COM 5120 Communication Theory

Final Exam

December 27, 2022
15:30 $\sim$ 17:20

Note: There are **6** problems with total 100 points within **2** pages, please write your answer with detail in the answer sheet.

No credit without detail. Closed books. You may use scientific calculator.

1. (12%) Let X be a geometrically distributed random variable, i.e.,

$$P(X = k) = p(1-p)^{k-1}, k = 1, 2, 3, ...$$

   (a) Find the entropy of X.
   (b) Given that $X > K$, where K is a positive integer, what is the entropy of X?

2. (12%) For the channel shown in Figure 1 and given that $P(A) = 1 - p,\ P(B) = P(C) = \frac{p}{2}$, find the channel capacity and the input distribution that achieves capacity.

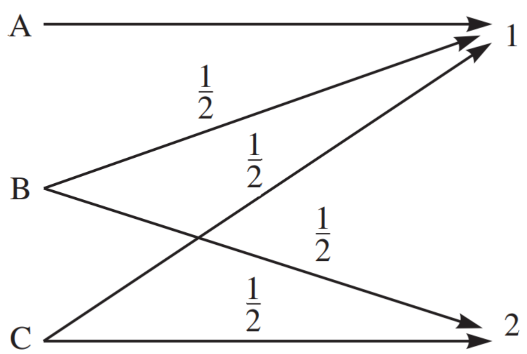

Figure 1: channel model

3. (12%) A voice-band telephone channel passes the frequencies in the band from 300 to 3300 Hz. It is desired to design a modem that transmits at a symbol rate of 2400 symbols/s, with the objective of achieving 9600 bits/s.
   Select an appropriate QAM signal constellation, carrier frequency, and the roll-off factor of a pulse with a raised cosine spectrum that utilizes the entire frequency band.
   Sketch the spectrum of the transmitted signal pulse and indicate the important frequencies.

4. (24%) Three equiprobable messages m_1, m_2 and m_3 are to be transmitted over an AWGN channel with noise power spectral density $\frac{1}{2}N_0$. The messages are

$$s_1(t) = \begin{cases} 1 & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases} \qquad s_2(t) = -s_3(t) \begin{cases} 1 & 0 \le t \le \dfrac{1}{2}T \\ -1 & \dfrac{1}{2}T < t \le T \\ 0 & \text{otherwise} \end{cases}$$

(a) What is the dimensionality of the signal space?
(b) Find an appropriate basis for the signal space.
(c) Please Draw the signal constellation of the messages m_1, m_2, m_3 and the optimal decision regions R_1, R_2, R_3 for this problem.
(d) Which of the three messages is most vulnerable to errors and why? In other words, which of $P(\text{error} \mid m_i \text{ transmitted})$, $i = 1, 2, 3$, is largest?

5. (24%) Two discrete memoryless information sources X and Y each have an alphabet with six symbols, $\mathcal{X} = \mathcal{Y} = \{1, 2, 3, 4, 5, 6\}$. The probabilities of the letters for X are $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{32}\}$. The source Y has a uniform distribution.
(a) Find the entropy of both X and Y.
(b) Design Huffman codes for each source. Which Huffman code is more efficient?
(***Hint***: Efficiency of a Huffman code is defined as the ratio of the source entropy $H(X)$ to the average codeword length $\bar{R}$.)
(c) If Huffman codes were designed for the second extension of these sources (i.e., two letters at a time), for which source X or Y would you expect a performance improvement compared to the single-letter Huffman code and why?

6. (16%) The transmission of a signal pulse with a raised cosine spectrum through a channel results in the following **(noise-free)** sampled output from the demodulator:

$$x_m = \begin{cases} -0.5 & m = -2 \\ 0.1 & m = -1 \\ 1 & m = 0 \\ -0.2 & m = 1 \\ 0.05 & m = 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Design a three-tap zero-forcing linear equalizer so that the output is

$$q_m = \begin{cases} 1 & m = 0 \\ 0 & m = \pm 1 \end{cases}$$

(b) Determine q_m for $m = \pm 2, \pm 3$ by convolving the impulse response of the equalizer with the channel response.