COM 5120 Communication Theory

Midterm#1 Solution

1.

(a)

$$P_I(t) = x(t)\cdot P_{Td}(t)$$

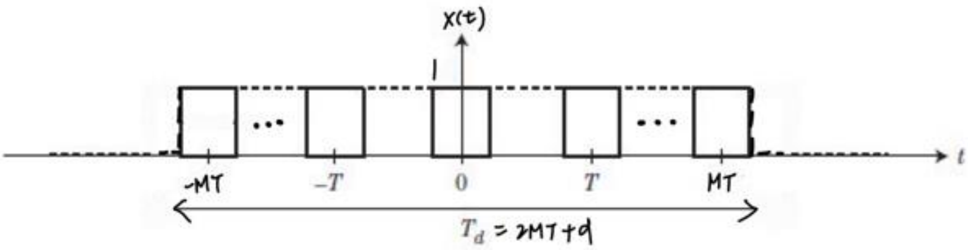
$$x(t) = \sum_{m=-\infty}^{\infty} p(t-mT) = p(t) * \sum_{m=-\infty}^{\infty} \delta(t-mT)$$

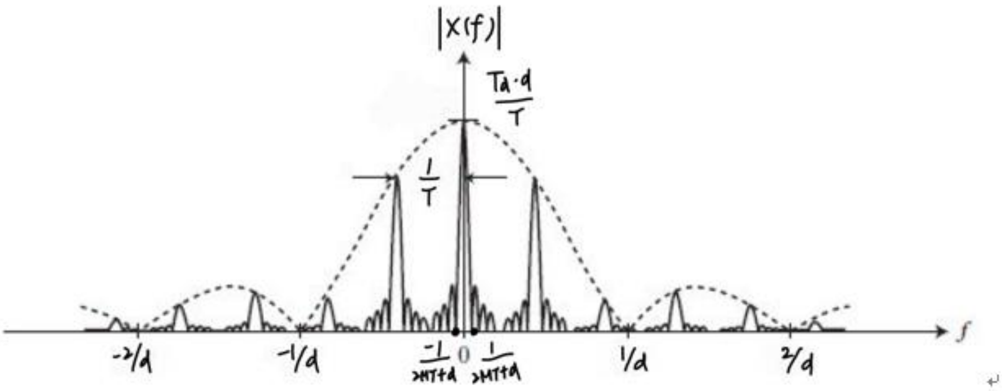
$$P_{Td}(t) = \begin{cases} 1, & -MT - \frac{d}{2} \le t \le MT + \frac{d}{2} \\ 0, & otherwise \end{cases}$$

$$\therefore P_I(f) = [\frac{d}{T}\sum_{m=-\infty}^{\infty} \sin c(dm\cdot\frac{1}{T})\cdot\delta(f - m\cdot\frac{1}{T})] * T_d \sin c(fT_d)$$

$$= \frac{T_d\cdot d}{T}\sum_{m=-\infty}^{\infty} \sin c(dm\cdot\frac{1}{T})\cdot \sin c[(f - m\cdot\frac{1}{T})T_d] \ \ , T_d = 2MT + d$$

(b)





(c) $\left\lfloor \frac{2T}{d} \right\rfloor + 1$ 個

(d)

$$x(t) = p_I(t)\cos(2\pi f_c t)$$

$$\rightarrow F.T.$$

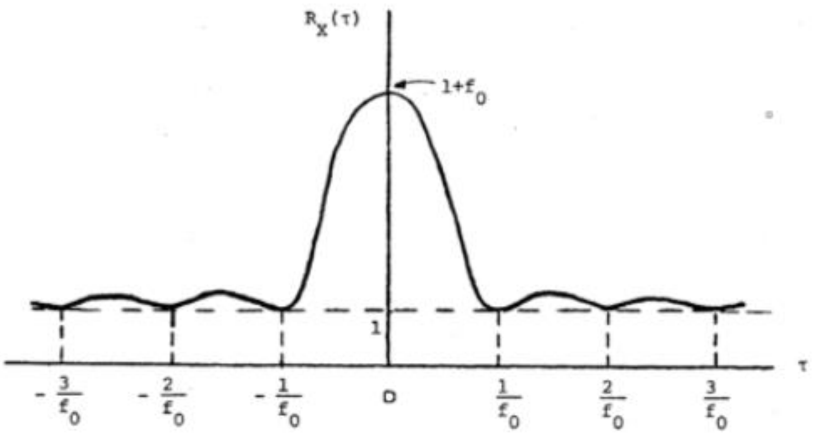
$$\rightarrow X(f) = P_I(f) * \frac{1}{2}[\delta(f-f_c)+\delta(f+f_c)] = \frac{1}{2}[P_I(f-f_c)+P_I(f+f_c)]$$

$$= \frac{1}{2}\frac{T_d \cdot d}{T}\sum_{m=-\infty}^{\infty} \sin c(d\cdot\frac{m}{T})\cdot\left[\sin c[(f-f_c-\frac{m}{T})T_d] + \sin c[(f+f_c-\frac{m}{T})T_d]\right] \text{ , } T_d = 2MT + d$$

2.

