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# 通訊系統 (II)

國立清華大學電機系暨通訊工程研究所

蔡育仁

台達館 821 室

Tel: 62210

E-mail: [yrtsai@ee.nthu.edu.tw](mailto:yrtsai@ee.nthu.edu.tw)

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## Chapter 8 Multichannel Modulation

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# Capacity of AWGN Channel

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## Capacity of AWGN Channel

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- According to **Shannon's information capacity law**, the capacity of an AWGN channel is defined by

$$C = B \log_2 \left[ 1 + P / (N_0 B) \right] = B \log_2 [1 + \text{SNR}] \quad \text{bits/sec}$$

- where  $B$  is the channel bandwidth in hertz and SNR is measured at the **channel output**

- Equivalently, the capacity  $C$  in bits per channel use is

$$C' = \frac{1}{2} \log_2 (1 + P / \sigma^2) = \frac{1}{2} \log_2 (1 + \text{SNR}) \quad \text{bits/transmission}$$

- In practice, we usually find that a **physically realizable encoding system** must transmit data at a rate  $R$  **less than** the maximum possible rate  $C$  for **reliable** reception.

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# Signal-to-Noise Ratio Gap

- For an **implementable** system operating at a certain low enough probability of symbol error
  - Actual SNR  $\Rightarrow$  capacity  $C$
  - We introduce an **signal-to-noise ratio gap** (or just **gap**), denoted by  $\Gamma$ , which is defined by
 
$$\Gamma = \frac{2^{2C} - 1}{2^{2R} - 1} = \frac{\text{SNR}}{2^{2R} - 1}$$
    - Attainable capacity  $R \Rightarrow$  equivalent SNR
    - Depends on the encoding system
  - $C$ : the capacity of the **ideal encoding system**
  - $R$ : the capacity of the corresponding **implementable encoding system**
- It is a function of the **permissible probability of symbol error**  $P_e$  and the **encoding system** of interest
- It provides a measure of the “**efficiency**” of an encoding system
  - with respect to the **ideal transmission system**

## Signal-to-Noise Ratio Gap (Cont.)

- A **small (large)** gap corresponds to an **efficient (inefficient)** encoding system
- Then, we have the attainable transmit data rate
 
$$R = \frac{1}{2} \log_2 (1 + \text{SNR}/\Gamma) \quad \text{bits/transmission}$$
- For example: the desired **probability of symbol error**  $P_e = 10^{-6}$ 
  - For an uncoded PAM or QAM system, the gap is **8.8 dB**
  - Through the use of channel coding (e.g., trellis codes), the gap may be reduced to as low as **1 dB**
- Because  $\text{SNR} = P/N_0B$ , the attainable **data rate** is defined as

$$R = \frac{1}{2} \log_2 \left( 1 + \frac{P}{\Gamma N_0 B} \right) \quad \text{bits/transmission}$$

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# Continuous-Time Channel Partitioning

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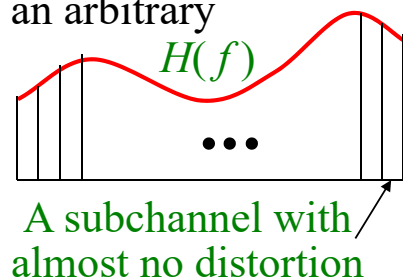
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## Continuous-Time Channel Partitioning

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- Consider a linear **wideband channel** with an arbitrary frequency response  $H(f)$ .
  - The magnitude response  $|H(f)|$  is approximated by a **staircase** function
  - $\Delta f$ : the width of each **subchannel**
- In each step, the channel may be assumed to operate as an AWGN channel **free from inter-symbol interference**.
  - Transmitting a **wideband signal** is transformed into the transmission of a set of **narrowband orthogonal signals**
  - Each orthogonal **narrowband signal**, with **its own carrier**, is generated using a modulation technique, e.g.,  $M$ -ary QAM
    - AWGN is the only transmission impairment (with a **constant response** for each subchannel)



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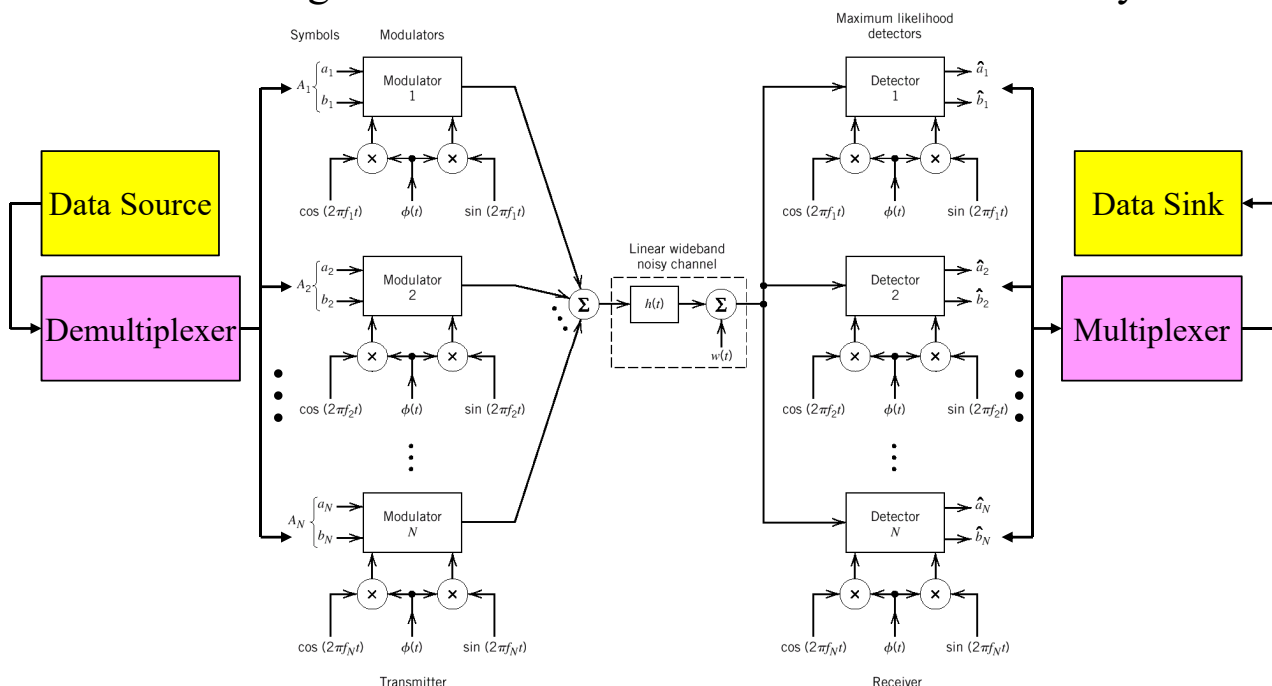
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## Continuous-Time Channel Partitioning (Cont.)

