Communication System Homework#1 (Spring 2020)

2.3 Any function $g(t)$ can be split unambiguously into an *even part* and an *odd part*, as shown by

$$g(t) = g_e(t) + g_o(t)$$

The even part is defined by

$$g_e(t) = \frac{1}{2}[g(t) + g(-t)]$$

and the odd part is defined by

$$g_o(t) = \frac{1}{2}[g(t) - g(-t)]$$

(a) Evaluate the even and odd parts of a rectangular pulse defined by

$$g(t) = A\,\mathrm{rect}\left(\frac{t}{T} - \frac{1}{2}\right)$$

(b) What are the Fourier transforms of these two parts of the pulse?

2.6 The Fourier transform of a signal $g(t)$ is denoted by $G(f)$. Prove the following properties of the Fourier transform:

(a) If a real signal $g(t)$ is an even function of time t, the Fourier transform $G(f)$ is purely real. If a real signal $g(t)$ is an odd function of time t, the Fourier transform $G(f)$ is purely imaginary.

(b)

$$t^n g(t) \rightleftharpoons \left(\frac{j}{2\pi}\right)^n G^{(n)}(f)$$

where $G^{(n)}(f)$ is the nth derivative of $G(f)$ with respect to f.

(c) $$\int_{-\infty}^{\infty} t^n g(t)\,dt = \left(\frac{j}{2\pi}\right)^n G^{(n)}(0)$$

(d) $$g_1(t)g_2^*(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda)G_2^*(\lambda - f)\,d\lambda$$

(e) $$\int_{-\infty}^{\infty} g_1(t)g_2^*(t)\,dt = \int_{-\infty}^{\infty} G_1(f)G_2^*(f)\,df$$

2.10 A signal $x(t)$ of finite energy is applied to a square-law device whose output $y(t)$ is defined by

$$y(t) = x^2(t)$$

The spectrum of $x(t)$ is limited to the frequency interval $-W \le f \le W$. Hence, show that the spectrum of $y(t)$ is limited to $-2W \le f \le 2W$. *Hint*: Express $y(t)$ as $x(t)$ multiplied by itself.

2.14

(a) The *root mean-square (rms) bandwidth* of a low-pass signal $g(t)$ of finite energy is defined by

$$W_{\text{rms}} = \left[\frac{\int_{-\infty}^{\infty} f^2 |G(f)|^2 \, df}{\int_{-\infty}^{\infty} |G(f)|^2 \, df} \right]^{1/2}$$

where $|G(f)|^2$ is the energy spectral density of the signal. Correspondingly, the *root mean-square (rms) duration* of the signal is defined by

$$T_{\text{rms}} = \left[\frac{\int_{-\infty}^{\infty} t^2 |g(t)|^2 \, dt}{\int_{-\infty}^{\infty} |g(t)|^2 \, dt} \right]^{1/2}$$

Using these definitions, show that

$$T_{\text{rms}} W_{\text{rms}} \geq \frac{1}{4\pi}$$

Assume that $|g(t)| \to 0$ faster than $1\sqrt{|t|}$ as $|t| \to \infty$.

(b) Consider a Gaussian pulse defined by

$$g(t) = \exp(-\pi t^2)$$

Show that, for this signal, the equality

$$T_{\text{rms}} W_{\text{rms}} \equiv \frac{1}{4\pi}$$

can be reached.

Hint: Use Schwarz's inequality:

$$\left\{ \int_{-\infty}^{\infty} [g_1^*(t) g_2(t) + g_1(t) g_2^*(t)] dt \right\}^2 \leq 4 \int_{-\infty}^{\infty} |g_1(t)|^2 \, dt \times \int_{-\infty}^{\infty} |g_2(t)|^2 \, dt$$

in which we set

$$g_1(t) = t g(t)$$

and

$$g_2(t) = \frac{dg(t)}{dt}$$

2.19 A tapped-delay-line filter consists of N weights, where N is odd. It is symmetric with respective to the center tap, that is, the weights satisfy the condition

$$w_n = w_{N-1-n} \qquad 0 \leq n \leq N-1$$

(a) Find the amplitude response of the filter.

(b) Show that this filter has a linear phase response.

