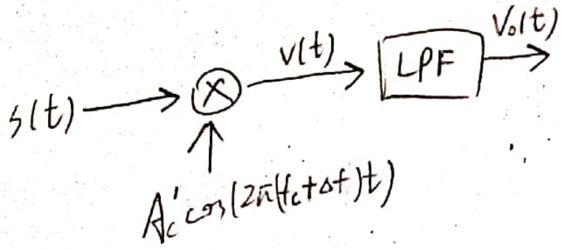
105021202 楊曄之

通一 HW2

3.18.



m(t) : baseband message signal

$$M_r(f) = \begin{cases} M(f) & \text{if } f>0 \\ \frac{1}{2}M(0) & \text{if } f=0 \\ 0 & \text{else} \end{cases}$$

$$M_l(f) = \begin{cases} M(f) & \text{if } f<0 \\ \frac{1}{2}M(0) & \text{if } f=0 \\ 0 & \text{else} \end{cases}$$

$$V(t) = s(t) A_c' \cos(2\pi(f_c+\Delta f)t)$$

$$\Rightarrow V_o(f) = \frac{A_c'}{2}\left(S(f-f_c-\Delta f) + S(f+f_c+\Delta f)\right)$$

Note that $\hat{m}(t)$ is the signal generated by Hilbert filter with input $m(t)$

and $s(t) = m(t) A_c \cos(2\pi f_c t) \pm \hat{m} \sin(2\pi f_c t)$, "+" for lower sideband, "−" for upper sideband

(a) s is upper sideband

$$V_o(f) = \frac{A_c'}{2}\left(S(f-f_c-\Delta f) + S(f+f_c+\Delta f)\right) = \frac{A_cA_c'}{4}\left(M_l(f-\Delta f) + M_r(f+\Delta f)\right)$$

$\Rightarrow$ If $\Delta f > 0$, then $V_o(t)$ is lower sideband signal generated by $m(t)$ with carrier frequency Δf

$$V_o(t) = \frac{A_cA_c'}{4}\left(m(t)\cos(2\pi\Delta f t) + \hat{m}\sin(2\pi\Delta f t)\right)$$

Conversely, if $\Delta f < 0$ then $V_o(t)$ is upper sideband

$$V_o(t) = \frac{A_cA_c'}{4}\left(m(t)\cos(2\pi\Delta f t) - \hat{m}\sin(2\pi\Delta f t)\right)$$

(b) s(t) is lower sideband

Similarly $V_o(f) = \frac{A_cA_c'}{4}\left(M_l(f+\Delta f) + M_r(f-\Delta f)\right)$

$\Rightarrow$ If $\Delta f > 0$, then $V_o(t)$ is upper sideband

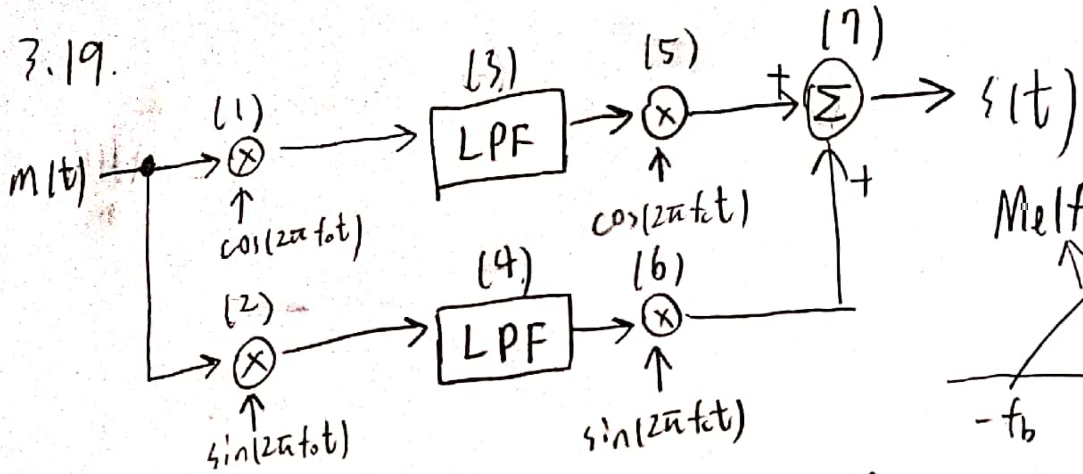
$$V_o(t) = \frac{A_cA_c'}{4}\left(m(t)\cos(2\pi\Delta f t) - \hat{m}\sin(2\pi\Delta f t)\right)$$

