COM5120 Communications Theory

HW#6

Due: For your own practice

1. For as set of N parallel Gaussian channels, the capacity is given
$$\mathrm{Y_i} = \mathrm{X_i} + \mathrm{N_i}$$
where $\mathrm{i} = 0,1,2,\ldots,\mathrm{N}-1$ and $\mathrm{N_i}$ are white gaussian noises obey the distribution $\mathcal{N}(0,\ \mathrm{i}^2)$. There is a power constraint $\sum_{i=0}^{N-1} X_i^2 \leq 5$
   A. (5%) Find the channel capacity for $\mathrm{N} = 2$
   B. (5%) Find the channel capacity for $\mathrm{N} = 4$
   C. (5%) Find the channel capacity for $\mathrm{N} \rightarrow \infty$

2. Following the previous question, if the power constraint is changed to
$\sum_{i=0}^{N-1} \frac{X_i^2}{i} \leq 5$
   A. (5%) Find the channel capacity for $\mathrm{N} = 2$
   B. (5%) Find the channel capacity for $\mathrm{N} = 4$
   C. (5%)Find the channel capacity for $\mathrm{N} \rightarrow \infty$

3. **(10%)** Consider an OFDM system with 16 QAM, 0.5 code rate, 64 subcarriers which 48 of them are used to transfer data and 16 of them are used as channel estimation or protection. Each subcarrier is spaced 312.5 kHz and use $\frac{1}{4}$ frame as guard interval. Find the data rate.

4. **(10%)** W is the DFT matrix in OFDM. Show that $\frac{1}{N}\mathrm{ww}^{\mathrm{H}} = I_N$

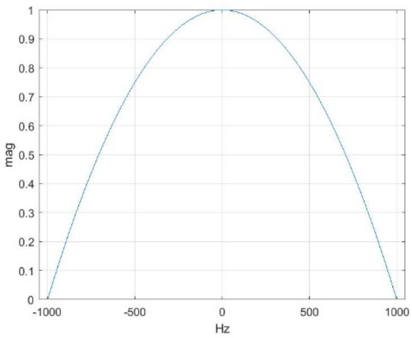
5. Consider a linear channel with bandwidth W. The channel is equally divided into three sub-channels which has squared magnitude response $|\mathrm{H(f)}|^2$ ดl in the piecewise-linear form with $|\mathrm{H(f)}|^2 = 1, \frac{1}{3}, \frac{1}{5}$ for sub-channels i =1,2 and 3 respectively. Assume the system transmits data at the rate equal to the Shannon's channel capacity and the noise variance $\sigma_{\mathrm{i}}^2 = 1, \frac{1}{2}, \frac{1}{3}$ for subchannels i = 1,2 and 3 respectively.
   A. **(10%)** Let the total transmit power be constrained such that $P = \mathrm{p}_1 + p_2 + p_3$ and P is constant. Please derive the formulas for the optimum powers p_1, p_2, and p_3 allocated to the three sub-channels of frequency

bands such that the overall channel capacity of the entire system can be maximized

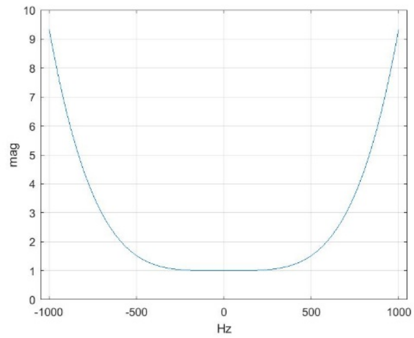
B. **(10%)** Given the total transmit power $P = 2$, and subchannel bandwidth $\Delta f = 2$, please calculate the corresponding values of p_1, p_2, and p_3

6. **(10%)** Compare zero-forcing and **MMSE** (minimum mean-square error) equalization in term of their objective.

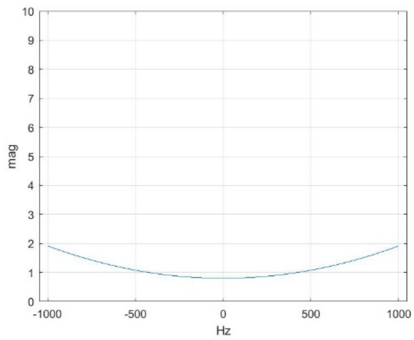
7. Fig (a) shows the magnitude of channel's frequency response. Assume noise is AWGN and transmitted signal is confined within the band [-1, 1] kHz.



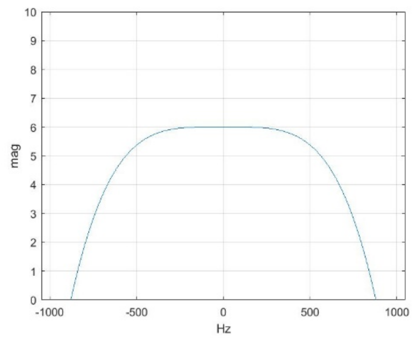
(a)



(b)



(b)



(d)

Fig (b), (c) and (d) show the frequency magnitude response of three filter.

A. **(10%)** Which one of Fig (b), (c) or (d) might represents the frequency magnitude response of a zero-forcing equalizer? Why?

B. **(10%)** Which one represent an MMSE equalizer? Why?