COM 5120 Communication Theory
Homework #5 Solution

1.

To achieve 8000 bps with the symbol rate R = 1/T= 2000 symbols per second, the number of information bits per symbol should be:

$$k = \frac{8000}{2000} = 4$$

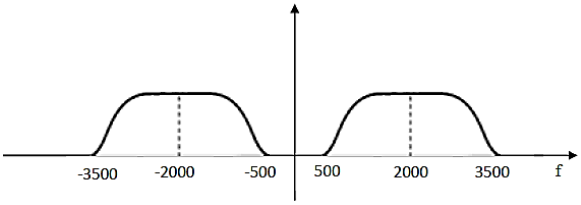
Hence, a 2^4 = 16QAM signal constellation is needed. The carrier frequency is the mid-frequency of the frequency band, which is

$$f_c = \frac{500 + 3500}{2} = 2000Hz$$

To apply the raised cosine pulse shaping with bandwidth W= 3500-500 = 3000Hz, the roll-off factor can be computed as

$$\frac{1}{2T}(1+\beta) = \frac{W}{2} = 1500 \Rightarrow \beta = 0.5$$

And the sketch of the spectrum of the transmitted signal pulse is shown below:



2.

(a)

Data sequence D_n		1	0	0	1	0	1	1	0	0	1
Precoded sequence P_n	0	1	1	1	0	0	1	0	0	0	1
Transmitted sequence I_n	-1	1	1	1	-1	-1	1	-1	-1	-1	1
Received sequence B_n		0	2	2	0	-2	0	0	-2	-2	0
Decoded sequence D_n		1	0	0	1	0	1	1	0	0	1

(b)

Data sequence D_n			1	0	0	1	0	1	1	0	0	1
Precoded sequence P_n	0	0	1	0	1	1	1	0	0	0	0	1
Transmitted sequence I_n	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	1
Received sequence B_n			2	0	0	2	0	-2	-2	0	0	2
Decoded sequence D_n			1	0	0	1	0	1	1	0	0	1

3.

(a) No. R=2500>2W=2400Hz ,ISI will happen

(b) $\mathrm{T}=\frac{1}{\mathrm{R}}=\frac{1}{800}\Rightarrow\frac{1}{2\mathrm{T}}(1+\beta)=\frac{\mathrm{W}}{2}=600\Rightarrow\beta=0.5$

4.

$$y_k=(I_{k-1}+I_{k+1})+v_k$$

$$\rightarrow b_n=Tx(-nT)=\begin{cases}T & ,n=0,-1\\ 0 & ,\text{otherwise}\end{cases}$$

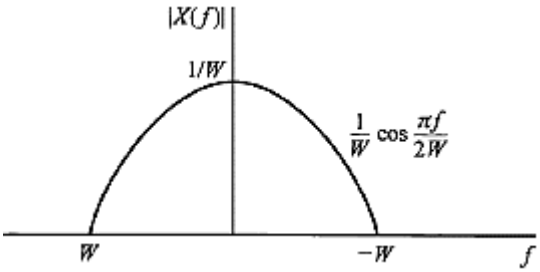
$$\rightarrow B(f)=\sum_{m=-\infty}^{\infty}X(f+\frac{m}{T})=\sum_{m=-\infty}^{\infty}b_n e^{j2\pi nfT}=T+Te^{-j2\pi fT}$$

For $\frac{1}{T}=2W$

$$X(f)=\begin{cases}T+Te^{-j2\pi fT} & ,|f|\le W\\ 0 & ,\text{otherwise}\end{cases}$$

$$=\begin{cases}\frac{1}{2W}e^{-j\pi f\frac{1}{2W}}(e^{j\pi f\frac{1}{2W}}+e^{-j\pi f\frac{1}{2W}}), & |f|\le W\\ 0 & ,\ \text{otherwise}\end{cases}$$

$$=\begin{cases}\frac{1}{W}e^{-j\pi f\frac{1}{2W}}\cos(\frac{\pi f}{2W}) & ,|f|\le W\\ 0 & ,\text{otherwise}\end{cases}$$



5.

$$y_k = (I_{k-1} - I_{k+1}) + v_k$$

$$\rightarrow b_n = Tx(-nT) = \begin{cases} -T & ,n = -1 \\ T & ,n = 1 \\ 0 & ,\text{otherwise} \end{cases}$$

$$\rightarrow B(f) = \sum_{m=-\infty}^{\infty} X(f + \frac{m}{T}) = \sum_{m=-\infty}^{\infty} b_n e^{j2\pi nfT} = Te^{j2\pi fT} - Te^{-j2\pi fT}$$

For $\frac{1}{T} = 2W$

$$X(f) = \begin{cases} Te^{j2\pi fT} - Te^{-j2\pi fT} & ,|f| \le W \\ 0 & ,\text{otherwise} \end{cases}$$

$$= \begin{cases} j2T\sin(2\pi fT) & ,|f| \le W \\ 0 & ,\text{otherwise} \end{cases}$$

$$= \begin{cases} \frac{j}{W}\sin(\frac{\pi f}{W}) & ,|f| \le W \\ 0 & ,\text{otherwise} \end{cases}$$