通訊系統 (II)

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課程要求

- 課程要求
  - Homework: 30 %
  - Midterm Exam: 35 %
  - Final Exam: 35 %
- 教科書：
  - Communication Systems, Simon Haykin **(4th Ed./5th Ed.)** John Wiley & Sons, Inc.
- 講義位置：(140.114.26.93) http://nyquist.ee.nthu.edu.tw/ (PW:2020commsysIIEE4640)
- 助教時間：每週二10:00~12:00
- 助教：TWNTHUEE4640@gmail.com

課程內容

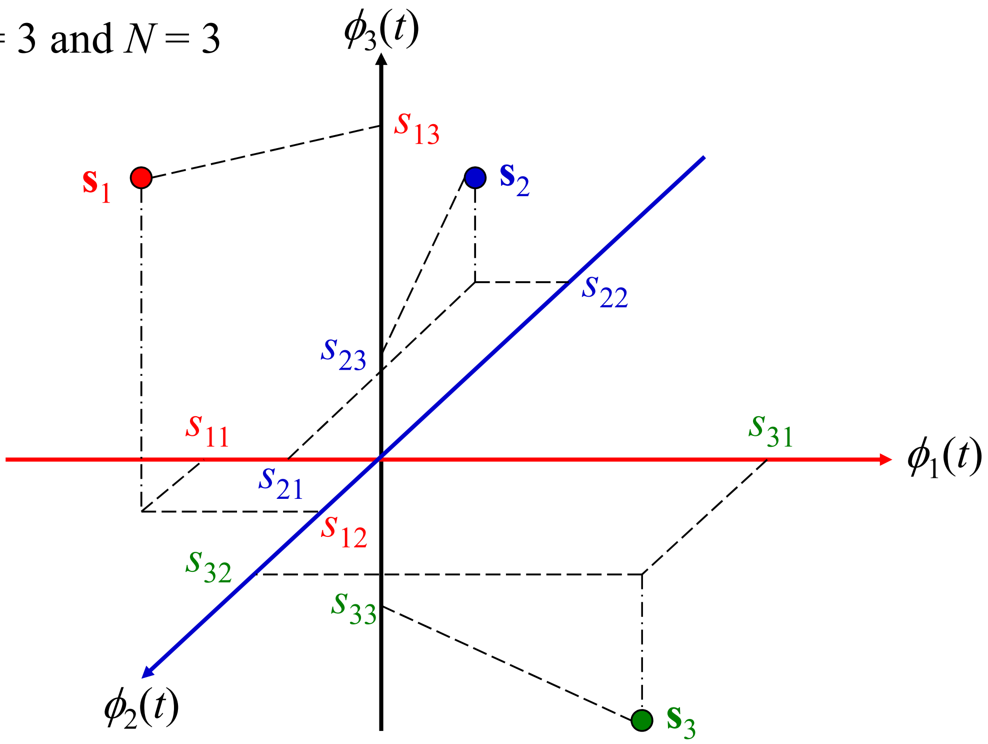
- Preliminaries
- Ch. 1: Signal-Space Analysis
- Ch. 2: Phase-Shift Keying Modulation
- Ch. 3: Hybrid Amplitude/Phase Modulation
- Ch. 4: Frequency-Shift Keying Modulation
- Ch. 5: Detection of Signals with Unknown Phase (Non-coherent Detection)
- Ch. 6: Comparison of Digital Modulation Schemes Using a Single Carrier ← 期中考試
- Ch. 7: Information Theory
- Ch. 8: Multichannel Modulation
- Ch. 9: Error-Control Coding
- Ch. 10: Spread-Spectrum Modulation ← 期末考試

Introductory Courses

- Signals and Systems
  - Signals and Systems
  - Linear Time-Invariant Systems
  - Fourier Analysis
- Probability Theory
  - Probability
  - Statistic
- Communications System I

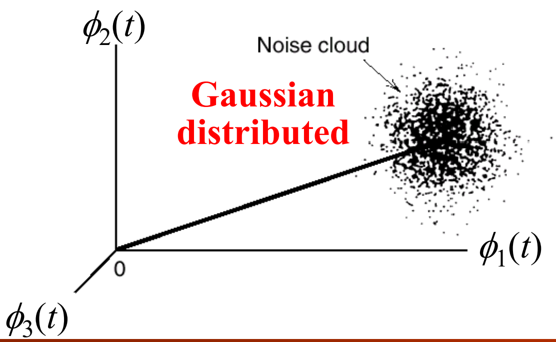
Ch. 1 – Signal-Space Analysis

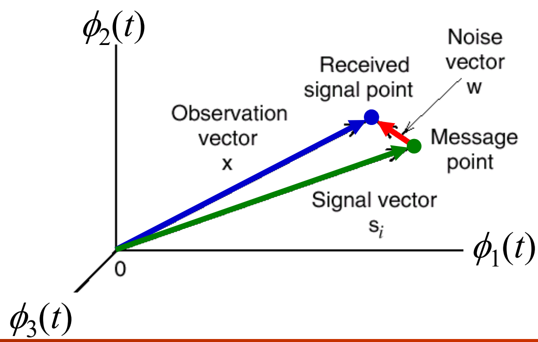
- $M = 3$ and $N = 3$



Ch. 1 – Signal-Space Analysis

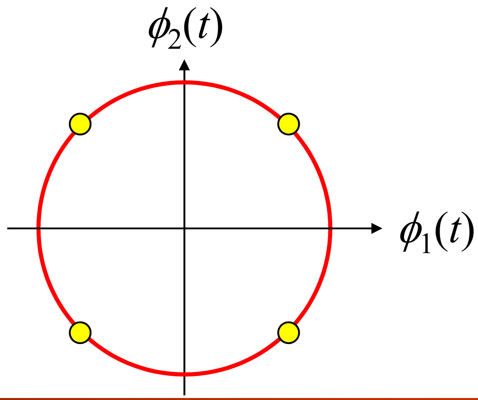
- Signal Detection – **MAP** (**maximum *a posteriori* probability)** and **ML** (**maximum likelihood**) decision rules
- The observation vector $\mathbf{x}$ (**received signal point**) differs from the transmitted signal vector $\mathbf{s}_i$ by a **random noise vector w**
- Given the observation vector $\mathbf{x}$, perform a mapping from $\mathbf{x}$ to an **estimate** $\hat{m}$ of the transmitted symbol m_i





