COM 5120 Communication Theory
Homework #1 solution

1.

(a)

$x(t) = P_I(t) \cdot P_{T_d}(t)$

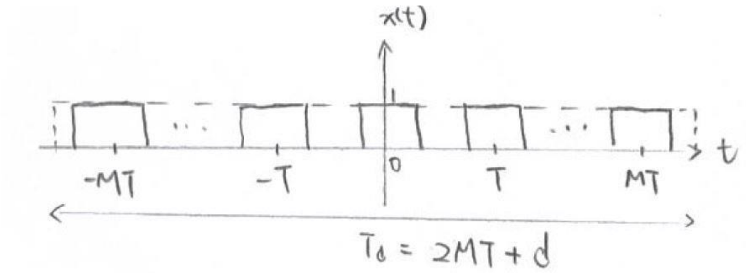
$P_I(t) = \sum_{m=-\infty}^{\infty} p(t-mT) = p(t) * \sum_{m=-\infty}^{\infty} \delta(t-mT)$

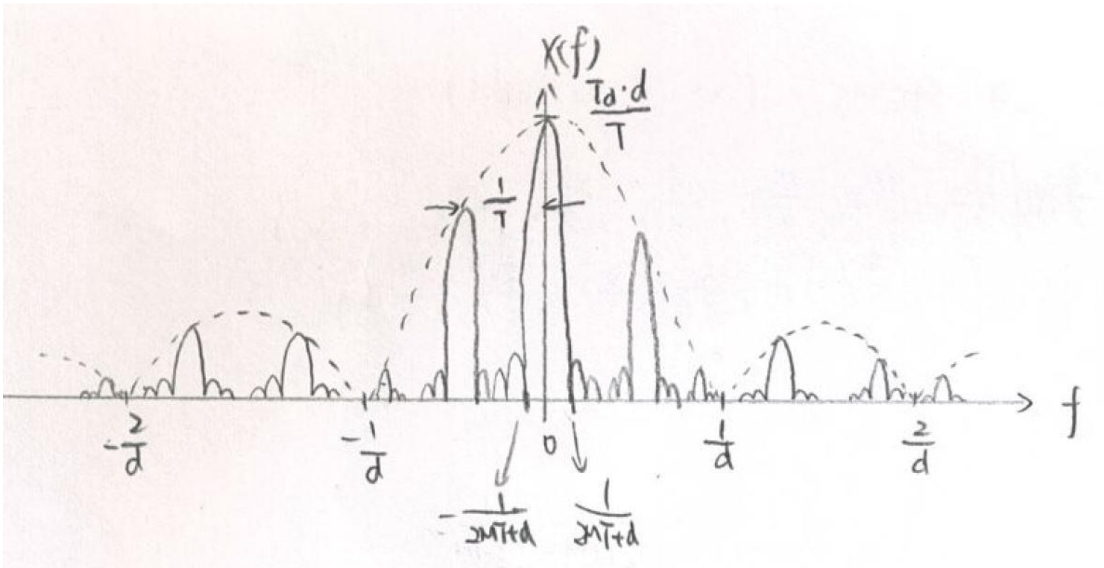
$$P_{T_d}(t) = \begin{cases} 1, & -MT - \frac{d}{2} \le t \le MT + \frac{d}{2} \\ 0, & otherwise \end{cases}$$

$$\therefore X(f) = \left\{\frac{d}{T}\sum_{k=-\infty}^{\infty} \mathrm{sinc}\left(dk \cdot \frac{1}{T}\right) \cdot \delta\left(f - k \cdot \frac{1}{T}\right)\right\} * T_d sinc(fT_d)$$

$$= \frac{T_d \cdot d}{T}\sum_{k=-\infty}^{\infty} \mathrm{sinc}\left(dk \cdot \frac{1}{T}\right) \cdot sinc[(f - k \cdot \tfrac{1}{T})T_d] \qquad (T_d = 2MT + d)$$

(b)





2.

(a)

$$X(f) = 2rect\left(\frac{f}{F}\right) * \left\{\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)\right\}$$

$$= rect\left(\frac{f - f_0}{F}\right) + rect\left(\frac{f + f_0}{F}\right)$$

(b)

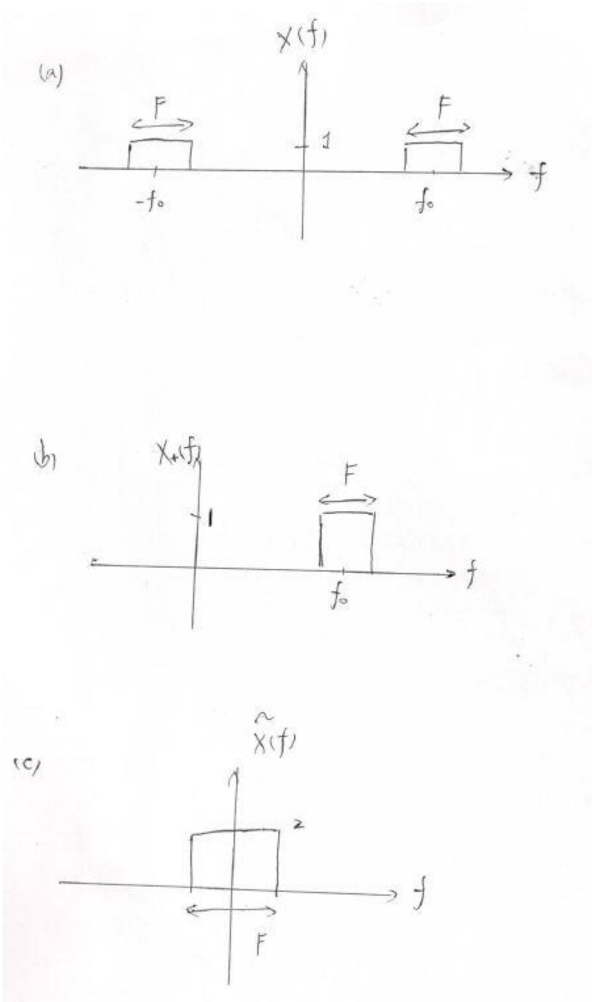
$$X_+(f) = \frac{1}{2}X(f) + \frac{1}{2}j[-jsgn(f)]X(f)$$

