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2.3.

(a)

$$g(t)=A\,\text{rect}\left(\frac{t}{T}-\frac{1}{2}\right)$$

$$\Rightarrow g_e(t)=\frac{1}{2}\left(g(t)+g(-t)\right)=\frac{A}{2}\left(\text{rect}\left(\frac{t}{T}-\frac{1}{2}\right)+\text{rect}\left(\frac{-t}{T}-\frac{1}{2}\right)\right)$$

$$g_o(t)=\frac{1}{2}\left(g(t)-g(-t)\right)=\frac{A}{2}\left(\text{rect}\left(\frac{t}{T}-\frac{1}{2}\right)-\text{rect}\left(\frac{-t}{T}-\frac{1}{2}\right)\right)$$

#

(b) let $X(t)=\text{rect}\left(\frac{t}{T}\right)$ (Note that $X(t)=X(-t)$)

$$\Rightarrow X(jw)=\int_{-\frac{T}{2}}^{\frac{T}{2}} e^{-jwt}\,dt=\frac{2\sin(w\cdot\frac{T}{2})}{w}=T\,\text{sinc}\left(\frac{wT}{2}\right)$$ (where $w=2\pi f$ which is used in the course)

$$\Rightarrow g_e(t)=\frac{A}{2}\left(X(t-\tfrac{1}{2}T)+X(-t-\tfrac{1}{2}T)\right)$$

$$g_o(t)=\frac{A}{2}\left(X(t-\tfrac{1}{2}T)-X(-(t+\tfrac{1}{2}T))\right)$$

$$\Rightarrow G_e(jw)=\frac{AT}{2}\left(e^{-jw\frac{T}{2}}\text{sinc}\left(\frac{wT}{2}\right)+e^{jw\frac{T}{2}}\text{sinc}\left(\frac{wT}{2}\right)\right)$$

$$=ATe^{-jw\frac{T}{2}}\text{sinc}\left(\frac{wT}{2}\right)=G(jw)$$

$$G_o(jw)=0$$

#

Ïhý uú

2.6.

(a) let $g(t)$ be real and even

then $G^*(f) = \left(\int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt\right)^* = \int_{-\infty}^{\infty} g(t) e^{j2\pi ft} dt = G(-f)$ since $g(t)$ is real

$G(-f) = \int_{-\infty}^{\infty} g(-t) e^{-j2\pi ft} dt = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt = G(f)$ since $g(t)$ is even

$\Rightarrow G(f) = G(-f) = G^*(f)$ which implies $G(f)$ is real

let $g(t)$ be real and odd

then $G(f) = G^*(-f)$ since $g(t)$ is real

$G(-f) = \int_{-\infty}^{\infty} g(-t) e^{-j2\pi ft} dt = -G(f)$ since $g(t)$ is odd

$\Rightarrow G(f) = -G(-f) = -G^*(f)$ which implies $G(f)$ is purely imaginary. #

(b) $G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt$

$\Rightarrow \frac{d}{df}(G(f)) = -j2\pi \int_{-\infty}^{\infty} t g(t) e^{-j2\pi ft} dt$

$\Rightarrow \frac{j}{2\pi} \frac{d}{df}(G(f)) = \int_{-\infty}^{\infty} t g(t) e^{-j2\pi ft} dt$

$\Rightarrow t g(t) \rightleftharpoons \frac{j}{2\pi} G'(f)$

By induction, $t^n g(t) \rightleftharpoons \left(\frac{j}{2\pi}\right)^n G^{(n)}(f)$ obviously. #

(c) According to (b)

$$\left(\frac{j}{2\pi}\right)^n G^{(n)}(f) = \int_{-\infty}^{\infty} t^n g(t) e^{-j2\pi ft} dt$$

$$\Rightarrow \left(\frac{j}{2\pi}\right)^n G^{(n)}(0) = \int_{-\infty}^{\infty} t^n g(t)\, dt$$

#

Ïhý uú

(d) We have $g_1(t)\,g_2(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda)\, G_2(f-\lambda)\, d f$ for any g_1, g_2

$$\Rightarrow g_1(t)\, g_2^*(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda)\, G_2^*(\lambda - f)\, d\lambda$$

(Since ub $g(t) \rightleftharpoons G(f)$ then $g^*(t) \rightleftharpoons G^*(-f)$) #

