1. **(6%)** In communications system, the Nyquist criterion describes the conditions satisfied by a channel achieves zero ISI. Given a pulse $x(t)$ having raised-cosine spectrum, show that $x(t)$ satisfies the Nyquist criterion for any roll-off factor β.

(sol)

the raised cosine spectrum is given (lecture 9)

$$x(t) = \frac{\mathrm{sinc}\left(\frac{t}{T}\right)\cos\left(\frac{\pi\beta t}{T}\right)}{1-\frac{4\beta^2 t^2}{T^2}}$$

The function $\mathrm{sinc}\left(\frac{t}{T}\right)$ is 1 when $t = 0$ and when $t = nT$. Therefore, the

Nyquist criterion will be satisfied as long as the function $g(t)$ is:

$$g(t) = \frac{\mathrm{sinc}\left(\frac{t}{T}\right)\cos\left(\frac{\pi\beta t}{T}\right)}{1-\frac{4\beta^2 t^2}{T^2}} = \begin{cases} 1 & t = 0 \\ \text{Bounded} & \text{otherwise} \end{cases}$$

$$\lim_{\beta t\to\frac{T}{2}} g(t) = \lim_{\beta t\to\frac{T}{2}} \frac{\mathrm{sinc}\left(\frac{t}{T}\right)\cos\left(\frac{\pi\beta t}{T}\right)}{1-\frac{4\beta^2 t^2}{T^2}} = \lim_{x\to 1}\frac{\cos\left(\frac{\pi}{2}x\right)}{1-x} = \lim_{x\to 1}\frac{\pi}{2}\sin\left(\frac{\pi}{2}x\right) = \frac{\pi}{2}$$

Hence the pulse $x(t)$ satisfies the Nyquist criterion.

2. The sample of a channel's impulse response are $h(-2T) = 0.01$, $h(-T) = 0.1, h(0) = 1.0, h(T) = 0.2, h(2T) = -0.02, h(kT) = 0$ for $k \neq -2,-1,0,1,2$

   A. **(5%)** Determine the tsp coefficients for a three-tap zero-forcing equalizer.

   B. **(5%)** If the equalizer of A. is applied, determine the output sampled of the overall impulse response which combines channel and equalizer.

(sol)

A. $$\begin{pmatrix} 1.0 & 0.1 & -0.01 \\ 0.2 & 1.0 & 0.1 \\ -0.02 & 0.2 & 1.0 \end{pmatrix}\begin{pmatrix} w_{-1} \\ w_0 \\ w_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} w_{-1} \\ w_0 \\ w_1 \end{pmatrix} = \begin{pmatrix} -0.106 \\ 1.0423 \\ -0.21 \end{pmatrix}$$

B. $h(t) = -0.01\delta(t+2T) + 0.1\delta(t+T) + 1\delta(t) + 0.2\delta(t-T) - 0.026\delta(t-2T)$

$$y(t) = -0.106h(t+T) + 1.0423h(t) - 0.21h(t-T)$$
$$y(-3T) = -0.106h(-2T) = 0.00106$$
$$y(-2T) = -0.106h(-T) + 1.0423h(-2T) = -0.021$$
$$y(-T) = -0.106h(0) + 1.0423h(-T) - 0.21h(-2T) = 0$$
$$y(0) = -0.106h(T) + 1.0423h(0) - 0.21h(-T) = 1$$

$$y(T) = -0.106h(2T) + 1.0423h(T) - 0.21h(0) = 0$$
$$y(2T) = 1.0423h(2T) - 0.21h(T) = -0.0628$$
$$y(3T) = -0.21h(2T) = 0.0042$$

3. **(12%)** If an M-ary PAM is used for transmitted which frequency range 300Hz < f < 3000Hz. Shape function is a raised-cosine, please determine the roll-off factor to achieve ~~5200 bits per second~~.
 9600 bits per second

(sol)