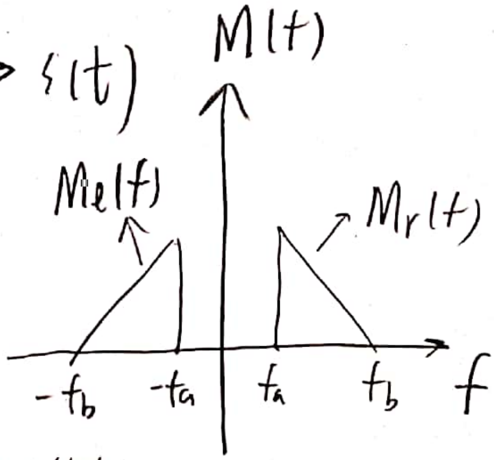
Conversely, if $\Delta f < 0$ then $V_o(t)$ is lower sideband

$$V_o(t) = \frac{A_cA_c'}{4}\left(m(t)\cos(2\pi\Delta f t) + \hat{m}\sin(2\pi\Delta f t)\right)$$

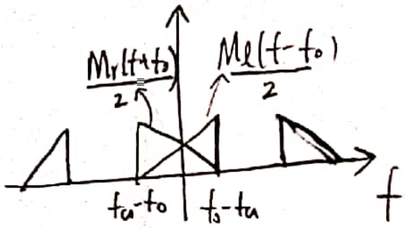
#

3.19.

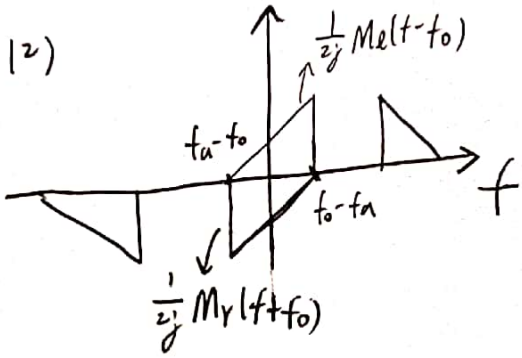




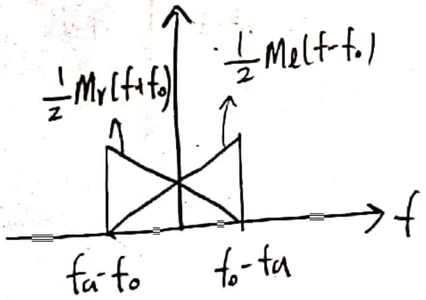
(1)



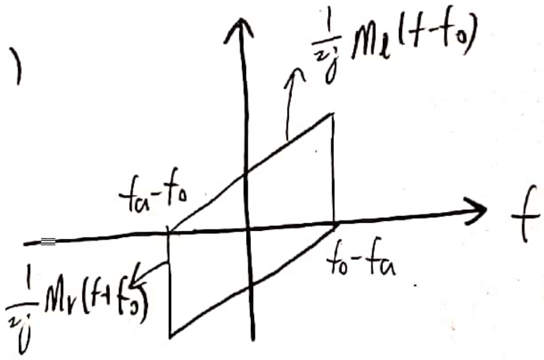
(2)