- Data transmission over each subchannel can be **optimized** by invoking **Shannon's information capacity law**
  - The optimization of each subchannel is performed **independently** of all the others
- The need for **complicated equalization** of a **wideband channel** (due to the **non-constant** response) is replaced by
  - The need of **demultiplexing** and **multiplexing**
    - **Demultiplexing**: Demultiplex the incoming data stream into multiple subchannels
    - **Multiplexing**: Multiplex the demodulated data from multiple subchannels to a single data stream

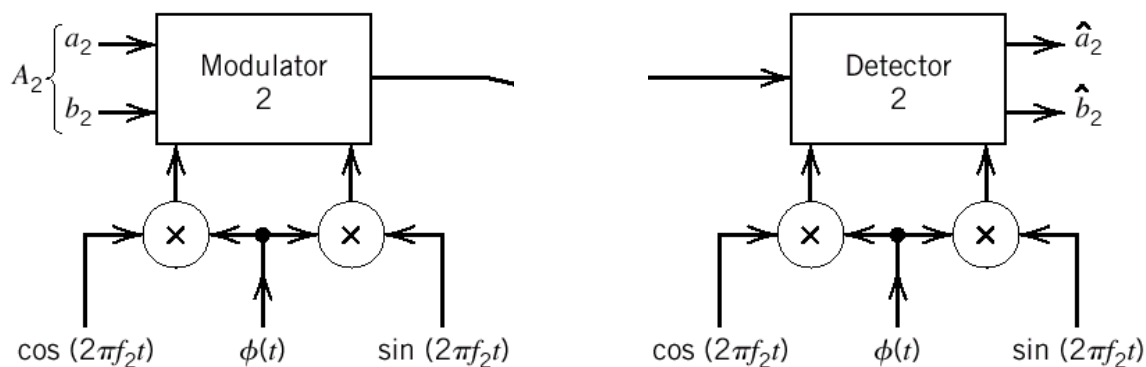
## Continuous-Time Channel Partitioning (Cont.)

- A block diagram of the **multichannel** data transmission system



## Continuous-Time Channel Partitioning (Cont.)

- The incoming data stream is first applied to a **demultiplexer**
  - Produce a set of  $N$  **substreams**
  - Each substream represents a sequence of **two-element** subsymbols,  $(a_n, b_n)$ ,  $n = 1, 2, \dots, N$ , for **QAM modulation**
- The detected data of the  $N$  substreams are finally applied to a **multiplexer** to restore an output data stream



## Geometric Signal-to-Noise Ratio

- In the **multichannel** transmission system, **each subchannel** is characterized by an SNR of its own.
  - However, it is highly desirable to derive a **single performance measure** of the **entire system**
- We assume that all of the subchannels are represented by one-dimensional constellations
  - The average channel capacity of the entire system is

$$\begin{aligned}
 R &= \frac{1}{N} \sum_{n=1}^N R_n = \frac{1}{2N} \sum_{n=1}^N \log_2 \left( 1 + \frac{P_n}{\Gamma \sigma_n^2} \right) = \frac{1}{\underline{2N}} \log_2 \left[ \prod_{n=1}^N \left( 1 + \frac{P_n}{\Gamma \sigma_n^2} \right) \right] \\
 &= \frac{1}{2} \log_2 \left[ \prod_{n=1}^N \left( 1 + \frac{P_n}{\Gamma \sigma_n^2} \right) \right]^{\underline{1/N}} \quad \text{bits/transmission}
 \end{aligned}$$

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## Geometric Signal-to-Noise Ratio (Cont.)

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- Let  $(\text{SNR})_{\text{overall}}$  denote the **overall SNR** of the entire system.
  - Then, we may express the rate  $R$  as

$$R = \frac{1}{2} \log_2 \left[ 1 + \frac{(\text{SNR})_{\text{overall}}}{\Gamma} \right] \text{ bits/transmission}$$

- Accordingly, the overall SNR is

$$(\text{SNR})_{\text{overall}} = \Gamma \left[ \prod_{n=1}^N \left( 1 + \frac{P_n}{\Gamma \sigma_n^2} \right)^{1/N} - 1 \right]$$

- If the SNR is large enough, we have the approximation

$$(\text{SNR})_{\text{overall}} \approx \prod_{n=1}^N \left( \frac{P_n}{\sigma_n^2} \right)^{1/N}$$

- It is the **geometric mean** of the SNRs of the individual subchannels and is **independent** of the gap  $\Gamma$ .

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## Loading of the Multichannel Transmission System

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# Power Loading

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- Define the magnitude response  $g_n = |H(f_n)|$ ,  $n = 1, 2, \dots, N$
- Assuming that the number of subchannels  $N$  is large enough
  - $g_n$  is a **constant** over the entire bandwidth  $\Delta f$
  - The average channel capacity is

$$R = \frac{1}{2N} \sum_{n=1}^N \log_2 \left( 1 + \frac{g_n^2 P_n}{\Gamma \sigma_n^2} \right) \text{ bits/transmission}$$

- where  $g_n$  and  $\Gamma$  are usually fixed; noise variance is  $\Delta f N_0$ ,  $\forall n$
- Goal: Optimize the overall bit rate  $R$  through a proper allocation of the **total transmit power** among the various subchannels
  - Subject to the total transmit power constraint

$$P = \sum_{n=1}^N P_n$$

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## Power Loading (Cont.)

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- **Maximize** the bit rate  $R$  through an **optimal sharing** of the total transmit power  $P$  between the  $N$  subchannels
  - Subject to the total transmit power constraint  $P$
- Through the **method of Lagrange multipliers**, the solution to the **constrained optimization problem** is

$$P_n + \frac{\Gamma \sigma_n^2}{g_n^2} = K, \quad n = 1, 2, \dots, N$$

- where  $K$  is a prescribed constant to meet the total transmit power constraint  $P$
- The process of allocating the transmit power  $P$  to the individual subchannels is called **loading**.