3.8 Consider a message signal $m(t)$ with the spectrum shown in Figure P3.8. The message bandwidth $W = 1$ kHz. This signal is applied to a product modulator, together with a carrier wave $A_c \cos(2\pi f_c t)$, producing the DSB-SC modulated signal $s(t)$. The modulated signal is next applied to a coherent detector. Assuming perfect synchronism between the carrier waves in the modulator and detector, determine the spectrum of the detector output when: (a) the carrier frequency $f_c = 1.25$ kHz and (b) the carrier frequency $f_c = 0.75$ kHz. What is the lowest carrier frequency for which each component of the modulated signal $s(t)$ is uniquely determined by $m(t)$?

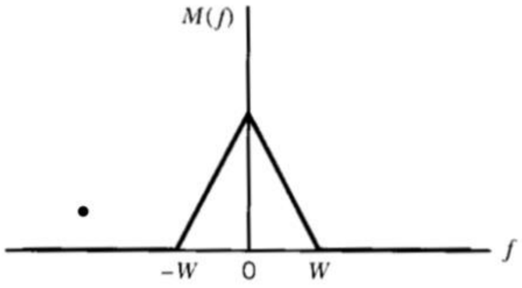

Figure P3.8

3.9 Figure P3.9 shows the circuit diagram of a *balanced modulator*. The input applied to the top AM modulator is $m(t)$, whereas that applied to the lower AM modulator is $-m(t)$; these two modulators have the same amplitude sensitivity. Show that the output $s(t)$ of the balanced modulator consists of a DSB-SC modulated signal.

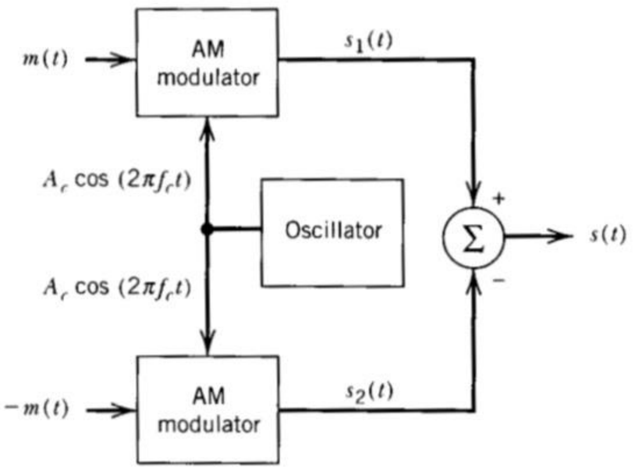

Figure P3.9

3.11 A DSB-SC modulated signal is demodulated by applying it to a coherent detector.

(a) Evaluate the effect of a frequency error Δf in the local carrier frequency of the detector, measured with respect to the carrier frequency of the incoming DSB-SC signal.

(b) For the case of a sinusoidal modulating wave, show that because of this frequency error, the demodulated signal exhibits *beats* at the error frequency. Illustrate your answer with a sketch of this demodulated signal.

3.14 Suppose that in the receiver of the quadrature–carrier multiplex system of Figure 3.16 the local carrier available for demodulation has a phase error ϕ with respect to the carrier source used in the transmitter. Assuming a distortionless communication channel between transmitter and receiver, show that this phase error will cause *cross-talk* to arise between the two demodulated signals at the receiver outputs. By cross-talk we mean that a portion of one message signal appears at the receiver output belonging to the other message signal, and vice versa.

3.15 A particular version of *AM stereo* uses quadrature multiplexing. Specifically, the carrier $A_c \cos(2\pi f_c t)$ is used to modulate the sum signal

$$m_1(t) = V_0 + m_l(t) + m_r(t)$$

where V_0 is a dc offset included for the purpose of transmitting the carrier component, m_ℓ is the left-hand audio signal, and $m_r(t)$ is the right-hand audio signal. The quadrature carrier $A_c \sin(2\pi f_c t)$ is used to modulate the difference signal

$$m_2(t) = m_l(t) - m_r(t)$$

(a) Show that an envelope detector may be used to recover the sum $m_r(t) + m_l(t)$ from the quadrature-multiplexed signal. How would you minimize the signal distortion produced by the envelope detector?

(b) Show that a coherent detector can recover the difference $m_l(t) - m_r(t)$.

(c) How are the desired $m_l(t)$ and $m_r(t)$ finally obtained?