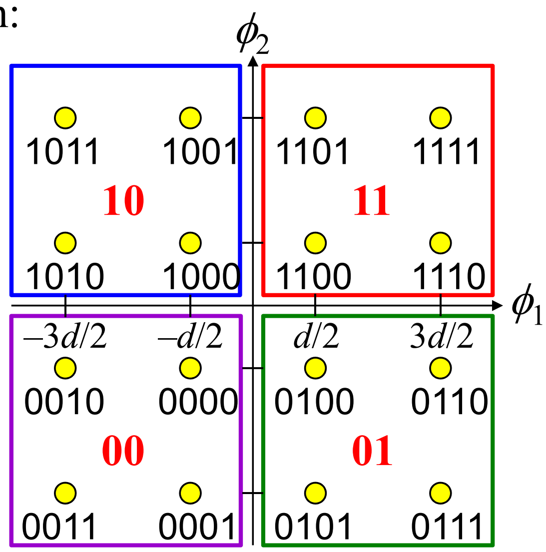
Ch. 2 – Phase-Shift Keying Modulation

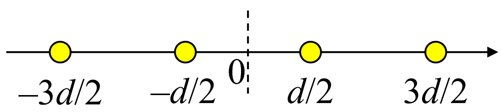
- In an ***M*-ary** PSK modulation scheme, multiple bits are transmitted in a symbol
- The signal are generated by changing the phase of a sinusoidal carrier in ***M* discrete steps**
- In QPSK, the phase of the carrier takes on one of four **equally spaced** values, such as **π/4, 3π/4, 5π/4, and 7π/4**



Ch. 3 – Hybrid Amplitude/Phase Modulation

- *M*-ary **Quadrature Amplitude Modulation** (**QAM**) is a **two-dimensional** generalization of *M*-ary PAM (**Pulse-Amplitude Modulation**)
- Consider a 16-QAM constellation: 4 bits per symbol

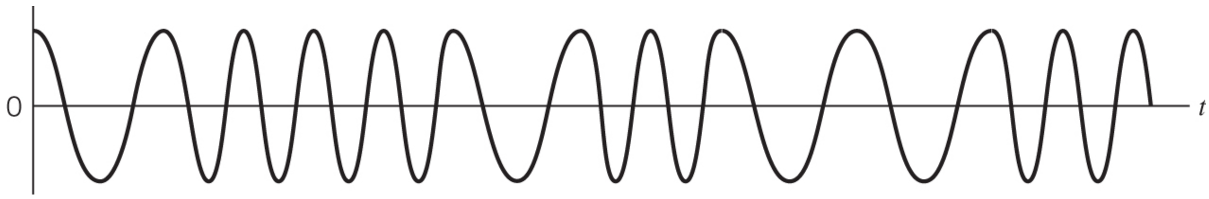




One-dimensional 4-ary PAM constellation

Ch. 4 – Frequency-Shift Keying Modulation

- In an ***M*-ary** FSK modulation scheme, multiple bits are transmitted in a symbol
- The signal are generated by changing the frequency of a sinusoidal carrier in ***M* discrete steps**
- In **binary FSK**, symbols 1 and 0 are distinguished from each other by transmitting one of two sinusoidal waves that **differ in frequency by a fixed amount**



Ch. 5–Detection of Signals with Unknown Phase

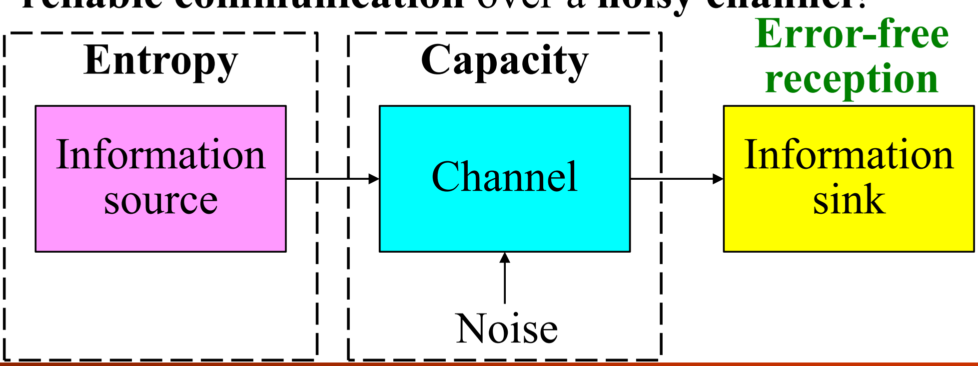
- In previous study, we assume that the receiver is **perfectly synchronized** (in both **frequency** and **phase**) to the transmitter
  - The only **channel impairment** is **AWGN**
- In practice, there is also uncertainty due to the randomness of certain signal parameters; for example, a **time-variant channel**
- The **phase** may change in a way that the receiver cannot follow
  - The receiver **cannot estimate** the received carrier phase
  - The carrier phase may **change too rapidly** for the receiver to **track**
- A digital communication receiver with no provision made for **carrier phase recovery** is said to be **noncoherent**
  - **Noncoherent detection**

Ch. 6–Comparison of Digital Modulation Schemes

- The popular digital modulation schemes are classified into **two categories**, depending on the method of **detection** used at the receiver:
  - **Class I, Coherent detection**:
    - Binary PSK: two symbols, single frequency
    - Binary FSK: two symbols, two frequencies
    - QPSK: four symbols, single frequency—includes the QAM as a special case
    - MSK: four symbols, two frequencies
  - **Class II, Noncoherent detection**:
    - DPSK: two symbols, single frequency
    - Binary FSK: two symbols, two frequencies

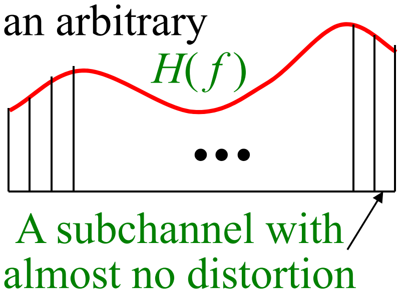
Ch. 7 – Information Theory

- In communications, **information theory** deals with modeling and analysis of a **communication system**
- In particular, it provides answers to two fundamental questions:
  - **Signal Source:** What is the irreducible **complexity**, below which a signal **cannot be compressed**?
  - **Channel:** What is the ultimate **transmission rate** for **reliable communication** over a **noisy channel**?