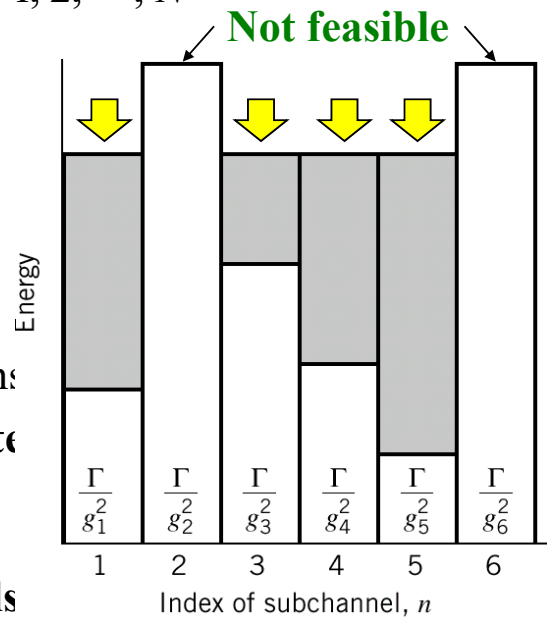


# Water-Filling Interpretation

- The **optimal** power allocation must satisfy the condition

$$P_n + \frac{\Gamma \sigma_n^2}{g_n^2} = K, \quad n = 1, 2, \dots, N$$

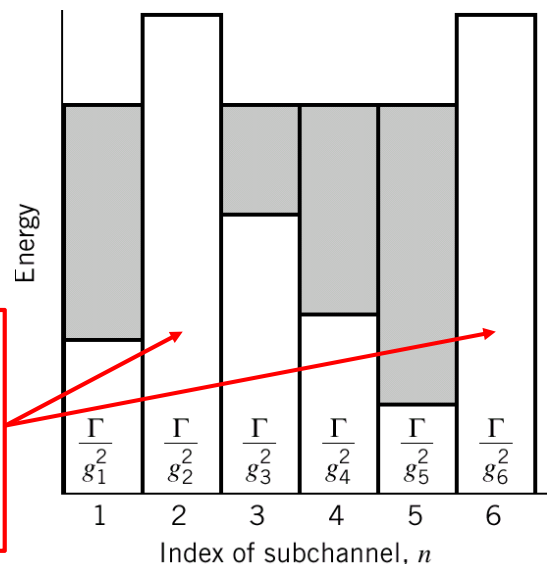
- Consider the case with  $N = 6$ 
  - The gap  $\Gamma$  is assumed to be constant over all subchannels
  - The average noise power is set to  $\sigma_n^2 = N_0 \Delta f = 1$
- We make the following observations:
  - With  $\sigma_n^2 = 1$ , the sum of **allocated power**  $P_n$  and the **scaled noise power**  $\Gamma/g_n^2$  is equal to a constant  $K$  for **four subchannels**



## Water-Filling Interpretation (Cont.)

- The sum of power allocations to these four subchannels consumes **all the available transmit power**  $P$ .
- The remaining two subchannels have been eliminated from consideration
  - Because they would each require **negative power** to satisfy the condition (i.e.,  $P_n < 0$ )

The channel response  $g_n$  is **too small**  
 $\Rightarrow$  The **scaled noise power**  $\Gamma/g_n^2$  is **very large**  
 $\Rightarrow$  Allocating power to these two channels is **inefficient**



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## Water-Filling Interpretation (Cont.)

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- The optimum solution for **loading** is referred to as **water-filling solution**
- This terminology follows from analogy of our optimization problem with
  - **A fixed amount of water**—standing for transmit power
  - Being **poured into a container** with a number of connected regions
  - **Each having a different depth**—standing for noise power
- In such a scenario, the water distributes itself in such a way that
  - A **constant water level** is attained across the whole container, hence the term “**water filling**”

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## Process of Loading

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- The allocation of the fixed transmit power  $P$  among the various subchannels can be formularized as follows:
  - There are a total of  $(N + 1)$  unknowns and  $(N + 1)$  equations

$$P_n + \frac{\Gamma \sigma_n^2}{g_n^2} = K, \quad n = 1, 2, \dots, N$$

$$\sum_{i=1}^N P_n = P$$

**Unknowns:**  
 $P_n$  and  $K$

$$\begin{bmatrix} 1 & 1 & \cdots & 1 & 0 \\ 1 & 0 & \cdots & 0 & -1 \\ 0 & 1 & \cdots & 0 & -1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ K \end{bmatrix} = \begin{bmatrix} P \\ -\Gamma \sigma^2 / g_1^2 \\ -\Gamma \sigma^2 / g_2^2 \\ \vdots \\ -\Gamma \sigma^2 / g_N^2 \end{bmatrix} \Rightarrow \mathbf{Mu} = \mathbf{c}$$

## Process of Loading (Cont.)

- Multiplying the inverse of  $\mathbf{M}$  on both sides of the equation
  - The unknowns  $P_1, P_2, \dots, P_N$ , and  $K$  can be obtained

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ K \end{bmatrix} = \mathbf{u} = \mathbf{M}^{-1} \mathbf{c}$$

- $K$  is always positive
- It is possible for some of the  $P_n$  values to be negative
  - In such a situation, the negative  $P_n$  values are discarded

