phi)| = \begin{vmatrix} \frac{\partial x}{\partial a} & \frac{\partial x}{\partial \phi} \\ \frac{\partial y}{\partial a} & \frac{\partial y}{\partial \phi} \end{vmatrix} = \begin{vmatrix} \cos\phi & -a\sin\phi \\ \sin\phi & a\cos\phi \end{vmatrix} = a\left(\cos^2\phi + \sin^2\phi\right) = a$$

$$f_{X,Y}(x,y)\Big|_{\substack{x=a\cos\phi \\ y=a\sin\phi}} = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{a^2\cos^2\phi + a^2\sin^2\phi}{2\sigma^2}\right)$$

$$\Rightarrow f_{A,\Phi}(a,\phi) = \frac{a}{2\pi\sigma^2}\exp\left(-\tfrac{a^2}{2\sigma^2}\right)$$

Need to find marginal pdf's of A and Φ, i.e., $f_A(a) = ?$ $f_\Phi(\phi) = ?$

Statistical Multipath Fading Model (cont'd)

For $0 < a < \infty$, $0 < \phi < 2\pi$

$$① \; f_A(a) = \int_0^{2\pi} f_{A,\Phi}(a,\phi)d\phi = \int_0^{2\pi} \frac{a}{2\pi\sigma^2} \exp^{\left(-\frac{a^2}{2\sigma^2}\right)} d\phi = \frac{a}{\sigma^2} e^{-\frac{a^2}{2\sigma^2}}$$

$$② \; f_\Phi(\phi) = \int_0^{\infty} f_{A,\Phi}(a,\phi)da = \int_0^{\infty} \frac{a}{2\pi\sigma^2} e^{-\frac{a^2}{2\sigma^2}} da = \frac{1}{2\pi}$$

① is the PDF of Rayleigh distribution

② is the PDF of uniform distribution

For a Rayleigh distributed r.v., the average power is $\Omega = \mathbb{E}[a^2] = \mathbb{E}[\sum_{k=1}^{N} \alpha_k^2] = 2\sigma^2$ [recall (1)], and thus

$$f_A(a) = \frac{2a}{\Omega} \exp\left(-\frac{a^2}{\Omega}\right)$$

For normalized average power, i.e., $\Omega = 1$, $f_A(a) = 2a\exp(-a^2)$.

Statistical Multipath Fading Model (cont'd)

If a LOS path is present (i.e., one path dominates the rest), we need to reconsider our Gaussian approximation. Without loss of generality, assume LOS path is the *first* path. From (1), link

$$\alpha = \sum_{k=1}^{N} \alpha_k e^{j\theta_k} = \underbrace{\alpha_1 e^{j\theta_1}}_{\text{LOS component}} + \sum_{k=2}^{N} \alpha_k e^{j\theta_k} = x_1 + jy_1 + \tilde{x} + j\tilde{y}$$

$$= \underbrace{(x_1 + \tilde{x})}_{=x} + \underbrace{j(y_1 + \tilde{y})}_{=y}.$$

Now, α can be modeled as a non-zero mean complex Gaussian with means μ_x and μ_y, respectively.

$$x \sim \mathcal{N}(\mu_x, \sigma^2), \quad y \sim \mathcal{N}(\mu_y, \sigma^2)$$

$$f_{X,Y}(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(x - \mu_x)^2 + (y - \mu_y)^2}{2\sigma^2}\right)$$

Statistical Multipath Fading Model (cont'd)

Let $x \triangleq a\cos\phi$ and $y \triangleq a\sin\phi$

$$f_{A,\Phi}(a,\phi) = |J(a,\phi)| f_{X,Y}(x,y)\Big|_{\substack{x=a\cos\phi \\ y=a\sin\phi}}$$

$$|J(a,\phi)| = a,\ f_{X,Y}(x,y)\Big|_{\substack{x=a\cos\phi \\ y=a\sin\phi}} = \tfrac{1}{2\pi\sigma^2} e^{\left(\frac{a^2+\mu_x^2+\mu_y^2-2a(\mu_x\cos\phi+\mu_y\sin\phi)}{2\sigma^2}\right)}.$$