Ch. 8 – Multichannel Modulation

- Consider a linear **wideband channel** with an arbitrary frequency response $H(f)$.
  - The magnitude response $|H(f)|$ is approximated by a **staircase** function
  - Δf: the width of each **subchannel**



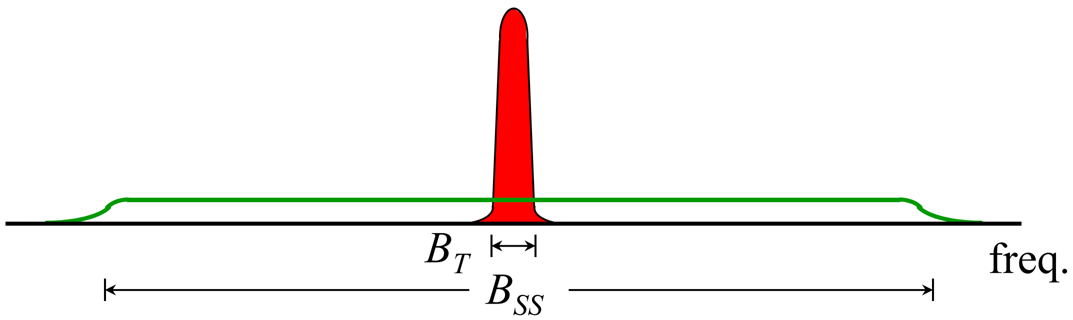
- In each step, the channel may be assumed to operate as an AWGN channel **free from inter-symbol interference**.
- **Power Loading** is to **maximize** the bit rate R through an **optimal sharing** of the total transmit power P between the N subchannels
  - Subject to the total transmit power constraint

Ch. 9 – Error-Control Coding

- **Error-control coding**: At the transmitter, incorporate a fixed number of **redundant bits** into the structure of a **codeword**
- It is feasible to provide **reliable communication** over a noisy channel
  - Provided that **Shannon's code theorem** is satisfied
- In effect, **channel bandwidth** is traded off for **reliability** in communications.
- Another practical motivation for the use of coding is to **reduce the required E_b/N_0** for a fixed BER. This reduction in E_b/N_0 may, in turn, be exploited to
  - **Reduce** the **required transmitted power**
  - **Reduce** the **hardware costs** by requiring a **smaller antenna size** (antenna gain) in the case of **radio communications**

Ch. 10 – Spread-Spectrum Modulation

- **Spread-spectrum** modulation refers to any modulation scheme that produces a spectrum for the transmitted signal **much wider** than the bandwidth of the information being transmitted
- The **demodulation** must be accomplished, in part, by correlating **the received signal** with **a replica of the signal** that is used in the transmitter to spread the information signal



Preliminaries

Probability and Random Variables

Probability

- Consider an experiment with a number of possible outcomes (e.g. the rolling of a die)
- The **sample space** $\boldsymbol{S} = \{\zeta_1, \zeta_2, \zeta_3, \ldots, \zeta_k, \ldots\}$ consists of the set of all possible outcomes (e.g., $\boldsymbol{S} = \{1, 2, 3, 4, 5, 6\}$)
  - The **sample points** of the experiment are the possible outcomes
  - An **event** E is a subset of $\boldsymbol{S}$, and may consist of any number of sample points (e.g., $E = \{2, 4\}$)
- The **σ-field** $\mathcal{F}$ is defined as the class of all subsets of $\boldsymbol{S}$.
- The **probability measure** is a set function $\mathbf{P}[\cdot]$ that assigns to every event $E \subset \boldsymbol{S}$ a number $\mathbf{P}[E]$ called the probability of E.
- The three objects $(\boldsymbol{S}, \mathcal{F}, \mathbf{P})$ form a triplet called a **probability space** $\mathcal{P}$.

Probability (Cont.)

- The probability measure satisfies the following three axioms:
  - $0 \le \mathbf{P}[A] \le 1$
  - $\mathbf{P}[\boldsymbol{S}] = 1$
  - If A and B are two mutually exclusive events, then
$$\mathbf{P}\left[A \cup B\right] = \mathbf{P}\left[A\right] + \mathbf{P}\left[B\right]$$
- The following properties can be derived from the above axioms:
  - $\mathbf{P}\left[\overline{A}\right] = 1 - \mathbf{P}\left[A\right]$, where $\overline{A}$ is the **complement** of the event A
  - When events A and B are not mutually exclusive, then the probability of the union event "A or B" satisfies
$$\mathbf{P}\left[A \cup B\right] = \mathbf{P}\left[A\right] + \mathbf{P}\left[B\right] - \mathbf{P}\left[A \cap B\right]$$
  - If $A_1, A_2, \ldots, A_m$ are mutually exclusive events that include all possible outcomes of the random experiment, then
$$\mathbf{P}\left[A_1\right] + \mathbf{P}\left[A_2\right] + \cdots + \mathbf{P}\left[A_m\right] = 1$$