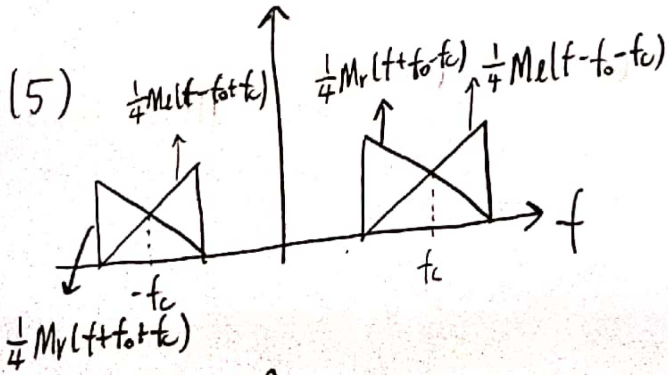
(3)



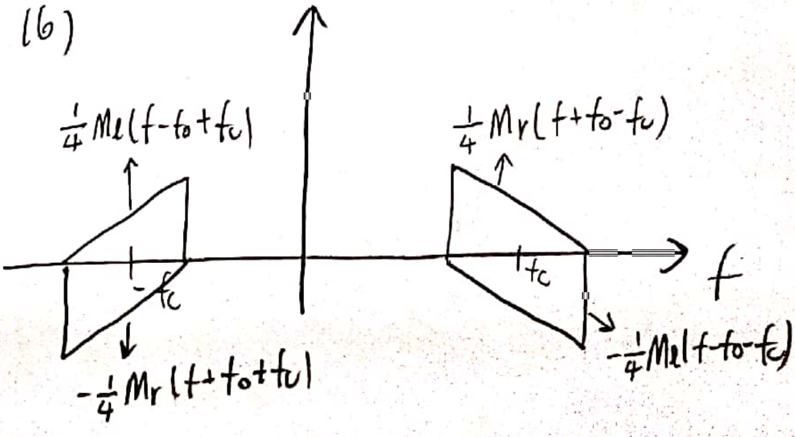
(4)



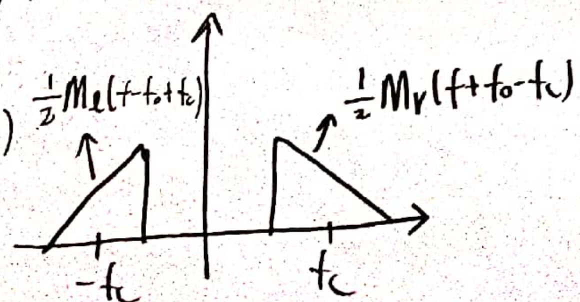
(5)



(6)



(7)



It's easy to see the result of (a) and (b) by the graphs (1) ~ (7)

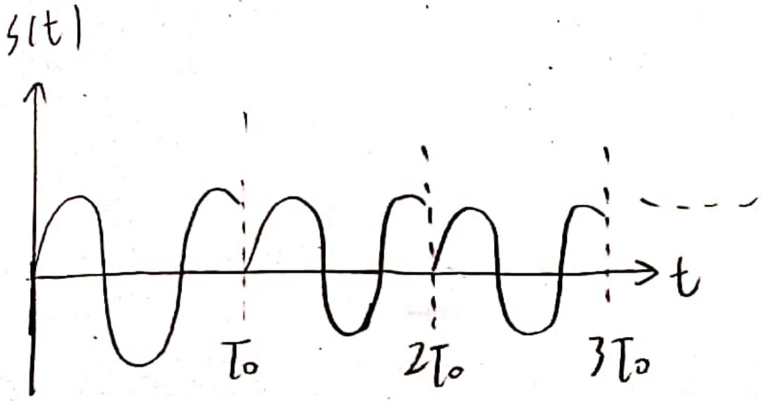
#

4.1

For PM, let $s_n(t) = A_c\cos[2\pi f_c t + k_p m(t)] = A_c\cos\left[2\pi f_c t + \frac{k_p A}{T_0}(t - nT_0)\right]$ for $nT_0 \le t \le (n+1)T_0$ where $n \in \mathbb{N} \cup \{0\}$

