COM 5120 Communication Theory
Homework #3
Due: 11/23/2020 (Monday)

1. (10%) Consider the set of three (M=3) finite-energy signaling waveforms in $0 \leq \mathrm{t} \leq 1$:

$$s_1(t) = 1 \qquad 0 \leq t \leq 1$$
$$s_2(t) = cos(8\pi t) \qquad 0 \leq t \leq 1$$
$$s_3(t) = 2 * cos^2(4\pi t) \qquad 0 \leq t \leq 1$$

The channel is AWGN with PSD of $\frac{N_0}{2} = 10^{-1}\ W/Hz$. Find the conditional error probability P_{e3}, assuming that $s_3(t)$ was sent.

2. (15%) Consider a digital communication system that transmits information via QAM over a voice-band telephone channel at a rate of 2400 symbols/s. The additive noise is assumed to be white and Gaussian.

(a) (5%) Determine the $\frac{\varepsilon_b}{N_0}$ required to achieve an error probability of 10^{-5} at 2400 bits/s.

(b) (5%) Repeat part 1 for a rate of 4800 bits/s.

(c) (5%) Repeat part 1 for a rate of 9,600 bits/s.

3. (20%) Consider a communication system where three equiprobable messages m_1, m_2, m_3 are transmitted. Let m_1, m_2, m_3 be encoded by signals $s_1(t), s_2(t), s_3(t)$ respectively given by

$$s_1(t) = 3\sqrt{2}\cos 2\pi t, \qquad s_2(t) = 2\sqrt{2}\sin 2\pi t, \qquad s_3(t) = -2\sqrt{2}\sin 2\pi t$$

where the signal duration is $0 \leq t \leq 1$ and each signal is zero outside this interval. Assume that the signals are transmitted over an additive white Gaussian noise channel.

(a) (5%) Find a set of orthonormal basis function to represent the set of signals, and then draw the corresponding signal constellation.

(b) (5%) Determine the optimum decision regions.

(c) (10%) Determine an equivalent minimum-energy signal set that would yield the same probability of error as the signal set described above. Draw the corresponding signal constellation and optimum decision regions.

4. (25%) Consider a one-dimensional discrete communication model shown below. The transmitted symbol $X \in \{+a, -a\}$ where a>0 is a deterministic and known value. The noise N is dependent on X. Specifically, given $X = +a$, N is Gaussian distributed with zero mean and variance σ_1^2, and given $X = -a$, N is Gaussian distributed with zero mean and variance σ_2^2, where σ_1^2 and σ_2^2 are known. Assume that Prob$[X =$

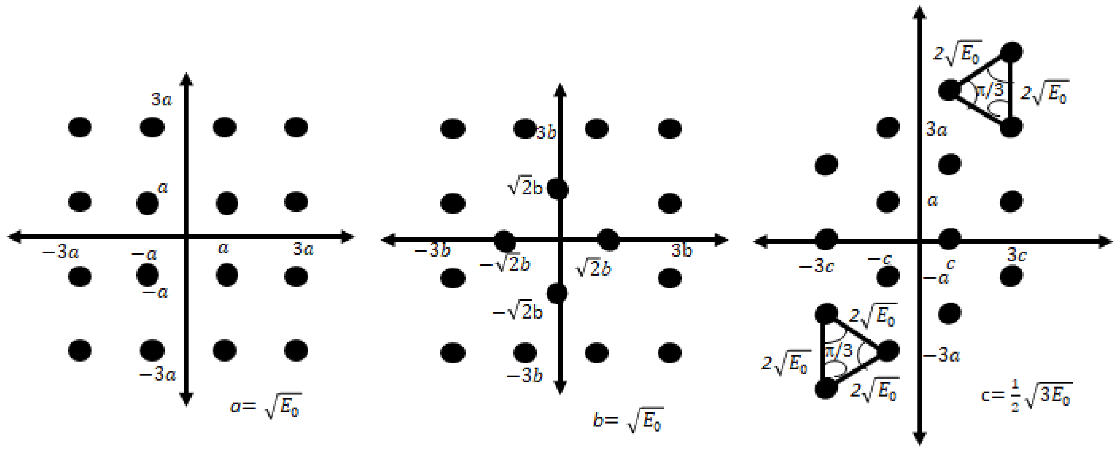
$+a] = p_1$ and Prob$[X = -a] = p_2$.

(a) (10%) Derive a maximum a posteriori probability (MAP) receiver for detecting X.

(b) (10%) Suppose $\sigma_1^2 = 1,\ \sigma_2^2 = 2,\ a = 1,\ p_1 = p_2 = 0.5$. Find the decision regions for $X = +a$ and $X = -a$.

(c) (5%) Find the probability of error for the values specified in (b).

5. (20%) Consider three M-ary QAMs (A), (B) and (C), where $M = 16$ as shown in the following figure with the symbol period of T_s.



(A) (B) (C)

(a) (6%) Please find the average energy per symbol of the these QAM schemes.

(b) (6%) Please compare the CFM (Constellation Figure of Merits) of these QAM schemes.

(c) (8%) Find the average probability of symbol error of these QAM over additive white Gaussian noise (AWGN) channel with PSD of $\frac{N_0}{2}$ in case optimal detection is used.

6. (10%) A M-ary PSK signal set is given that $s_m(t) = g(t)\cos\left(2\pi f_c t + \frac{2\pi}{M}(m-1)\right)$, $m = 1, \ldots, M$. $E_s = \|s_m(t)\|^2 = \frac{1}{2}E_g$, where $E_g = \int_0^T g^2(t)\,dt$. $\emptyset_1(t) = \sqrt{\frac{2}{E_g}}\, g(t)cos(2\pi f_c t);$

$$\emptyset_2(t) = -\sqrt{\frac{2}{E_g}}\, g(t)sin(2\pi f_c t)$$

$$\underline{s}_m = \left[\sqrt{E_s}\cos\left(\frac{2\pi}{M}(m-1)\right),\ \sqrt{E_s}\sin\left(\frac{2\pi}{M}(m-1)\right)\right]^T, m = 1, \ldots, M$$

Please derive the probability of error for M-ary PSK.