Spring 2022 – Cooperative Communication and Networks

Assignment 3

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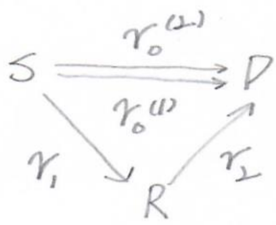
– Numerical Questions –

Q1) Derive the outage probability of Selection Decode-and-Forward (SDF) when the average SNR of the first hop is identical to that of the direct channel. Show the diversity gain of SDF is equal to two. (Hint: Use the rate expression given in the Appendix of lecture notes.)

$$C^{SDF} = \begin{cases} \frac{1}{2}\log_2(1+\gamma_0^{(1)}+\gamma_2), & \gamma_1 \geq T = 2^{R-1} \\ \frac{1}{2}\log_2(1+\gamma_0^{(1)}+\gamma_0^{(2)}), & \gamma_1 < T \end{cases}$$

$$\gamma_c^{SDF} = \begin{cases} \gamma_0^{(1)}+\gamma_2, & \gamma_1 \geq T \\ \gamma_0^{(1)}+\gamma_0^{(2)}, & \gamma_1 < T \end{cases}$$



$$P_{out}^{SDF} = P[C^{SDF} < R]$$

$$= P[\tfrac{1}{2}\log_2(1+\gamma_0^{(1)}+\gamma_0^{(2)}) < R]$$

$$+ P[\tfrac{1}{2}\log_2(1+\gamma_1) \geq R] \cdot P[\tfrac{1}{2}\log_2(1+\gamma_0^{(1)}+\gamma_2)]$$

$$= P[\gamma_0^{(1)}+\gamma_0^{(2)} < T] + P[\gamma_1 \geq T]\, P[\gamma_0^{(1)}+\gamma_2 < T]$$

$$= P[\gamma_0^{(1)}+\gamma_0^{(2)} < T] + e^{-\frac{T}{\bar{\gamma}_1}} \cdot P[\gamma_0^{(1)}+\gamma_2 < T]$$

?

$$\boxed{\begin{aligned} &P(X \le x) = F_X(x) = \begin{cases} 1-\left[\frac{\bar{Y}}{\bar{Y}-\bar{Z}} e^{-\frac{x}{\bar{Y}}} + \frac{\bar{Z}}{\bar{Z}-\bar{Y}} e^{-\frac{x}{\bar{Z}}}\right], & \bar{Y} \neq \bar{Z} \\ 1-\left[1+\frac{x}{\bar{Y}}\right] e^{-\frac{x}{\bar{Y}}}, & \bar{Y} = \bar{Z} \end{cases} \\ &X = Y + Z \end{aligned}}$$

$$= \begin{cases} 1-\left(\frac{\bar{\gamma}_0^{(1)}}{\bar{\gamma}_0^{(1)}-\bar{\gamma}_0^{(2)}} e^{-\frac{T}{\bar{\gamma}_0^{(1)}}} + \frac{\bar{\gamma}_0^{(2)}}{\bar{\gamma}_0^{(2)}-\bar{\gamma}_0^{(1)}} \cdot e^{-\frac{T}{\bar{\gamma}_0^{(2)}}}\right), & \bar{\gamma}_0^{(1)} \neq \bar{\gamma}_0^{(2)} \\ 1-\left(1+\frac{T}{\bar{\gamma}_0^{(1)}}\right) e^{-\frac{T}{\bar{\gamma}_0^{(1)}}}, & \bar{\gamma}_0^{(1)} = \bar{\gamma}_0^{(2)} \end{cases}$$

$$+ e^{-\frac{T}{\bar{\gamma}_1}} \cdot \begin{cases} 1-\left(\frac{\bar{\gamma}_0^{(1)}}{\bar{\gamma}_0^{(1)}-\bar{\gamma}_2} e^{-\frac{T}{\bar{\gamma}_0^{(1)}}} + \frac{\bar{\gamma}_2}{\bar{\gamma}_2-\bar{\gamma}_0^{(1)}} e^{-\frac{T}{\bar{\gamma}_2}}\right), & \bar{\gamma}_0^{(1)} \neq \gamma_2 \\ 1-\left(1+\frac{T}{\bar{\gamma}_0^{(1)}}\right) e^{-\frac{T}{\bar{\gamma}_0^{(1)}}}, & \bar{\gamma}_0^{(1)} = \gamma_2 \end{cases}$$

$$\bar{\gamma}_0 = \bar{\gamma}_1 \rightarrow \bar{\gamma}_0^{(1)} \underset{?}{=} \bar{\gamma}_0^{(2)} = \gamma_1$$

$$\Rightarrow P_{out}^{SDF} = 1-\left(1+\frac{T}{\bar{\gamma}_1}\right) e^{-\frac{T}{\bar{\gamma}_1}}, \quad \bar{\gamma}_0^{(1)} = \bar{\gamma}_0^{(2)} = \bar{\gamma}_1$$

