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通二
HW4

6.20

$$X=\begin{pmatrix}x_1\\x_2\end{pmatrix}\quad s_1=\begin{pmatrix}\sqrt{E}\\0\end{pmatrix}\quad s_2=\begin{pmatrix}0\\\sqrt{E}\end{pmatrix}$$ wrt the basis ϕ_1, ϕ_2

then $X^T s_1 > X^T s_2 \iff \sqrt{E}\,x_1 > \sqrt{E}\,x_2 \iff x_1 > x_2$

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6.22.

(a)

$$\begin{cases} s_1(t) = A_c\cos\left(2\pi\left(f_c+\frac{\Delta f}{2}\right)t\right), & 0\le t\le T_b \\ s_2(t) = A_c\cos\left(2\pi\left(f_c-\frac{\Delta f}{2}\right)t\right), & 0\le t\le T_b \end{cases}$$

then $\int_0^{T_b} s_1^2(t)\,dt = A_c^2\int_0^{T_b}\cos^2\left(2\pi\left(f_c+\frac{\Delta f}{2}t\right)\right)dt = \frac{A_c^2}{2}\int_0^{T_b} 1+\cos\left(4\pi\left(f_c+\frac{\Delta f}{2}\right)t\right)dt$

$$= \frac{A_c^2}{2}\left(T_b + \frac{\sin\left(4\pi\left(f_c+\frac{\Delta f}{2}\right)T_b\right)}{4\pi\left(f_c+\frac{\Delta f}{2}\right)}\right)$$

$$\approx \frac{T_b}{2}A_c^2 \quad (\text{due to narrow pass-band})$$

Similarly, $\int_0^{T_b} s_2^2(t)\,dt \approx \frac{T_b}{2}A_c^2$

$$\int_0^{T_b} s_1(t)s_2(t)\,dt = A_c^2\int_0^{T_b}\cos\left(2\pi\left(f_c+\frac{\Delta f}{2}\right)t\right)\cos\left(2\pi\left(f_c-\frac{\Delta f}{2}\right)t\right)dt$$

$$= \frac{A_c^2}{2}\int_0^{T_b}\cos(4\pi f_c t)+\cos(2\pi\Delta f t)\,dt$$

$$= \frac{A_c^2}{2}\left(\frac{\sin(4\pi f_c T_b)}{4\pi f_c} + \frac{\sin(2\pi\Delta f T_b)}{2\pi\Delta f}\right)$$

$$\approx \frac{A_c^2}{2}\left(\frac{\sin(2\pi\Delta f T_b)}{2\pi\Delta f}\right) \quad (\text{due to narrow pass-band})$$

$$\Rightarrow \rho = \frac{\int_0^{T_b} s_1(t)s_2(t)\,dt}{\left(\int_0^{T_b}s_1^2(t)\,dt\right)^{1/2}\left(\int_0^{T_b}s_2^2(t)\,dt\right)^{1/2}} \approx \frac{\int_0^{T_b}s_1(t)s_2(t)\,dt}{\int_0^{T_b}s_1^2(t)\,dt} \approx \frac{\sin(2\pi\Delta f T_b)}{2\pi\Delta f T_b} = \operatorname{sinc}(2\pi\Delta f T_b)$$

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(b) $s_1(t)$ and $s_2(t)$ are orthogonal $\Leftrightarrow \int_0^{T_b} s_1(t)s_2(t)\,dt = 0 \Leftrightarrow \rho = \text{sinc}(2\pi\Delta f T_b) = 0$

$\Rightarrow \frac{1}{2T_b}$ is the minimum value of Δf s.t. $s_1(t)$ and $s_2(t)$ are orthogonal

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(c) Let $\{\phi_1, \phi_2\}$ be an orthonormal basis s.t.

$s_1(t) = s_{11}\phi_1(t)$, where $s_{11} = \left(\int_0^{T_b} s_1^2(t)\,dt\right)^{1/2} = \sqrt{E_b}$

$s_2(t) = s_{21}\phi_1(t) + s_{22}\phi_2(t)$

Then $\rho = \frac{\int_0^{T_b} s_1(t)s_2(t)\,dt}{\int_0^{T_b} s_1^2(t)\,dt} = \frac{\int_0^{T_b}(s_{11}\phi_1(t))(s_{21}\phi_1(t)+s_{22}\phi_2(t))\,dt}{E_b} = \frac{s_{11}s_{21}}{E_b}$

$\Rightarrow s_{11}s_{21} = E_b\rho \Rightarrow s_{21} = \sqrt{E_b}\,\rho$

Otherwise $s_{21}^2 + s_{22}^2 = E_b \Rightarrow s_{22} = \sqrt{(1-\rho^2)E_b}$ $\Rightarrow s_1 = \begin{pmatrix}\sqrt{E_b}\\ 0\end{pmatrix}$ $s_2 = \begin{pmatrix}\rho\sqrt{E_b}\\ \sqrt{(1-\rho^2)E_b}\end{pmatrix}$

