

6.2.

HW2

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t) \quad 0 \leq t \leq T_b$$

$$s_1(t) = \sqrt{E_b} \phi_1(t)$$

$$s_2(t) = -\sqrt{E_b} \phi_1(t)$$

$$\phi_e(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t + \varphi), \text{ local coherent reference signal}$$

$$X = \int_0^{T_b} x(t) \phi_e(t) dt = \int_0^{T_b} (s_1(t) + n(t)) \cdot (\cos \varphi \phi_1(t) - \sqrt{\frac{2}{T_b}} \sin \varphi \sin(2\pi f_c t)) dt$$

$$= \cos \varphi s_{11} + \sqrt{\frac{1}{2T_b}} \int_0^{T_b} (\cos \varphi n_{11}(t) + \sin \varphi n_{21}(t)) dt$$

$\Rightarrow X$ is a Gaussian distributed random variable with mean $\mu = \cos \varphi s_{11}$

$$\text{variance: } \sigma^2 = \frac{N_0}{2}$$

(Because $n_{11}(t)$ and $n_{21}(t)$ are independent zero-mean Gaussian process with PSD N_0 for $-\frac{B}{2} < f < \frac{B}{2}$)

$$\sigma^2 = \frac{1}{2T_b} \times (\cos^2 \varphi N_0 B + \sin^2 \varphi N_0 B) = \frac{1}{2T_b} N_0 B = \frac{N_0}{2}$$

$$\Rightarrow P_{01} = \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^0 e^{-\frac{1}{N_0} (X - \cos \varphi \sqrt{E_b})^2} dx$$

$$= \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{\cos^2 \varphi E_b}{N_0}} \right)$$

$$\text{Similarly, } P_{10} = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{\cos^2 \varphi E_b}{N_0}} \right)$$

Thus, the average error probability is $\frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{\cos^2 \varphi E_b}{N_0}} \right)$

6.5.

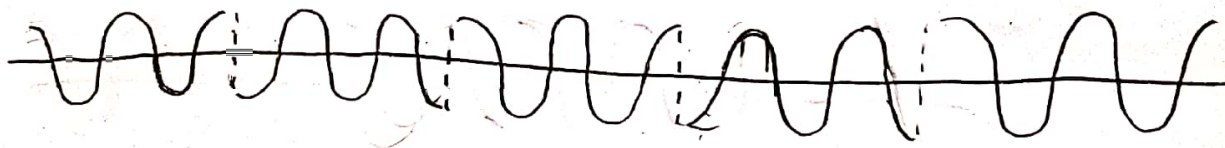
$$s_n(t) = \sqrt{\frac{2E}{T}} \cos\left[(2n-1)\frac{\pi}{4}\right] \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin\left[(2n-1)\frac{\pi}{4}\right] \sin(2\pi f_c t)$$

(a)

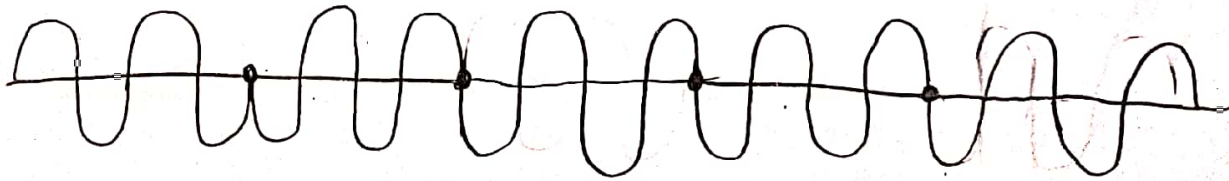
binary
sequence

: 1 1 0 0 1 0 0 0 1 0

in-phase
(first bit)

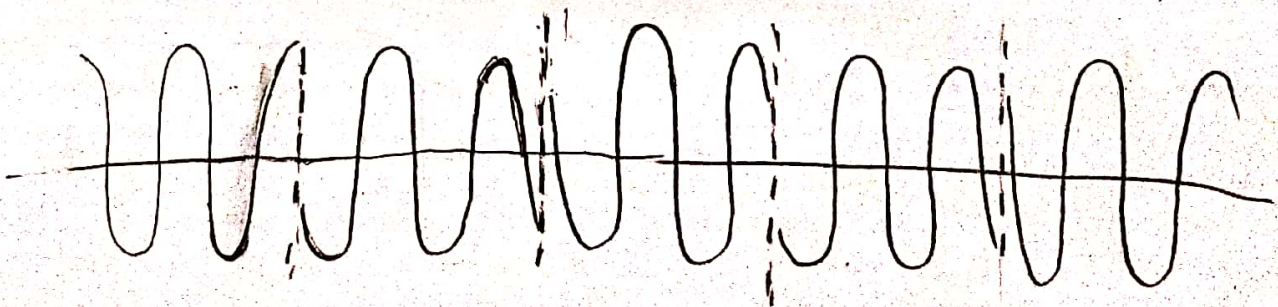


quadrature
(second bit)



(b)

QPSK



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a.b.

We know that the outputs of in-phase and quadrature components are independent

Thus, the average probability of symbol correct overall system is

$$P_c = (1 - P_{eI})(1 - P_{eQ}) = 1 - P_{eI} - P_{eQ} + P_{eI}P_{eQ}$$

$$\Rightarrow P_e = 1 - P_c = P_{eI} + P_{eQ} - P_{eI}P_{eQ} \quad \#$$

6.10.

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Step k	Phase θ_{k-1}	Input Dibit	Phase Change $\Delta\theta_k$	Transmitted Phase θ_k
1	0	01	$3\pi/4$	$3\pi/4$
2	$3\pi/4$	10	$-\pi/4$	$\pi/2$
3	$\pi/2$	10	$-\pi/4$	$\pi/4$
4	$\pi/4$	00	$\pi/4$	$\pi/2$

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