COM 5120 Communication Theory

Midterm Exam

November 10, 2022
15:30 $\sim$ 17:20

Note: There are **7** problems with total 100 points within **3** pages, please write your answer with detail in the answer sheet.

No credit without detail, except for question 1. No calculator. Closed books.

1. (13%) A transmitter transmits signal s via AWGN model $r = s + n$, where r is the received signal at the receiver, n is the white noise with $N(0, \sigma^2)$, which of the following is **NOT** a **sufficient statistic** with respect to the estimation of s at the receiver? Assuming the same signal s is transmitted 3 times. The received signals are denoted as r_1, r_2, r_3. **(Single choice, no derivation required)**

   (a) $\{r_1, r_2, r_3\}$

   (b) $\sum_{i=1}^{3} r_i$

   (c) $\frac{1}{2}\sum_{i=1}^{2} r_i$

   (d) $\{\sum_{i=1}^{3} r_i\}^3$

   (e) $\frac{1}{3}\sum_{i=1}^{3} r_i$

   (f) $\frac{1}{2}\sum_{i=1}^{3} r_i$

2. (15%) We observed N i.i.d. Bernoulli experiments, $x[n]$, $n = 1 \sim N$, with $P_r\{x[n] = 1\} = p$, $P_r\{x[n] = 0\} = 1 - p$. Derive and find the **maximum likelihood estimator** of p.

3. (15%) Let $X(t)$ denote a (real, zero-mean, WSS) bandpass process with autocorrelation function $R_X(\tau)$ and power spectral density $S_X(f)$, where $S_X(0) = 0$, and let $\hat{X}(t)$ denote the Hilbert transform of $X(t)$. Then $\hat{X}(t)$ can be viewed as the output of a filter, with impulse response $\frac{1}{\pi t}$ and transfer function $-j\text{sgn}(f)$, whose input is $X(t)$. Recall that when $X(t)$ passes through a system with transfer function $H(f)$ and the output is $Y(t)$, we have $S_Y(f) = S_X(f)|H(t)|^2$ and $S_{XY}(f) = S_X(f)H^*(f)$.
   (a) Prove that $R_{\hat{X}}(\tau) = R_X(\tau)$
   (b) Prove that $R_{X\hat{X}}(\tau) = -\hat{R}_X(\tau)$
   (c) If $Z(t) = X(t) + j\hat{X}(t)$, determine $S_Z(f)$ in terms of $S_X(f)$

4. (14%) Consider the three waveforms $f_n(t)$ shown in Figure 1.
(a) Show that these waveforms are **orthonormal**.
(b) Express the waveform $x(t)$ as a **linear combination** of $f_n(t)$, $n = 1, 2, 3$ if

$$x = \begin{cases} -2, & 0 \le t < 1 \\ 6, & 1 \le t < 3 \\ 4, & 3 \le t < 4 \end{cases}$$

and determine the **weighting coefficients**.

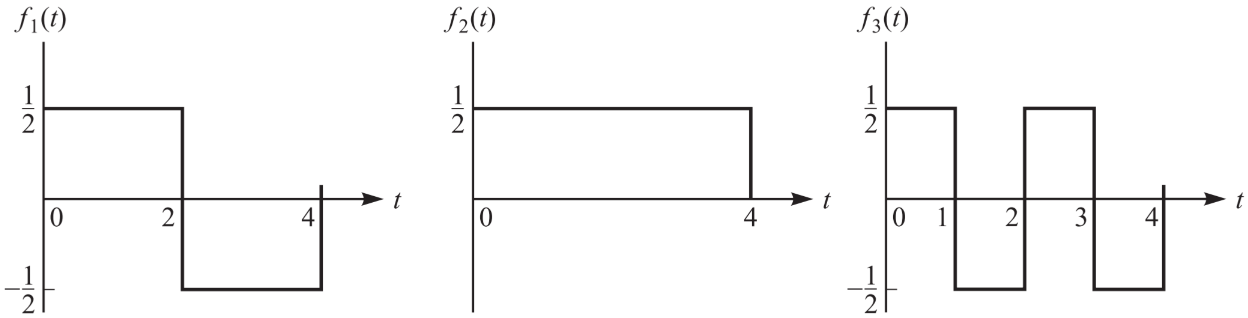

Figure 1: three waveforms $f_1(t)$, $f_2(t)$, $f_3(t)$

5. (14%) Consider the octal signal point constellations shown in Figure 2.
(a) The nearest-neighbor signal points in the 8-QAM signal constellation are separated in distance by A units. Determine the **radii** a and b of the inner and outer circles, respectively.
(b) The adjacent signal points in the 8-PSK are separated by a distance of A units. Determine the **radius** r of the circle.

