Communication system Homework#2 (Spring 2020)

3.18 The local oscillator used for the demodulation of an SSB signal $s(t)$ has a frequency error Δf measured with respect to the carrier frequency f_c used to generate $s(t)$. Otherwise, there is perfect synchronism between this oscillator in the receiver and the oscillator supplying the carrier wave in the transmitter. Evaluate the demodulated signal for the following two situations:

(a) The SSB signal $s(t)$ consists of the upper sideband only.
(b) The SSB signal $s(t)$ consists of the lower sideband only.

3.19 Figure P3.19 shows the block diagram of *Weaver's method* for generating SSB modulated waves. The message (modulating) signal $m(t)$ is limited to the frequency band $f_a \le |f| \le f_b$. The auxiliary carrier applied to the first pair of product modulators has a frequency f_0, which lies at the center of this band, as shown by

$$f_0 = \frac{f_a + f_b}{2}$$

The low-pass filters in the upper and lower branches are identical, each with a cutoff frequency equal to $(f_b - f_a)/2$. The carrier applied to the second pair of product modulators has a frequency f_c that is greater than $(f_b - f_a)/2$. Sketch the spectra at the various points in the modulator of Figure P3.19, and hence show that:

(a) For the lower sideband, the contributions of the upper and lower branches are of opposite polarity, and by adding them at the modulator output, the lower sideband is suppressed.
(b) For the upper sideband, the contributions of the upper and lower branches are of the same polarity, and by adding them, the upper sideband is transmitted.

4.1 Sketch the PM and FM waves produced by the sawtooth wave shown in Figure P4.1.

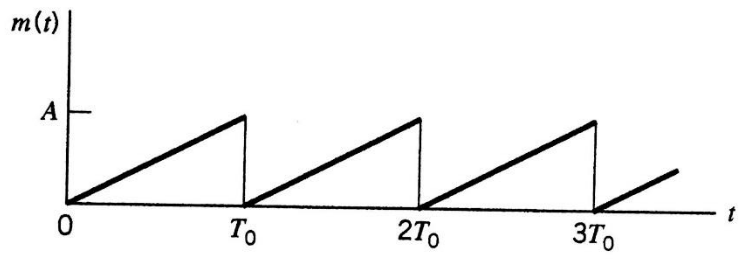

Figure P4.1

4.5 The sinusoidal modulating wave

$$m(t) = A_m \cos(2\pi f_m t)$$

is applied to a phase modulator with phase sensitivity k_p. The unmodulated carrier wave has frequency f_c and amplitude A_c.

(a) Determine the spectrum of the resulting phase-modulated signal, assuming that the maximum phase deviation $\beta_p = k_p A_m$ does not exceed 0.3 radians.

(b) Construct a phasor diagram for this modulated signal, and compare it with that of the corresponding narrow-band FM signal.

4.8 A carrier wave is frequency-modulated using a sinusoidal signal of frequency f_m and amplitude A_m.

(a) Determine the values of the modulation index β for which the carrier component of the FM signal is reduced to zero. See the Appendix for calculating $J_0(\beta)$.

(b) In a certain experiment conducted with $f_m = 1$ kHz and increasing A_m (starting from 0 volts), it is found that the carrier component of the FM signal is reduced to zero for the first time when $A_m = 2$ volts. What is the frequency sensitivity of the modulator? What is the value of A_m for which the carrier component is reduced to zero for the second time?

4.10 A carrier wave of frequency 100 MHz is frequency-modulated by a sinusoidal wave of amplitude 20 volts and frequency 100 kHz. The frequency sensitivity of the modulator is 25 kHz per volt.

(a) Determine the approximate bandwidth of the FM signal, using Carson's rule.

(b) Determine the bandwidth by transmitting only those side frequencies whose amplitudes exceed 1 percent of the unmodulated carrier amplitude. Use the universal curve of Figure 4.9 for this calculation.

(c) Repeat your calculations, assuming that the amplitude of the modulating signal is doubled.

(d) Repeat your calculations, assuming that the modulation frequency is doubled.

