Spring 2022 – Cooperative Communications and Networking

Assignment 2

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Q1) [Outage probability, 40%] Derive the outage probability of selection combining (SC) for N receive antennas over Rayleigh fading channels. Let highest SNR is given by $\gamma_{\max} = \max_{n=1,\cdots,N}\left(|\alpha_n|^2 \frac{E}{N_0}\right)$

The outage probability of selection combining with N receive antennas is given by

$$P_{\text{out}} = \mathbb{P}[\gamma_{\max} < \gamma_0] \tag{1}$$

where γ_0 is a prescribed threshold. By considering $\gamma_{\max}$ as a random variable, (1) can be solved by finding the cumulative distribution function (CDF) of t$\gamma_{\max}$.

For notation simplicity, define the average SNR of the nth branch as $\bar{\gamma}_n = \mathbb{E}[|\alpha_n|^2 \frac{E}{N_0}] = \frac{E}{N_0}\mathbb{E}[|\alpha_n|^2]$ where α_n represents the fading coefficient of the nth branch. For Rayleigh fading channels, $|\alpha_n|^2$ is exponentially distributed. Because γ_n is a constant multiple of $|\alpha_n|^2$, it is also exponentially distributed with the probability density function (PDF)

$$f_{\gamma_n}(\gamma) = \frac{1}{\bar{\gamma}_n} e^{-\gamma/\bar{\gamma}_n}.$$

With the above information, we are ready to derive the distribution of $\gamma_{\max}$, which is nothing but the maximum of N i.i.d. exponential random variables. Thus

$$\begin{aligned} P_{\text{out}} &= \mathbb{P}[\max\ \{\gamma_n\}_{n=1}^{N} < \gamma_0] \\ &= \prod_{n=1}^{N} \mathbb{P}[\gamma_n < \gamma_0] \\ &= \prod_{n=1}^{N} \int_0^{\gamma_0} f_{\gamma_n}(\gamma) d\gamma \\ &= \prod_{n=1}^{N} \left(1 - e^{-\gamma_0/\bar{\gamma}_n}\right) \end{aligned} \tag{2}$$

Q2) [High-SNR approximation, 20%] The outage probability derived in Q2 is a function of the average SNR of each branch. If the average SNR of the branches are the same and approaching to infinity, show that the outage probability is proportional to the average SNR to the power of $-N$.

Under the condition $\bar{\gamma}_1 = \bar{\gamma}_2 = \cdots = \bar{\gamma}_N = \bar{\gamma}$, (2) reduces to

$$P_{\text{out}} = \left(1 - e^{-\gamma_0/\bar{\gamma}}\right)^N. \tag{3}$$

When $\bar{\gamma} \to \infty$ and using the first-order Taylor expansion $e^{-x} = 1 - x$, we have

$$P_{\text{out}} \approx \left(\frac{\gamma_0}{\bar{\gamma}}\right)^N \propto \bar{\gamma}^{-N}. \tag{4}$$

Simulation

Q3) [SC, 40%]

Perform Monte Carlo simulation for the outage probability of selection combining (SC).

Fig. 1 shows the results.

Diversity gain: The term "diversity" means we use independent channels (e.g., time, frequency, antennas) to transmit the same information. This is different from "multiplexing", which use independent channels to transmit different information. Clearly, the purpose of a diversity scheme (such as selection combining) is to increase the SNR at the receiver so as to increase the chance of successful information recovery. As to multiplexing, its goal is to deliver information faster. Since a diversity scheme can boost the SNR, a good diversity scheme can reduce more transmission errors by increasing the transmission power. Thus the diversity gain tells us how many errors can be reduced per unit of transmission power. In other words, the diversity gain can be determined by measuring the slope of the error curve with respect to SNR.

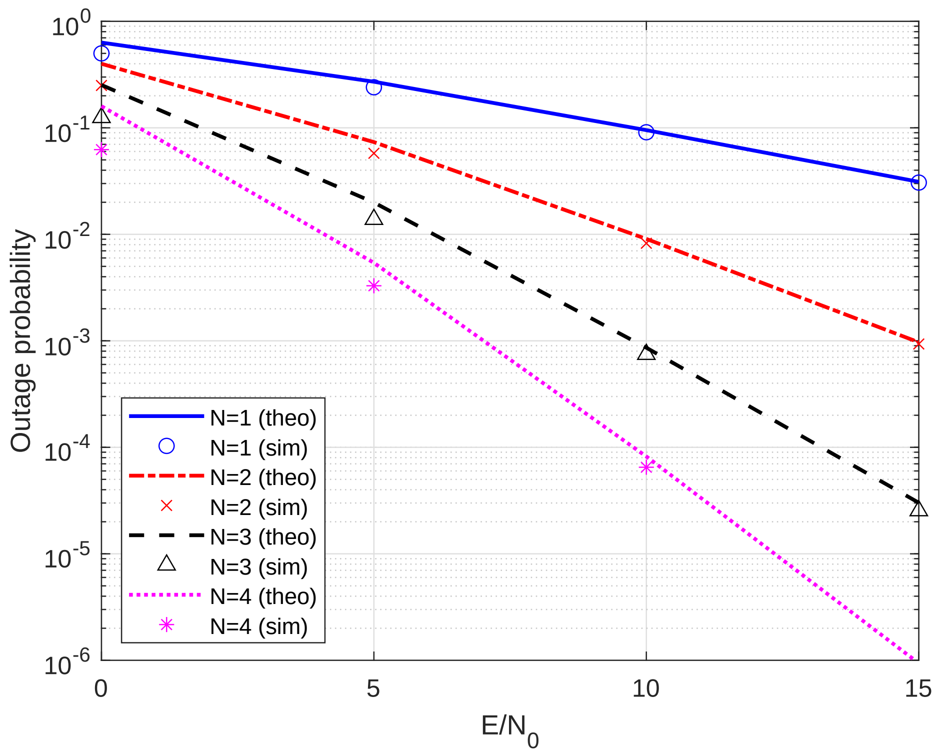

Figure 1: Outage probability of selection combining in i.i.d. Rayleigh fading channels.