COM 5120 Communication Theory
Homework #2
Due: 11/02/2020

1. (20%) Consider the set of three(M=3) finite-energy signaling waveforms in $0 \le \mathrm{t} \le 1$:
   $s_1(t) = 1 \qquad\qquad 0 \le \mathrm{t} \le 1$
   $s_2(t) = \cos(8\pi t) \qquad 0 \le \mathrm{t} \le 1$
   $s_3(t) = 2cos^2(4\pi t)\; 0 \le \mathrm{t} \le 1$
   The channel is AWGN with PSD $N_0/2 = 10^{-1}$W/Hz.
   (a) Find a set of orthonormal basis function to represent the set of signal. (10%)
   (b) Draw the corresponding signal constellation. (10%)

2. (20%) Consider the BFSK signals,
   $s_1(t) = Acos2\pi f_1 t,\; s_2(t) = Acos(2\pi f_2 t + \theta)$,
   where f_1 and f_2 are frequency, θ is phase difference. The period of signal is 10ms.
   (a) If these two signals should be coherent orthogonal(if θ=0), then determine the minimum $f_2 - f_1$ to demodulate without distortion. (10%)
   (b) If these two signals should be non-coherent orthogonal(if $\theta \neq 0$), then determine the minimum $f_2 - f_1$ to demodulate without distortion. (10%)

3. (20%) The power density spectrum of the cyclostationary process

$$\mathrm{v(t)} = \sum_{n=-\infty}^{\infty} I_n\, g(t - nT)$$

   can be derived by averaging the autocorrelation function $R_v(t+\tau,\; t)$ over the period T of the process and then evaluating the Fourier transform of the average autocorrelation function. An alternative approach is to change the cyclostatwnary process into a stationary process $v_\Delta(t)$ by adding a random variable Δ, uniformly distributed over $0 \le \Delta < \mathrm{T}$, so that

$$v_\Delta(\mathrm{t}) = \sum_{n=-\infty}^{\infty} I_n\, g(t - nT - \Delta)$$

   And defining the spectral density of v(t) as the Fourier transform of the autocorrelation function of the stationary process $v_\Delta(\mathrm{t})$. Derive the following equation by evaluating the autocorrelation function of $v_\Delta(\mathrm{t})$ and its Fourier transform.

$$S_{v_\Delta v_\Delta}(f) = \frac{1}{T}|G(f)|^2 S_{ii}(f)$$

4. (20%) Consider the PAM signal $S(t) = \sum_n b_n h(t - nT)$. Suppose that h(t) = 1, $0 \le t \le T$ and h(t) = 0 otherwise, and $b_n = a_n - a_{n-2}$, where $\{a_n = \pm 1\}$ is a sequence of uncorrelated random variables with $P_r[a_n = 1] = P_r[a_n = -1] = \frac{1}{2}$.
   (a) Determine the autocorrelation function of the sequence $\{b_n\}$. (10%)
   (b) Determine the power spectral density of S(t). (10%)

5. (20%) Given the M-ary PAM signal of

$$d(t) = \sum_n I_n g(t - nT), \qquad nT \le t \le (n+1)T$$

where the binary sequence b_n is mapped into $I_n \in \{\pm 1, \pm 3, \ldots, \pm(M-1)\}$. Let

$$g(t) = \begin{cases} \frac{1}{2T}, & 0 \le t \le T \\ 0, & otherwise \end{cases} \quad and \quad q(t) = \int_0^t g(\tau)d\tau$$

To construct a M-ary CPFSK from the M-ary PAM signal, each M-PAM signal I_n is mapped into frequency $I_n \in \{\pm 1, \pm 3, \ldots, \pm(M-1)\}$, and the CPFSK signal is constructed as

$$S(t) = \sqrt{\frac{2E}{T}} \cos[2\pi f_c t + \theta(t, I_n)]$$

where $\theta(t, I_n) = 4\pi f_d T \int_{-\infty}^{t} d(\tau)d\tau$. Let $\theta(nT) = 2\pi f_d T \sum_{k=-\infty}^{n-1} I_k$.

   (a) Please identify the expression of $\theta(t, I_n)$ in terms of $\theta(nT)$, I_n and $q(t)$.(10%)
   (b) Define $h = 2f_d T$. Please determine the number of terminal phase states in the state trellis diagram for h = 3/5 and M = 4.(5%)
   (c) Following from (b), please sketch the phase trellis of the signal s(t) for t=0, T, 2T, 3T, 4T.(5%)