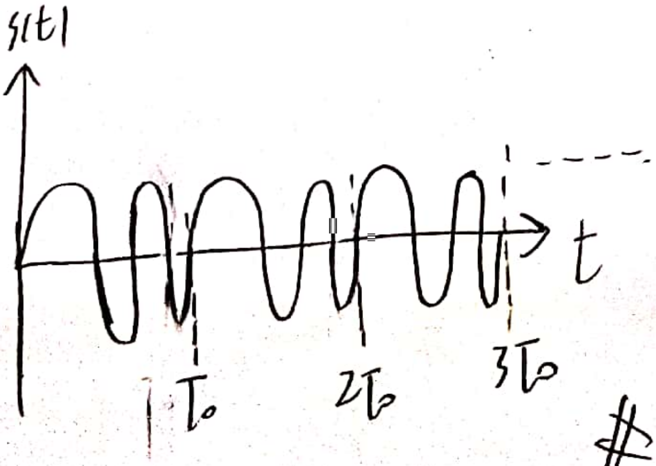
Then $s(t) = \sum_{n=0}^{\infty} s_n(t)$ and $f_i(t) = f_c + \frac{k_p A}{2\pi T_0}$ is constant

To simplify the graph, assume $f_c T_0 = 1$



For FM let $f_n(t) = f_c + \frac{k_f A}{T_0}(t - nT_0)$ for $nT_0 \le t \le (n+1)T_0$, where $n \in \mathbb{N} \cup \{0\}$

Then $f_i(t) = \sum_{n=0}^{\infty} f_n(t)$, $s(t) = A_c\cos[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau]$



#

4.5

$$m(t)=A_m\cos(2\pi f_m t)$$

(a) $$s(t)=A_c\cos\left(2\pi f_c t+k_p m(t)\right)=A_c\cos\left(2\pi f_c t+\beta_p\cos(2\pi f_m t)\right)$$

$$=A_c\cos(2\pi f_c t)\cos\left(\beta_p\cos(2\pi f_m t)\right)-A_c\sin(2\pi f_c t)\sin\left(\beta_p\cos(2\pi f_m t)\right)$$

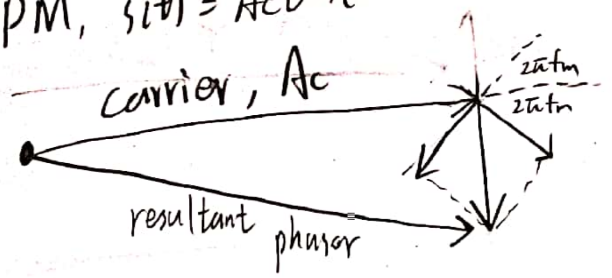
$$\approx A_c\cos(2\pi f_c t)-A_c\sin(2\pi f_c t)\left(\beta_p\cos(2\pi f_m t)\right)$$

$$=A_c\cos(2\pi f_c t)-\frac{A_c\beta_p}{2}\sin(2\pi(f_c+f_m)t)-\frac{A_c\beta_p}{2}\sin(2\pi(f_c-f_m)t)$$

$$\Rightarrow S(f)=\frac{A_c}{2}\left(\delta(f-f_c)+\delta(f+f_c)\right)-\frac{A_c\beta_p}{4j}\left[\delta(f-f_c-f_m)-\delta(f+f_c+f_m)+\delta(f-f_c+f_m)-\delta(f+f_c-f_m)\right]$$

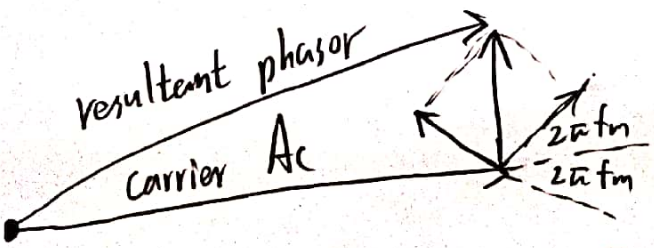
#

(b) PM, $$s(t)=A_c\cos(2\pi f_c t)-\frac{A_c\beta_p}{2}\left[\sin(2\pi(f_c+f_m)t)+\sin(2\pi(f_c-f_m)t)\right]$$