## Example

- Consider a linear channel whose squared magnitude response  $|H(f)|^2$  has the **piecewise linear form**
- To simplify the example, we have set the gap  $\Gamma = 1$  and the noise variance  $\sigma_n^2 = 1$
- Under this set of values, we have

$$P_1 + P_2 = P$$

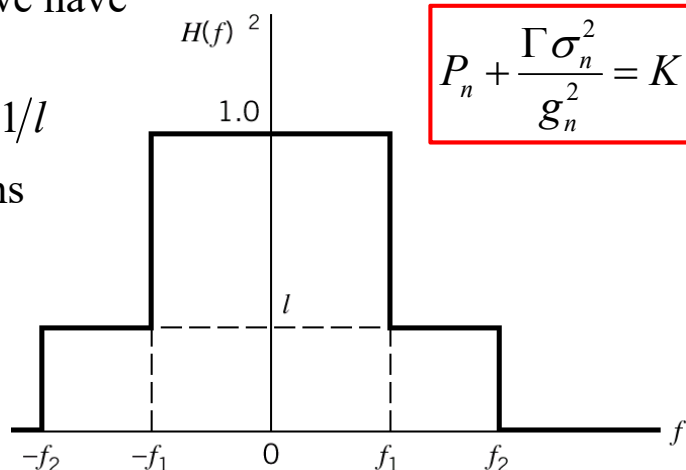
$$P_1 - K = -1; \quad P_2 - K = -1/l$$

- Solving the three equations for  $P_1, P_2$ , and  $K$

$$P_1 = (P - 1 + 1/l)/2$$

$$P_2 = (P + 1 - 1/l)/2$$

$$K = (P + 1 + 1/l)/2$$

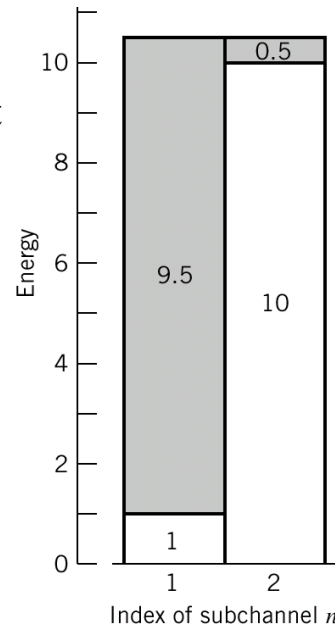


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## Example (Cont.)

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- Since  $0 < l < 1$ , it follows that  $P_1 > 0$
- But it is possible for  $P_2$  to be **negative**
  - It happens if  $l < 1/(P + 1)$
  - Correspondingly,  $P_1$  exceeds the transmit power  $P$  ( $P_1 > P$ )
- Therefore, it follows that, in this example, the only acceptable solution is to have  $1/(P + 1) < l < 1$ .
- Let  $P = 10$  and  $l = 0.1$ 
  - The desired solution is  $P_1 = 9.5$   
 $P_2 = 0.5$   
 $K = 10.5$



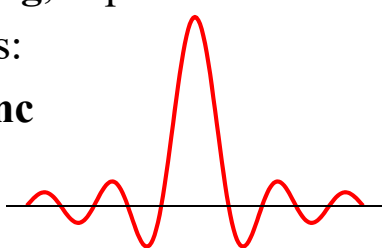
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## Discrete Multitone

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# Discrete Multitone

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- In the **multichannel modulation system**, **orthogonality** of the basis functions and the **channel partitioning**, is preserved.
- However, the system has two shortcomings:
  - The **passband basis functions** use a **sinc function** (rectangular function in the frequency domain)
    - Which is nonzero for an **infinite time interval**, resulting in a **non-causal** system
    - Practical considerations favor a **finite** time interval
  - For a **finite** number of subchannels  $N$ , the system is **suboptimal**
    - Optimality is assured only when  $N$  approaches **infinity** (to fit in with the frequency response  $H(f)$ )

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## Discrete Multitone (Cont.)

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- We may overcome these shortcomings by using **discrete multitone (DMT)**, which transforms a **wideband channel** into a set of  $N$  subchannels **operating in parallel**.
- The transformation of DMT is performed in **discrete time** as well as **discrete frequency**.
- The transmitter **input–output behavior** of the entire system admits a **linear matrix representation**
  - The **discrete Fourier transform (DFT)** can be used for implementation of DMT
  - The DFT is the result of discretizing the **Fourier transform** both in **time** and **frequency**.

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# Signal Generation

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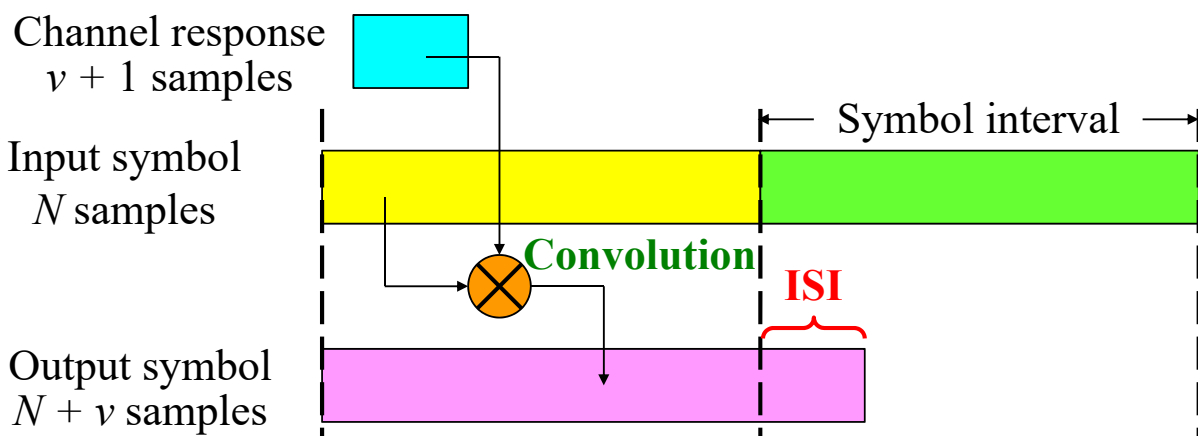
- Let the **channel impulse response** be  $h(t)$ , confined to a finite interval  $[0, T_h]$ 
  - The sequence  $h_0, h_1, \dots, h_v$  denote the **baseband equivalent impulse response** of the channel sampled at the rate  $1/T_s$ 
$$T_h = (1 + v)T_s$$
- The sampling rate  $1/T_s$  is chosen to be greater than **twice the highest frequency component** of interest (Nyquist rate)
  - In accordance with the **sampling theorem**
- Let  $s[n] = s(nT_s)$  denote a sample of the transmitted symbol  $s(t)$ ,  $w[n] = w(nT_s)$  denote a sample of the channel noise  $w(t)$ , and  $x[n] = x(nT_s)$  denote the sample of the channel output.

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## Signal Generation (Cont.)