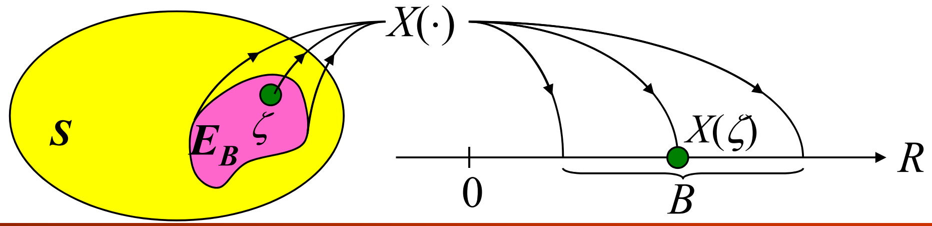
Conditional Probability

- The conditional probability of B given A is defined by
$$\mathbf{P}\left[B|A\right] = \mathbf{P}\left[A \cap B\right] / \mathbf{P}\left[A\right] \quad \Rightarrow \quad \mathbf{P}\left[A \cap B\right] = \mathbf{P}\left[B|A\right]\mathbf{P}\left[A\right]$$
- We may also write
$$\mathbf{P}\left[A \cap B\right] = \mathbf{P}\left[A|B\right]\mathbf{P}\left[B\right]$$
- The Bayes' rule
$$\mathbf{P}\left[B|A\right] = \frac{\mathbf{P}\left[A|B\right]\mathbf{P}\left[B\right]}{\mathbf{P}\left[A\right]}$$
- If $\mathbf{P}\left[B|A\right] = \mathbf{P}\left[B\right]$, the probability of the joint event $A \cap B$ is
$$\mathbf{P}\left[A \cap B\right] = \mathbf{P}\left[A\right]\mathbf{P}\left[B\right], \quad \text{and} \quad \mathbf{P}\left[A|B\right] = \mathbf{P}\left[A\right]$$
  - Event A and B that satisfy this condition are said to be **statistically independent**

Random Variables

- Consider an experiment having a sample space $\boldsymbol{S}$ and the random outcome $\zeta \in \boldsymbol{S}$.
- For every ζ we define a **function $X(\zeta)$**, called a **random variable (RV)**, which has
  - The **domain**: the sample space $\boldsymbol{S}$
  - The **range**: a set of real number
- For the example of rolling a die
  - $X(\zeta) = \zeta$: the outcomes are mapped into $\{1, 2, 3, 4, 5, 6\}$
  - $X(\zeta) = \zeta^2$: the outcomes are mapped into $\{1, 4, 9, 16, 25, 36\}$

$E_B \subset \boldsymbol{S}$



Random Variables (Cont.)

- The event $\{\zeta: X(\zeta) \le x\}$ corresponds to an assigned probability
- For a continuous RV, the probability that $\{X \le x\}$ is called the **cumulative distribution function (CDF)**

$$F_X(x) = \mathbf{P}[X \le x], \quad \text{and} \quad F_X(x_1) \le F_X(x_2) \text{ if } x_1 < x_2$$

- The **probability density function (PDF)** of RV X is

$$f_X(x) = \tfrac{d}{dx} F_X(x) \Rightarrow F_X(x) = \int_{-\infty}^{x} f_X(u)\, du$$

- Furthermore, we have

$$\mathbf{P}[x_1 < X \le x_2] = \mathbf{P}[X \le x_2] - \mathbf{P}[X \le x_1] = \int_{x_1}^{x_2} f_X(u)\, du$$

- The set $\{\zeta: X(\zeta) \in B\}$ must correspond to an event $E_B \subset \boldsymbol{S}$
- In order to assign certain desirable continuity properties to the function $F_X(x)$ at $x = \pm\infty$, we require that the events **$\{X = \infty\}$** and **$\{X = -\infty\}$** have probability **zero**

Multiple Random Variables

- Consider two random variables X and Y. We define the **joint distribution function** as

$$F_{X,Y}(x,y) = \mathbf{P}\left[X \le x, Y \le y\right]$$

- The **joint probability density function** is defined as

$$f_{X,Y}(x,y) = \tfrac{\partial^2}{\partial x \partial y} F_{X,Y}(x,y) \quad \text{and} \quad \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(\xi,\eta)\, d\xi\, d\eta = 1$$

- Furthermore, we have

$$F_X(x) = \int_{-\infty}^{\infty}\int_{-\infty}^{x} f_{X,Y}(\xi,\eta)\, d\xi\, d\eta$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,\eta)\, d\eta$$

- The **conditional probability density function** of Y given that $X = x$ is defined by

$$f_Y(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$$

Multiple Random Variables (Cont.)

- Accordingly, it must satisfy all the requirements of a pdf, i.e.,

$$f_Y(y|x) \ge 0 \quad \text{and} \quad \int_{-\infty}^{\infty} f_Y(y|x)\, dy = 1$$

- If the random variables X and Y are **statistically independent**, then knowledge of the outcome of X can in no way affect the distribution of Y.
  - The conditional pdf reduces to the **marginal density**

$$f_Y(y|x) = f_Y(y)$$

  - Furthermore, the joint pdf can be expressed as

$$f_{X,Y}(x,y) = f_X(x) f_Y(y)$$

- Equivalently, if the joint pdf of the random variables X and Y equals the **product of their marginal densities**, then X and Y are **statistically independent**

Statistical Averages

- The **expected value (mean)** of a random variable X

$$\mu_X = E[X] = \int_{-\infty}^{\infty} x \, f_X(x) \, dx$$

- Considering a real-valued function $g(X)$, the quantity obtained by letting the argument be a random variable is also a random variable, denoted as $Y = g(X)$
- The expected value of Y is

$$\mu_Y = E[Y] = \int_{-\infty}^{\infty} y \, f_Y(y) \, dy$$

- A simple procedure to obtain μ_Y is

$$\mu_Y = E[g(X)] = \int_{-\infty}^{\infty} g(X) f_X(x) \, dx$$

Statistical Averages (Cont.)

- By setting $g(X) = X^n$, the n-th **moment** of a random variable X is defined by

$$E[X^n] = \int_{-\infty}^{\infty} x^n f_X(x) \, dx$$

  - First moment: mean
  - Second moment: mean-square value
- The n-th **central moments** of a random variable X is defined by

$$E[(X - \mu_X)^n] = \int_{-\infty}^{\infty} (x - \mu_X)^n \; f_X(x) \, dx$$

  - First central moment: 0
  - Second central moment: **variance**

$$\sigma_X^2 = E[X^2] - \mu_X^2$$

  - σ_X is called the **standard deviation** of a random variable X

Gaussian (Normal) Distribution

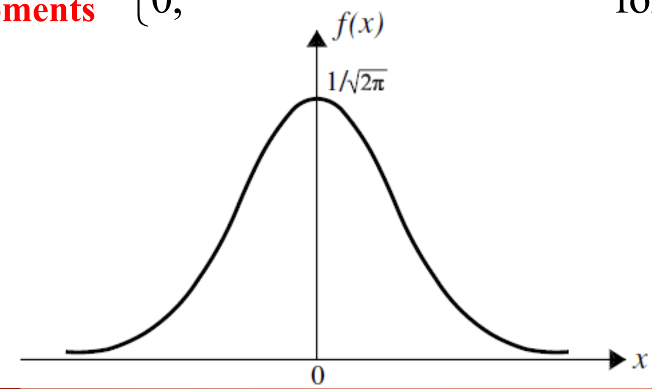
- The pdf of a Gaussian random variable X is defined by

