#1 (10)

(c)

#2 (20)

$Pr\{x[n]=1\}=p\ ;\ Pr\{x[n]=0\}=1-p$

$P\{x[n]\ n=1,2,\dots,N \mid p\}$

$= p^{\sum x[n]}(1-p)^{N-\sum x[n]}$

Let $\bar{x}=\frac{1}{N}\sum x[n] \Rightarrow \sum x[n]=N\bar{x}$

$= p^{N\bar{x}}(1-p)^{N-N\bar{x}} =$

$$\frac{dP\{x[n]\ n=1,2,\dots,N \mid p\}}{dp} = (N\bar{x})\cdot\cancel{p}^{N\bar{x}-1}(1-\cancel{p})^{N-N\bar{x}} - \cancel{p}^{N\bar{x}}\cdot(N-N\bar{x})(1-\cancel{p})^{N-N\bar{x}-1} = 0$$

$$= N\bar{x}\cdot(1-p)-(N-N\bar{x})\cdot p \Rightarrow N\bar{x}-Np \Rightarrow \hat{p}=\bar{x}=\frac{1}{N}\sum_{n=1}^{N}x[n]$$

#3 (15)

$$x(t) \longrightarrow \boxed{\text{Hibert}} \longrightarrow \hat{x}(t)$$

(a) $S_{\hat{x}}(f)=|-j\,sgn(f)|^2 S_x(f)=S_x(f) \rightarrow R_{\hat{x}}(\tau)=R_x(\tau)$

(b) $S_{x\hat{x}}(f)=S_x(f)\cdot(-j\,sgn(f))^*=j\,sgn(f)\,S_x(f) \xrightarrow{F} R_{x\hat{x}}(\tau)=-\hat{R}_x(\tau)$

(c) $z(t)=x(t)+j\hat{x}(t)$

$R_z(\tau)=E[(x(t)+j\hat{x}(t))(x(t+\tau)-j\hat{x}(t+\tau))^*]$

$= R_x(\tau)+R_{\hat{x}}(\tau)-j[R_{x\hat{x}}(\tau)-R_{\hat{x}x}(\tau)]$

clue

$\underline{R_{\hat{x}x}(\tau)=R_{x\hat{x}}(-\tau)}=-R_{x\hat{x}}(\tau)=-\hat{R}_x(\tau)$

$= 2R_x(\tau)-j\,2R_{x\hat{x}}(\tau)=2(R_x(\tau)+j\,R_{x\hat{x}}(\tau))$

$\downarrow F$

$= 2(S_x(f)+j(-j\,sgn(f)S_x(f)))=2(1+j\,sgn(f))S_x(f)$

#4 (5)

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(a) ① $|f_1(t)| = 1$; $|f_2(t)| = 1$; $|f_3(t)| = 1$

② $\int_0^4 f_1(t)\, f_2(t)\, dt = 0$; $\int_0^4 f_2(t)\, f_3(t)\, dt = 0$; $\int_0^4 f_1(t)\, f_3(t)\, dt = 0$

(b) $$\boxed{x(t) = \begin{cases} - & 0 < t < 1 \\ + & 1 \le t < 3 \\ + & 3 \le t < 4 \end{cases}}$$ design $\longrightarrow$

① $a_1 = \int_0^4 x(t)\, f_1(t)\, dt = -\frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} = 0$

② $a_2 = \int_0^4 x(t)\, f_2(t)\, dt = \frac{-1}{2} + \frac{1}{2} + \frac{1}{2} - \frac{1}{2} = 0$

③ $a_3 = \int_0^4 x(t)\, f_3(t)\, dt = \frac{-1}{2} + \frac{-1}{2} + \frac{1}{2} + \frac{1}{2} = 0$

#5 (8)

(a) $A^2 = 2a^2 \quad a = \frac{A}{\sqrt{2}}$

(b) $a^2 + b^2 - 2ab\cos 45^\circ = A^2$

$b^2 - 2 \cdot \frac{A}{\sqrt{2}} \cdot b \cdot \frac{\sqrt{2}}{2} - \frac{A^2}{2} = 0 \quad b^2 - bA - \frac{A^2}{2} = 0$

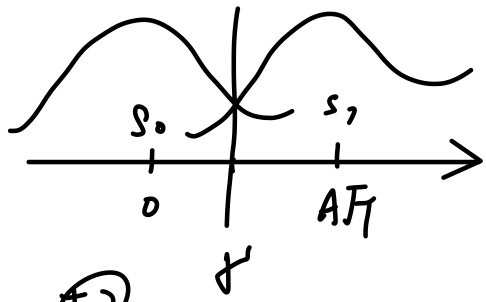
$b = \frac{A \pm \sqrt{A^2 + 2A^2}}{2} = \left(\frac{1+\sqrt{3}}{2}\right) A$

