COM 5120 Communication Theory
Homework #3
Due: 2021/12/02

1. (15%) Consider a communication system where three equiprobable messages m1,m2,m3 are transmitted. Let m1,m2,m3 be encoded by signals $s_1(t)$,$s_2(t)$,$s_3(t)$ respectively given by

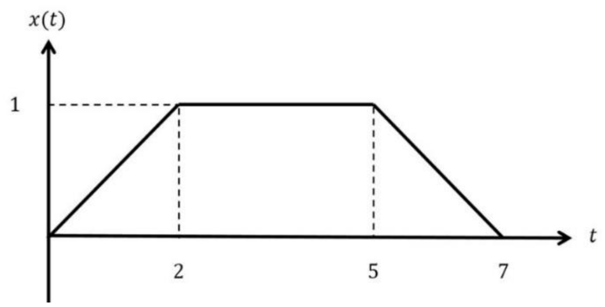
$$s_1(t) = 3\sqrt{2}\cos(2\pi t)\ ,\ s_2(t) = 2\sqrt{2}\sin(2\pi t)\ ,\ s_3(t) = -2\sqrt{2}\sin(2\pi t)$$

where the signal duration is $0 \le t \le 1$ and each signal is zero outside this interval. Assume that the signals are transmitted over an additive white Gaussian noise channel.
(a) Find a set of orthonormal basis function to represent the set of signals, and then draw the corresponding signal constellation. (5%)
(b) Determine the optimum decision regions. (5%)
(c) Determine an equivalent minimum-energy signal set that would yield the same probability of error as the signal set described above. Draw the corresponding signal constellation and optimum decision regions. (5%)

2. (20%) A binary communication scheme uses two equiprobable signals $s_1(t)$,$s_2(t)$ where

$$\begin{cases} s_1(t) = x(t) \\ s_2(t) = x(t-1) \end{cases}$$

and x(t) is shown as follows




The signal is transmitted over the AWGN channel with power spectral density of the noise $\frac{N_0}{2}$.
(a) Design an optimal matched filter receiver for this system. Carefully label the diagram and determine all the required parameters. (10%)
(b) Determine the error probability for this communication system. (You should answer in Q function form.) (10%)

3. (25%) Consider a one-dimensional discrete communication model shown below.
The transmitted symbol $X \in \{+a, -a\}$ where a>0 is a deterministic and known value.

The noise $N$ is dependent on $X$. Specifically, given $X = +a$, $N$ is Gaussian distributed with zero mean and variance $\sigma_1^2$, and given $X = -a$, $N$ is Gaussian distributed with zero mean and variance $\sigma_2^2$, where $\sigma_1^2$ and $\sigma_2^2$ are known. Assume that Prob$[X = +a] = p$ and Prob$[X = -a] = 1 - p$.

(a) Derive a maximum $a$ posteriori probability (MAP) receiver for detecting $X$. (10%)

(b) Suppose $\sigma_1^2 = 1,\ \sigma_2^2 = 2,\ a = 1,\ p = 0.5$. Find the decision regions for $X = +a$ and $X = -a$. (10%)

(c) Find the probability of error for the values specified in (b).(You should answer in Q function form.) (5%)

4. (20%) Consider a signal detector with an input r = s + n , where the transmit signal equals to +A or -A with equal probability.

(a) Let the noise variable n is characterized by the Gaussian PDF as

$$p(n) = \frac{1}{\sqrt{2\pi}\sigma}\ e^{-(\frac{n}{\sqrt{2}\sigma})^2}$$

Please determine the probability of error as a function of the parameters A and σ. (You should answer in Q function form.) (7%)

(b) If the noise variable n is characterized by the Laplacian PDF as

$$p(n) = \frac{1}{\sqrt{2}\sigma}\ e^{-|n|\frac{\sqrt{2}}{\sigma}}$$

Please determine the probability of error as a function of the parameters A and σ. (7%)

(c) Determine the SNR required to achieve an error probability of $10^{-5}$ for a Laplacian PDF. (6%)

5. (20%) In a binary FSK system, the signals $s_1(t)$ and $s_2(t)$, representing symbols 1 and 0 with equal probability. $s_1(t)$ and $s_2(t)$ are defined by

$$s_1(t) = \mathrm{A}\cos[2\pi(f_c + \tfrac{\Delta f}{2})t]\ ,\ and\ s_2(t) = \mathrm{A}\cos[2\pi(f_c - \tfrac{\Delta f}{2})t]\ ,\ 0 \le t \le T.$$

where $\Delta f = \frac{1}{2T}$. The signals are transmitted over the AWGN channel with zero mean and variance $\frac{N_0}{2}$.

(a) With coherent detection, based on MAP detection rule, please derive the error probability of this BFSK as a function of $E_b/\ N_0$. (You should answer in Q function form.) (7%)

(b) With non-coherent detection, based on MAP detection rule, please derive the error probability of this BFSK as a function of $E_b/\ N_0$. (7%)

(c) Please compare error probability and sketch $P_e$ vs. $E_b/\ N_0$ curve of the coherent and the non-coherent BFSK receivers. (6%)