FM

$$s(t)=A_c\cos(2\pi f_c t)+\frac{A_c\beta}{2}\left(\cos(2\pi(f_c+f_m)t)-\cos(2\pi(f_c-f_m)t)\right)$$



The difference between the phasor of PM and FM is their phase.
Their phase is the inverse of each other

#

4.8

(a) To make the carrier component of the FM signal reduce to zero

we need $J_0(\beta)=0$ $\left(s(t)=A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(2\pi(f_c+nf_m)t) \right)$

According to Appendix and interpolation

$J_0(\beta)=0$ at $\beta = 2.4626$, 5.4501, 8.8223, 11.6751

(b) $\beta = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m} = \frac{k_f A_m}{1000}$

$\frac{2k_f}{1000} = 2.4626 \Rightarrow k_f = 1.2313 \times 10^3$

The second time $J_0(\beta)=0$ is at $\beta=5.4501$

$\Rightarrow A_m = \frac{1000}{k_f} \cdot 5.4501 \approx 4.4263$

#

4.10. $m(t) = A_m \cos(2\pi f_m t)$, $A_m = 20 \,(\text{volt})$ $f_m = 100 \,(kHz)$

$s(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau)$, $k_f = 25 \,(kHz/volt)$

$\Delta f = A_m k_f$

(a) According to Carson's rule

$B_T = 2\Delta f + 2 f_m = 1000 + 200 = 1200 \,(kHz)$ #

(b) $\beta = \frac{\Delta f}{f_m} = 5 \Rightarrow 2n_{max} = 16$

$B_T = 2n_{max} f_m = 1600 \,(kHz)$ #

(c) $A_m = 40 \,(\text{volt}) \Rightarrow \Delta f = 1000 \,(kHz)$, $f_m = 100 \,(kHz)$ $\beta = 10$

Then the B_T derived by Carson's rule is

$B_T = 2200 \,(kHz)$.

And according to 1% transmission BW

$\beta = 10 \Rightarrow 2n_{max} = 28$.

$\Rightarrow B_T = 2800 \,(kHz)$ #

(d) $f_m = 200 \,(kHz) \Rightarrow \Delta f = 500 \,(kHz)$, $\beta = 2.5$

Then the B_T derived by Carson's rule is

$B_T = 1400 \,(kHz)$

And according to 1% transmission BW

$\frac{B_T}{\Delta f} = 4$

$\Rightarrow B_T = 2000 \,(kHz)$

#

4.12.

let $x(t) = g(t)s(t) = g(t)\cos(2\pi f_c t - \pi k t^2)$

$h(t) = \cos(2\pi f_c t + \pi k t^2)$

$\Rightarrow \tilde{x}(t) = g(t)e^{-j\pi k t^2}$

$\tilde{h}(t) = e^{j\pi k t^2}$

Then for the output $y(t)$, we have

$$\tilde{y}(t) = \frac{1}{2}\tilde{x}(t) * \tilde{h}(t)$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \tilde{x}(\tau)\tilde{h}(t-\tau)\,d\tau$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} g(\tau)e^{-j\pi k \tau^2} e^{j\pi k (t-\tau)^2}\,d\tau$$

$$= \frac{1}{2}e^{j\pi k t^2}\int_{-\infty}^{\infty} g(\tau)e^{-j2\pi k t \tau}\,d\tau$$

$$= \frac{1}{2}e^{j\pi k t^2} G(kt)$$

#

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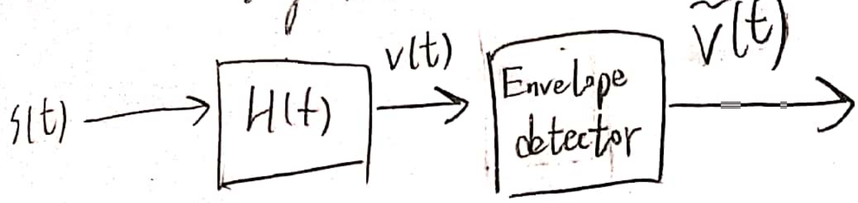
4.16.