(b) $A^2 = r^2 + r^2 - 2r^2 \cos 45^\circ$

$(2 - \sqrt{2})\, r^2 = A^2 \quad r = \frac{1}{\sqrt{2-\sqrt{2}}} A$

#6 (20)

(a) $E = \int_0^T A^2 dt = A^2 T$



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MAP - ML

$$p(r|s_1) \underset{s_0}{\overset{s_1}{\gtrless}} p(r|s_0)$$

$$\Rightarrow \frac{\cancel{1}}{\sqrt{2\pi}} e^{-\frac{(r-A\sqrt{T})^2}{N_0}} \underset{s_0}{\overset{s_1}{\gtrless}} \frac{\cancel{1}}{\sqrt{2\pi}} e^{-\frac{r^2}{N_0}}$$

$\ln()$

$$\Rightarrow -r^2 + 2A\sqrt{T}r - A^2T + r^2 \underset{s_0}{\overset{s_1}{\gtrless}} 0 \Rightarrow r \underset{s_0}{\overset{s_1}{\gtrless}} \frac{1}{2}A\sqrt{T}$$

(b)

$$P_e = \frac{1}{2}P(r|s_0) + \frac{1}{2}P(r|s_1)$$

$$= \frac{1}{2}\int_{\frac{1}{2}A\sqrt{T}}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{r^2}{N_0}} dr + \frac{1}{2}\int_{-\infty}^{\frac{1}{2}A\sqrt{T}} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(r-A\sqrt{T})^2}{N_0}} dr$$

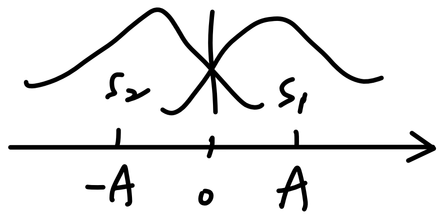
$$\frac{x^2}{2} = \frac{r^2}{N_0} \quad r = \sqrt{\frac{N_0}{2}}x \quad dr = \sqrt{\frac{N_0}{2}}dx \quad \frac{x^2}{2} = \frac{(r-A\sqrt{T})^2}{N_0}$$

$$= \frac{1}{2}\int_{\frac{1}{2}A\sqrt{T}\sqrt{\frac{2}{N_0}}}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{x^2}{2}} \cdot \sqrt{\frac{N_0}{2}}dx + \frac{1}{2}\int_{-\infty}^{-\frac{1}{2}A\sqrt{T}\sqrt{\frac{2}{N_0}}} e^{-\frac{x^2}{2}} \cdot \sqrt{\frac{N_0}{2}}dx$$

$$= \underline{Q\left(\frac{1}{2}A\sqrt{T} \cdot \sqrt{\frac{2}{N_0}}\right)} = Q(SNR)$$

#7 (20)

$r = \pm A + n$



(a) MAP $\rightarrow$ ML

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$$P(r|A) \underset{s_0}{\overset{s_1}{\gtrless}} P(r|-A)$$

$$\Rightarrow \cancel{\frac{1}{\sqrt{2\pi}}} e^{-\frac{\sqrt{2}}{\sigma}|r-A|} \underset{s_0}{\overset{s_1}{\gtrless}} \cancel{\frac{1}{\sqrt{2\pi}}} e^{-\frac{\sqrt{2}}{\sigma}|r+A|}$$

$\ln()$

$$\Rightarrow (r-A)^2 - (r+A)^2 \underset{s_0}{\overset{s_1}{\gtrless}} 0 \qquad r \underset{s_0}{\overset{s_1}{\gtrless}} 0$$

(b)

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$$\begin{aligned}
P(e) &= \frac{1}{2}P(e|A) + \frac{1}{2}P(e|-A) \\
&= \frac{1}{2}\int_{-\infty}^{0} f(r|A)dr + \frac{1}{2}\int_{0}^{\infty} f(r|-A)dr \\
&= \frac{1}{2}\int_{-\infty}^{0} \lambda_2 e^{-\lambda|r-A|}dr + \frac{1}{2}\int_{0}^{\infty} \lambda_2 e^{-\lambda|r+A|}dr \\
&= \frac{\lambda}{4}\int_{-\infty}^{-A} e^{-\lambda|x|}dx + \frac{\lambda}{4}\int_{A}^{\infty} e^{-\lambda|x|}dx \\
&= \frac{1}{2}e^{-\lambda A} = \frac{1}{2}e^{\frac{-\sqrt{2}A}{\sigma}}
\end{aligned}$$

$$= \frac{1}{2} e^{-\sqrt{\frac{2}{\sigma}}A}$$

$$2 \cdot \frac{\lambda}{4}\int_{A}^{\infty} e^{-\lambda x} dx$$

$$= \frac{\lambda}{2} \cdot \frac{-1}{\lambda} e^{-\lambda x}\Big|_{A}^{\infty}$$

$$= \frac{-1}{2}(0 - e^{-\lambda A}) = \frac{1}{2} e^{-\lambda A}.$$

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