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma_X}\exp\left[-\frac{(x-\mu_X)^2}{2\sigma_X^2}\right], \quad -\infty < x < \infty$$

- The higher-order moments are uniquely determined by μ_X and σ_X

$$E[(X-\mu_X)^n] = \begin{cases} 1\times 3\times 5\times\cdots\times(n-1)\sigma_X^n, & \text{for } n \text{ even} \\ 0, & \text{for } n \text{ odd} \end{cases}$$

Central moments



Fourier Theory and Signal Representation

Fourier Transform

- Let $g(t)$ denote a **non-periodic** deterministic signal
- The **Fourier transform** of the signal $g(t)$ is given by

$$G(f) = \int_{-\infty}^{\infty} g(t) \exp(-j2\pi ft)\, dt = F\left[g(t)\right]$$

- The original signal $g(t)$ is recovered by the **inverse Fourier transform**

$$g(t) = \int_{-\infty}^{\infty} G(f) \exp(j2\pi ft)\, df = F^{-1}\left[G(f)\right]$$

- The **Fourier-transform pair**

$$g(t) \rightleftharpoons G(f)$$

Fourier Transform (Cont.)

- In general, the Fourier transform $G(f)$ is a complex function

$$G(f) = \left|G(f)\right| \exp\left[j\theta(f)\right]$$

  – $|G(f)|$ is called the **continuous amplitude spectrum** of $g(t)$
  – $|\theta(f)|$ is called the **continuous phase spectrum** of $g(t)$
- For a **real-valued** function $g(t)$, the Fourier transform has the property

$$G(-f) = G^*(f) \Rightarrow \left|G(-f)\right| = \left|G(f)\right| \text{ and } \theta(-f) = -\theta(f)$$

- The spectrum of a real-valued signal: **conjugate symmetry**
  – The amplitude spectrum of a real-valued signal is an **even function** of the frequency
  – The phase spectrum of a real-valued signal is an **odd function** of the frequency

Properties of Fourier Transform

- Linearity: $ag_1(t) + bg_2(t) \rightleftharpoons aG_1(f) + bG_2(f)$
- Time scaling: $g(at) \rightleftharpoons \frac{1}{|a|} G(f/a)$ **Time-compressed for *a* > 1**
- Duality: $g(t) \rightleftharpoons G(f) \quad \Rightarrow \quad G(t) \rightleftharpoons g(-f)$
- Time shifting: $g(t - t_0) \rightleftharpoons G(f)\exp(-j2\pi f t_0)$
- Frequency shifting: $\exp(j2\pi f_c t) g(t) \rightleftharpoons G(f - f_c)$
- Area under $g(t)$: $\int_{-\infty}^{\infty} g(t)\, dt = G(0)$
- Area under $G(f)$: $\int_{-\infty}^{\infty} G(f)\, df = g(0)$
- Differentiation in the time domain: $\frac{d}{dt} g(t) \rightleftharpoons j2\pi f G(f)$

Properties of Fourier Transform (Cont.)

- Integration in the time domain:

$$\int_{-\infty}^{t} g(\tau)\, d\tau \rightleftharpoons \frac{1}{j2\pi f} G(f) + \frac{G(0)}{2}\delta(f)$$

- Conjugate functions: $g(t) \rightleftharpoons G(f) \quad \Rightarrow \quad g^*(t) \rightleftharpoons G^*(-f)$
- Multiplication in the time domain:

$$g_1(t) g_2(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda) G_2(f - \lambda)\, d\lambda$$

- Convolution in the time domain:

$$\int_{-\infty}^{\infty} g_1(\tau) g_2(t - \tau)\, d\tau \rightleftharpoons G_1(f) G_2(f)$$

- Rayleigh's energy theorem:

$$\int_{-\infty}^{\infty} |g(t)|^2\, dt = \int_{-\infty}^{\infty} |G(f)|^2\, df$$

Autocorrelation and Power Spectral Density of a Stationary Process

Autocorrelation Function

- The **autocorrelation function** of a process $X(t)$ is defined as the expectation of the product of two RVs, $X(t_1)$ and $X(t_2)$.

$$R_X(t_1, t_2) = E[X(t_1)\, X(t_2)]$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2 f_{X(t_1),X(t_2)}(x_1 x_2)\, dx_1\, dx_2$$

- If $X(t)$ is **stationary** to second order, the autocorrelation function depends only on the **time difference** $t_2 - t_1$.

$$R_X(t_1, t_2) \overset{\text{stationary}}{=} R_X(t_2 - t_1) = R_X(\tau) \quad \text{for all } t_1 \text{ and } t_2$$

- For a stationary process $X(t)$:

$$R_X(\tau) = E[X(t+\tau)\, X(t)]$$

- The mean-square value (average power) of $X(t)$:

$$R_X(0) = E[X^2(t)]$$

Power Spectral Density

- The power spectral density shows the **frequency-domain** characteristics of a signal
  - Bandwidth and the distribution of energy in different frequency components
- Define the **power spectral density** of a stationary process $X(t)$ as the **Fourier transform of autocorrelation function** $R_X(\tau)$