(a)The autocorrelation function of X(t) is $R_X(\tau) = 1 + f_0\text{sinc}(f_0\tau)^2$

Which is sketched below:



(b)Since $R_X(\tau)$ contains a constant component of amplitude 1, it follows that the dc power contained in X(t) is 1.

(c) the sampling rate is $\frac{f_0}{n}$ where n is an integer, the samples are uncorrelated

NO, uncorrelated$\neq$ independent

They would be statistically independent if X(t) were a Gaussian process

3.

(a)E[z(t)z(t+τ)] = E{[x(t+τ)+jy(t+τ)][x(t)+jy(t)]} = E[x(t)x(t+τ) – E[y(t)y(t+τ)]
+ jE[x(t)y(t+τ)] + E[y(t)x(t+τ)] = $R_{xx}(\tau)$ - $R_{yy}(\tau)$ + j[$R_{yx}(\tau)$ + $R_{xy}(\tau)$]

But $R_{xx}(\tau) = R_{yy}(\tau)$ and $R_{yx}(\tau) = -R_{xy}(\tau)$. Therefore :

$$E[z(t)z(t+\tau)] = 0$$

(b)V = $\int_0^T z(t)\,dt$

E(V^2) = $\int_0^T \int_0^T E[z(a)z(b)]\,dadb = 0$

From the result in (a) above. Also :

E[VV^*] = $\int_0^T \int_0^T E[z(a)z^*(b)]\,dadb = \int_0^T \int_0^T N_0\delta(a-b)\,dadb = \int_0^T N_0\,da =$

N_0T

4.

S1=<1,1,0>;S2=<1,0,1>;S3=<-1,2,2>;S4=<1,-1,-1>

(a)

$\Phi 1 = \frac{\mathrm{S1}}{\sqrt{E_{S1}}} = <\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0>$

F2=S2-(S2 • Φ1)Φ1

$\Phi 2 = \frac{\mathrm{F2}}{\sqrt{E_{F2}}} = <\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}>$

F3=S3-(S3 • Φ1)Φ1-(S3 • Φ2)Φ2

$\Phi 3 = \frac{\mathrm{F3}}{\sqrt{E_{F3}}} = <\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}>$

(b)

$S1=\sqrt{2}\Phi 1 \quad \approx 1.4\Phi 1$

$S2=\frac{\sqrt{2}}{2}\Phi 1+\frac{\sqrt{6}}{2}\Phi 2 \quad \approx 0.7\Phi 1+1.2\Phi 2$

$S3=\frac{\sqrt{2}}{2}\Phi 1+\frac{\sqrt{6}}{6}\Phi 2+\frac{5\sqrt{3}}{3}\Phi 3 \quad \approx 0.7\Phi 1+0.4\Phi 2+2.9\Phi 3$

$S4=-\sqrt{3}\Phi 3 \quad \approx -1.7\Phi 3$

圖略(圖有畫出相對關係就給分)

(c)

$S_{AVG}=(S1+S2+S3+S4)/4=\frac{\sqrt{2}}{2}\Phi 1+\frac{\sqrt{6}}{6}\Phi 2+\frac{\sqrt{3}}{6}\Phi 3$

$S1'=S1-S_{AVG}=\frac{\sqrt{2}}{2}\Phi 1-\frac{\sqrt{6}}{6}\Phi 2-\frac{\sqrt{3}}{6}\Phi 3 \approx 0.7\Phi 1-0.4\Phi 2-0.3\Phi 3$

$S2'=S2-S_{AVG}=\frac{\sqrt{6}}{3}\Phi 2-\frac{\sqrt{3}}{6}\Phi 3 \approx 0.8\Phi 2-0.3\Phi 3$