(e)

$$\int_{-\infty}^{\infty} g_1(t) g_2^*(t)\, dt = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(f) e^{j2\pi f} df\; g_2^*(t)\, dt$$

$$= \int_{-\infty}^{\infty} G(f) \int_{-\infty}^{\infty} g_2^*(t) e^{j2\pi f} dt\, df$$

$$= \int_{-\infty}^{\infty} G(f) \left(\int_{-\infty}^{\infty} g_2(t)\, G^{-j2\pi f} dt \right)^* df$$

$$= \int_{-\infty}^{\infty} G(f)\, G^*(f)$$

#

Ïhý uú

2.10. Let $y(t) = x^2(t)$, $x(t) \rightleftharpoons X(f)$

then $y(t) = x(t)\,x(t) \rightleftharpoons Y(f) = X(f) * X(f)$

$\Rightarrow Y(f) = \int_{-\infty}^{\infty} X(\lambda)\, X(f-\lambda)\, d\lambda = \int_{-W}^{W} X(\lambda)\, X(f-\lambda)\, d\lambda$

In the integration $X(\lambda) X(f-\lambda)$ has nonzero values iff $\begin{cases} -W < \lambda < W \\ -W+\lambda < f < W+\lambda \end{cases}$

$\Rightarrow Y(f)$ has nonzero value only at $0 \le |f| \le 2W$

#

Ïhý uú

2.14.

(a) According to Rayleigh's Energy thm, we have $\int_{-\infty}^{\infty}|g(t)|^2 dt = \int_{-\infty}^{\infty}|G(f)|^2 df$.

WLOG, we can assume $\int_{-\infty}^{\infty}|g(t)|^2 dt = \int_{-\infty}^{\infty}|G(f)|^2 df = 1$

$$\left(\begin{array}{l} \text{If } \int_{-\infty}^{\infty}|g(t)|^2 = k \neq 1,\ k>0, \text{ then consider } g(t) = k\,\tilde{g}(t) \text{ where } \int_{-\infty}^{\infty}|\tilde{g}(t)|^2 dt = 1 \\ \Rightarrow \dfrac{\int_{-\infty}^{\infty} t^2 |g(t)|^2 dt}{\int_{-\infty}^{\infty}|g(t)|^2 dt} = \dfrac{\int_{-\infty}^{\infty} t^2 |\tilde{g}(t)|^2 dt}{\int_{-\infty}^{\infty}|\tilde{g}(t)|^2 dt} = \int_{-\infty}^{\infty} t^2 |\tilde{g}(t)|^2 dt \end{array}\right.$$

$$\Rightarrow T_{rms} W_{rms} = \left(\int_{-\infty}^{\infty} t^2 |g(t)|^2 dt \int_{-\infty}^{\infty} f^2 |G(f)|^2 df\right)^{1/2}$$

Consider $L = 4\int_{-\infty}^{\infty} t^2 |g(t)|^2 dt \int_{-\infty}^{\infty} |g'(t)|^2 dt)$

By Schwarz's inequality

$$L \geq \left(\int_{-\infty}^{\infty} t g^*(t) g'(t) + t g(t) (g'(t))^* dt\right)^2$$

$$= \left(t|g(t)|^2 \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} |g(t)|^2 dt\right)^2$$

$$= \left(\int_{-\infty}^{\infty} |g(t)|^2 dt\right)^2 = 1$$

By Rayleigh's Energy thm

Otherwise $\int_{-\infty}^{\infty}|g'(t)|^2 dt = \int_{-\infty}^{\infty}|2\pi f X(f)|^2 df = 4\pi^2 \int_{-\infty}^{\infty} f^2 |X(f)|^2 df$

$$\Rightarrow L = 16\pi^2 \int_{-\infty}^{\infty} t^2 |g(t)|^2 dt \int_{-\infty}^{\infty} f^2 |G(f)|^2 df \geq 1$$

$$\Rightarrow T_{rms} W_{rms} = \left(\frac{L}{16\pi^2}\right)^{1/2} \geq \frac{1}{4\pi}$$

#

(b) According to Schwarz's inequality, the equality holds iff $g'(t) = c\,t\,g(t)$ for some $c \in \mathbb{C}$

Now $g(t) = e^{-\pi t^2} \Rightarrow g'(t) = 2te^{-\pi t^2} = 2t g(t)$

$\Rightarrow$ the equality holds

$$\Rightarrow T_{rms} W_{rms} = \frac{1}{4\pi}$$

#

2.19.

$W_n = W_{N-1-n} \quad 0 \le n \le N-1$

$$H(e^{jw}) = \sum_{n=0}^{N-1} W_n e^{-jwn} = \left(\sum_{n=0}^{N-1} W_n e^{-jw(n-\frac{N-1}{2})}\right) e^{-jw\frac{N-1}{2}}$$

$$= \left[W_{\frac{N-1}{2}} + 2\sum_{k=0}^{\frac{N-1}{2}-1} W_k \left(e^{-jw(k-\frac{N-1}{2})} + e^{jw(k-\frac{N-1}{2})}\right)\right] e^{-jw\frac{N-1}{2}}$$

$$= \left[W_{\frac{N-1}{2}} + 4\sum_{k=0}^{\frac{N-1}{2}-1} W_k \cos(k-\tfrac{N-1}{2})\right] e^{-jw\frac{N-1}{2}}$$