$$H(f) = \frac{j2\pi fCR}{1+j2\pi fCR} \simeq j2\pi fCR \quad , \quad s(t) = A_c\cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right]$$



$$v(t) = H(f)s(f) = j2\pi fCR\, s(f)$$

$$\Rightarrow v(t) = CR\frac{ds(t)}{dt} = -A_cCR\sin\left[2\pi f_c t + 2\pi k_f\int_0^t m(\tau)\,d\tau\right]\left(2\pi f_c + 2\pi k_f m(t)\right)$$

$$= -2\pi A_cCR\left(f_c + k_f m(t)\right)\Big[\sin\left(2\pi k_f\int_0^t m(\tau)\,d\tau\right)\cdot\cos(2\pi f_c t) + \cos\left(2\pi k_f\int_0^t m(\tau)\,d\tau\right)\sin(2\pi f_c t)\Big]$$

$$\Rightarrow \tilde{v}(t) = 2\pi A_cCR\big(\underbrace{f_c + k_f m(t)}_{>0}\big) \quad \#$$

4.19.

(a) $s(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau)$, $m(\tau) = C$ is constant

$= A_c \cos(2\pi f_c t + 2\pi k_f C t)$

$= A_c \cos(2\pi(f_c + C)t)$

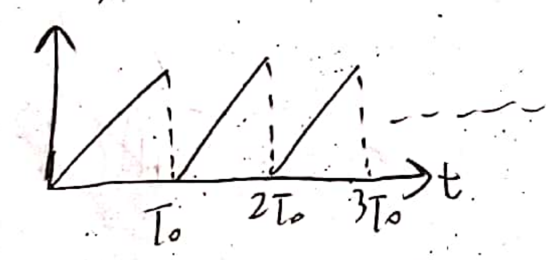
$\Rightarrow$ Then in the time interval $[\frac{t-\frac{T_1}{2}}{T_1}, \frac{t+\frac{T_1}{2}}{T_1}]$

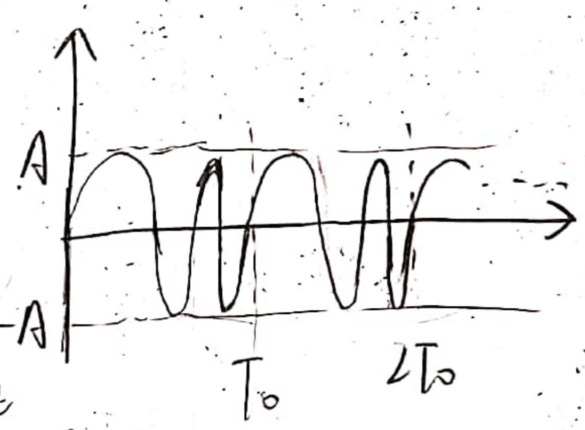
the phase difference is $2\pi(f_c+C)$

$\Rightarrow \frac{2\pi(f_c+C)}{\pi} = 2(f_c+C)$ is the number of zero crossing

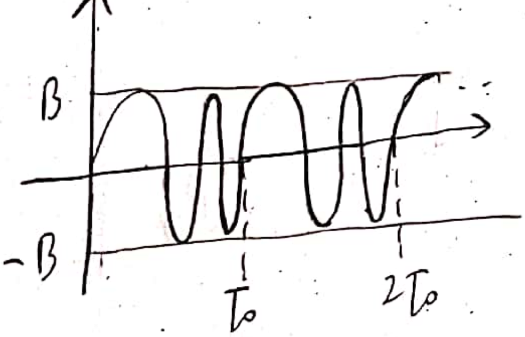
and (f_c+C) is exctaly the instantaneous frequency $f_i(t)$ #

