COM5120 Communication theory

Homework #4

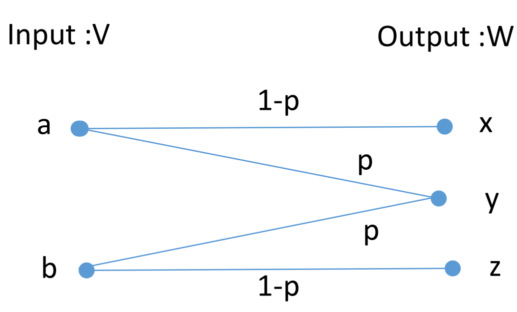
Due: 12/07/2020 (Monday)

1. (20%) Let X be a geometrically distributed random variable,
$$\mathrm{P}(\mathrm{X}=\mathrm{k})=p(1-p)^{k-1},\ \ k=1,2,3\ \ldots.$$
   (1) Find the entropy of X ?
   (2) Given that X>K , where K is positive integer, find $\mathrm{H}(\mathrm{X}|\mathrm{X}>\mathrm{K})$?

2. (20%) Random variables X, Y are distributed according to the joint distributions:
$$\mathrm{P}(\mathrm{X}=0,\mathrm{Y}=0)=\frac{1}{14}\ ,\mathrm{P}(\mathrm{X}=0,\mathrm{Y}=1)=\frac{2}{7},$$
$$\mathrm{P}(\mathrm{X}=1,\mathrm{Y}=0)=\frac{3}{14}\ ,\mathrm{P}(\mathrm{X}=1,\mathrm{Y}=1)=\frac{3}{7},$$
   Compute $\mathrm{H}(\mathrm{X})$, $\mathrm{H}(\mathrm{Y})$, $\mathrm{H}(\mathrm{X}|\mathrm{Y})$, $\mathrm{H}(\mathrm{Y}|\mathrm{X})$, H(X, Y)

3. (20%) Find the differential entropy of the continuous random variable X in the following case:
   (1) X is an exponential random variable with $\lambda>0$
$$\mathrm{p}(\mathrm{x})=\begin{cases}\dfrac{1}{\lambda}e^{-x/\lambda}\ , & x>0\\ 0\ , & otherwise\end{cases}$$
   (2) X is an Laplacian random variable with $\lambda>0$
$$\mathrm{p}(\mathrm{x})=\frac{1}{2\lambda}e^{-|x|/\lambda}$$

4. (20%) Consider a channel with transition probability
   $\mathrm{P}(\mathrm{x}|\mathrm{a})=\mathrm{P}(\mathrm{z}|\mathrm{b})=1-\mathrm{p}$ and $\mathrm{P}(y|\mathrm{a})=\mathrm{P}(\mathrm{y}|\mathrm{b})=\mathrm{p}$ as shown in the figure :
   (1) Determine the average mutual information I(V;W).
   (2) Determine the channel capacity.



5. (20%) Consider a BSC with crossover probability of p. Suppose that R is the number of bits in a source codeword that represents one of 2^R possible levels at the output of a quantizer.

   (1) Determine the probability that a codeword transmitted over the BSC is received correctly.

   (2) Determine the probability of having at least 1 bit error in a codeword transmitted over the BSC.

   (3) Determine the probability of having n_e, or fewer bit errors in a codeword.

   (4) Evaluate the probabilities in (1)(2)(3), for R = 5, p = 0.1, and $n_e = 5$