1. (13%)

If $I_k = I_{k-1}$, then $y_k = I_{k-1} + \alpha I_{k-1} + n_k = (1+\alpha) I_k + n_k$

$$d = (1+\alpha)\sqrt{E_b} \Rightarrow P_{e,I_k = I_{k-1}} = Q[(1+\alpha)\sqrt{\frac{2E_b}{N_0}}]$$

If $I_k \neq I_{k-1}$, then $y_k = I_{k-1} + \alpha I_{k-1} + n_k = (1-\alpha) I_k + n_k$

$$d = (1-\alpha)\sqrt{E_b} \Rightarrow P_{e,I_k \neq I_{k-1}} = Q[(1-\alpha)\sqrt{\frac{2E_b}{N_0}}]$$

Therefore, $P_e = \frac{1}{2} P_{e,I_k = I_{k-1}} + \frac{1}{2} P_{e,I_k \neq I_{k-1}} = \frac{1}{2}\left\{ Q[(1+\alpha)\sqrt{\frac{2E_b}{N_0}}] + Q[(1-\alpha)\sqrt{\frac{2E_b}{N_0}}] \right\}$

2. (25%)

(1) (5%)

$$H(f) = tri(\frac{f}{W}) \leftrightarrow h(t) = W \sin c^2(Wt)$$

$$v(t) = s(t) * h(t) + n(t)$$

$$v(t) = s(t) * \left[W \sin c^2(Wt) \right] + n(t)$$

(2) (5%)

$$y(t) = v(t) * g(-t) = \int_{-\infty}^{\infty} v(\tau) g(\tau - t) d\tau$$

$$s(t) = \sum_{n=-\infty}^{\infty} I_n g(t - nT)$$

$$\begin{aligned} v(t) &= s(t) * h(t) + n(t) \\ &= s(t) * \left[W \sin c^2(Wt) \right] + n(t) \\ &= W \sum_{n=-\infty}^{\infty} I_n g(t - nT) * \sin c^2(Wt) + n(t) \end{aligned}$$

$$v(\tau) = W \sum_{n=-\infty}^{\infty} I_n g(t - nT) * \sin c^2(W\tau) + n(\tau)$$

$$\begin{aligned} y_k &= y(kT) = \int_{-\infty}^{\infty} v(\tau) g(\tau - kT) d\tau \\ &= W \sum_{n=-\infty}^{\infty} I_n \int_{-\infty}^{\infty} g(\tau - nT) * \sin c^2(W\tau) g(\tau - kT) d\tau + \int_{-\infty}^{\infty} n(\tau) g(\tau - kT) d\tau \end{aligned}$$

$$\left(or \quad = \frac{1}{2T} \sum_{n=-\infty}^{\infty} I_n \int_{-\infty}^{\infty} g(\tau - nT) g(\tau - kT) d\tau * \sin c^2(\frac{k}{2}) + \int_{-\infty}^{\infty} n(\tau) g(\tau - kT) d\tau \right)$$

(3) (5%)

When n=k,the ISI pattern is :

$$W\sum_{n\neq k} I_n \int_{-\infty}^{\infty} g(\tau-nT)*\sin c^2(W\tau)g(\tau-kT)d\tau$$

(4) (5%)

(題目為 $G(f)=\begin{cases}1 & ,0\leq f\leq W\\ 0 & ,otherwise\end{cases}$)

$$\sum_m X(f+\frac{m}{T})=1$$

However, $X(f)=G(f)H(f)G(-f)=0$

It is impossible

(題目為 $G(f)=\begin{cases}1 & ,|f|\leq W\\ 0 & ,otherwise\end{cases}$)

$$\sum_m X(f+\frac{m}{T})=1$$

However, $X(f)=G(f)H(f)G(-f)\neq 0$

It is possible

the Nyquist pulse criterion should be satisfied when 1/T = W

(5) (5%)

let $x(t)=g(t)*h(t)*g(-t)$

$X(f)=G(f)H(f)G(-f)=1$

$\Rightarrow H(f)|G(f)|^2=1$

$$\Rightarrow G(f)=\frac{1}{\sqrt{H(f)}}=\begin{cases}\dfrac{1}{\sqrt{1-\dfrac{|f|}{W}}} & ,|f|\leq W\\ 0 & ,otherwise\end{cases}$$

3. (24%)

(1) (6%)

$$P_m = D_m + P_{m-2} \mod 4, D_m \in \{0,1,2,3\} \to P_m \in \{0,1,2,3\}$$

$$I_m = 2P_m - 3 \to I_m \in \{\pm 1, \pm 3\}$$

(2) (6%)

$y(t) = [\sum_m (I_m - I_{m-2})\delta(t - mT) * g(t) * c(t) + n(t)] * g(t)$

$= (\sum_m I_m\delta(t - mT) - \sum_m I_{m-2}\delta(t - mT)) * g(t) * c(t) * g(t) + n(t) * g(t)$

