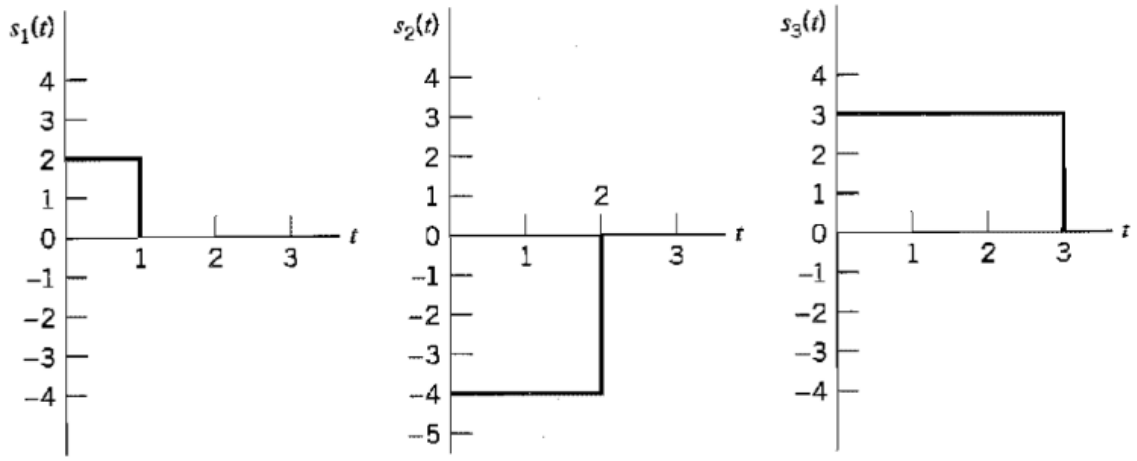
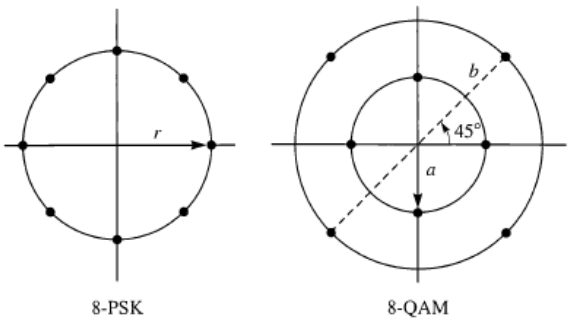
COM 5120 Communication Theory
Homework #2
Due: 11/4/2021

1. (20%) The following figure displays the waveforms of three signals $s_1(t)$,$s_2(t)$,and $s_3(t)$.
   (a) Using the Gram-Schmidt orthogonalization procedure, find a set of orthonormal basis functions to represent the three signals.(15%)
   (b) Express each of these signals in terms of the set of basis functions found in part(a).(5%)



2. (20%) Consider the octal signal point constellations



   (a) The nearest-neighbor signal points in the 8-QAM signal constellation are separated in distance by *A* units. Determine the radius *a* and *b* of the inner and outer circles, respectively.(5%)
   (b) The adjacent signal points in the 8-PSK are separated by a distance of *A* units. Determine the radius *r* of the circle.(5%)
   (c) Determine the average transmitter powers for the two signal constellations, and compare the two powers. What is the relative power advantage of one constellation over the other? (Assume all signal points are equally probable.)(10%)

3. (20%) Given the M-ary PAM signal of

$$d(t) = \sum_n I_n g(t - nT)$$

where the binary sequence b_n is mapped into $I_n \in$ {±1, ±3, … , ±(M-1)}.
Let

$$g(t) = \begin{cases} \frac{1}{2T}, & 0 \le t \le T \\ 0, & otherwise \end{cases} \quad and\ q(t) = \int_0^t g(\tau)d\tau$$

To construct a M-ary CPFSK from the M-ary PAM signal, each M-PAM signal I_n is mapped into frequency {$\pm f_d$,$\pm 3f_d$, … ,±(M−1)f_d}, and the n^{th} CPFSK signal is constructed as

$$s(t) = \sqrt{\frac{2E}{T}}\ cos\ [2\pi f_c t + \theta(t, I_n)], \qquad nT \le t \le (n+1)T$$

where $\theta(t, I_n) = 4\pi f_d T \int_{-\infty}^{t} d(\tau)d\tau$. Let $\theta(nT) = 2\pi f_d T \sum_{k=-\infty}^{n-1} I_k$.

(a) Please identify the expression of $\theta(t, I_n)$ in terms of $\theta(nT)$, I_n ,and $q(t)$.(10%)
(b) Define $h = 2f_d T$. Please determine the number of terminal phase states in the state trellis diagram for h = 3/5 and M = 4.(5%)
(c) Following from (b), please sketch the phase trellis of the signal s(t) for t=0, T, 2T, 3T, 4T.(5%)

4. (20%) Consider a QPSK modulated signal represented for the following baseband signal

$$d(t) = \sum_n I_n g(t - nT)$$

where I_n takes one of the four possible values $\pm\frac{1}{2} \pm \frac{1}{2}j$ with equal probability. The sequence of $\{I_n\}$ is statistically independent.
(a) Let

$$g(t) = \begin{cases} 1, & 0 \le t \le T \\ 0, & otherwise \end{cases}$$

Please determine the power spectral density spectrum of d(t).(10%)
(b) Let

$$g(t) = \begin{cases} \sin\left(\frac{\pi t}{T}\right), & 0 \le t \le T \\ 0, & otherwise \end{cases}$$

Please determine the power spectral density spectrum of d(t).(10%)

5. (20%) Consider the PAM signal

$$s(t) = \sum_n b_n \mathrm{h}(t - nT)$$

where $b_n = a_n - a_{n-2}$, $\{a_n = \pm 1\}$ is a sequence of uncorrelated random variables with $\mathrm{P_r}[a_n = 1] = \mathrm{P_r}[a_n = -1] = \frac{1}{2}$, and

$$\mathrm{h}(t) = \begin{cases} 1, & 0 \le t \le T \\ 0, & otherwise \end{cases}$$

(a) Determine the autocorrelation function of the sequence $\{b_n\}$. (10%)

(b) Determine the power spectral density of s(t). (10%)