Bandwidth: 3000-300=2700 (Hz)

9600/4 < 2700 < 9600/2 (to achieve the bit rate, $k = 4$ in PAM)

$$(1+\beta) \times \left(\frac{\frac{2400}{4}}{2}\right) = \left(\frac{2700}{2}\right) \rightarrow \beta = 0.125$$

4. Consider a received signal $r(t) = s(t) + \alpha s(t-T) + n(t)$, where $s(t)$ is the the transmited signal, $s(t-T)$ is contributed by delay path, α is attenuation ($\alpha < 1$), and $n(t)$ is **AWGN**.
   A. **(5%)** Calculate the output at t = 2T that applies filter matched to s(t).
   B. **(5%)** If transmitted signal $s(t)$ is binary antipodal and detector ignores ISI.

(sol)

A. $y(2T) = \int_0^{2T} r(\tau)\, s(\tau) d\tau$

$$= \int_0^{2T} \{s(\tau) + \alpha s(\tau - T) + n(t)\}\, s(\tau) d\tau$$

$$= \alpha^2 \int_0^{T} s^2(\tau) d\tau + \int_0^{T} s(\tau) n(\tau) d\tau$$

B. $y_k = I_k E_S + \alpha I_{k-1} E_S + n_k$ $\left(E_S = \int_0^T s^2(\tau) d\tau,\ n_k = \int_{(k-1)T}^{kT} s(\tau) n(\tau) d\tau\right)$

$$\Pr(\text{error}) = \frac{1}{2}\Pr(\text{error}|I_n = 1, I_{n-1} = 1) + \frac{1}{2}\Pr(\text{error}|I_n = 1, I_{n-1} = -1)$$

$$= \frac{1}{2}\Pr\left((1+\alpha)E_S + n_k < 0\right) + \frac{1}{2}\Pr\left((1-\alpha)E_S + n_k < 0\right)$$

$$= \frac{1}{2} Q\left(\sqrt{\frac{2(1+\alpha)^2 E_S}{N_o}}\right) + \frac{1}{2} Q\left(\sqrt{\frac{2(1-\alpha)^2 E_S}{N_o}}\right)$$

5. **(6%)** A band-limited channel introduces **ISI** over three successive symbols. The output of matched filter is sampled at the period T.

$$\int_{-\infty}^{\infty} s(t)s(t-kT)dt = \begin{cases} E_b & k=0 \\ 0.5E_b & k=\pm 1 \\ 0.01E_b & k=\pm 2 \\ 0 & \text{otherwisw} \end{cases}$$

Determine the three-tap equalizer that equalizes the channel to partial-response (duobinary) signal.

$$y_k = \begin{cases} E_b & k=0,1 \\ 0 & \text{otherwise} \end{cases}$$

$$q_m = w_{-1}x_{m-(-1)} + w_0 x_m + w_1 x_{m-1}$$

$$\begin{pmatrix} 1.0E_b & 0.5E_b & 0.01E_b \\ 0.5E_b & 1.0E_b & 0.5E_b \\ 0.01E_b & 0.5E_b & 1.0E_b \end{pmatrix}\begin{pmatrix} w_{-1} \\ w_0 \\ w_1 \end{pmatrix} = \begin{pmatrix} 0 \\ E_b \\ E_b \end{pmatrix}$$

$$(w_{-1}, w_0, w_1) = (-0.501, 1, 0.501)$$

6.

A. **(12%)** The binary sequence 10010110010 is the input to a precoder whose output is used to modulate a duobinary transmitting filter. (Construct a table contain I_N, B_N, D_N, P_N).

B. **(6%)** Describe the necessity of precoding for such a duobinary signaling scheme.

C. **(6%)** Describe the disadvantage of using duobinary signaling scheme.

(sol)

A.