$$- e^{-\frac{T}{\bar{\gamma}_1}} \cdot \begin{cases} -1+\left(\frac{\bar{\gamma}_1}{\bar{\gamma}_1-\bar{\gamma}_2} e^{-\frac{T}{\bar{\gamma}_1}} + \frac{\bar{\gamma}_2}{\bar{\gamma}_2-\bar{\gamma}_1} e^{-\frac{T}{\bar{\gamma}_2}}\right), & \bar{\gamma}_1 \neq \bar{\gamma}_2 \\ -1+\left(1+\frac{T}{\bar{\gamma}_1}\right) e^{-\frac{T}{\bar{\gamma}_1}}, & \bar{\gamma}_1 = \bar{\gamma}_2 \end{cases}$$

$$= \begin{cases} 1-e^{-\frac{T}{\bar{\gamma}_1}}\left(0+\frac{T}{\bar{\gamma}_1}+\frac{\bar{\gamma}_1}{\bar{\gamma}_1-\bar{\gamma}_2} e^{-\frac{T}{\bar{\gamma}_1}} + \frac{\bar{\gamma}_2}{\bar{\gamma}_2-\bar{\gamma}_1} e^{-\frac{T}{\bar{\gamma}_2}}\right), & \bar{\gamma}_1 \neq \bar{\gamma}_2 \\ 1-e^{-\frac{T}{\bar{\gamma}_1}}\left(0+\frac{T}{\bar{\gamma}_1}+e^{-\frac{T}{\bar{\gamma}_1}}+\frac{T}{\bar{\gamma}_1} e^{-\frac{T}{\bar{\gamma}_1}}\right), & \bar{\gamma}_1 = \bar{\gamma}_2 \end{cases}$$

high SNR approximation

$\bar{\gamma}_1 = \frac{P_1}{N_0} E(\alpha_1^2) = SNR \cdot \sigma_1^2$, when $SNR \rightarrow \infty$, $\frac{1}{\bar{\gamma}_1} \rightarrow 0$, $e^{-x} \approx 1-x$

$$\Rightarrow P_{out}^{SDF} \approx \begin{cases} 1-(1-\frac{T}{\bar{\gamma}_1})(0+1\cdot(1-\frac{T}{\bar{\gamma}_1})-0), & \bar{\gamma}_1 \neq \bar{\gamma}_2 \\ 1-(1-\frac{T}{\bar{\gamma}_1})(0+(1-\frac{T}{\bar{\gamma}_1})+0), & \bar{\gamma}_1 = \bar{\gamma}_2 \end{cases}$$

$$= \begin{cases} 1-(1-\frac{T}{\bar{\gamma}_1})(1-\frac{T}{\bar{\gamma}_1}) \\ 1-(1-\frac{T}{\bar{\gamma}_1})(1-\frac{T}{\bar{\gamma}_1}) \end{cases} = 1-(1-\frac{2T}{\bar{\gamma}_1}+(\frac{T}{\bar{\gamma}_1})^2)$$

$$= \frac{2T}{\bar{\gamma}_1}-(\frac{T}{\bar{\gamma}_1})^2 = \frac{2T}{\sigma_1^2}\cdot\frac{1}{SNR}-(\frac{T}{\sigma_1^2}\cdot\frac{1}{SNR})^2$$

$$= -(\frac{T}{SNR})^2(\frac{1}{\sigma_1})^2+\frac{T}{SNR}\cdot\frac{1}{\sigma_1^2} \propto SNR^{-2}$$

∴ with one relay and the existence of direct link, SDF achieves diversity gain of 2 #

– Simulation –

Write a computer program to simulate the variable-gain AF (VAF) following the steps:

- Generate fading coefficients for the source-relay channel and the relay-destination channel, respectively.
- Generate the complex Gaussian noise.
- Suppose the signal power and the source transmission power are both equal to one. Then determine the noise power for a given signal-to-noise ratio (SNR).
- Assume the maximum transmission power of relay is also one. Compute the amplification gain used by the relay.
- Determine the receive SNR at the destination.
- When the receive SNR is less than a threshold with the value of one, the transmission is in outage.
- Count the number of outage events among all the simulation trials

Q2) The SNR mentioned above is the ratio between transmission power P and noise power N_0. Suppose P = 1 Watt. For a given SNR in decibel, show the formula you used to determine the noise power in Watt.

$$SNR = \frac{P}{N_0} \Rightarrow SNR(dB) = 10log_{10}(SNR) = 10log_{10}\left(\frac{P}{N_0}\right)(dB)$$

