COM 5120 Communication Theory

Homework #2

Due at 23:59, December 20, 2022

1. For the QAM signal constellation shown in Figure 1, determine the optimum decision boundaries for the detector, assuming that the SNR is sufficiently high that errors occur only between adjacent points.

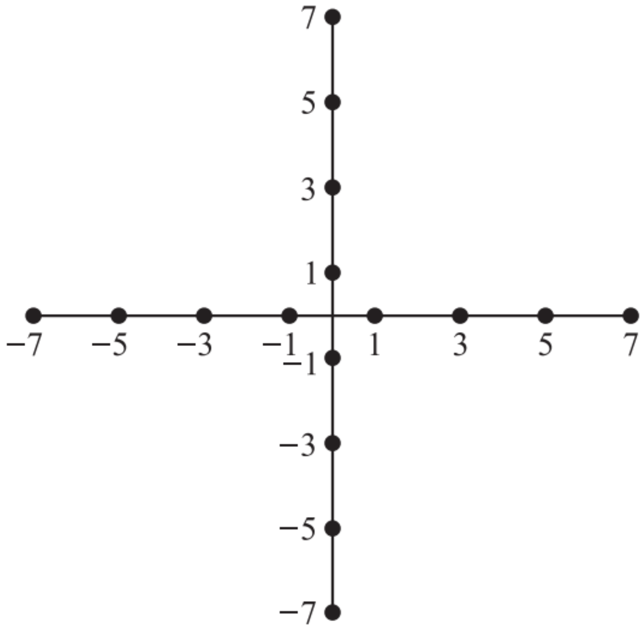

Figure 1: QAM signal constellation

2. Consider a digital communication system that transmits information via QAM over a voice-band telephone channel at a rate of 2400 symbols/s. The additive noise is assumed to be white and Gaussian.
(a) Determine the ε_b/N_0 required to achieve an error probability of 10^{-5} at 4800 bits/s.
(b) Repeat part (a) for a rate of 9600 bits/s.
(c) Repeat part (a) for a rate of $19,200$ bits/s.
(d) What conclusions do you reach from these results?

3. Let X be a geometrically distributed random variable, i.e.,

$$P(X = k) = p(1-p)^{k-1}, k = 1, 2, 3, ...$$

(a) Find the entropy of X.
(b) Given that $X > K$, where K is a positive integer, what is the entropy of X?

4. A DMS has an alphabet of eight letters x_i, $i = 1, 2, ..., 8$, with probabilities 0.25, 0.20, 0.15, 0.12, 0.10, 0.08, 0.05, and 0.05.
(a) Use the Huffman encoding procedure to determine a binary code for the source output.
(b) Determine the average number $\bar{R}$ of binary digits per source letter.
(c) Determine the entropy of the source and compare it with $\bar{R}$.

5. A discrete memoryless source has an alphabet of size 7, $\mathcal{X} = \{x_1,\ x_2,\ x_3,\ x_4,\ x_5,\ x_6,\ x_7\}$, with corresponding probabilities $\{0.02,\ 0.11,\ 0.07,\ 0.21,\ 0.15,\ 0.19,\ 0.25\}$.
(a) Determine the entropy of this source.
(b) Design a Huffman code for this source, and find the average codeword length of the Huffman code.
(c) A new source $\mathcal{Y} = \{y_1,\ y_2,\ y_3\}$ is obtained by grouping the outputs of the source $\mathcal{X}$ as

$$
\begin{aligned}
y_1 &= \{x1,\ x2,\ x5\} \\
y_2 &= \{x3,\ x7\} \\
y_3 &= \{x4,\ x6\}
\end{aligned}
$$

Determine the entropy of $\mathcal{Y}$.
(d) Which source is more predictable, $\mathcal{X}$ or $\mathcal{Y}$? Why?

6. A memoryless source has the alphabet $A = \{-5,\ -3,\ -1,\ 0,\ 1,\ 3,\ 5\}$, with corresponding probabilities $\{0.05,\ 0.1,\ 0.1,\ 0.15,\ 0.05,\ 0.25,\ 0.3\}$.
(a) Find the entropy of the source.
(b) Assuming that the source is quantized according to the quantization rule

$$
\begin{cases}
q(-5) = q(-3) = -4 \\
q(-1) = q(0) = q(1) = 0 \\
q(3) = q(5) = 4
\end{cases}
$$

find the entropy of the quantized source.
