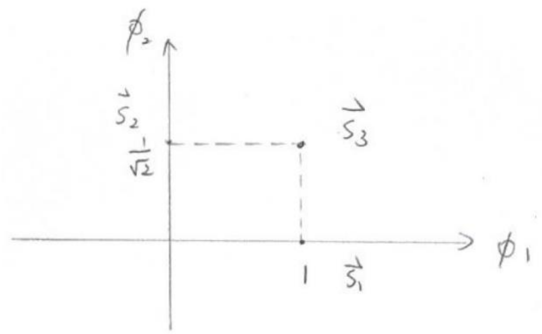
COM 5120 Communication Theory

Homework #2 solution

1.

(a) $\varphi_1(t) = 1, 0 \le \mathrm{t} \le 1$；$\varphi_2(t) = \sqrt{2}\cos(8\pi t), 0 \le \mathrm{t} \le 1$

(b) $s_1(t) = \varphi_1(t)$； $s_2(t) = \frac{1}{\sqrt{2}}\varphi_2(t)$； $s_3(t) = 1 + \cos(8\pi t) = \varphi_1(t) + \frac{1}{\sqrt{2}}\varphi_2(t)$



2. (沒過程都至少扣一半)

(a)

$$\int_0^T s_1(t)s_2(t)\,dt = \int_0^T A^2\cos(2\pi f_1 t)\cos(2\pi f_2 t)\,dt\ ;\quad \theta = 0$$

$$= \frac{A^2}{2}\int_0^T \cos(2\pi(f_1 - f_2)t) + \cos(2\pi(f_1 + f_2)t)\,dt$$

$$= \frac{A^2}{2}\left\{\frac{1}{2\pi(f_1 - f_2)}\sin(2\pi(f_1 - f_2)T) + \frac{1}{2\pi(f_1 + f_2)}\sin(2\pi(f_1 + f_2)T)\right\} = 0$$

$\rightarrow\ 2\pi(f_1 - f_2)T = n\pi,\ 2\pi(f_1 + f_2)T = m\pi\ ; n, m \in Z$

$\rightarrow\ f_1 - f_2 = \frac{n}{2T},\quad f_1 + f_2 = \frac{m}{2T} \rightarrow |f_1 - f_2|_{min} = \frac{1}{2T}$

(b)

$$\int_0^T s_1(t)s_2(t)\,dt = \int_0^T A^2\cos(2\pi f_1 t)\cos(2\pi f_2 t + \theta)\,dt\ ;\quad \theta \ne 0$$

$$= \frac{A^2}{2}\int_0^T \cos(2\pi(f_1 - f_2)t - \theta) + \cos(2\pi(f_1 + f_2)t + \theta)\,dt$$

$$= \frac{A^2}{2}\left\{\frac{1}{2\pi(f_1 - f_2)}[\sin(2\pi(f_1 - f_2)T - \theta) - \sin(-\theta)]\right.$$

$$\left. + \frac{1}{2\pi(f_1 + f_2)}[\sin(2\pi(f_1 + f_2)T + \theta) - \sin(\theta)]\right\} = 0$$

$\rightarrow\ 2\pi(f_1 - f_2)T = 2n\pi,\ 2\pi(f_1 + f_2)T = 2m\pi\ ; n, m \in Z$

$\rightarrow\ f_1 - f_2 = \frac{n}{T},\quad f_1 + f_2 = \frac{m}{T} \rightarrow |f_1 - f_2|_{min} = \frac{1}{T}$

3.

$R_{v_\Delta v_\Delta}(t) = E[v_\Delta(t+\tau)v_\Delta^*(t)]$

$= \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} E(I_m I_n^*)E[g(t+\tau-mT-\Delta)g^*(t-nT-\Delta)]$

$= \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} R_{ii}(m-n)E[g(t+\tau-mT-\Delta)g^*(t-nT-\Delta)]$

$= \sum_{m=-\infty}^{\infty} R_{ii}(m)\sum_{n=-\infty}^{\infty} E[g(t+\tau-mT-nT-\Delta)g^*(t-nT-\Delta)]$

$= \sum_{m=-\infty}^{\infty} R_{ii}(m)\sum_{n=-\infty}^{\infty} \int_0^T \frac{1}{T} g(t+\tau-mT-nT-\Delta)g^*(t-nT-\Delta)d\Delta$

