COM 5120 Communication Theory

Midterm Make-up Exam

November 17, 2022
15:30 ∼ 17:20

Note: There are **6** problems with total 100 points within **3** pages, please write your answer with detail in the answer sheet.

No credit without detail. No calculator. Closed books.

1. (25%) Consider the four waveforms shown in Figure 1.
   (a) Determine the dimensionality of the waveforms and a set of basis functions.
   (b) Use the basis functions to represent the four waveforms by vectors s_1, s_2, s_3, and s_4.
   (c) Determine the minimum distance between any pair of vectors.

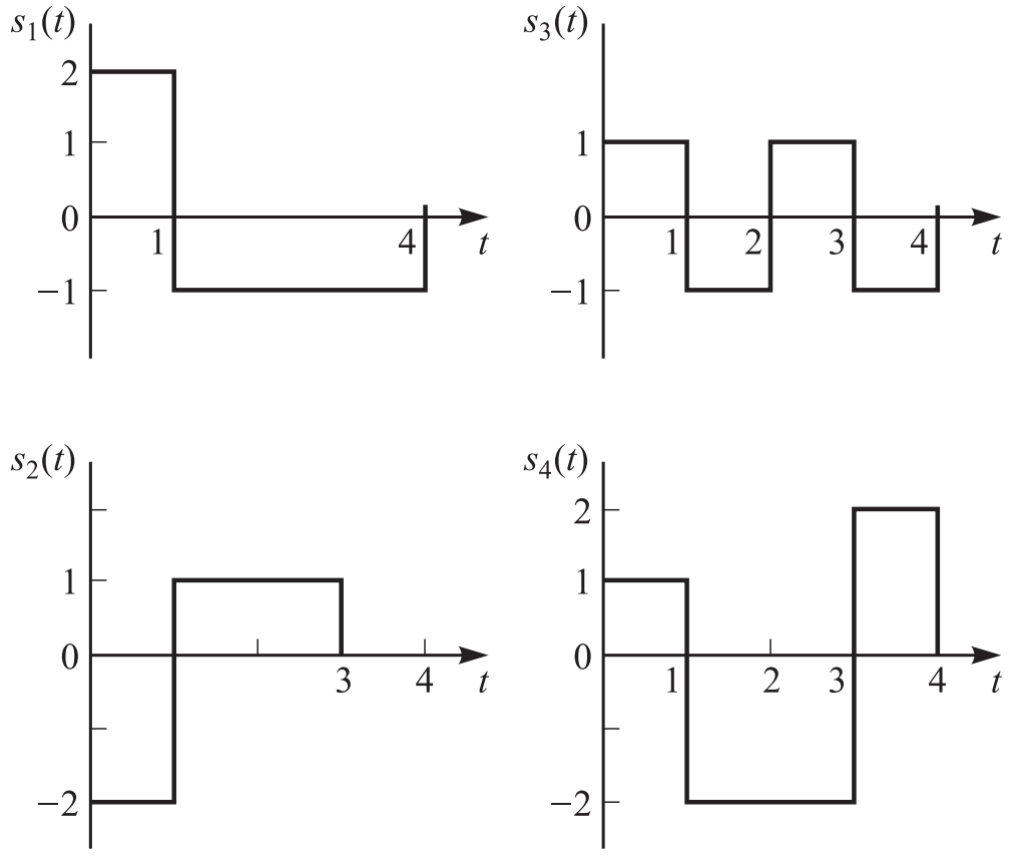

Figure 1: four waveforms $s_1(t)$, $s_2(t)$, $s_3(t)$, $s_4(t)$

2. (25%) Suppose that X is a Gaussian random variable with zero mean and unit variance. Let

$$Y = aX^3 + b, \quad a > 0$$

Determine and plot the PDF of Y.

3. (10%) Consider the octal signal point constellations shown in Figure 2.
   (a) The nearest-neighbor signal points in the 8-QAM signal constellation are separated in distance by A units. Determine the **radii** a and b of the inner and outer circles, respectively.
   (b) The adjacent signal points in the 8-PSK are separated by a distance of A units. Determine the **radius** r of the circle.

4. (20%) Consider the 8-point QAM signal constellation shown in Figure 2.
   (a) Is it possible to assign 3 data bits to each point of the signal constellation such that the nearest (adjacent) points differ in only 1 bit position?
   (b) Determine the symbol rate if the desired bit rate is 90 Mbits/s.

