COM 5120 Communication Theory
Homework #1

Due: 10/14/2020

1. (20%) Consider a finite periodic pulse signal $x(t)$ with period T, i.e.

$$x(t) = \sum_{m=-M}^{M} p(t - mT),$$

where $p(t)$=1 for –d/2≤ t ≤ d/2, otherwise $p(t)$=0, and d < T.

   (a) Please determine $X(f)$, the Fourier transform of $x(t)$.

   (b) Please sketch $X(f)$ and mark the major null frequencies.

2. (20%) Consider the signal $x(t) = 2F sinc(Ft)\cos(2\pi f_0 t)$, where $f_0 \gg F$.

   (a) Find and sketch the spectrum of $x(t)$. (6%)

   (b) Find and sketch the spectrum of $x_+(t) = \frac{1}{2}x(t) + \frac{j}{2}\hat{x}(t)$, where $\hat{x}(t)$ is the Hilbert transform of $x(t)$. (6%)

   (c) Find the complex envelope $\tilde{x}(t)$, where $x_+(t) = \frac{1}{2}\tilde{x}(t) \cdot e^{j2\pi f_0 t}$, and also find and sketch its spectrum. (8%)

3. (20%) Let $y(t) = \int_{t-T}^{t} x(\tau)d\tau$, that is $y(t)$ is the integrator output of $x(t)$, where T is the integration period. The $x(t)$ and $y(t)$ are sample functions of stationary processes $X(t)$ and $Y(t)$, respectively. Let $X(t)$ be the integrator input, please show that the power spectral density $S_Y(f)$ of the integrator output $Y(t)$ is related to that of the integrator input $S_x(f)$ as

$$S_Y(f) = T^2 sinc^2(fT) S_x(f)$$

4. (20%) Let $X(t)$, $Y(t)$ be independent random processes, θ be a random variable uniformly distributed over [0, 2π) and independent of both $X(t)$ and $Y(t)$, φ be a fixed constant in [0, 2π), f_c be a constant frequency($f_c > 0$). Please indicate whether the following statements (a), (b), (c), (d) are true or false and explain why.

   (a) If $X(t)$ is wide sense stationary, then $X(t)\cos(2\pi f_c t + \varphi)$ is cyclo-stationary and $X(t)\cos(2\pi f_c t + \theta)$ is wide sense stationary. (5%)

   (b) Let $X_1(t) = X(t)\cos(2\pi f_c t + \theta)$, $X_2(t) = X(t)\sin(2\pi f_c t + \theta)$, then $X_1(t)$ and $X_2(t)$ both have zero mean. (5%)

   (c) The $X_1(t)$ and $X_2(t)$ in (b) are uncorrelated random processes. (5%)

   (d) Let $Y_2(t) = Y(t)\sin(2\pi f_c t + \theta)$, then $X_1(t)$ in (b) and $Y_2(t)$ are uncorrelated random processes. (5%)

5. (20%) A pair of noise random processes $n_1(t)$ and $n_2(t)$ are related by

$$n_2(t) = n_1(t)\cos(2\pi f_c t + \theta) - n_1(t)\sin(2\pi f_c t + \theta)$$

where f_c is a constant, and θ is the value of a random variable ϑ whose probability density function is defined by

$$f_\vartheta(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \le \theta \le 2\pi \\ 0, & otherwise \end{cases}$$

The noise process $n_1(t)$ is stationary and its power spectral density is as shown in Figure 1. Find and plot the corresponding power spectral density of $n_2(t)$. (20%)

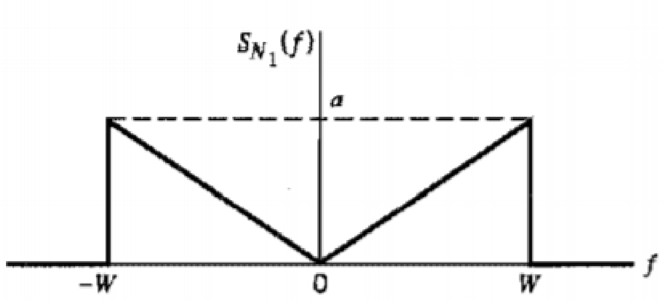

Figure 1.