$\Rightarrow \|s_1 - s_2\| = \sqrt{2(1-\rho)E_b}$

$\Rightarrow P_e = \frac{1}{2}\,\text{erfc}\left(\frac{\|s_1-s_2\|}{2\sqrt{N_0}}\right) = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{(1-\rho)E_b}{2N_0}}\right)$

$\Rightarrow P_e$ has minimum value $\Leftrightarrow \rho$ has minimum value

ρ has minimum -0.2172 at $\Delta f = \frac{0.7151}{T_b}$ (These are approximately value evaluated by Matlab

$\Rightarrow \Delta f = \frac{0.7151}{T_b}$ minimizes the average probability of symbol error

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(d) The average probability of symbol error of BPSK is given by $P_e = \frac{1}{2}\,\text{erfc}\sqrt{\frac{E_b}{N_0}}$

To make this FSK has the same noise performance, it requires $\frac{E_b}{N_0}$ increasing by the factor $\frac{2}{1-\rho}$, where $\rho = -0.2172$

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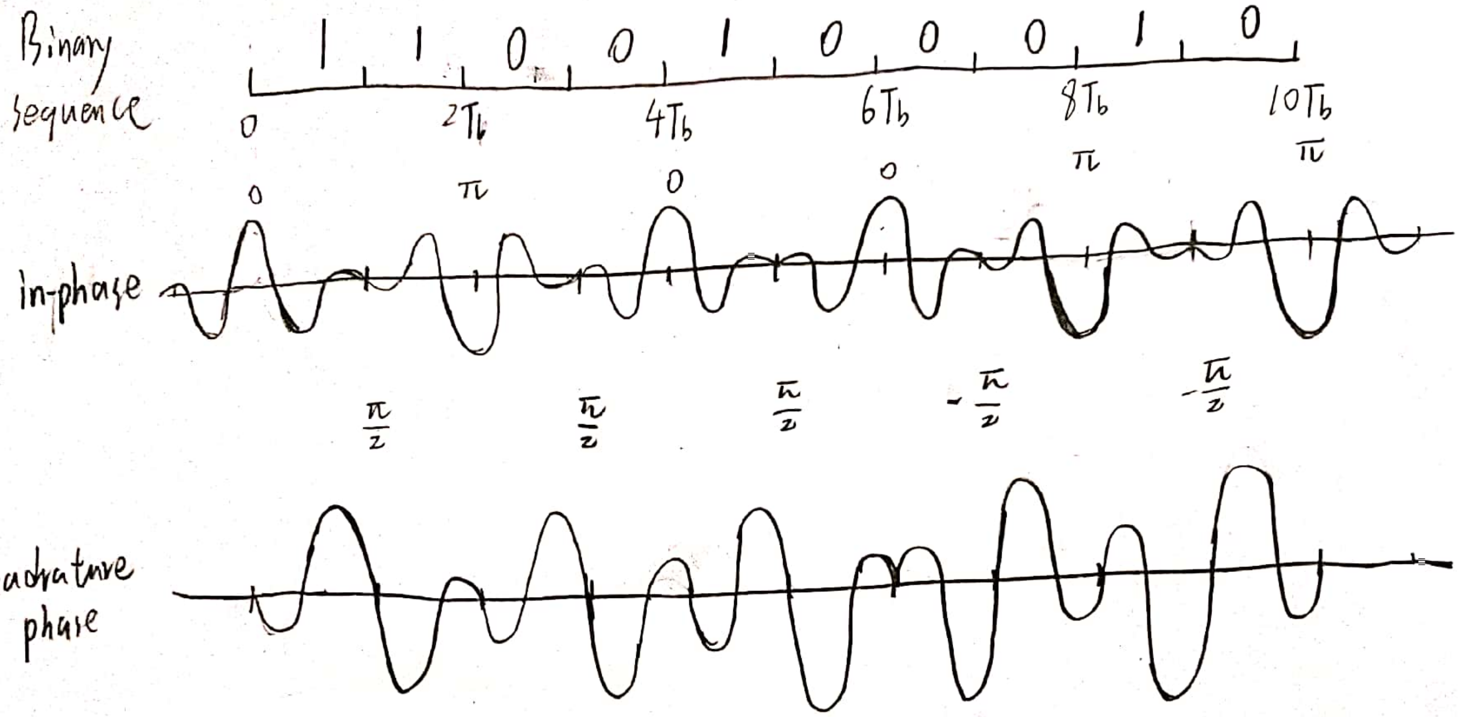
6.27

(a)

Assume the initial phase is 0



(b)