4.12 Figure P4.12 shows the block diagram of a real-time *spectrum analyzer* working on the principle of frequency modulation. The given signal $g(t)$ and a frequency-modulated signal $s(t)$ are applied to a multiplier and the output $g(t)s(t)$ is fed into a filter of impulse response $h(t)$. The $s(t)$ and $h(t)$ are *linear FM signals* whose instantaneous frequencies vary at opposite rates, as shown by

$$s(t) = \cos(2\pi f_c t - \pi k t^2),$$
$$h(t) = \cos(2\pi f_c t + \pi k t^2)$$

where k is a constant. Show that the envelope of the filter output is proportional to the amplitude spectrum of the input signal $g(t)$ with kt playing the role of frequency f. *Hint:* Use the complex notations described in Chapter 2 for the analysis of band-pass signals and band-pass filters.

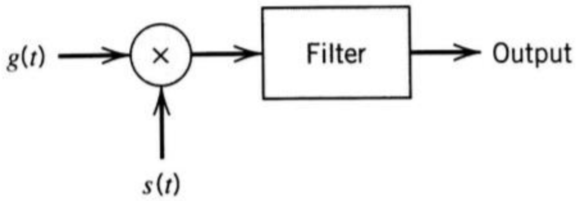

Figure P4.12

4.16 The FM signal

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt\right]$$

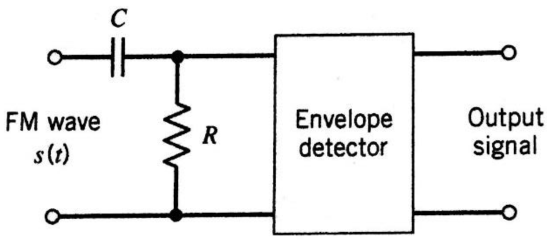

Figure P4.16

is applied to the system shown in Figure P4.16 consisting of a high-pass RC filter and an envelope detector. Assume that (a) the resistance R is small compared with the reactance of the capacitor C for all significant frequency components of $s(t)$ and (b) the envelope detector does not load the filter. Determine the resulting signal at the envelope detector output, assuming that $k_f|m(t)| < f_c$ for all t.

4.19 Figure P4.19 shows the block diagram of a *zero-crossing detector* for demodulating an FM signal. It consists of a limiter, a pulse generator for producing a short pulse at each zero-crossing of the input, and a low-pass filter for extracting the modulating wave.

(a) Show that the instantaneous frequency of the input FM signal is proportional to the number of zero crossings in the time interval $t - (T_1/2)$ to $t + (T_1/2)$, divided by T_1. Assume that the modulating signal is essentially constant during this time interval.

(b) Illustrate the operation of this demodulator, using the sawtooth wave of Figure P4.1 as the modulating wave.

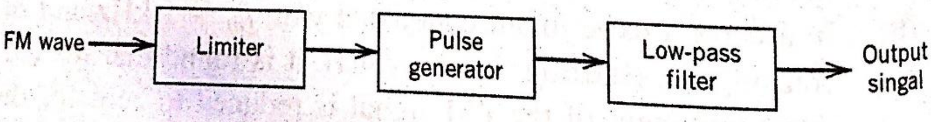

Figure P4.19

4.21

(a) Let the modulated wave $s(t)$ in Problem 4.20 be applied to an ideal amplitude *limiter*, whose output $z(t)$ is defined by

$$z(t) = \operatorname{sgn}[s(t)] = \begin{cases} +1, & s(t) > 0 \\ -1, & s(t) < 0 \end{cases}$$

Show that the limiter output may be expressed in the form of a Fourier series as follows:

$$z(t) = \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\cos[2\pi f_c t(2n+1) + (2n+1)\phi(t)]$$

(b) Suppose that the limiter output is applied to a band-pass filter with a passband amplitude response of one and bandwidth B_T centered about the carrier frequency f_c, where B_T is the transmission bandwidth of the FM signal in the absence of amplitude modulation. Assuming that f_c is much greater than B_T, show that the resulting filter output equals

$$y(t) = \frac{4}{\pi}\cos[2\pi f_c t + \phi(t)]$$

By comparing this output with the original modulated signal $s(t)$ defined in Problem 4.20, comment on the practical usefulness of the result.