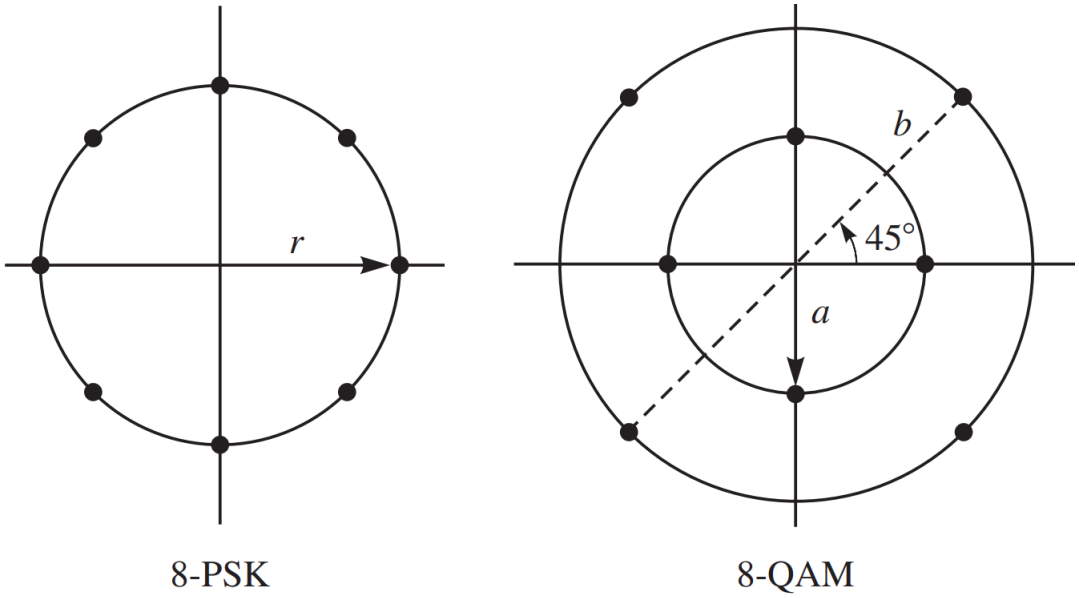

Figure 2: 8-PSK and 8-QAM

5. (10%) A binary digital communication system employs the signals

$$s_0(t) = 0,\ 0 \le t \le T$$
$$s_1(t) = A,\ 0 \le t \le T$$

   for transmitting the information. This is called *on-off signaling*. The demodulator crosscorrelates the received signal $r(t)$ with $s(t)$ and samples the output of the correlator at $t + T$.
   (a) Determine the **optimum detector** for an AWGN channel and the **optimum threshold**, assuming that the signals are equally probable.
   Given that the correlation type demodulator employes a filter:

$$f(t) = \begin{cases} \dfrac{1}{\sqrt{T}}, & 0 \le t < T \\ 0, & otherwise \end{cases}$$

   (b) Determine the **probability of error** as a function of the SNR. How does on-off signaling compare with antipodal signaling?

6. (10%) Consider the three waveforms $f_n(t)$ shown in Figure 3.
   (a) Show that these waveforms are **orthonormal**.
   (b) Express the waveform $x(t)$ as a **linear combination** of $f_n(t)$, $n = 1, 2, 3$ if

$$x = \begin{cases} -2, & 0 \le t < 1 \\ 6, & 1 \le t < 3 \\ 4, & 3 \le t < 4 \end{cases}$$

   and determine the **weighting coefficients**.

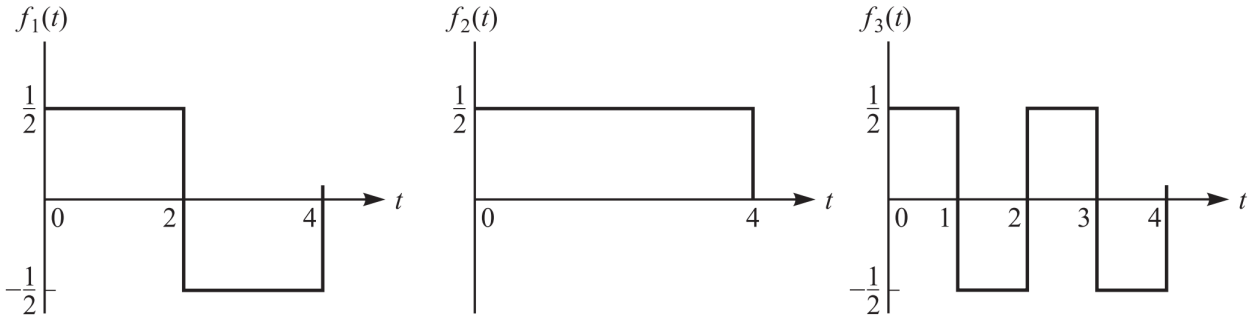

Figure 3: three waveforms $f_1(t)$, $f_2(t)$, $f_3(t)$