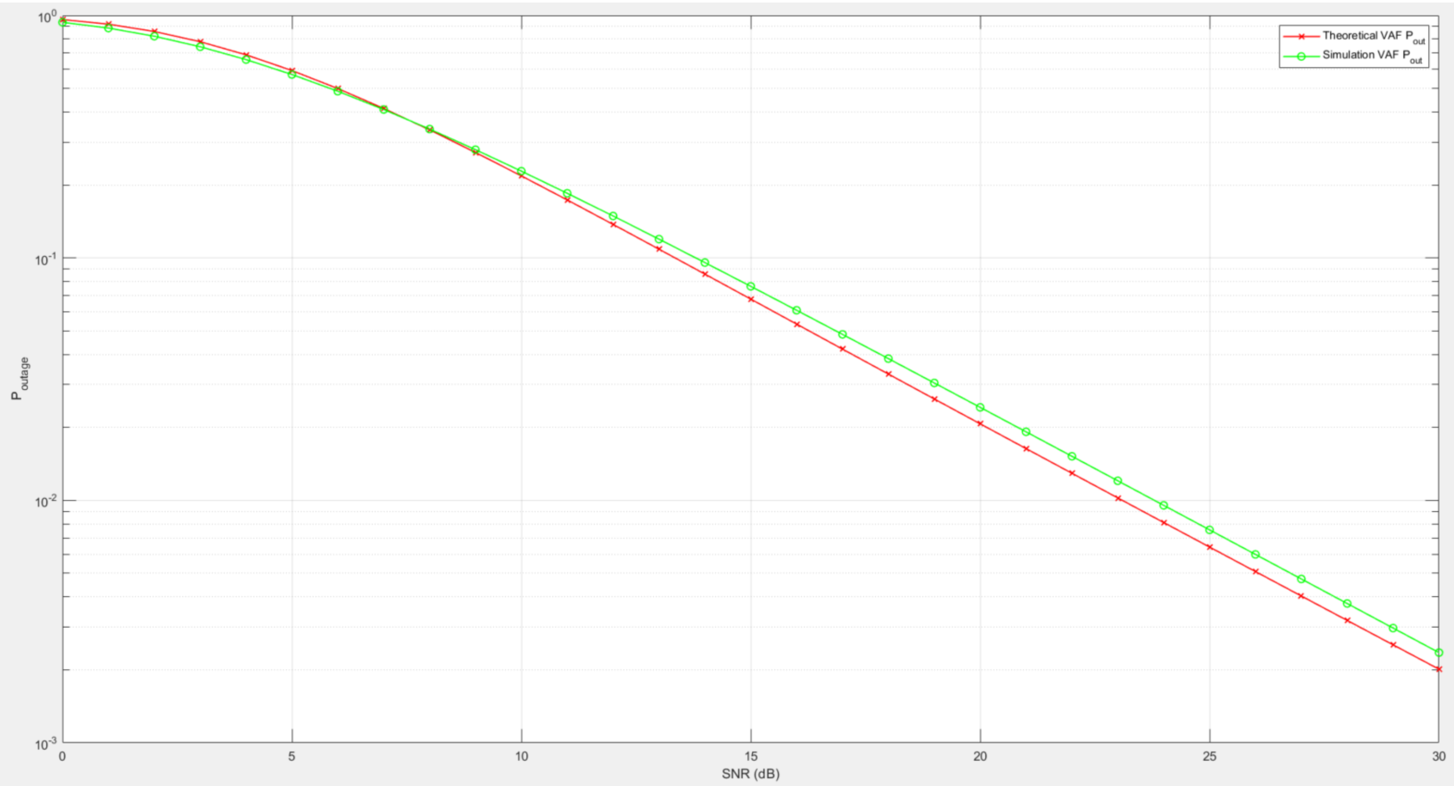
$$\Rightarrow with\ Tx\ power\ P = 1\ Watt, the\ noise\ power\ N_0 = 10^{-\frac{SNR(dB)}{10}}\ Watt.$$

Q3) Plot both the simulated and theoretical outage probabilities as a function of SNR in one figure.

- SNR range: 0~30 dB.
- Clearly label each curve (simulated or analytical).
- Set y-axis as logarithm scale.
- Colors are not differentiable when printed in black and white. Use different line styles and symbols to represent different curves.

– Consider Case I. No Direct Link VAF

– Using high SNR approximation, results in Eq.(9) $P_{out}^{VAF} \approx 1 - e^{-T(\frac{1}{\bar{\gamma}_1}+\frac{1}{2})}$



Q4) Plot the average squared gain versus SNR (dB). Explain how the average squared gain varies with the noise power. With the trend you observed, does the noise amplification problem get worse at higher SNR?

$$Average\ squared\ gain\ \mathrm{G}^2 = \frac{P_2}{P_1\alpha_1^2 + N_0} = \frac{1}{\alpha_1^2 + N_0}\ of\ VAF, with\ P_1 = P_2 = fixed\ power\ P = 1\ Watt.$$

$No.\ The\ noise\ power\ would\ decreases\ as\ SNR\ increases.\ It\ could\ be\ easily\ seen\ from\ the\ above\ equation\ that\ gain\ G\ is\ inversely\ proportional\ to\ noise\ \mathrm{N}_0, which\ means\ higher\ the\ SNR, lower\ the\ noise\ \mathrm{N}_0, and\ finally\ resulted\ in\ higher\ amplification\ gain, in\ theory.$