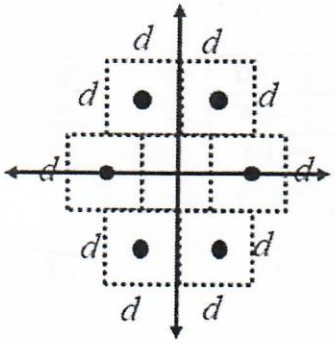
COM5120 Communication Theory
Midterm II

Fall, 2021

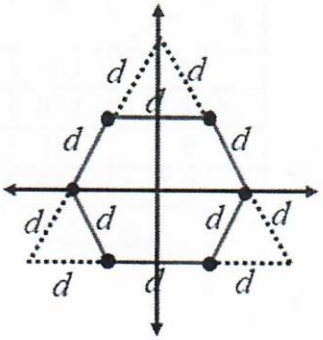
Date: 12/16/2021 (*Thursday*), 18:30pm – 21:00pm

Note: Total of 120 points for the Midterm Exam of **6** problems in two pages. Closed book(s). Open one sheet of A4 size note (two sides). Non-programmable calculator is allowed if you feel necessary.

1. (20%) Consider the following two M-ary QAM constellations, A and B, with $M = 6$. In constellation A, the modulation symbols are at the center of squares with length d on each side. In constellation B, the modulation symbols are on the vertex of regular triangles with length d on each side. The modulations are transmitted over the AWGN channel with noise distribution function ~ $N(0, N_0/2)$.



Contellation A



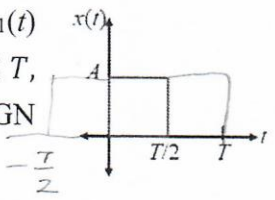
Contellation B

(a)(7%) Let $d = \sqrt{E_0}$, please determine the constellation figure of merit (CFM) of both constellations.

(b)(7%) Please determine and compare the symbol error probabilities of A and B.

(c) (6%) Pl8ease determine the average SNR per bit for A and B.

2. (20%) A binary communication scheme uses two equiprobable signals $s_1(t)$ $s_2(t)$, where $s_1(t) = x(t), 0 \le t \le T$, $s_2(t) = x(t - T/2)$, $0 \le t \le T$, and $x(t)$ is shown as follows. The power spectral density of the AWGN noise is $N_0/2$.



(a) (7%) Design an optimal matched filter receiver for this system. Carefully label the diagram and determine all the required parameters.

(b) (7%) Determine the error probability for this communication system.

(c) (6%) Show that the receiver can be implemented using only one matched filter.

3. (20%) Two equiprobable messages m_1 and m_2 are transmitted through a channel with input X and output Y relate by $Y = \alpha X + N$, where N is zero-mean AWGN with variance $N_0/2$ and α is a random variable independent of noise.

(a) (5%) Assume X is antipoal signaling, $X = \pm A$ with equal probability and $\alpha = 0.5$ with probability = 1. What is the optimal decision rule and the resulting error probability?

(b) (5%) Following (a), but $\alpha = \pm 1$ with equal probability. What is the optimal decision rule and the resulting error probability?

(c) (5%) Following (a), but $\alpha = 0$ or 1 with equal probability. What is the optimal decision rule and the resulting error probability?

(d) (5%) Assuming equally probable on-off signaling (i.e. X= 0 or A) and $\alpha = 0$ or 1 with equal probability. What is the optimal decision rule?

4. (24%) Given random variables X and Y and let the joint probability distribution $p(x, y)$ takes the following table.

X \ Y	0	1	2
0	1/16	1/32	1/32
1	1/16	1/16	1/8
2	1/4	1/16	1/8
3	1/16	1/16	1/16

Please find

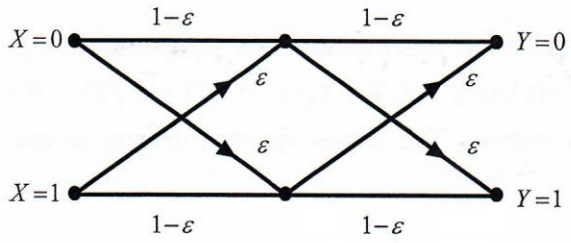
(a) H(X), H(Y), and H(X, Y).

(b) H(X|Y), H(Y |X).

(c) I(X; Y)

(d) Suppose that Z = X + Y mod 4. Find I(X; Y |Z).

5. (24%) Consider a relay transmission system that employs cascading the binary symmetric channels with the same cross-over error probability as follows. The prior probability of $P(X=0) = p$, and $P(X=1) = 1-p$.



(a) (8%) Determine the average error probability at the destination.

(b) (8%) Determine the distribution of $P(X=0)$ and $P(X=1)$ that maximizes the capacity,

(c) (8%) Please calculate the capacity of this channel.

6. (12%) Consider an AWGN channel of power spectral density $N_0/2$. The bandwidth B = 1MHz and N_0 = 1μW/Hz. Find the minimum energy per bit for reliable communications at bit rate R_b = 2Mbps.