(b)

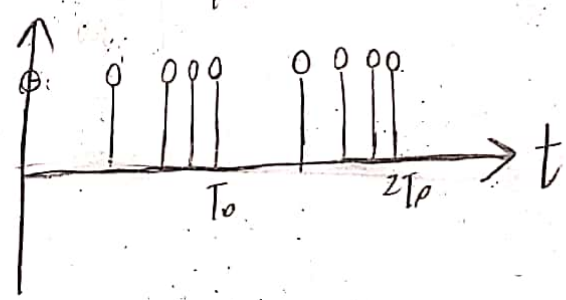




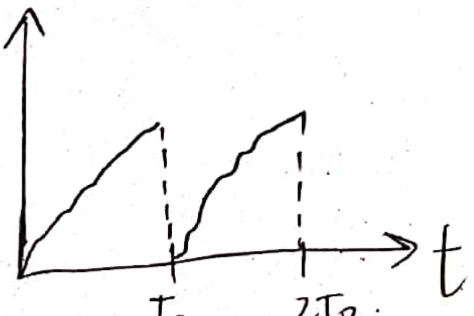
Output of limiter, where B<A



Output of pulse generator



Output of LPF



#

4.21.

(a)

$$z(t) = sgn(s(t)) = sgn(a(t)\cos(2\pi f_c t + \phi(t))) = sgn[\cos(2\pi f_c t + \phi(t))]$$

let $\varphi = 2\pi f_c t + \phi(t)$

Then $z(t) = sgn(\cos(\varphi)) = \sum_{n=-\infty}^{\infty} C_n e^{jnw_0\varphi} = \sum_{n=-\infty}^{\infty} C_n e^{jn\varphi}$, $w_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$

$$\Rightarrow C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} sgn(\cos\varphi) e^{-jnw_0\varphi} d\varphi = \frac{1}{2\pi}\left(-\int_{-\pi}^{-\frac{\pi}{2}} e^{-jn\varphi} d\varphi + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{-jn\varphi} d\varphi - \int_{\frac{\pi}{2}}^{\pi} e^{-jn\varphi} d\varphi\right)$$

$$\Rightarrow C_n = \frac{-1}{j2\pi n}\left(-e^{j\frac{n\pi}{2}} + e^{jn\pi} + e^{-j\frac{n\pi}{2}} - e^{j\frac{n\pi}{2}} - e^{-jn\pi} + e^{-j\frac{n\pi}{2}}\right)$$

$$= \frac{-1}{j2\pi n}\left(j2\left(\sin(n\pi) - 2\sin\left(\frac{n\pi}{2}\right)\right)\right)$$

$$= \frac{1}{\pi n}\left(2\sin\left(\frac{n\pi}{2}\right) - \sin(n\pi)\right) \quad \text{for } n \neq 0$$

$$\Rightarrow C_n = \begin{cases} \frac{2}{\pi n}(-1)^{\frac{n-1}{2}} & \text{if } n \text{ is odd} \\ 0 & \text{else} \end{cases} \quad (\text{Note that } C_n = C_{-n}\ \forall n)$$

$$\Rightarrow z(t) = sgn(\cos(\varphi)) = \sum_{n=-\infty}^{\infty} C_n e^{jn\varphi} = \sum_{k=0}^{\infty} 2C_{2k+1}\cos((2k+1)\varphi)$$

$$= \frac{4}{\pi}\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}\cos(2\pi f_c t(2k+1) + (2k+1)\phi(t))$$

#

(b) Each term of $z(t)$ is centered at the frequency $(2k+1)f_c$ and $f_c \gg B_T$

Thus, the output of the band-pass filter is

$$y(t) = \frac{4}{\pi}\cos(2\pi f_c t + \phi(t))$$

This process remove $a(t)$, changing of amplitude, from $s(t)$

#