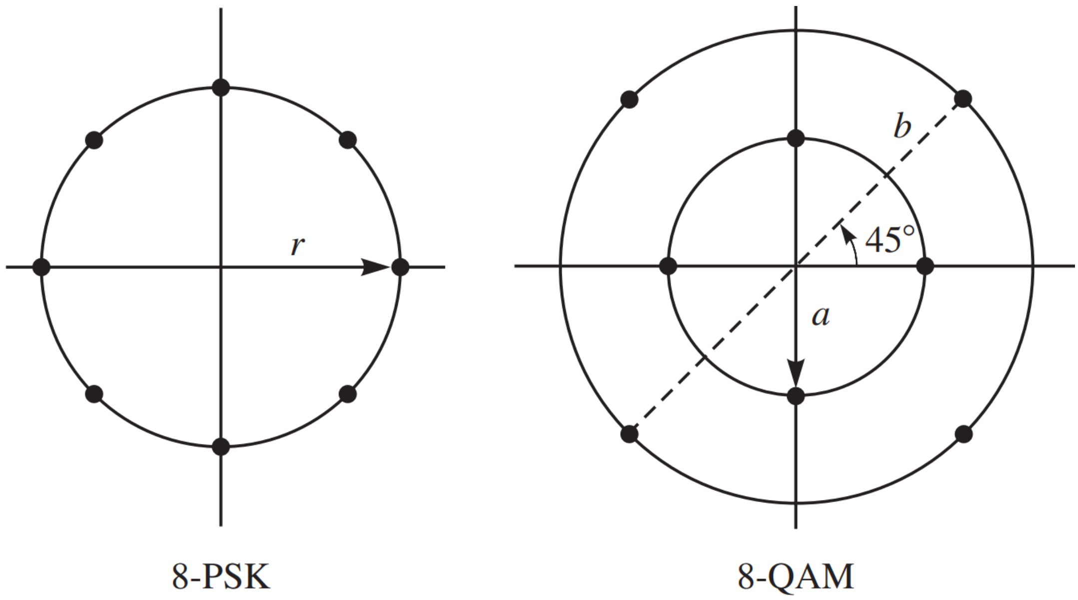

Figure 2: 8-PSK and 8-QAM

6. (14%) A binary digital communication system employs the signals

$$s_0(t) = 0,\ 0 \le t \le T$$
$$s_1(t) = A,\ 0 \le t \le T$$

for transmitting the information. This is called *on-off signaling.* The demodulator crosscorrelates the received signal $r(t)$ with $s(t)$ and samples the output of the correlator at $t + T$.
(a) Determine the **optimum detector** for an AWGN channel and the **optimum threshold**, assuming that the signals are equally probable.
Given that the correlation type demodulator employes a filter:

$$f(t) = \begin{cases} \dfrac{1}{\sqrt{T}}, & 0 \le t < T \\ 0, & otherwise \end{cases}$$

(b) Determine the **probability of error** as a function of the SNR. How does on-off signaling compare with antipodal signaling?

7. (15%) Consider a signal detector with an input

$$r = \pm A + n$$

where $+A$ and $-A$ occur with equal probability and the noise variable n is characterized by the (Laplacian) PDF shown in Figure 3, where $p(n) = \frac{1}{\sqrt{2}\sigma} e^{-|n|\sqrt{\frac{2}{\sigma}}}$. Determine the **probability of error** as a function of the parameters A and σ.

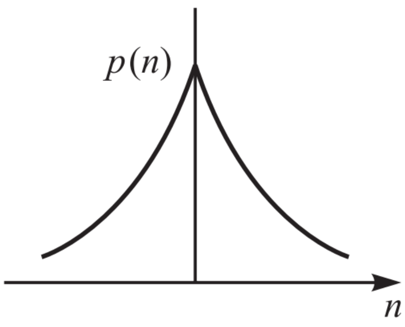

Figure 3: Laplacian PDF, $p(n) = \frac{1}{\sqrt{2}\sigma} e^{-|n|\sqrt{\frac{2}{\sigma}}}$