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau) \exp(-j2\pi f\tau)\, d\tau, \quad \text{(W/Hz)}$$

$$R_X(\tau) = \int_{-\infty}^{\infty} S_X(f) \exp(j2\pi f\tau)\, df$$

- The **average total power** is obtained by integrating the power spectral density over **all frequency components**

$$R_X(0) = \int_{-\infty}^{\infty} S_X(f)\, df$$

Sinusoidal Signal Representation

Sinusoidal Signal Representation

- Consider a general sinusoidal signal

$$s(t) = A(t)\cos\left[2\pi f_c t + \phi(t)\right]$$

  - $A(t)$ is the time-varying envelope
  - f_c is the carrier frequency
  - $\phi(t)$ is the time-varying phase
- The **time-varying phase** also implies that the **instantaneous frequency** is time-varying

$$\boxed{\exp(j\theta) = \cos(\theta) + j\sin(\theta)}$$

- The signal can also be represented as

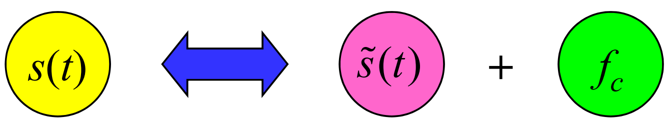
$$\begin{aligned} s(t) &= \mathrm{Re}\left\{A(t)\exp\left[j\left(2\pi f_c t + \phi(t)\right)\right]\right\} \\ &= \mathrm{Re}\left\{A(t)\exp\left[j\phi(t)\right]\exp\left[j2\pi f_c t\right]\right\} \\ &= \mathrm{Re}\left\{\tilde{s}(t)\exp\left[j2\pi f_c t\right]\right\} \end{aligned}$$

  - $\tilde{s}(t)$ is known as the **complex envelope** (a **low-pass** signal)

Sinusoidal Signal Representation (Cont.)

- For a specific **known** carrier frequency f_c, the signal $s(t)$ can be **completely** represented by the **complex envelope** $\tilde{s}(t)$

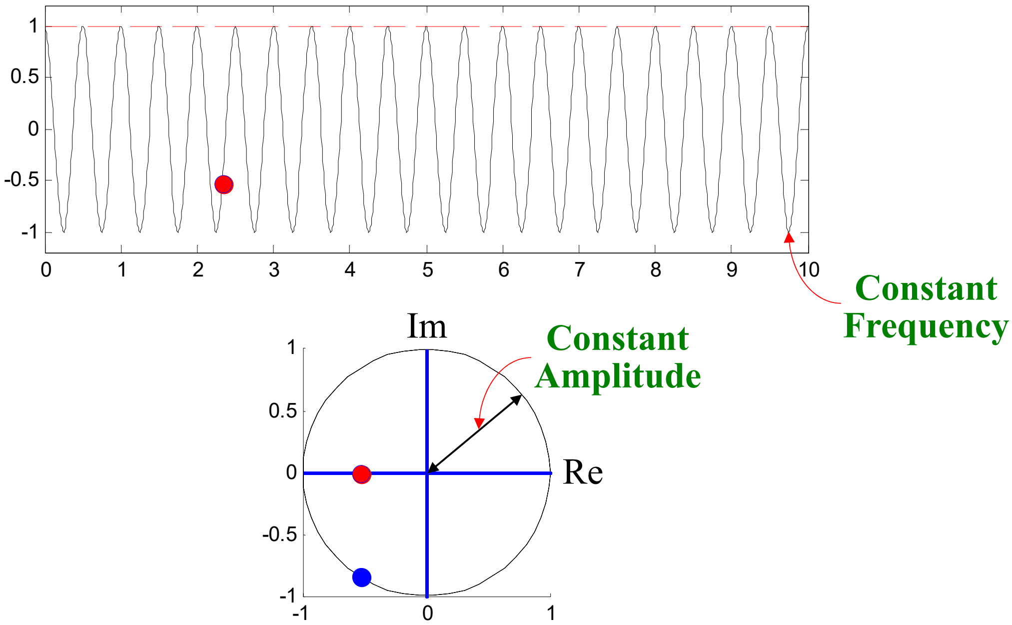
$$\begin{aligned} s(t) &= A(t)\cos\left(2\pi f_c t + \phi(t)\right) \\ &= \mathrm{Re}\left\{A(t)\exp\left[j\left(2\pi f_c t + \phi(t)\right)\right]\right\} \\ &= \mathrm{Re}\left\{\tilde{s}(t)\exp\left[j2\pi f_c t\right]\right\} \end{aligned}$$



Constant Envelope and Zero Phase

- When the envelope $A(t)$ is constant ($A(t) = 1$) and $\phi(t) = 0$

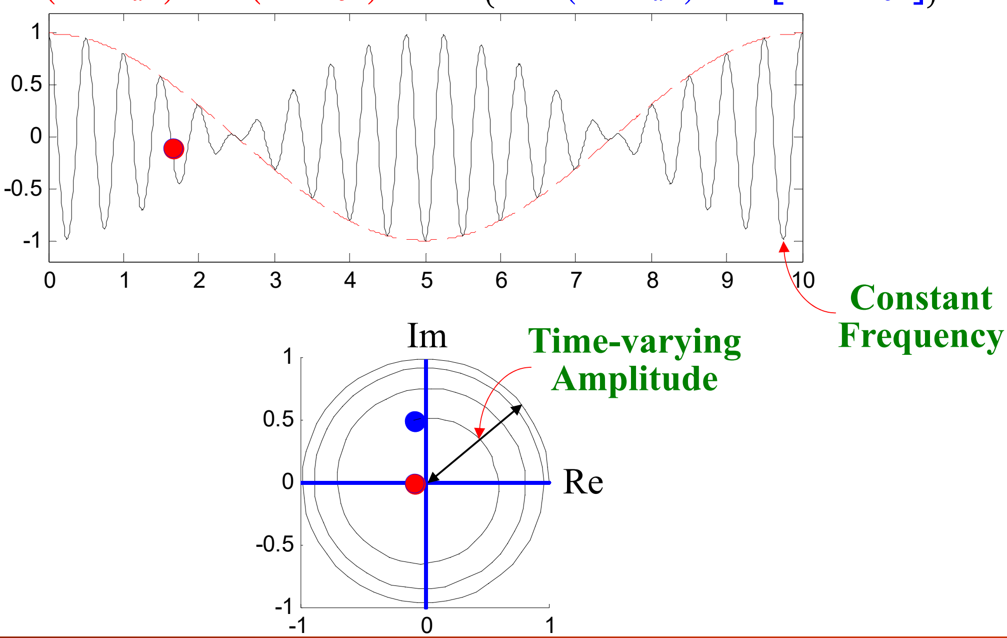
$$s(t) = \cos\left(2\pi f_c t\right) = \mathrm{Re}\left\{\exp\left[j2\pi f_c t\right]\right\}$$