$$= \frac{1}{2}[1 + sgn(f)]X(f) = rect(\frac{f - f_0}{F})$$

(c)

$$\tilde{X}(f) = 2X_+(f + f_0) = 2rect\left(\frac{f}{F}\right)$$

$$\tilde{x}(t) = 2Fsinc(Ft)$$



3.

We are given $y(t) = \int_{t-T}^{t} x(\tau)d\tau$

For $x(t) = \delta(t)$, the impulse response of this running integrator is, by definition,

$h(t) = \int_{t-T}^{t} \delta(\tau)d\tau$

$= 1$ for $t - T \leq 0 \leq t$ or, equivalently, $0 \leq t \leq T$

Correspondingly, the frequency response of the running integrator is

$\mathrm{H(f)} = \int_{-\infty}^{\infty} h(t)\exp(-j2\pi ft)\,dt$

$= \int_{0}^{T} \exp(-j2\pi ft)\,dt$

$= \frac{1}{j2\pi f}[1 - \exp(-j2\pi fT)]$

$= Tsinc(fT)\exp(-j\pi fT)$

Hence the power spectral density $S_Y(f)$ is defined in terms of the power spectral density $S_X(f)$ as follows

$$S_Y(f) = |H(f)|^2 S_X(f)$$
$$= T^2 sinc^2(fT) S_X(f)$$

4.

(a)

False,

$$E[X(t)\cos(2\pi f_c t + \varphi)] = E[X(t)]\cos(2\pi f_c t + \varphi)$$

which is related to t.

(b)

True,

$$E[X_1(t)] = E[X(t)]E[\cos(\cancel{2\pi f_c} t + \theta)] = 0$$
$$E[X_2(t)] = E[X(t)]E[\sin(\cancel{2\pi f_c} t + \theta)] = 0$$

(c)

False,

$$E[X_1(t_1)X_2(t_2)] = E[X(t_1)X(t_2)]E[\cos(2\pi f_c t_1 + \theta)\, sin(2\pi f_c t_2 + \theta)]$$
$$\neq E[X_1(t)]E[X_2(t)] = 0$$

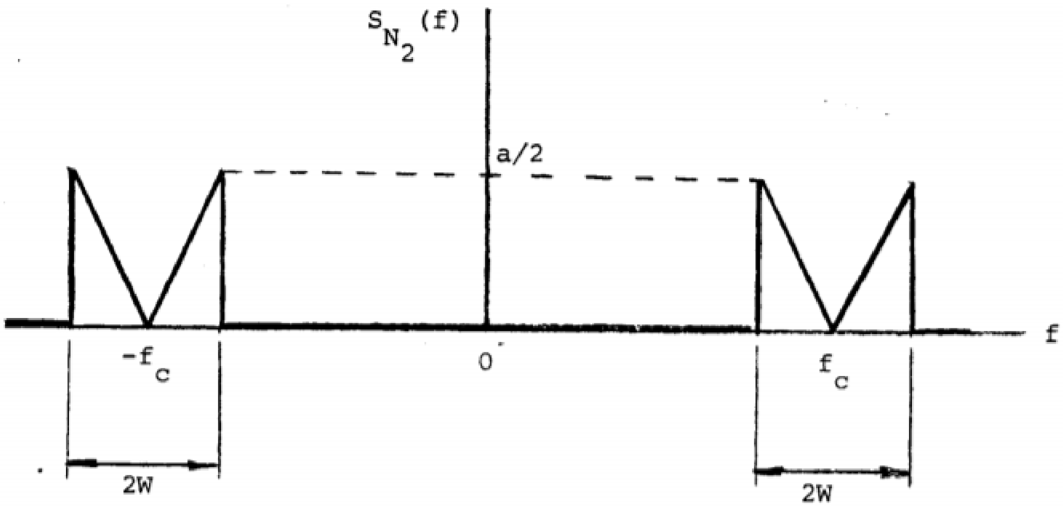
(d)

False,

$$E[X_1(t_1)Y_2(t_2)] = E[X(t_1)Y(t_2)]E[\sin(2\pi f_c t_2 + \theta)\, cos(2\pi f_c t_1 + \theta)]$$
$$\neq E[X_1(t_1)]E[Y_2(t_2)]$$

5.

$$n_2(t) = n_1(t)\{\cos(2\pi f_c t + \theta) - \sin(2\pi f_c t + \theta)\}$$
$$= \sqrt{2}n_1(t)\cos\left(2\pi f_c t + \frac{\pi}{4} + \theta\right)$$
$$R_{N_2}(\tau) = E[n_2(t)n_2(t-\tau)] = R_{N_1}(\tau)\cos(2\pi f_c \tau)$$
$$S_{N_2}(f) = \frac{1}{2}S_{N_1}(f - f_c) + \frac{1}{2}S_{N_1}(f + f_c)$$



$S_{N_2}(f)$
a/2
f
$-f_c$
0
f_c
2W
2W