Data seq. D_n:		1	0	0	1	0	1	1	0	0	1	0
Precoded seq. P_n:	0	1	1	1	0	0	1	0	0	0	1	1
Transmitted seq. I_n:	-1	1	1	1	-1	-1	1	-1	-1	-1	1	1
Received seq. B_n:		0	2	2	0	-2	0	0	-2	-2	0	2
Decoded seq. D_n:		1	0	0	1	0	1	1	0	0	1	0

B. 有寫到 error propagation 並簡簡述成因可拿滿分

C. 3-level output，在同樣條件下，欲達相同的 BER E_b 要更高

7. A bandlimited signal can be represented as

$$\mathrm{x(t)} = \sum_{n=-\infty}^{\infty} \frac{\mathrm{x_n} \sin\left(2\pi W\left(t-\frac{n}{W}\right)\right)}{2\pi W\left(t-\frac{n}{W}\right)}$$

A. **(6%)**

$$\mathrm{x_n} = \begin{cases} -1 & \mathrm{n} = 0 \\ 2 & \mathrm{n} = \pm 1 \\ 0 & \text{otherwisw} \end{cases}$$

Determine the spectrum $\mathrm{X(f)}$ and plot $|\mathrm{X(f)}|$.

B. **(2%)** Following A. , plot $\mathrm{x(t)}$.

請見 Problem 9.10

8. For a radio system with multipath channel response $\mathrm{r(t)}$ of a signal $\mathrm{s(t)}$ being $\mathrm{r(t)} = \mathrm{c_1} s(t-t_1) + c_2 s(t-t_2)$

A. **(3%)** Determine the frequency response of channel

B. **(3%)** If equalization with no constraint on the equalizer structure. Find the frequency response in term of $\mathrm{c_1}, \mathrm{c_2}, t_1, t_2$

C. **(12%)** Consider linear equalization where equalizer has **three-tap filter** structure $\mathrm{y(t)} = e_0 r(t) + e_1 r(t-T) + e_2 r(t-2T)$. Assume $\mathrm{c_1} \gg c_2$ and $\mathrm{t_1} \ll t_2$. Find the coefficients $e_0, \mathrm{e_1}, e_2$ in term of $\mathrm{c_1}, \mathrm{c_2}, t_1, t_2$

D. **(6%)** Following C. , assuming that overall system transfer function is equal to $K_0 \exp(-j2\pi f \tau_o)$, where K_0, τ_o are set as desired, e.g. $K_0 = c_1, \tau_o = t_1$, determine the coefficients $e_0, \mathrm{e_1}, e_2$ in term of $\mathrm{c_1}, \mathrm{c_2}, t_1, t_2$.

(sol)

A. $\mathcal{F}\{\mathrm{r(t)}\} = \mathrm{R(f)} = \mathrm{c_1}\mathcal{F}\{\mathrm{s(t)}\}\exp(-\mathrm{j}2\pi\mathrm{ft_1}) + \mathrm{c_2}\mathcal{F}\{\mathrm{s(t)}\}\exp(-\mathrm{j}2\pi\mathrm{ft_2})$

$$\mathrm{H(f)} = \frac{\mathrm{R(f)}}{\mathcal{F}\{\mathrm{s(t)}\}}$$

$$= \mathrm{c_1}\exp(-\mathrm{j}2\pi\mathrm{ft_1}) + \mathrm{c_2}\exp(-\mathrm{j}2\pi\mathrm{ft_2})$$

B. $\mathrm{H_{eq}(f)} = \frac{1}{\mathrm{H(f)}} = \frac{1}{\mathrm{c_1}\exp(-\mathrm{j}2\pi\mathrm{ft_1}) + \mathrm{c_2}\exp(-\mathrm{j}2\pi\mathrm{ft_2})}$