$(Let \quad h(t) = g(t) * c(t) * g(t), n'(t) = n(t) * g(t))$

$= (\sum_m I_m\delta(t - mT) - \sum_m I_m\delta(t - (m + 2)T)) * h(t) + n'(t)$

$= \sum_m I_m(\delta(t - mT) - \delta(t - (m + 2)T)) * h(t) + n'(t)$

$= \sum_m I_m(h(t - mT) - h(t - (m + 2)T)) + n'(t)$

$\to x(t) = h(t) - h(t - 2T), h(t) = g(t) * c(t) * g(t)$

$(g(t) = \frac{1}{T}sinc(\frac{t}{T}), c(t) = \frac{1}{T}sinc(\frac{t}{T}))$

(3) (6%)

$\mathrm{Method} \quad 1:$

$\mathrm{R_x(\tau) = E[x(t + \tau)x(t)] = E[(h(t + \tau) - h(t + \tau - 2T))(h(t) - h(t - 2T))]}$

$\mathrm{(h(t) = g(t) * c(t) * g(t), g(t) = \frac{1}{T}sinc(\frac{t}{T}), c(t) = \frac{1}{T}sinc(\frac{t}{T}))}$

$\mathrm{S_x(f) = \sum_{n=-\infty}^{\infty} R_x(\tau)e^{-j2\pi nf\tau}}$

$\mathrm{Method} \quad 2:$

$\mathrm{x_k = \delta[k] - \delta[k - 2]}$

$\mathrm{R_{xx}[m] = E[x_{k+m}x_k] = 2\delta[m] - \delta[m - 2] - \delta[m + 2]}$

$\mathrm{S_{xx}(f) = 2 - e^{-j4\pi fT} - e^{j4\pi fT} = 2 - 2\cos(4\pi fT)}$

(4) (6%)

$$y_m = B_m + n_m = I_m - I_{m-2} + n_m$$

$$= 2P_m - 3 - (2P_{m-2} - 3) + n_m$$

$$= 2(P_m - P_{m-2}) + n_m$$

$$D_m = (P_m - P_{m-2}) \mod 4$$

$$\hat{D}_m = \begin{cases} 3 & , if \quad |y_m| \geq 6 \\ 2 & , if \quad 4 \leq |y_m| < 6 \\ 1 & , if \quad 2 \leq |y_m| < 4 \\ 0 & , if \quad 0 \leq |y_m| < 2 \end{cases}$$

4. (24%)

(1) (6%)

The equivalent discrete-time impulse response of the channel is:

$$h(t) = \sum_{n=-1}^{1} h_n \delta(t - nT) = 0.2\delta(t+T) + 0.4\delta(t) + 0.2\delta(t-T)$$

We denote $\{c_n\}$ as the coefficients of the FIR equalizer, then the equalized signal is :

$$q_m = \sum_{n=-1}^{1} c_n h_{m-n}$$

Which in matrix notation is written as:

$$\begin{bmatrix} 0.4 & 0.2 & 0 \\ 0.2 & 0.4 & 0.2 \\ 0 & 0.2 & 0.4 \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 3.75 & -2.5 & 1.25 \\ -2.5 & 5 & -2.5 \\ 1.25 & -2.5 & 3.75 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2.5 \\ 5 \\ -2.5 \end{bmatrix}$$

$$\left(or \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} \frac{15}{4} & \frac{-5}{2} & \frac{5}{4} \\ \frac{-5}{2} & 5 & \frac{-5}{2} \\ \frac{5}{4} & \frac{-5}{2} & \frac{15}{4} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{-5}{2} \\ 5 \\ \frac{-5}{2} \end{bmatrix} \right)$$

(2) (6%)

$h_0 = 0.4, \quad h_1 = 0.2, \quad h_{-1} = 0.2$

$c_0 = 5, \quad c_1 = -2.5, \quad c_{-1} = -2.5$

Using the formula $q_m = \sum_{n=-1}^{1} c_n h_{m-n}$,we have

$$q_2 = c_{-1}h_3 + c_0 h_2 + c_1 h_1 = -0.5$$
$$q_{-2} = c_{-1}h_{-1} + c_0 h_{-2} + c_1 h_{-3} = -0.5$$
$$q_3 = c_{-1}h_4 + c_0 h_3 + c_1 h_2 = 0$$
$$q_{-3} = c_1 h_{-2} + c_0 h_{-3} + c_1 h_{-4} = 0$$
$$\Rightarrow q_2 = -0.5, \quad q_{-2} = -0.5, \quad q_3 = 0, \quad q_{-3} = 0$$

(3) (6%)

