COM 5120 Communication Theory

Homework #1,

Due:10/14/2021

1. (20%) Consider the rectangular pulse signal $p(t) = A \cdot \Pi(\frac{t}{\tau_0})$ and let the pulse train

$$x(t) = \sum_{-\infty}^{\infty} p(t - nT_0)$$

(1) (5%) Find the magnitude spectrum $|X(f)|$ of $x(t)$

(2) (7%) Find the power spectrum density $S_X(f)$ of $x(t)$

(3) (8%) Find the time -average autocorrelation function $R_X(\tau)$ of $x(t)$

2. (20%) Consider a finite periodic pulse signal $x(t)$ with period T , i.e.

$$x(t) = \sum_{m=-M}^{M} p(t - mT)$$

where p(t)=1 for $-d/2 \le t \le d/2$, otherwise p(t)=0, and d<T.

(1) (10%) Please determine $X(f)$, the Fourier transform of $x(t)$

(2) (10%) Please sketch $X(f)$ and mark the major null frequencies

3. (20%) The Hilbert transform is given by $\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau$. Prove the following properties:

(1) (10%) If $x(t) = x(-t)$, then $\hat{x}(t) = -\hat{x}(-t)$

(2) (10%) If $x(t) = \cos\omega_0 t$,then $\hat{x}(t) = \sin\omega_0 t$

4.(20%) Consider a random process $x(t) = A\cos(2\pi f_0 t + \Theta)$,where A and f_0 are constants and Θ is a random variable with the pdf

$$f_\Theta(\theta) = \begin{cases} \frac{1}{\pi}, |\theta| \leqq \frac{\pi}{2} \\ 0, otherwise \end{cases}$$

(1) (10%) Is $x(t)$ a stationary random process? Explain your answer.

(2) (10%) Is $x(t)$ ergodic? Explain your answer.

5. (20)% Let X and Y be statistically independent Gaussian-distributed random variables, each with zero mean and unit variance. Define the Gaussian process

$$Z(t) = X\cos(2\pi t) + Y\sin(2\pi t)$$

Is the process Z(t) is WSS? Please prove it.