# Practice #2

No need to turn it in, this is for practice purpose only.

Practice and get familiar with the questions are encouraged.

Some types of the questions might appear on the test.

**4.5** A communication system transmits one of the three messages $m_1, m_2$, and $m_3$ using signals $s_1(t)$, $s_2(t)$, and $s_3(t)$. The signal $s_3(t) = 0$, and $s_1(t)$ and $s_2(t)$ are shown in Figure P4.5. The channel is an additive white Gaussian noise channel with noise power spectral density equal to $N_0/2$.

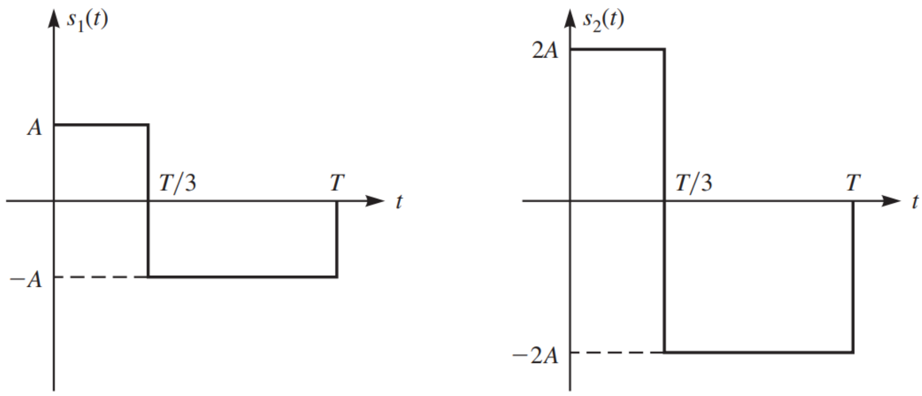



**FIGURE P4.5**

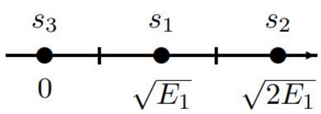
1. Determine an orthonormal basis for this signal set, and depict the signal constellation.
2. If the three messages are equiprobable, what are the optimal decision rules for this system? Show the optimal decision regions on the signal constellation you plotted in part 1.
3. If the signals are equiprobable, express the error probability of the optimal detector in terms of the average SNR per bit.
4. Assuming this system transmits 3000 symbols per second, what is the resulting transmission rate (in bits per second)?

**Problem 4.5**

1. Note that $s_2(t) = 2s_1(t)$ and $s_3(t) = 0s_1(t)$, hence the system is PAM and a singular basis function of the form $\phi_1(t) = \frac{1}{A\sqrt{T}}s_1(t)$ would work

$$\phi(t) = \begin{cases} \frac{1}{\sqrt{T}} & 0 < t \leq T/3 \\ -\frac{1}{\sqrt{T}} & T/3 \leq t < T \end{cases}$$

Assuming $E_1 = A^2T$, we have $s_3 = 0$, $s_1 = \sqrt{E_1}$, $s_2 = 2\sqrt{E_1}$. The constellation is shown below.



2. For equiprobable messages the optimal decision rule is the nearest neighbor rule and the perpendicular bisectors are the boundaries of the decision regions as indicated in the figure.

3. This is ternary PAM system with the distance between adjacent pints in the constellation being $d = \sqrt{E_1} = A\sqrt{T}$. The average energy is $E_{\text{avg}} = \frac{1}{3}(0 + A^2T + 4A^2T) = \frac{5}{3}A^2T$, and $E_{\text{bavg}} = E_{\text{avg}}/\log_2 3 = \frac{5}{3\log_2 3}A^2T$, from which we obtain

$$d^2 = \frac{3\log_2 3}{5}E_{\text{bavg}} \approx 0.951E_{\text{bavg}}$$

The error probability of the optimal detector is the average of the error probabilities of the three signals. For the two outer signals error probability is $P(n > d/2) = Q\left(\frac{d/2}{\sqrt{N_0/2}}\right)$ and for the middle point $s_1$ it is $P(|n| > d/2) = 2Q\left(\frac{d/2}{\sqrt{N_0/2}}\right)$. From this,

$$P_e = \frac{4}{3}Q\left(\sqrt{\frac{d^2}{2N_0}}\right) = \frac{4}{3}Q\left(\sqrt{\frac{0.951E_{\text{bavg}}}{2N_0}}\right) = 43Q\left(\sqrt{0.475\frac{E_{\text{bavg}}}{2N_0}}\right)$$

4. $R = R_s \log_2 M = 3000 \times \log_2 3 \approx 4755$ bps.

**4.6** Suppose that binary PSK is used for transmitting information over an AWGN with a power spectral density of $\frac{1}{2}N_0 = 10^{-10}$ W/Hz. The transmitted signal energy is $\mathcal{E}_b = \frac{1}{2}A^2T$, where $T$ is the bit interval and $A$ is the signal amplitude. Determine the signal amplitude required to achieve an error probability of $10^{-6}$ when the data rate is

1. 10 kilobits/s
2. 100 kilobits/s
3. 1 megabit/s

**Problem 4.6**

For binary phase modulation, the error probability is

$$P_2 = Q\left[\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right] = Q\left[\sqrt{\frac{A^2T}{N_0}}\right]$$

With $P_2 = 10^{-6}$ we find from tables that

$$\sqrt{\frac{A^2T}{N_0}} = 4.74 \Longrightarrow A^2T = 44.9352 \times 10^{-10}$$

If the data rate is 10 Kbps, then the bit interval is $T = 10^{-4}$ and therefore, the signal amplitude is

$$A = \sqrt{44.9352 \times 10^{-10} \times 10^4} = 6.7034 \times 10^{-3}$$

Similarly we find that when the rate is $10^5$ bps and $10^6$ bps, the required amplitude of the signal is $A = 2.12 \times 10^{-2}$ and $A = 6.703 \times 10^{-2}$ respectively.