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- By **linear convolution**, the input symbol sequence  $\{s[n]\}$  of length  $N$  produces an output sequence  $\{x[n]\}$  of length  $N + v$ .
  - The extension of  $v$  **samples** is due to the **inter-symbol interference** produced by the channel (the channel impulse response  $h(t)$ )

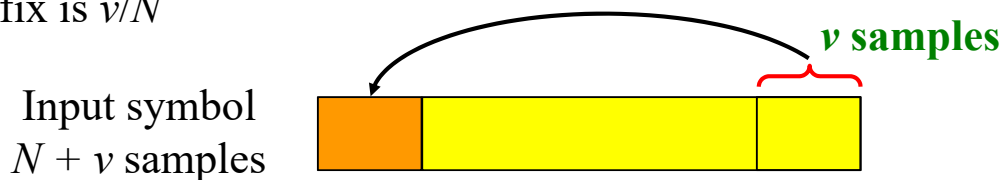


## Signal Generation (Cont.)

- To overcome the effect of ISI, we create a **cyclically extended guard interval**
  - Each symbol sequence is **preceded** by a **periodic extension** of the sequence itself
  - The **periodic extension** is called as a **cyclic prefix (CP)**
- Specifically, the **last**  $v$  samples of the symbol sequence are repeated at the beginning of the sequence being transmitted

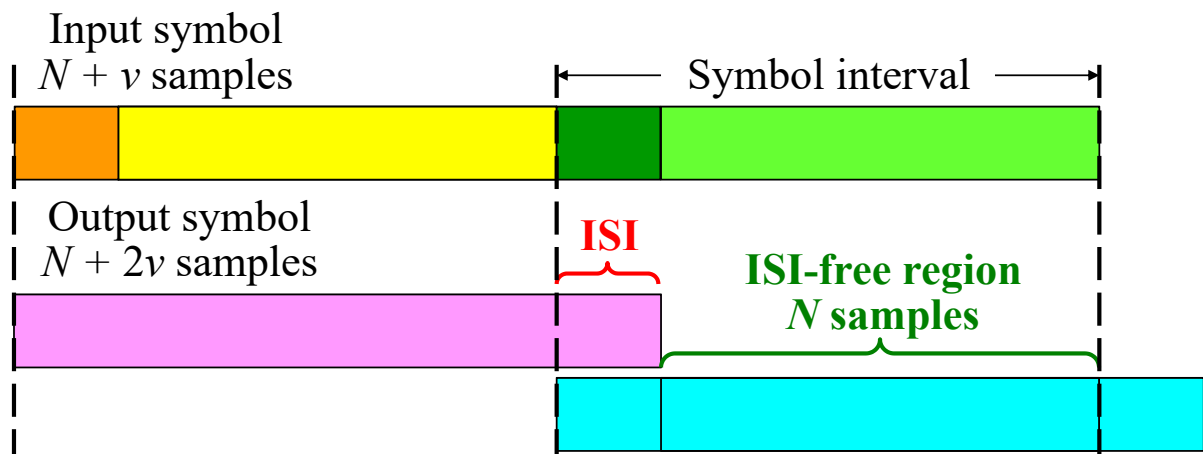
$$s[-k] = s[N - k], \quad k = 1, 2, \dots, v$$

- The **excess bandwidth factor** due to the inclusion of the cyclic prefix is  $v/N$



## Signal Generation (Cont.)

- With insertion of the  $v$  samples of CP, we have
  - The output symbol contains  $N + 2v$  samples
  - The length of ISI is still  $v$  samples
  - The ISI-free region contains  $N$  samples



## Signal Generation (Cont.)

- With insertion of the CP, the matrix description of the channel output takes the new form:

$$\begin{bmatrix} x[N-1] \\ x[N-2] \\ \vdots \\ x[v] \\ x[v-1] \\ \vdots \\ x[0] \end{bmatrix} = \begin{bmatrix} h_0 & h_1 & h_2 & \cdots & h_{v-1} & h_v & 0 & \cdots & 0 \\ 0 & h_0 & h_1 & \cdots & h_{v-2} & h_{v-1} & h_v & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & h_0 & h_1 & \cdots & h_v \\ h_v & 0 & 0 & \cdots & 0 & 0 & h_0 & \cdots & h_{v-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ h_1 & h_2 & h_3 & \cdots & h_v & 0 & 0 & \cdots & h_0 \end{bmatrix} \begin{bmatrix} s[N-1] \\ s[N-2] \\ \vdots \\ s[v] \\ s[v-1] \\ \vdots \\ s[0] \end{bmatrix} + \begin{bmatrix} w[N-1] \\ w[N-2] \\ \vdots \\ w[v] \\ w[v-1] \\ \vdots \\ w[0] \end{bmatrix}$$

Diagram illustrating the channel output matrix  $\mathbf{H}$  and the transmitted signal vector  $\mathbf{s}$ . The matrix  $\mathbf{H}$  is a circulant matrix where each row is a right-shift of the previous row. The channel response  $[h_0, h_1, \dots, h_v, 0, \dots, 0]^T$  is shown as the first row. The transmitted signal vector  $\mathbf{s}$  is shown below the matrix, with the cyclic prefix (CP) highlighted in pink. Red arrows indicate the mapping from the transmitted signal to the received signal.

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## Signal Generation (Cont.)

- The discrete-time representation of the channel output is
 
$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{w}$$
  - $\mathbf{s}$ : the transmitted symbol vector;  $\mathbf{w}$ : the channel noise vector;  $\mathbf{x}$ : the received signal vector; all are  $N$ -by-1 vectors
- The channel matrix  $\mathbf{H}$  is a **circulant matrix**:

- Constructed by the **channel response**  $[h_0, h_1, \dots, h_v, 0, \dots, 0]^T$
  - Every **row** of the matrix is obtained by **cyclically** applying a **right-shift** to the **previous row** by **one position**
- $$\mathbf{H} = \begin{bmatrix} h_0 & h_1 & h_2 & \cdots & h_{v-1} & h_v & 0 & \cdots & 0 \\ 0 & h_0 & h_1 & \cdots & h_{v-2} & h_{v-1} & h_v & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & h_0 & h_1 & \cdots & h_v \\ h_v & 0 & 0 & \cdots & 0 & 0 & h_0 & \cdots & h_{v-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ h_1 & h_2 & h_3 & \cdots & h_v & 0 & 0 & \cdots & h_0 \end{bmatrix}$$

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# Discrete Fourier Transform