C. $\mathrm{H_{eq}(f)} \sim \mathrm{e_0} + \mathrm{e_1}\exp(-\mathrm{j}2\pi\mathrm{T}) + \mathrm{e_2}\exp(-\mathrm{j}2\pi(2\mathrm{T}))$

$$\mathrm{H_{eq}(f)} = \frac{1}{\mathrm{c_1}\exp(-\mathrm{j}2\pi\mathrm{ft_1}) + \mathrm{c_2}\exp(-\mathrm{j}2\pi\mathrm{ft_2})}$$

$$= \frac{1}{\mathrm{c}_1 \exp(-\mathrm{j}2\pi\mathrm{f}\mathrm{t}_1)\left(1 + \frac{\mathrm{c}_2}{\mathrm{c}_1}\exp\left(-\mathrm{j}2\pi\mathrm{f}(\mathrm{t}_2 - \mathrm{t}_1)\right)\right)}$$

$$\begin{aligned} &= \left(\frac{1}{\mathrm{c}_1}\exp(\mathrm{j}2\pi\mathrm{f}\mathrm{t}_1)\right)\left[1 - \frac{\mathrm{c}_2}{\mathrm{c}_1}\exp(-\mathrm{j}2\pi\mathrm{f}\mathrm{T}) \right. \\ &\qquad + \left(\frac{\mathrm{c}_2}{\mathrm{c}_1}\right)^2 \exp(-\mathrm{j}2\pi\mathrm{f}2\mathrm{T}) \\ &\qquad \left. + \left(\frac{\mathrm{c}_2}{\mathrm{c}_1}\right)^3 \exp(-\mathrm{j}2\pi\mathrm{f}3\mathrm{T}) + \cdots \right] \text{ let } \mathrm{T} = \mathrm{t}_2 - \mathrm{t}_1 \end{aligned}$$

(use Taylor expansion to approximate the desired form)

$$\sim K\left[1 - \frac{\mathrm{c}_2}{\mathrm{c}_1}\exp(-\mathrm{j}2\pi\mathrm{f}\mathrm{T}) + \left(\frac{\mathrm{c}_2}{\mathrm{c}_1}\right)^2 \exp(-\mathrm{j}2\pi\mathrm{f}2\mathrm{T})\right]$$

$$(\mathrm{e}_0, \mathrm{e}_1, \mathrm{e}_2) = K\left(1, -\frac{\mathrm{c}_2}{\mathrm{c}_1}, \left(\frac{\mathrm{c}_2}{\mathrm{c}_1}\right)^2\right), K = \left(\frac{1}{\mathrm{c}_1}\exp(\mathrm{j}2\pi\mathrm{f}\mathrm{t}_1)\right)$$

D. Considering the system equivalent response is

$$\mathrm{H}_{\mathrm{eq}}(f) = k_O \frac{\exp(-j2\pi\mathrm{f}\mathrm{t}_1)}{H(f)} = \frac{\mathrm{c}_1 \exp(-\mathrm{j}2\pi\mathrm{f}\mathrm{t}_1)}{\mathrm{c}_1 \exp(-\mathrm{j}2\pi\mathrm{f}\mathrm{t}_1) + \mathrm{c}_2 \exp(-\mathrm{j}2\pi\mathrm{f}\mathrm{t}_2)}$$

$$= \frac{1}{1 + \frac{\mathrm{c}_2}{\mathrm{c}_1}\exp(-\mathrm{j}2\pi\mathrm{f}2\mathrm{T})}$$

$$\sim 1 - \frac{\mathrm{c}_2}{\mathrm{c}_1}\exp(-\mathrm{j}2\pi\mathrm{f}2\mathrm{T}) + \left(\frac{\mathrm{c}_2}{\mathrm{c}_1}\right)^2 \exp(-\mathrm{j}2\pi\mathrm{f}2\mathrm{T})$$

$$(\mathrm{e}_0, \mathrm{e}_1, \mathrm{e}_2) = \left(1, -\frac{\mathrm{c}_2}{\mathrm{c}_1}, \left(\frac{\mathrm{c}_2}{\mathrm{c}_1}\right)^2\right)$$