Time-varying Envelope and Zero Phase

- When the envelope $A(t)$ is time-varying and $\phi(t) = 0$

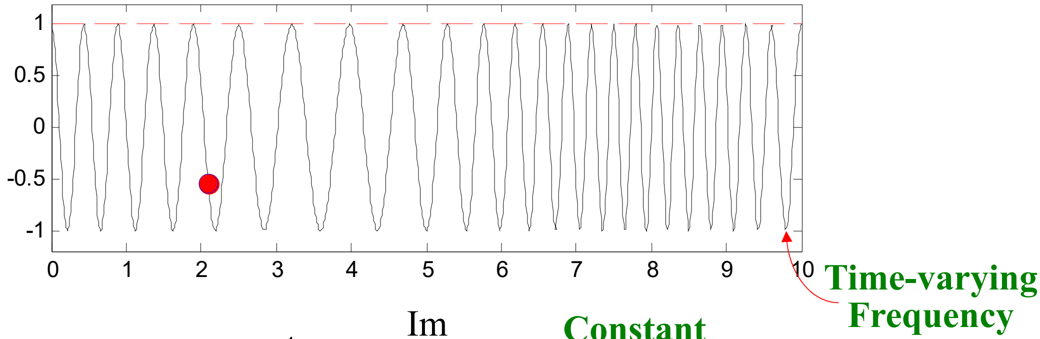
$$s(t) = \cos\left(2\pi f_a t\right)\cos\left(2\pi f_c t\right) = \mathrm{Re}\left\{\cos\left(2\pi f_a t\right)\exp\left[j2\pi f_c t\right]\right\}$$

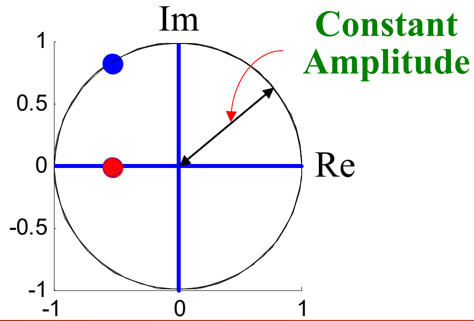


Constant Envelope and Time-varying Phase

- When the envelope $A(t)$ is constant and $\phi(t)$ is time-varying

$$s(t) = \cos\left(2\pi f_c t + \phi(t)\right) = \mathrm{Re}\left\{\exp\left[j2\pi f_c t + j\phi(t)\right]\right\}$$

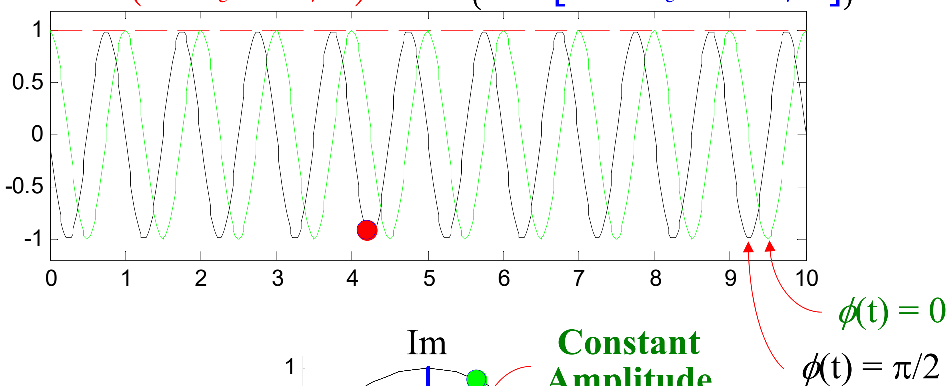


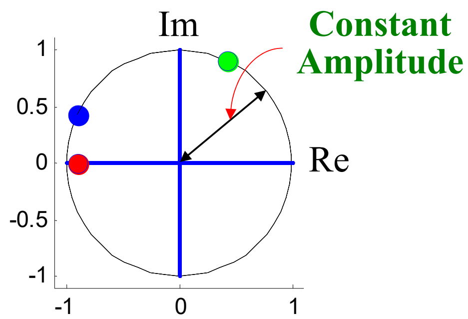


Constant Envelope and Constant Phase

- When both the envelope $A(t)$ and phase $\phi(t)$ (= π/2) are constant

$$s(t) = \cos\left(2\pi f_c t + \pi/2\right) = \mathrm{Re}\left\{\exp\left[j2\pi f_c t + j\pi/2\right]\right\}$$





Constant Envelope and Constant Phase (Cont.)

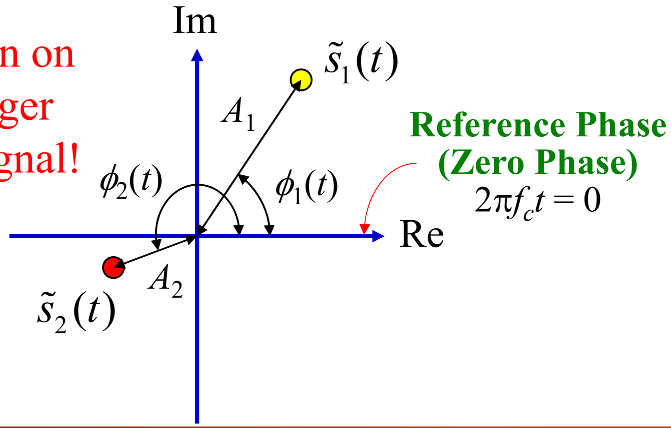
- In general, we can set $\mathbf{2\pi f_c t}$ as the **reference phase** (i.e., the zero phase)

$$s(t) = A\cos\left(2\pi f_c t + \phi\right)$$

- Then, a signal with constant envelope A and constant phase ϕ can be represented as a complex number (a point in the complex-plane)