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- Consider an  $N$ -by-1 vector  $\mathbf{x} = [x[N-1], x[N-2], \dots, x[0]]^T$
- Let the DFT of  $\mathbf{x}$  be  $\mathbf{X} = [X[N-1], X[N-2], \dots, X[0]]^T$ 
  - where the element is defined by

$$X[k] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n] \exp(-j2\pi kn/N), \quad k = 0, 1, \dots, N-1$$

- The term  $\exp(-j2\pi kn/N)$  is the **kernel** of the DFT
- Correspondingly, the IDFT (i.e., inverse DFT) of  $\mathbf{X}$  is

$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X[k] \exp(j2\pi kn/N), \quad n = 0, 1, \dots, N-1$$

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# Channel Representation

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- For a circulant matrix, it permits the **spectral decomposition**:

$$\mathbf{H} = \mathbf{Q}^\dagger \mathbf{\Lambda} \mathbf{Q}$$

- where the superscript  $\dagger$  denotes **Hermitian transposition**
- The matrix  $\mathbf{Q}$  is a **square matrix** defined in terms of the kernel of the  $N$ -point DFT as

$$\mathbf{Q} = \frac{1}{\sqrt{N}} \begin{bmatrix} \exp\left[-j\frac{2\pi}{N}(N-1)(N-1)\right] & \cdots & \exp\left[-j\frac{2\pi}{N}2(N-1)\right] & \exp\left[-j\frac{2\pi}{N}(N-1)\right] & 1 \\ \exp\left[-j\frac{2\pi}{N}(N-1)(N-2)\right] & \cdots & \exp\left[-j\frac{2\pi}{N}2(N-2)\right] & \exp\left[-j\frac{2\pi}{N}(N-2)\right] & 1 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \exp\left[-j2\pi(N-1)/N\right] & \cdots & \exp\left[-j2\pi \times 2/N\right] & \exp\left[-j2\pi/N\right] & 1 \\ 1 & \cdots & 1 & 1 & 1 \end{bmatrix}$$
$$q_{kl} = \frac{1}{\sqrt{N}} \exp\left[-j\frac{2\pi}{N}kl\right], \quad k, l \in \{0, 1, \dots, N-1\}$$

## Channel Representation (Cont.)

- The matrix  $\mathbf{Q}$  is an **orthonormal matrix** or **unitary matrix**

$$\mathbf{Q}^\dagger \mathbf{Q} = \mathbf{I}$$

- The matrix  $\mathbf{\Lambda}$  is a **diagonal matrix** that contains the  $N$  **DFT values** of the sequence  $h_0, h_1, \dots, h_v$
- Denoting these transform values by  $\lambda_{N-1}, \dots, \lambda_1, \lambda_0$ 
  - $[\lambda_{N-1}, \dots, \lambda_1, \lambda_0]^T = \text{DFT}([h_0, h_1, \dots, h_v, 0, \dots, 0]^T)$

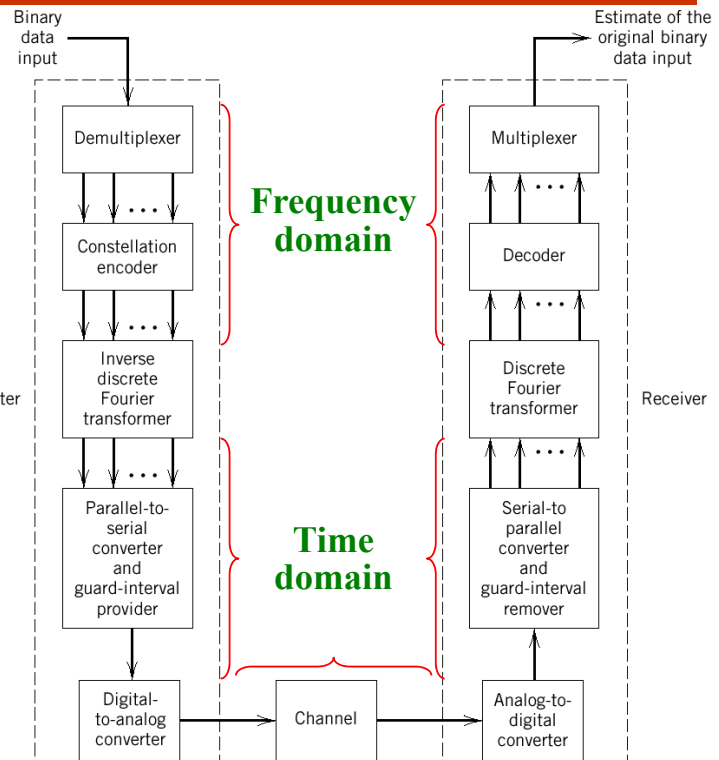
$$\mathbf{\Lambda} = \text{diag}(\lambda_{N-1}, \dots, \lambda_1, \lambda_0) = \begin{bmatrix} \lambda_{N-1} & 0 & \dots & 0 \\ 0 & \lambda_{N-2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_0 \end{bmatrix}$$

## Frequency–Domain Representation

- Define the **time-domain** transmit signal vector as **Time domain transmitted samples**  $\xrightarrow{\text{red arrow}} \mathbf{s} = \mathbf{Q}^\dagger \mathbf{S} \xleftarrow{\text{red arrow}}$  **Frequency domain parallel signals**
  - where  $\mathbf{S}$  is the **frequency-domain** transmit signal vector
  - Each element of the  $N$ -by-1 vector  $\mathbf{S}$  is a **complex-valued** point in a two-dimensional QAM signal constellation.
- Given the channel output vector  $\mathbf{x}$ , we define its corresponding **frequency-domain** representation as
 
$$\mathbf{X} = \mathbf{Q} \mathbf{x}$$
- Then, we may rewrite  $\mathbf{X}$  in the equivalent form
 
$$\mathbf{X} = \mathbf{Q} \mathbf{x} = \mathbf{Q} (\mathbf{H} \mathbf{s} + \mathbf{w}) = \mathbf{Q} (\mathbf{Q}^\dagger \mathbf{\Lambda} \mathbf{Q} \mathbf{Q}^\dagger \mathbf{S} + \mathbf{w}) = \mathbf{\Lambda} \mathbf{S} + \mathbf{W}$$
  - where  $\mathbf{W} = \mathbf{Q} \mathbf{w}$  is the **frequency-domain** noise vector
- That is,  $X_k = \lambda_k S_k + W_k, k = 0, 1, \dots, N-1 \xleftarrow{\text{red arrow}}$  **Parallel subchannels**

