Homework #1

Due 23:59, Oct. 27, 2022

2.10 Consider the three waveforms $f_n(t)$ shown in Figure P2.10. **(5%+5%)**

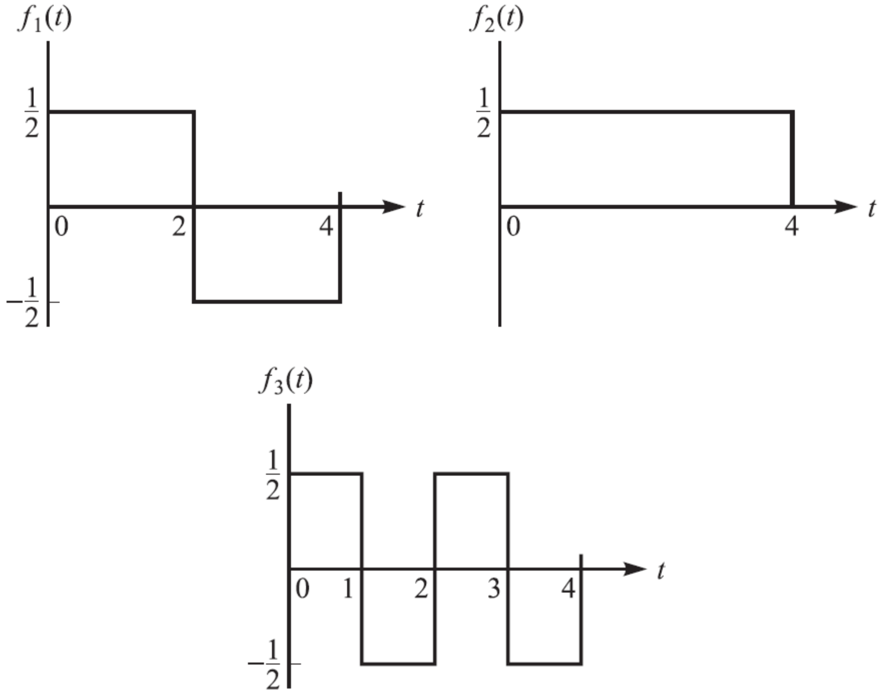

FIGURE P2.10

a. Show that these waveforms are orthonormal.
b. Express the waveform $x(t)$ as a linear combination of $f_n(t)$, $n = 1, 2, 3$, if

$$x(t) = \begin{cases} -1 & 0 \le t < 1 \\ 1 & 1 \le t < 3 \\ -1 & 3 \le t < 4 \end{cases}$$

and determine the weighting coefficients.

2.12 Determine a set of orthonormal functions for the four signals shown in Figure P2.12. **(10%)**

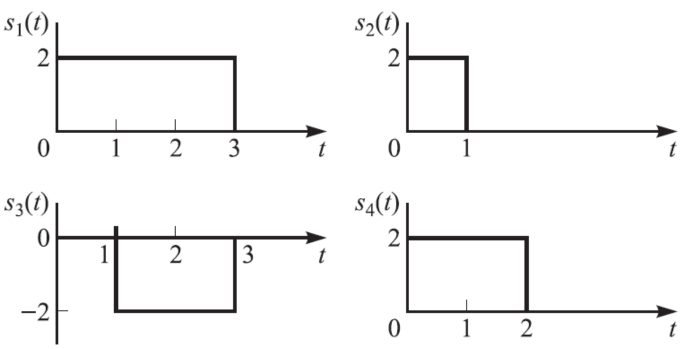

FIGURE P2.12

2.15 The random variables $X_i,\ i = 1, 2,\ \ldots,\ n$, have joint PDF $p(x_1, x_2, \ldots, x_n)$. Prove that

$$p(x_1, x_2, x_3, \ldots, x_n) = p(x_n|x_{n-1}, \ldots, x_1)p(x_{n-1}|x_{n-2}, \ldots, x_1) \cdots p(x_3|x_2, x_1)p(x_2|x_1)p(x_1)$$

(10%)

2.17 The PDF of a random variable X is $p(x)$. A random variable Y is defined as

$$Y = aX + b$$

where $a < 0$. Determine the PDF of Y in terms of the PDF of X. **(10%)**

2.20 X is a $\mathcal{N}(0, \sigma^2)$ random variable. This random variable is passed through a system whose input-output relation is given by $y = g(x)$. Find the PDF or the PMF of the output random variable Y in each of the following cases. **(5% + 5% + 5% + 5%)**

1. Square-law device, $g(x) = ax^2$.
2. Limiter,

$$g(x) = \begin{cases} -b & x \le -b \\ b & x \ge b \\ x & |x| < b \end{cases}$$

3. Hard limiter,

$$g(x) = \begin{cases} a & x > 0 \\ 0 & x = 0 \\ b & x < 0 \end{cases}$$

4. Quantizer, $g(x) = x_n$ for $a_n \le x < a_{n+1}$, $1 \le n \le N$, where x_n lies in the interval $[a_n, a_{n+1}]$ and the sequence $\{a_1, a_2, \ldots, a_{N+1}\}$ satisfies the conditions $a_1 = -\infty$, $a_{N+1} = \infty$ and for $i > j$ we have $a_i > a_j$.

2.39 A lowpass Gaussian stochastic process $X(t)$ has a power spectral density

$$\mathcal{S}(f) = \begin{cases} N_0 & |f| < B \\ 0 & \text{otherwise} \end{cases}$$

Determine the power spectral density and the autocorrelation function of $Y(t) = X^2(t)$.

(10%)

2.42 For the Nakagami PDF, given by Equation 2.3–67, define the normalized random variable $X = R/\sqrt{\Omega}$. Determine the PDF of X.

(10%)

2.46 Determine the mean, the autocorrelation sequence, and the power density spectrum of the output of a system with unit sample response

$$h(n) = \begin{cases} 1 & n = 0 \\ -2 & n = 1 \\ 1 & n = 2 \\ 0 & \text{otherwise} \end{cases}$$

when the input $x(n)$ is a white noise process with variance σ_x^2. **(10%)**

2.54 Determine the autocorrelation function of the stochastic process

$$X(t) = A\sin(2\pi f_c t + \Theta)$$

where f_c is a constant and Θ is a uniformly distributed phase, i.e.,

$$p(\theta) = \frac{1}{2\pi}, \qquad 0 \le \theta \le 2\pi$$

(10%)