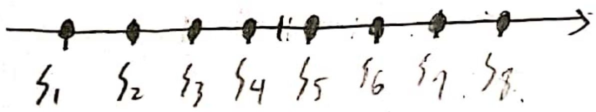
5.2.

$$s_i(t) = A_i \,\mathrm{rect}\left(\frac{t}{T} - \frac{1}{2}\right), \quad A_i = -9 + 2i \quad i = 1, \cdots 8$$



5.3.

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{\int_0^T s_1^2(t)\,dt}} = \sqrt{\frac{3}{T}}\, s_1(t)$$

$$s_{21} = \int_0^T s_2(t)\phi_1(t)\,dt = \sqrt{\frac{T}{3}}$$

$$g_2(t) = s_2(t) - \sqrt{\frac{T}{3}}\,\phi_1(t)$$

$$\Rightarrow \phi_2(t) = \frac{g_2(t)}{\sqrt{\int_0^T g_2^2(t)\,dt}} = \sqrt{\frac{3}{T}}\, g_2(t)$$

$$s_{31} = \int_0^T s_3(t)\phi_1(t) = 0$$

$$s_{32} = \int_0^T s_3(t)\phi_2(t) = \sqrt{\frac{T}{3}}$$

$$g_3(t) = s_3(t) - \sqrt{\frac{T}{3}}\,\phi_2(t)$$

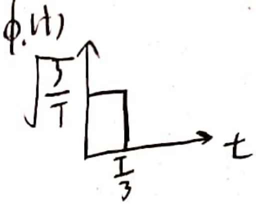
$$\Rightarrow \phi_3(t) = \frac{g_3(t)}{\sqrt{\int_0^T g_3^2(t)\,dt}} = \sqrt{\frac{3}{T}}\, g_3(t)$$

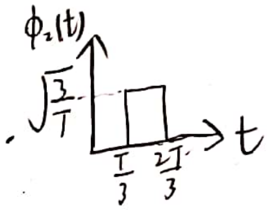
$$s_{41} = \int_0^T s_4(t)\phi_1(t)\,dt = \sqrt{\frac{T}{3}}$$

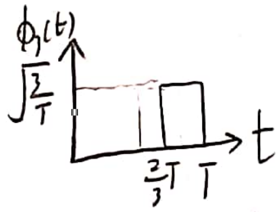
$$s_{42} = \int_0^T s_4(t)\phi_2(t)\,dt = \sqrt{\frac{T}{3}}$$

$$s_{43} = \int_0^T s_4(t)\phi_3(t)\,dt = \sqrt{\frac{T}{3}}$$

$$g_4(t) = s_4(t) - \sqrt{\frac{T}{3}}\phi_1(t) - \sqrt{\frac{T}{3}}\phi_2(t) - \sqrt{\frac{T}{3}}\phi_3(t) = 0$$





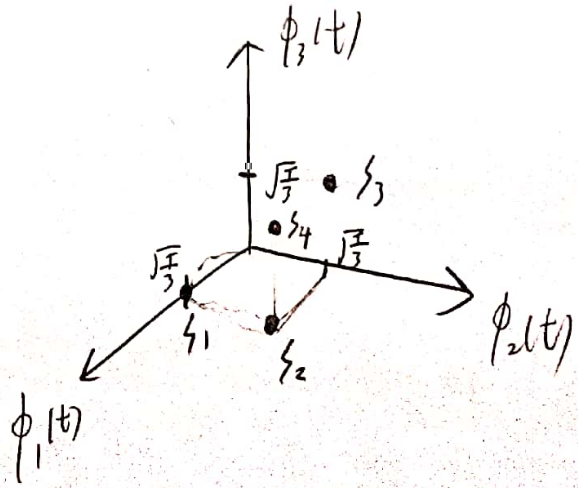


$$s_1(t) = \sqrt{\frac{T}{3}}\,\phi_1(t)$$

$$s_2(t) = \sqrt{\frac{T}{3}}\,\phi_1(t) + \sqrt{\frac{T}{3}}\,\phi_2(t)$$

$$s_3(t) = \sqrt{\frac{T}{3}}\,\phi_2(t) + \sqrt{\frac{T}{3}}\,\phi_3(t)$$

$$s_4(t) = \sqrt{\frac{T}{3}}\,\phi_1(t) + \sqrt{\frac{T}{3}}\,\phi_2(t) + \sqrt{\frac{T}{3}}\,\phi_3(t)$$



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5.9.

Let $\langle x,y\rangle$ denote the inner product of the vectors x and y and $\|x\|^2=\langle x,x\rangle$

Then for $x,y\neq 0$

$$0\le\|x-cy\|^2=\langle x-cy,x-cy\rangle=\langle x,x-cy\rangle-c\langle y,x-cy\rangle$$

$$=\langle x,x\rangle-c^*\langle x,y\rangle-c\langle y,x\rangle+cc^*\langle y,y\rangle \text{ —— } ①$$

Take $c=\frac{\langle x,y\rangle}{\langle y,y\rangle}$

Then $① = \|x\|^2-\frac{\langle x,y\rangle^*\langle x,y\rangle+\langle x,y\rangle\langle y,x\rangle-\langle x,y\rangle\langle x,y\rangle^*}{\langle y,y\rangle}$

$$=\|x\|^2-\frac{|\langle x,y\rangle|^2}{\|y\|^2}\ge 0 \qquad (\langle x,y\rangle^*=\langle y,x\rangle$$

$\Rightarrow |\langle x,y\rangle|^2\le\|x\|^2\|y\|^2$ and "=" holds iff $y=kx$ where k is constan