# DFT-Based DMT System

- Block diagram of the DFT-based DMT system
- The **transmitter** consists of the functional blocks:
  - Demultiplexer,
  - Constellation encoder,
  - IDFT, Parallel-to-serial converter,
  - Digital-to-analog converter (DAC)
- The **receiver** performs the **inverse operations** of the transmitter



## DFT-Based DMT System (Cont.)

- **Demultiplexer:** converts the incoming **serial data stream** into **parallel form** ( $N$  constellations in the **frequency-domain**)
- **Constellation encoder:** maps the parallel data into **all multibit subchannels** with each subchannel being represented by a **QAM signal constellation**
  - **Bit allocation** among the subchannels is performed in accordance with a **loading** algorithm.
- **IDFT:** transforms the **frequency-domain** parallel data at the constellation encoder output into parallel **time-domain** data
  - For efficient implementation of the IDFT using the **fast Fourier transform (FFT)** algorithm, we need to choose  $N = 2^k$ , where  $k$  is a positive integer

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## DFT-Based DMT System (Cont.)

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- **Parallel-to-serial converter:** converts the **parallel** time-domain samples into **serial** form (discrete time samples)
  - Guard intervals stuffed with **cyclic prefixes** are inserted into the serial data on a periodic basis before conversion into analog form.
- **Digital-to-analog converter (DAC):** converts the digital data into analog form ready for transmission over the channel

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## Orthogonal Frequency Division Multiplexing (OFDM)

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# OFDM Concept

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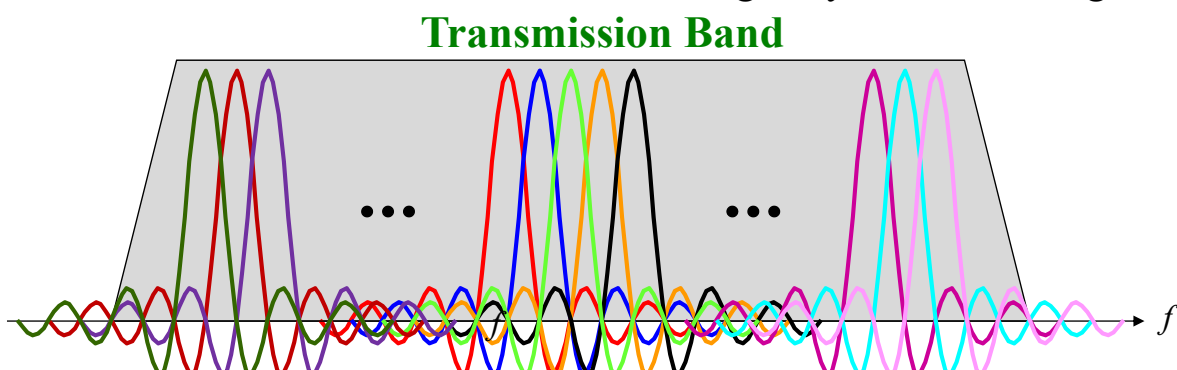
- **Orthogonal frequency division multiplexing (OFDM)** is a form of **multi-carrier modulation**.
  - OFDM is particularly well suited for **high data-rate transmission** over **delay-dispersive channels**.
- Specifically, a large number of closely spaced **orthogonal subcarriers (tones)** is used to support the transmission.
  - The incoming data stream is divided into a number of **low data-rate sub-streams**, one for each subcarrier
- In addition, two other changes have to be made for OFDM:
  - In the transmitter, an **upconverter** is included after the DAC to **translate** the signal to the **transmission band**
  - In the receiver, a **downconverter** is included before the ADC to **translate** the signal to the **baseband**

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## OFDM Concept (Cont.)

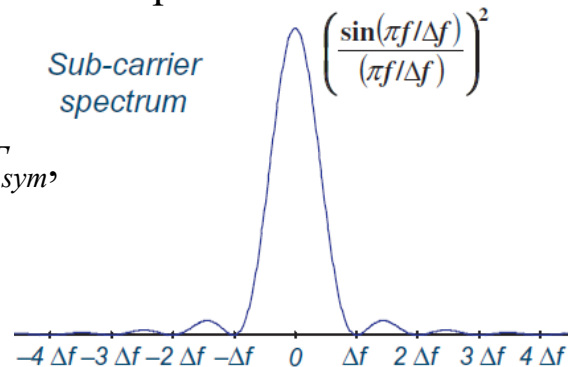
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- **Orthogonal frequency division multiplexing (OFDM)** is a promising technique because of its
  - High bandwidth efficiency and
  - Resistance to multipath fading
- **Orthogonality** is maintained among the subcarriers
- **Narrowband** transmission for each digitally modulated signal



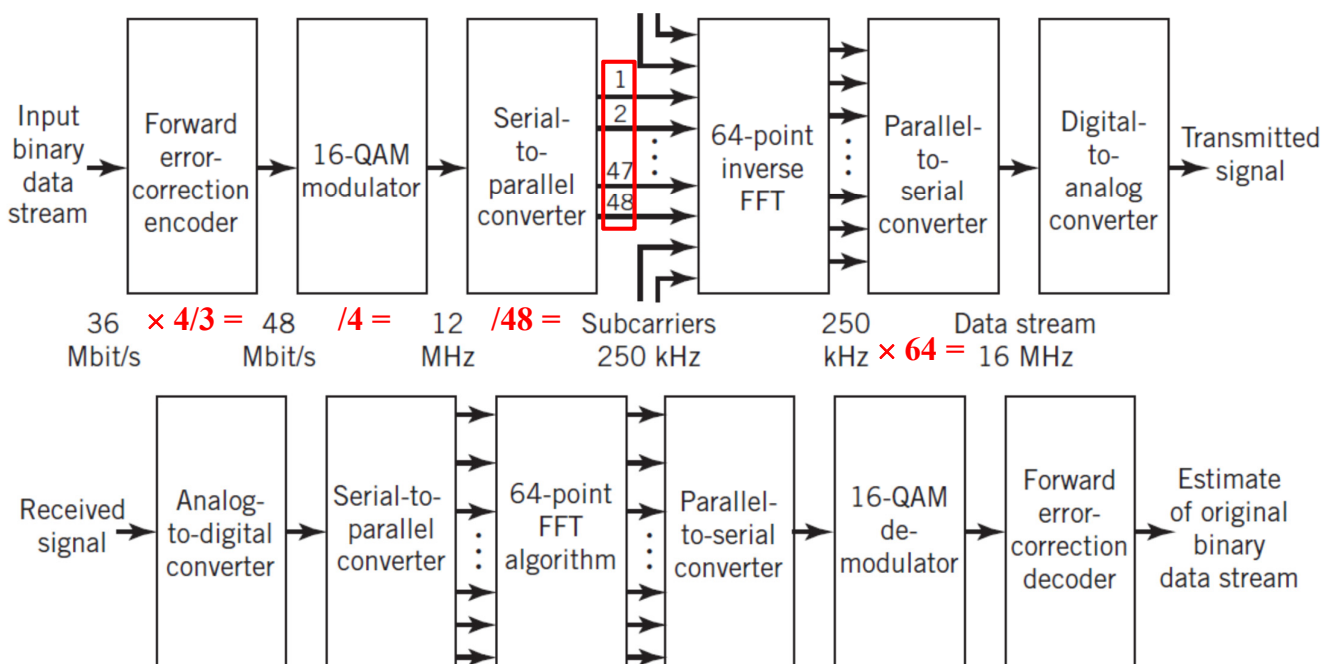
## OFDM Concept (Cont.)