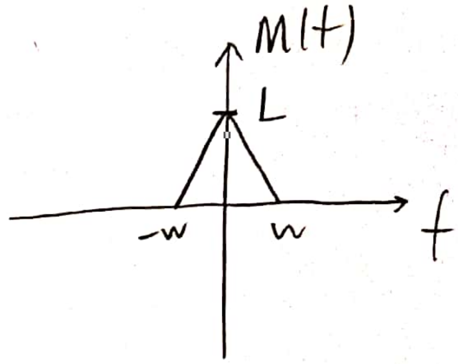
(a) $$|H(e^{jw})| = W_{\frac{N-1}{2}} + 4\sum_{k=0}^{\frac{N-1}{2}-1} W_k \cos(k-\tfrac{N-1}{2})$$

(b) $\measuredangle H(e^{jw}) = \frac{N-1}{2} w$

#

Ïhý uú

3.8.

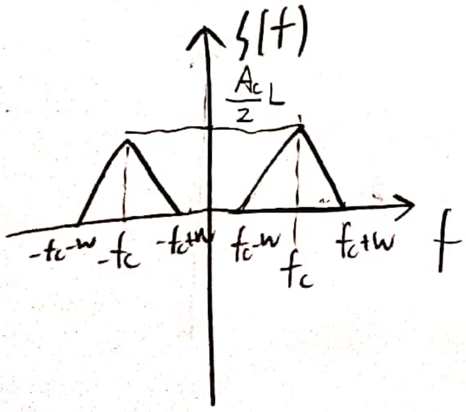


$$m(t) \xrightarrow{DSB\text{-}SC} s(t) = A_c \cos(2\pi f_c t)\, m(t)$$

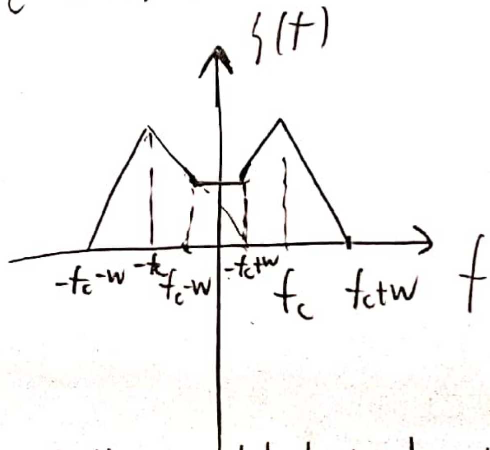
$$\Rightarrow S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right]$$

BW of $M(f)$ is $W = 1\,kHz$

(a) $f_c = 1.25\,kHz$



(b) $f_c = 0.75\,kHz$



The lowest f_c for which each component of the modulated signal $s(t)$ is uniquely determined by $m(t)$ is $1\,kHz$.

(i.e. there is no aliasing)

Ïhý uú

3.9.

$s_1(t) = A_c(1 + k_a m(t)) \cos(2\pi f_c t)$

$s_2(t) = A_c(1 - k_a m(t)) \cos(2\pi f_c t)$

$\Rightarrow s(t) = s_1(t) - s_2(t) = 2 k_a m(t) \cos(2\pi f_c t)$

$\Rightarrow s(t)$ is a DSB-SC modulated signal with $A_c = 2k_a$.

#

Ïhý uú

#

3.11.

(a) If there is no frequency error in the local carrier frequency

then $v(t) = s(t) A_c' \cos(2\pi f_c t + \phi)$

If there is an frequency error Δf in the local carrier frequency

then $\tilde{v}(t) = s(t) A_c' \cos(2\pi(f_c + \Delta f)t + \phi) = s(t) A_c' \cos(2\pi \Delta f t + 2\pi f_c t + \phi)$

The effect $E = \tilde{v}(t) - v(t) = s(t) A_c' [\cos(2\pi f_c t + \phi) - \cos(2\pi \Delta f t + 2\pi f_c t + \phi)]$

(b) For $s(t) = A_c \cos(2\pi f_c t) m(t)$

$\tilde{v}(t) = A_c \cos(2\pi f_c t) m(t) A_c' \cos(2\pi(f_c + \Delta f)t + \phi)$

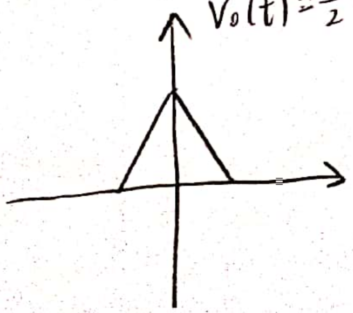
$= \frac{1}{2} A_c A_c' [2\pi(2f_c + \Delta f)t + \phi] m(t) + \frac{1}{2} A_c A_c' \cos(2\pi \Delta f t + \phi) m(t)$