Otherwise, for any functions $f:\mathbb{R}\to\mathbb{C}$, $g:\mathbb{R}\to\mathbb{C}$

We can define $\langle f,g\rangle=\int_{-\infty}^{\infty}g^*(t)f(t)\,dt=\int_{-\infty}^{\infty}f(t)\,g(t)$

Then $\langle f,g\rangle$ is an inner product of f and g

Thus, $\left|\int_{-\infty}^{\infty}s_1(t)s_2^*(t)\,dt\right|^2\le\int_{-\infty}^{\infty}|s_1(t)|^2dt\int_{-\infty}^{\infty}|s_2(t)|^2dt$

and "=" holds iff $s_2(t)=c\,s_1(t)$ where c is constant

\#

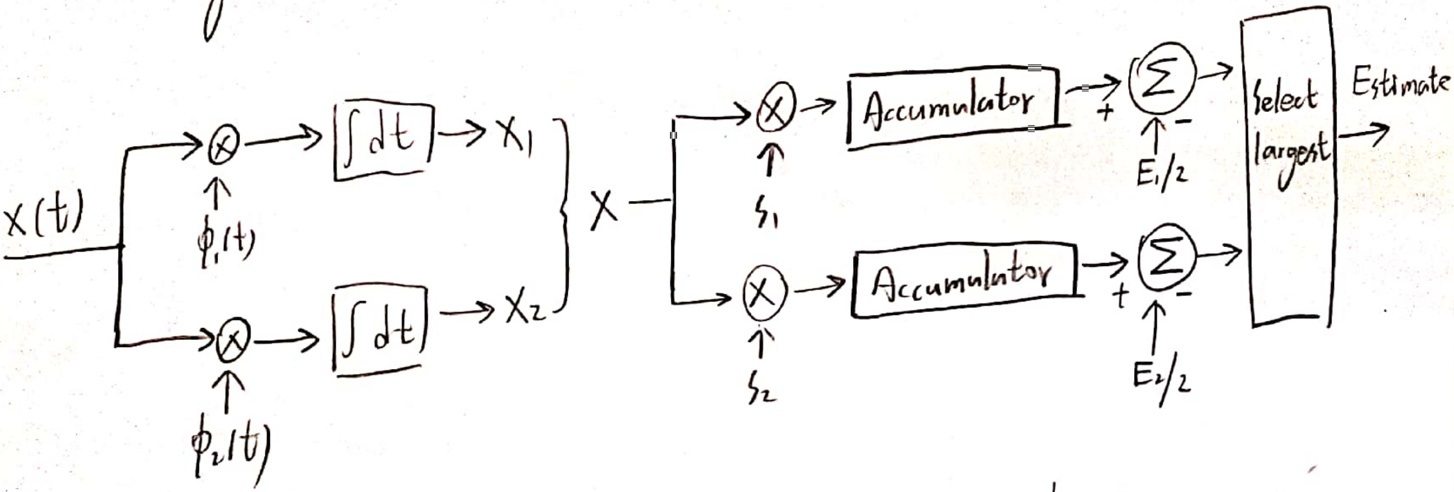
2.

(a)

Let $\phi_1(t) = \sqrt{\frac{1}{3T}} s_1(t)$ then $\{\phi_1(t), \phi_2(t)\}$ is an orthonormal basis.

$\phi_2(t) = \sqrt{\frac{1}{3T}} s_2(t)$

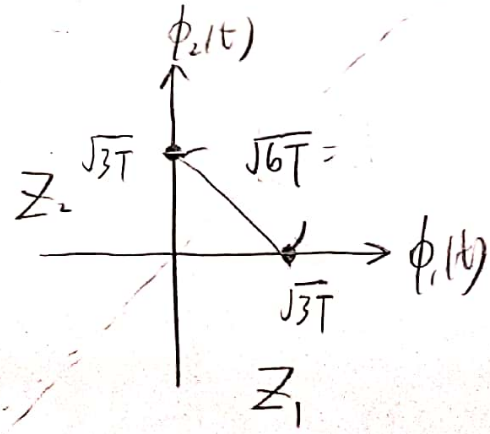
According to correlation receiver $(E = E_1 = E_2 = 3T)$



(b) $(\frac{E}{N_0} = 4,\ E = 3T,\ d_{12} = d_{21} = \sqrt{6T}$

Since we have only two signals

we can conclude that



$P_e = \frac{1}{2} P_2(s_1, s_2) + \frac{1}{2} P_2(s_2, s_1)$

$= \frac{1}{4} \operatorname{erfc}\left(\frac{d_{12}}{2\sqrt{N_0}}\right) + \frac{1}{4} \operatorname{erfc}\left(\frac{d_{21}}{2\sqrt{N_0}}\right)$

$= \frac{1}{2} \operatorname{erfc}\left(\frac{\sqrt{2E}}{2\sqrt{N_0}}\right)$

$= \frac{1}{2} \operatorname{erfc}(\sqrt{2})$ #

$\sqrt{6T}$

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5.17.

(a) We know that rotation and translation don't affect the probability of symbol error

And the constellation of (b) is actually derived by roatating (a) by 90° and right shift $\sqrt{2}a$

Thus, the two constellation have the same average probability of symbol error

(b)

For (a), each signal has equal energy $2\alpha^2$

$\Rightarrow$ Average energy of (a) is $2\alpha^2$

For (a), the total energy is $0+\alpha^2+\alpha^2+8\alpha^2=10\alpha^2$

$\Rightarrow$ Average energy of (b) is $\frac{5}{2}\alpha^2$

Thus, the constellation of (b) has minimum average energy

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