- However, the following characteristics distinguish OFDM from a straightforward multi-carrier extension:
  - The use of a typically **very large number** of relatively narrowband subcarriers (e.g., several hundred subcarriers)
  - Simple **rectangular pulse shaping** is used
  - ⇒ A sinc-square-shaped per-subcarrier spectrum
  - **Tight frequency-domain packing** of the subcarriers
  - ⇒ A subcarrier spacing  $\Delta f = 1/T_{sym}$ ,  
 $T_{sym}$  is the per-subcarrier modulation-symbol duration



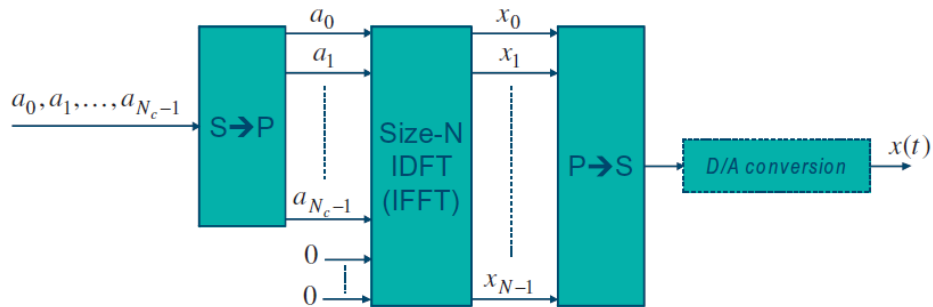
## OFDM Transmitter/Receiver

- Block diagrams of transmitter/receiver for a 36 Mbits/s system

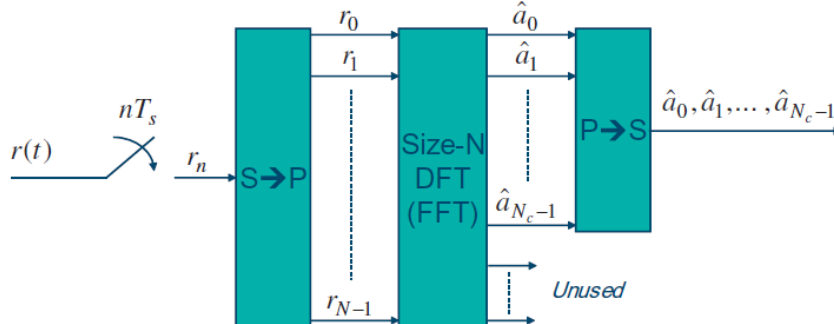


# OFDM Implementation

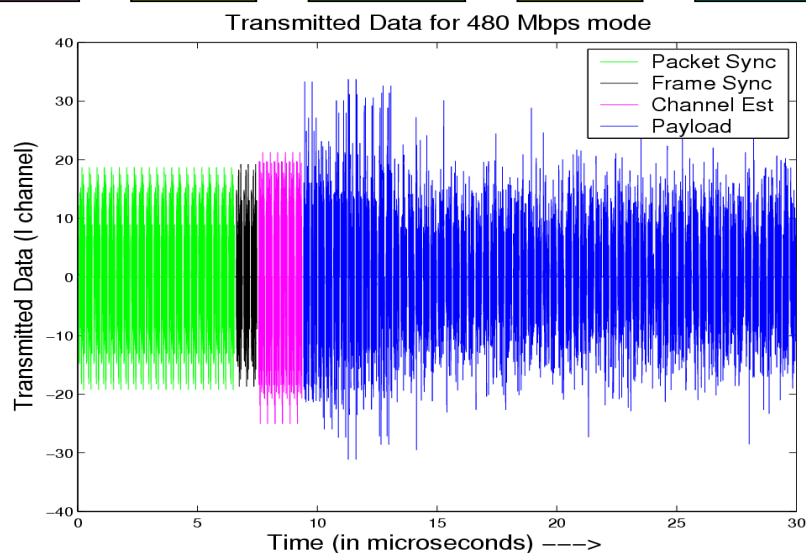
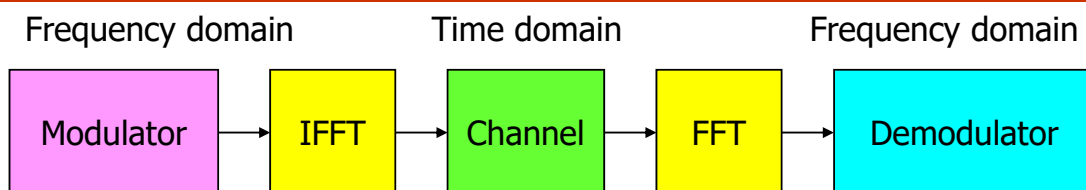
- OFDM **modulation** by means of **IFFT** processing



- OFDM **demodulation** by means of **FFT** processing



# OFDM Transmission



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# Homework

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- **You must give detailed derivations or explanations, otherwise you get no points.**
- Communication Systems, Simon Haykin (4<sup>th</sup> Ed.)
- 6.43;