Let $a = \Delta + nT$, $da = d\Delta$ and $a \in (-\infty, \infty)$, then

$R_{v_\Delta v_\Delta}(t) = \sum_{m=-\infty}^{\infty} R_{ii}(m)\sum_{n=-\infty}^{\infty} \int_{nT}^{(n+1)T} \frac{1}{T} g(t+\tau-mT-a)g^*(t-a)da$

$= \sum_{m=-\infty}^{\infty} R_{ii}(m)\frac{1}{T}\int_{-\infty}^{\infty} g(t+\tau-mT-a)g^*(t-a)da$

$= \frac{1}{T}\sum_{m=-\infty}^{\infty} R_{ii}(m)R_{gg}(\tau-mT)$

$\rightarrow\ S_{v_\Delta v_\Delta}(f) = \frac{1}{T}|G(f)|^2 S_{ii}(f)$

4.

(a)

$R_b(m) = E[b_{n+m}b_n] = E[(a_{n+m}-a_{n+m-2})(a_n-a_{n-2})]$

(i) m = 0, $R_b(0) = E[a_n^2] - 2E[a_n a_{n-2}] + E[a_{n-2}^2]$

$$= \left(\frac{1}{2}+\frac{1}{2}\right) - 0 + \left(\frac{1}{2}+\frac{1}{2}\right) = 2$$

(ii) m = 2, $R_b(2) = E[a_{n+2}a_n] - E[a_n^2] - E[a_{n+2}a_{n-2}] + E[a_n a_{n-2}]$

$$= 0 - \left(\frac{1}{2}+\frac{1}{2}\right) - 0 + 0 = -1$$

$$R_b(m) = \begin{cases} 2, & m = 0 \\ -1, & m = \pm 2 \\ 0, & else \end{cases}$$

(b)

$$R_s(\tau) = \frac{1}{T}\sum_{m=-\infty}^{\infty} R_b(m)R_h(\tau - mT)$$

$$= \frac{1}{T}\{2R_h(\tau) - R_h(\tau+2T) - R_h(\tau-2T)\}$$

$$S_s(f) = F\{R_s(\tau)\} = \frac{1}{T}\left\{2|H(f)|^2 - |H(f)|^2 e^{j4\pi fT} - |H(f)|^2 e^{-j4\pi fT}\right\}$$

$$= 4\mathrm{T}sinc^2(Tf)sin^2(2\pi fT)$$

5.

(a)

$$\theta(\mathrm{t}, I_n) = 4\pi f_d T \int_{-\infty}^{t} d(\tau) d\tau$$

$$= 4\pi f_d T \int_{-\infty}^{t} (\sum_n I_n g(\tau - nT)) d\tau$$

$$= 4\pi f_d T \sum_{k=-\infty}^{n-1} I_k \int_{-\infty}^{nT} g(\tau - kT) d\tau + 4\pi f_d T I_n \int_{nT}^{t} g(\tau - nT) d\tau$$

$$= 2\pi f_d T \sum_{k=-\infty}^{n-1} I_k + 4\pi f_d T I_n q(t - nT)$$

$$= \theta(nT) + 4\pi f_d T I_n q(t - nT) \ , \qquad \mathrm{nT} \le \mathrm{t} \le (\mathrm{n}+1)\mathrm{T}$$

(b)

h = $\frac{3}{5}$, where 3 and 5 are mutually prime integers, we have 2x5 = 10 states. (3 is odd)

$$\rightarrow \left\{0, \frac{3}{5}\pi, \frac{6}{5}\pi, \frac{9}{5}\pi, \frac{12}{5}\pi, \frac{15}{5}\pi, \frac{18}{5}\pi, \frac{21}{5}\pi, \frac{24}{5}\pi, \frac{27}{5}\pi\right\}$$

$$\rightarrow \left\{0, \frac{1}{5}\pi, \frac{2}{5}\pi, \frac{3}{5}\pi, \frac{4}{5}\pi, \pi, \frac{6}{5}\pi, \frac{7}{5}\pi, \frac{8}{5}\pi, \frac{9}{5}\pi\right\}$$

(c)

$M=4, \quad I_n \in \{\pm 1, \pm 3\}$