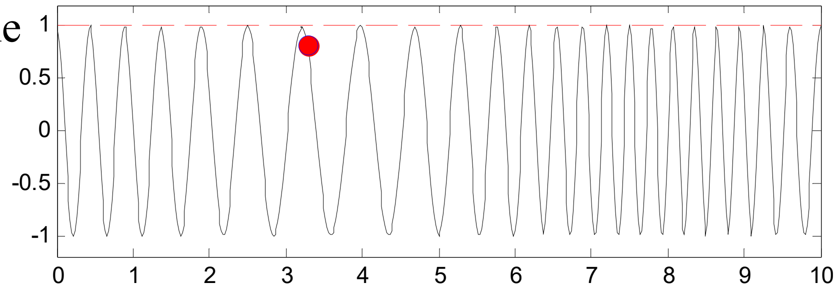
$$\tilde{s}(t) = A\exp\left[j\phi\right]$$

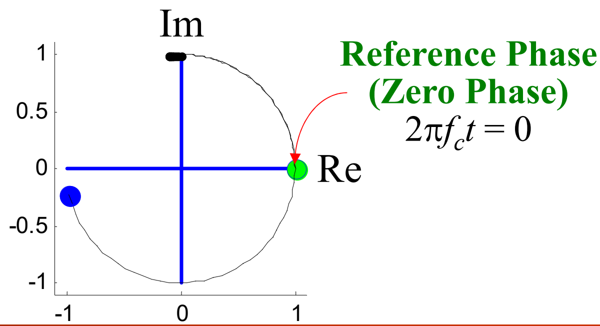
Note that the projection on the **Re** axis is no longer the amplitude of the signal!



Constant Envelope and Time-varying Phase

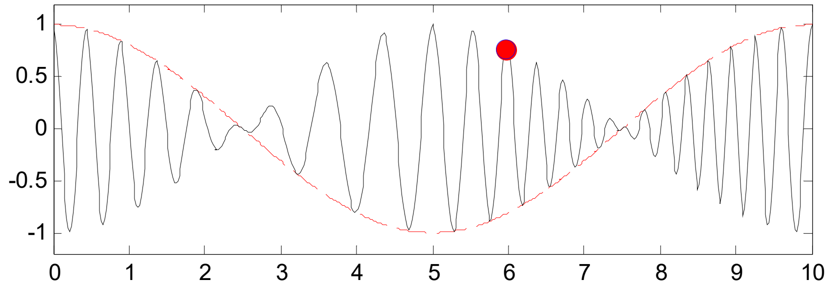
- Similarly, a signal with constant envelope $A(t)$ and time-varying phase $\phi(t)$ can be represented as a time-varying point in the complex-plane

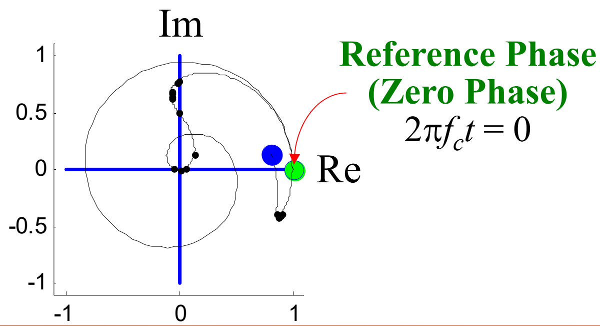




Time-varying Envelope and Phase

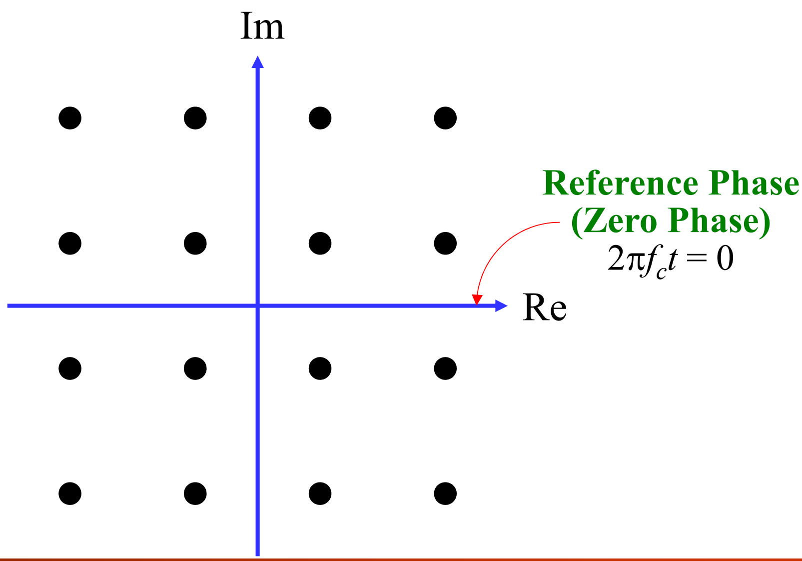
- Similarly, a signal with time-varying envelope $A(t)$ and phase $\phi(t)$ can be represented as a time-varying point in the complex-plane





Representation of 16QAM Signals

- QAM: a kind of **digital modulation** by using **different phases** and/or **different amplitudes** to represent different data
- 16QAM: 16 signal points (complex numbers) representing 4-bit data



Band-pass Signals

- A **band-pass signal** is sinusoidal with approximate frequency f_c and an amplitude varying with time
$$g(t) = a(t)\cos\left[2\pi f_c t + \phi(t)\right]$$
  - where $a(t)$ is the **envelope** and $\phi(t)$ is the **phase** of the signal
- The band-pass signal $g(t) = a(t)\cos\left[2\pi f_c t + \phi(t)\right]$ can be rewritten as
$$\begin{aligned} g(t) &= \mathrm{Re}\left[\underline{a(t)\exp(j\phi(t))}\exp(j2\pi f_c t)\right] \\ &= \mathrm{Re}\left[\tilde{g}(t)\exp(j2\pi f_c t)\right] \end{aligned}$$
  - where $\tilde{g}(t)$ is referred to as the **complex envelope** of the band-pass signal (a **low-pass equivalent** signal)
- The complex envelope can be represented as
$$\tilde{g}(t) = g_I(t) + jg_Q(t)$$
  - where $g_I(t) = a(t)\cos\phi(t), \quad g_Q(t) = a(t)\sin\phi(t)$