With $X(z) = 0.2z + 0.4 + 0.2z^{-1} = (f_0 + f_1 z^{-1})(f_0 + f_1 z)$,we choose

$f_0 = f_1 = \sqrt{0.2}$ with MSE criterion, $\sum_j c_j R_{vv}[l-j] = f_{-l}^*$,where

$$R_{vv}[l-j] = \begin{cases} x_{l-j} + N_0 \delta_{l-j} & ,|l-j| \le L \\ 0 & ,otherwise \end{cases}$$

Hence,we have

$$\begin{bmatrix} 0.4+0.2 & 0.2 & 0 \\ 0.2 & 0.4+0.2 & 0.2 \\ 0 & 0.2 & 0.4+0.2 \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} \sqrt{0.2} \\ \sqrt{0.2} \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 1.9048 & -0.7143 & 0.2381 \\ -0.7143 & 2.1429 & -0.7143 \\ 0.2381 & -0.7143 & 1.9048 \end{bmatrix} \begin{bmatrix} \sqrt{0.2} \\ \sqrt{0.2} \\ 0 \end{bmatrix} = \begin{bmatrix} 0.5324 \\ 0.6389 \\ -0.2130 \end{bmatrix}$$

$$\left(or \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} \frac{40}{21} & \frac{-5}{7} & \frac{5}{21} \\ \frac{-5}{7} & \frac{15}{7} & \frac{-5}{7} \\ \frac{5}{21} & \frac{-5}{7} & \frac{40}{21} \end{bmatrix} \begin{bmatrix} \sqrt{0.2} \\ \sqrt{0.2} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{5\sqrt{5}}{21} \\ \frac{2\sqrt{5}}{7} \\ \frac{-2\sqrt{5}}{21} \end{bmatrix} \right)$$

(4) (6%)

The residual distortion of the equalizer with the formula as $q_n = \sum_j c_j f_{n-j}$,we have

$$q_n = \begin{cases} 0 & ,n \le -2 \\ c_{-1} f_0 & ,n = -1 \\ c_1 f_0 + c_0 f_1 & ,n = 1 \\ c_1 f_1 & ,n = 2 \\ 0 & ,n \ge 3 \end{cases}$$

$$\Rightarrow q_n = \begin{cases} 0 & ,n \le -2 \\ 0.2381 & ,n = -1 \\ 0.1905 & ,n = 1 \\ -0.0953 & ,n = 2 \\ 0 & ,n \ge 3 \end{cases}$$

A measure of the residual intersymbol interference and assitive noise is obtained by evaluating the minimum value of J ,denote by $J_{\min}$ .We have the following procedure:

$$J_{\min} = 1 - q_0 = 1 - \sum_{j=-k}^{k} c_j f_{-j} = 1 - (c_{-1} f_1 + c_0 f_0) \cong 0.4762$$

Hence, we have the corresponding SINR,

$$\gamma_k = \frac{1 - J_{\min}}{J_{\min}} \cong 1.10005$$

5. (24%)

(1) (6%)

$$P = \sum_{n=1}^{4} P_n = \sum_{n=1}^{4} \Delta f_n p(f_n)$$

$$\Delta f_n = \Delta f = \frac{1}{T} = 1, \forall n$$

$$10 = k - \frac{1}{0.1} + k - \frac{1}{0.25} + k - \frac{1}{0.2} + k - \frac{1}{0.5} \rightarrow k = \frac{31}{4}$$

$$P_1 = \frac{31}{4} - \frac{1}{0.1} < 0 \rightarrow P_1 = 0$$

$$10 = k - \frac{1}{0.25} + k - \frac{1}{0.2} + k - \frac{1}{0.5} \rightarrow k = 7$$

$$P_2 = 7 - \frac{1}{0.25} = 3$$

$$P_3 = 7 - \frac{1}{0.2} = 2$$

$$P_4 = 7 - \frac{1}{0.5} = 5$$

(2) (6%)

$$Pe = E[Q(\sqrt{\frac{|h_n|^2 P_n}{\sigma_n^2}})] = E[Q(\sqrt{|h_n|^2 P_n})]$$

$$Water - filling:$$

$$Pe = \frac{1}{4}Q(\sqrt{0.1 \cdot 0}) + \frac{1}{4}Q(\sqrt{0.25 \cdot 3}) + \frac{1}{4}Q(\sqrt{0.2 \cdot 2}) + \frac{1}{4}Q(\sqrt{0.5 \cdot 5})$$

$$Equal - power:$$

$$Pe = \frac{1}{4}Q(\sqrt{0.1 \cdot 2.5}) + \frac{1}{4}Q(\sqrt{0.25 \cdot 2.5}) + \frac{1}{4}Q(\sqrt{0.2 \cdot 2.5}) + \frac{1}{4}Q(\sqrt{0.5 \cdot 2.5})$$

(3) (6%)

$$PAPR = N = 256 \quad (or \quad \frac{9}{5}N = \frac{9}{5} \cdot 256)$$

(4) (6%)

$$\frac{MN}{M^N} = NM^{1-N} = 256 \times 4^{-255}$$