$\tilde{v}(t) \xrightarrow{LPF} \tilde{v}_o(t) = \frac{1}{2} A_c A_c' \cos(2\pi \Delta f t + \phi) m(t)$

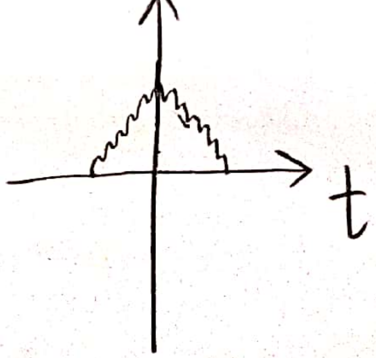
Since $\cos(2\pi \Delta f t + \phi)$ is nonconstant due to the term $2\pi \Delta f t$

$\therefore \tilde{v}_o(t)$ exhibits beats at Δf

$v_o(t) = \frac{1}{2} A_c A_c' \cos(\phi) m(t)$



$\tilde{v}_o(t)$



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Ïhý uú

3.14.

Let the input of receiver be $s(t) = m_1(t)\cos(2\pi f_c t) + m_2(t)\sin(2\pi f_c t)$

Now we has a phase error ϕ. (For convinient, I ignore A_c, A_c' here since they are just scalars.

Thus, cosider $\tilde{v}_1(t) = s(t)\cos(2\pi f_c t + \phi)$

$\Rightarrow \tilde{v}(t) = m_1(t)\cos(2\pi f_c t)\cos(2\pi f_c t + \phi) + m_2(t)\sin(2\pi f_c t)\cos(2\pi f_c t + \phi)$

$= \frac{1}{2}\cos(4\pi f_c t + \phi)\, m_1(t) + \frac{1}{2}\cos(\phi)\, m_1(t) + \frac{1}{2}\sin(4\pi f_c t + \phi)\, m_2(t) - \frac{1}{2}\sin(\phi)\, m_2(t)$

$\tilde{v}_1(t) \xrightarrow{LPF} \tilde{v}_{10}(t) = \frac{1}{2}\cos(\phi)\, m_1(t) - \frac{1}{2}\sin(\phi)\, m_2(t)$

Similarly, for $\tilde{v}_2(t) = s(t)\sin(2\pi f_c t + \phi)$, we derive

$\tilde{v}_2(t) \xrightarrow{LPF} \tilde{v}_{20}(t) = \frac{1}{2}\sin(\phi)\, m_1(t) + \frac{1}{2}\cos(\phi)\, m_2(t)$.

$\Rightarrow$ The phase error causes cross-talk between $\tilde{v}_{10}(t)$ and $\tilde{v}_{20}(t)$

#

Ïhý uú

3.15.

$$m_1(t) = V_0 + m_l(t) + m_r(t) \longrightarrow s_1(t) = A_c m_1(t)\cos(2\pi f_c t)$$

$$m_2(t) = m_l(t) - m_r(t) \longrightarrow s_2(t) = A_c m_2(t)\sin(2\pi f_c t)$$

(a)

We have $a(t)\cos(2\pi f_c t) + b(t)\cos(2\pi f_c t) \xrightarrow[\text{detector}]{\text{Envelope}} \sqrt{a^2(t) + b^2(t)}$

$$\Rightarrow s_1(t) \xrightarrow[\text{detector}]{\text{Envelope}} V_{1o}(t) = \sqrt{A_c^2 m_1^2(t)} = A_c m_1(t) = A_c(V_0 + m_l(t) + m_r(t))$$

To minimize the signal distortion, we need

$$\begin{cases} R_s C \ll 1/f_c & \text{(Charging time constant)} \\ 1/f_c \ll R_l C \ll 1/W & \text{(Discharging time constant)} \end{cases}$$

(b)

$$V_2(t) = s_2(t) A_c' \sin(2\pi f_c t + \phi)$$

$$= A_c m_2(t)\sin(2\pi f_c t) A_c' \sin(2\pi f_c t + \phi)$$

$$= \frac{-A_c A_c'}{2}\cos(4\pi f_c t + \phi) m_2(t) + \frac{A_c A_c'}{2}\cos(\phi) m_2(t)$$

$$V_2(t) \xrightarrow{LPF} V_{2o}(t) = A_c \cos(\phi) m_2(t) \quad (\text{set } A_c' = 2)$$

(c)

Construct $\tilde{V}_1(t) = \dfrac{V_{1o}(t)}{A_c} - V_0$

$$\tilde{V}_2(t) = \frac{V_{2o}(t)}{A_c}$$

Then $m_l(t) = \frac{1}{2}(\tilde{V}_1(t) + \tilde{V}_2(t))$

$$m_r(t) = \frac{1}{2}(\tilde{V}_1(t) - \tilde{V}_2(t))$$

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Ïhý uú