Let $s \triangleq \sqrt{\mu_x^2+\mu_y^2}$, $t \triangleq \arctan\left(\frac{\mu_y}{\mu_x}\right) \Rightarrow \mu_x = s\cos t, \mu_y = s\sin t$.

$$f_{X,Y}(x,y)\Big|_{\substack{x=a\cos\phi \\ y=a\sin\phi}} = \tfrac{1}{2\pi\sigma^2}\exp\left\{\frac{a^2+s^2-2as(\cos t\cos\phi+\sin t\sin\phi)}{2\sigma^2}\right\}$$

$$\Rightarrow f_{a,\phi}(a,\phi) = \frac{a}{2\pi\sigma^2}\exp\left\{\tfrac{a^2+s^2-2as\cos(\phi-t)}{2\sigma^2}\right\}.$$

Next, find marginal pdf of A.

Statistical Multipath Fading Model (cont'd)

$$
\begin{aligned}
f_A(a) &= \int_0^{2\pi} f_{A,\Phi}(a,\phi)d\phi \\
&= \frac{a}{2\pi\sigma^2}\exp\left\{-\tfrac{a^2+s^2}{2\sigma^2}\right\}\int_0^{2\pi}\exp\left\{-\tfrac{as\cos(\phi-t)}{\sigma^2}\right\}d\phi.
\end{aligned}
$$

Recall the definition of "zero-order modified Bessel function of the first-kind"

$$
I_0(x) = \tfrac{1}{2\pi}\int_0^{2\pi} e^{-x\cos\theta}d\theta
$$

$$
\Rightarrow f_A(a) = \frac{a}{\sigma^2}e^{-\frac{a^2+s^2}{2\sigma^2}}I_0\left(\tfrac{as}{\sigma^2}\right) \qquad \boxed{\text{Rician distribution}}
$$

This is known as *Rician fading*. Typically encountered in land mobile channels in rural areas and in (low-orbit) satellite channels where a direct component is present.

Statistical Multipath Fading Model (cont'd)

Define the **Rician parameter** as the ratio of power in the LOS and scattered components, $\boxed{K = \dfrac{s^2}{2\sigma^2}}$.

For a Rician distributed r.v., the average power is $\Omega = s^2 + 2\sigma^2$

$$s^2 = \frac{K}{1+K}\Omega, \qquad 2\sigma^2 = \frac{1}{1+K}\Omega.$$

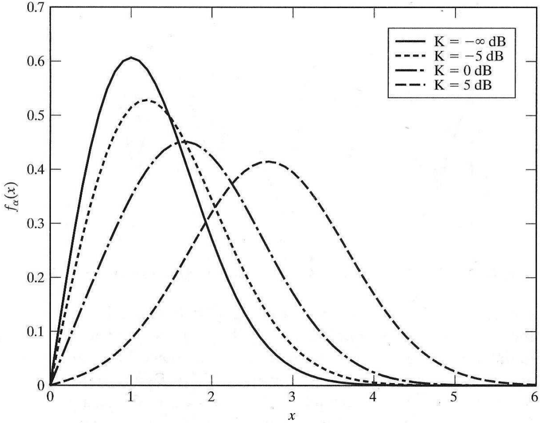
Now we can rewrite Rician distribution in terms of Rician parameter

$$f_A(a) = 2a\frac{1+K}{\Omega}e^{-K-\frac{1+K}{\Omega}a^2} I_0\left(2a\sqrt{\frac{K(1+K)}{\Omega}}\right).$$

For normalized average power, i.e., $\Omega = 1$,

$$f_A(a) = 2a(1+K)e^{-K-(1+K)a^2} I_0\left(2a\sqrt{K(1+K)}\right).$$

Rician vs. Rayleigh



$K = 0$ (no LOS) $\longrightarrow$ Rayleigh
$K \to \infty$ (LOS dominates) $\to$ Dirac impulse (i.e., no fading)

Summary

- Wireless transmissions suffer various kinds of channel impairments.
- Large-scale impairments: pathloss and shadowing
- Small-scale impairments: Rayleigh fading (w/o LOS) and Rician fading (with LOS)
- Mathematical models for different channel impairments introduced

Topics to be covered next week

Techniques to Overcome Fading Channels

The END