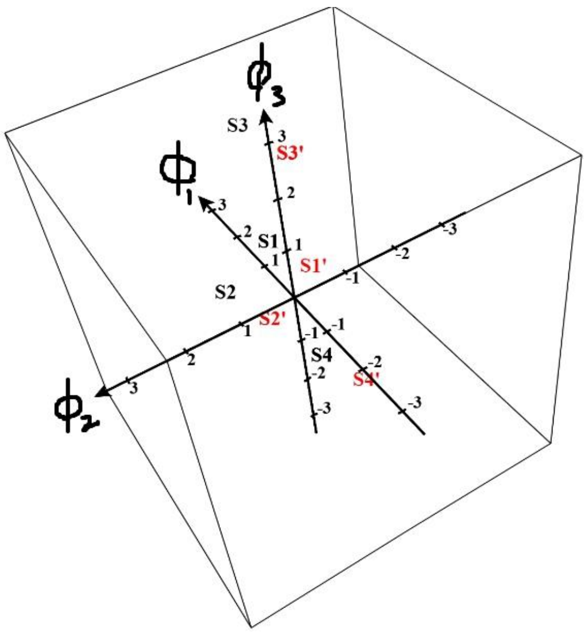
$S3'=S3-S_{AVG}=\frac{3\sqrt{3}}{2}\Phi 3 \approx 2.6\Phi 3$

$S4'=S4-S_{AVG}=-\frac{\sqrt{2}}{2}\Phi 1-\frac{\sqrt{6}}{6}\Phi 2-\frac{7\sqrt{3}}{6}\Phi 3 \approx -0.7\Phi 1-0.4\Phi 2-2\Phi 3$

$E_S \approx$1.4^2+0.7^2+1.2^2+0.7^2+0.4^2+2.9^2+1.7^2≈15.8

$E_{S'} \approx$0.7^2+0.4^2+0.3^2+0.8^2+0.3^2+2.6^2+0.7^2+0.4^2+2^2≈12.9

$E_{S'}$ is more energy efficient.



(圖有畫出相對關係就給分)

5.

(a)

8_APK:

Average power = (1/8) $(4A^2/2+4(4A^2)/2)$ = $5A^2/4$

(*where "/2" implies carrier's power)

8-PSK:

Average power = $R^2/2$

(*where "/2" implies carrier's power)

If the Average power per bit is the equivalent for the both schemes.

Therefore, Average power(8-APK) =Average power(8-PSK).

=> $5A^2/4=R^2/2$ => R ≈ 1.58 A

(b)

8_APK:

Average power = (1/8) $(4A^2/2+4(4A^2)/2)$ = $5A^2/4$

Symbol rate = 10M Hz

E_B= $E_S/3$= $(5A^2/4)$/ 30M Joule/b

8-PSK:

Average power = $R^2/2$

Symbol rate =10M Hz

E_B= $E_S/3$= $(R^2/2)$/ 30M Joule/b

(c)

For 8-APK, min distance = A

For 8-PSK min distance ≈ 2πR/8 ≈ (2πR/8) x 1.58 A = 1.24 A

Therefore 8-PSK is better for equal E_B/N_0.

(d)

$$S_{vv}(f)_a = \frac{1}{T} S_{II}(f)_a \,|G(f)|^2$$

$$S_{II}(f)_a = \frac{4A^2 + 4(4A^2)}{8} = \frac{20A^2}{8} = \frac{5A^2}{2}$$

Power Spectral Density $S_v(f)$ of v(t) for '8-APK':

$$S_{vv}(f) = \frac{1}{T}\frac{5A^2}{2}(sinc^2(Tf)) = \frac{5A^2}{2T} sinc^2(Tf)$$

6.

(1)

$R_{aa}[k]$= δ[k]

$R_{bb}[k]$= E[$b_{n+k} b_n$] = E[$(a_{n+k} + 2\alpha\, a_{n+k-2})(a_n + 2\alpha\, a_{n-2})$]

$R_{bb}[k]$= δ[k] + 2α δ[k+2]+ 2α δ[k-2]+ $4\alpha^2$δ[k]

=>$S_{bb}(f) = 1 + 4\alpha^2 + 4\alpha \cos(4\pi f T)$

(a)

$$G(f) = \int \sin\left(\frac{\pi t}{T}\right) e^{-j2\pi f t} dt = \int \frac{1}{2j}\left(e^{j\frac{\pi t}{T}} - e^{-j\frac{\pi t}{T}}\right) e^{-j2\pi f t} dt = \frac{2T\cos(\pi f T)}{\pi(1-4f^2T^2)} e^{-j\pi f T}$$

$$S_X(f) = \frac{1}{T} S_b(f) |G(f)|^2$$

$$= \frac{1}{T}\left(\frac{4T^2\cos^2(\pi f T)}{\pi^2(1-4f^2T^2)^2}\right)\left(1 + 4\alpha^2 + 4\alpha\cos(4\pi f T)\right)$$

(b)

$S_X\left(\frac{1}{6T}\right) = 0 \Rightarrow (1 + 4\alpha^2 + 4\alpha \cos(4\pi \frac{1}{6T} T) = 0 \Rightarrow 1+4\alpha^2-2\alpha = 0 \Rightarrow \alpha = \frac{1 \pm \sqrt{3} j}{4}$