COM 5120 Communication Theory
Homework #6
Due: 1/11/2022(Tue) 18:00

1. (20%) Consider transmission of a BPSK signal over an unequalized linear filter channel. The information sequence $\{I_n\}$ is independent binary random sequence taking values of ± 1. When $\{I_n\}$ is transmitted, the received signal is demodulated with matched filter and is passed through a whitening filter. The output of the whitening filter is $y_n = I_n - \frac{1}{4} I_{n-1} + \eta_n$ where $\{\eta_n\}$ is a real-valued white Gaussian noise sequence with zero-mean and variance σ^2.
   (a) (10%) Design a three-tap zero-forcing (ZF) equalizer with coefficients $c = \{c_{-1}, c_0, c_1\}$ so that the output is $q_k = \begin{cases} 1, & \text{k=0} \\ 0, & \text{k=}\pm 1 \end{cases}$
   (b) (10%) Please determine the residual ISI after the equalizer.

2. (20%) Consider a linear channel with bandwidth W. The channel is equally divided into three sub-channels which has squared magnitude response $|H(f)|^2$ in the piecewise-linear form with $|H(f_i)|^2 = 1, \frac{1}{3}, \frac{1}{9}$ for sub-channels $i = 1, 2$ and 3 respectively. Assume the system transmits data at the rate equal to the Shannon's channel capacity and the noise variance $\sigma_i^2 = 1, \frac{1}{2}, \frac{1}{4}$ for sub-channels $i = 1, 2$ and 3 respectively.
   (a) (10%) Let the total transmit power be constrained such that $P_1 + P_2 + P_3 = P$ is constant. Please derive the formulas for the optimum powers $P_1, P_2,$ and P_3 allocated to the three sub-channels of frequency bands such that the overall channel capacity of the entire system can be maximized.
   (b) (10%) Given the total transmit power P = 2, and subchannel bandwidth $\Delta f = 1$, please calculate the corresponding values of $P_1, P_2,$ and P_3.

3. (20%) Consider transmission of an independent binary random sequence $\{I_n\}$ taking values of $\pm\sqrt{E_b}$, at the period of T with equal probability. The channel is band-limited with distortion which introduces ISI over adjacent symbols. For an isolated transmitted signal $s(t)$, the noise free output of the demodulator $y_k = I_k + \frac{I_{k-1}}{4} + n_k$. The n_k is additive white Gaussian noise of zero-mean and power spectral density $N_0/2$. Please determine the average error probability of the system if ISI is not compensated.

4. (20%) In a binary PAM system, the input to the detector is $y_m = a_m + n_m + i_m$, where $a_m \pm 1$ is the desired signal, n_m is a zero-mean Gaussian random variable with variance σ_n^2 and i_m represents the ISI due to channel distortion. The ISI term is a random variable that takes the values $-\frac{1}{2}, 0$ and $\frac{1}{2}$ with probabilities $\frac{1}{4}, \frac{1}{2}$, and $\frac{1}{4}$, respectively. Determine the average probability of error as a function of σ_n^2.

5. (20%) Binary PAM is used to transmit information over an unequalized linear filter channel. When $a = 1$ is transmitted, the noise-free output of the demodulator is

$$x_m = \begin{cases} 0.3 & m = 1 \\ 0.9 & m = 0 \\ 0.3 & m = -1 \\ 0 & otherwise \end{cases}$$

   (a) (10%) Design a three-tap zero-forcing linear equalizer so that the output is

$$q_m = \begin{cases} 1 & m = 0 \\ 0 & m = \pm 1 \end{cases}$$

(b) (10%) Determine q_m for $m = \pm 2, \pm 3$, by convolving the impulse response of the equalizer with the channel response.