Stacks & Queues

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Template

Abstracted Bag Container

```
class Bag
{
public:
    Bag(int bagCapacity = 10);  // Constructor
   ~Bag();                      // Destructor

    int Size() const;           // Return the number of elements

    bool IsEmpty() const;       // Check if bag is empty

    int Element() const;        // Return an element in the bag

    void Push(const int);       // Insert an integer into the bag
    void Pop()                  // Delete an integer from the bag

private:
    int *array;                 // Integer array that stores the data
    int capacity;               // Capacity of array
    int top;                    // Position of top element
};
```


Bag Implementation

```
Bag::Bag( int bagCapacity):capacity( bagCapacity ) {
   if(capacity < 1) throw "Capacity must be > 0";
   array = new int [ capacity ];
   top = -1;}

Bag::~Bag(){ delete [] array; }

inline int Bag::Size() const { return top + 1; }

inline bool Bag::IsEmpty() const { return Size() == 0; }

inline int Bag::Element() const {
   if(IsEmpty()) throw "Bag is empty";
   return array [0];  // Always return the first element
}

void Bag::Push(const int x) {
   if(capacity == top+1) ChangeSize1D(array,capacity,2* capacity);
   capacity *= 2;
   array[++top]=x;}

void Bag::Pop( ) {
   if(IsEmpty()) throw "Bag is empty, cannot delete";
   int deletePos = top / 2; // Always delete the middle element
   copy (array+deletePos+1, array+top+1, array+deletePos);
   top--;
}
```

Abstracted Bag Container

```
template<class T>
class Bag
{
public:
    Bag(int bagCapacity = 10);  // Constructor
   ~Bag();                      // Destructor

    int Size() const;           // Return the number of elements

    bool IsEmpty() const;       // Check if bag is empty

    T& Element() const;         // Return an element in the bag

    void Push(const T&);        // Insert an element into the bag
    void Pop()                  // Delete an element from the bag

private:
    T *array;                   // Data array
    int capacity;               // Capacity of array
    int top;                    // Position of top element
};
```


Template Bag Implementation

```
template<class T>
Bag<T>::Bag( int bagCapacity):capacity( bagCapacity ) {
   if(capacity < 1) throw “Capacity must be > 0”;
   array = new T [ capacity ];
   top = -1;
}

template<class T>
void Bag<T>::Push(const T& x) {
   if(capacity == top+1) ChangeSize1D(array,capacity,2* capacity);
   capacity *= 2;
   array[++top]=x;
}

template<class T>
void Bag<T>::Pop() {
   if(IsEmpty()) throw “Bag is empty, cannot delete”;
   int deletePos = top / 2; // Always delete the middle emelent
   copy (array+deletePos+1, array+top+1, array+deletePos);
   array[top--].~T();
}
```

The Stack

Stack

- A stack is an ordered list
  - in which
    - insertions (or called additions or pushes)
    - deletions (or called removals or pops)
    - Both made at one end called the top
  - Operate in

Stack: ADT

```
template < class T >
class Stack // A finite ordered list
{
public:
        // Constructor
        Stack (int stackCapacity = 10);

        // Check if the stack is empty
        bool IsEmpty ( ) const;

        // Return the top element
        T& Top ( ) const;

        // Insert a new element at top
        void Push (const T& item);

        // Delete one element from top
        void Pop ( );
private:
         T* stack;
         int top;   // init. value = -1
         int capacity;
};
```


Stack Operations: Push & Pop

```
template < class T >
void Stack < T >::Push (const T& x)
{   // Add x to stack
    if(top == capacity – 1)
    {
       ChangeSize1D(stack, capacity, 2*capacity);
       capacity *= 2;
    }
    stack [ ++top ] = x;
}
```

```
template < class T >
void Stack < T >::Pop ( )
{   // Delete top element from stack
    if(IsEmpty()) throw “Stack is empty. Cannot delete.”;
    stack [ top-- ].~T();  // Delete the element
}
```

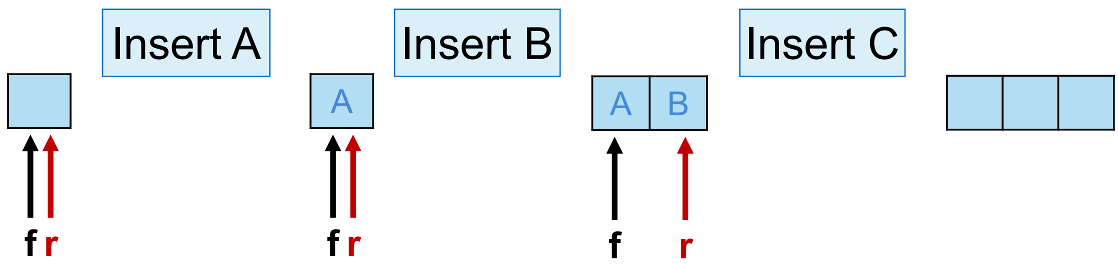
The Queue

Queue

- A queue is an ordered list
  - in which
    - insertions (or called additions or pushes)
    - deletions (or called removals or pops)
    - Made at different ends
  - New elements are inserted at rear end
  - Old elements are deleted at front end
  - Operate in

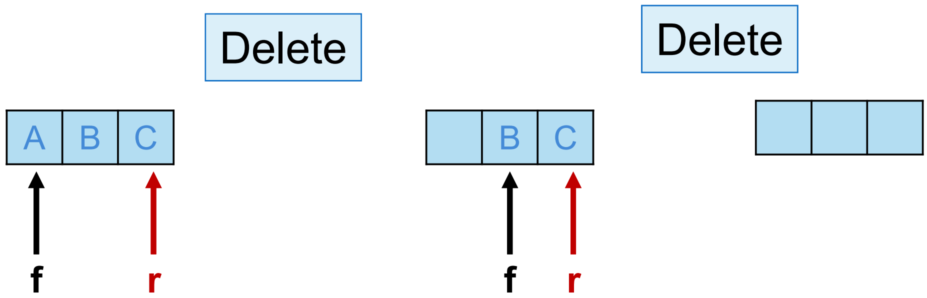
Queue Insertion

- Insert a new element into queue
  - f: front position
  - r: rear position



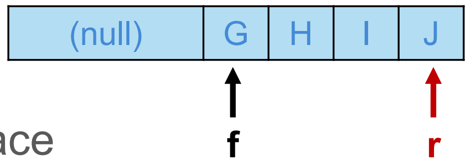
Queue Deletion

- Delete an old element from queue
  - f: front position
  - r: rear position

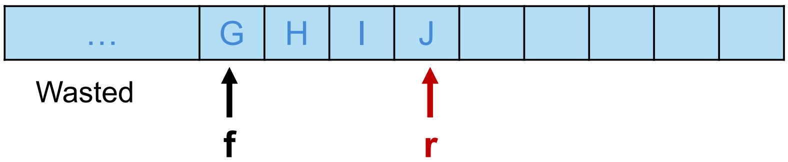


Problems

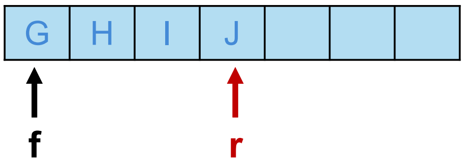
- What happen if rear == capacity-1 ?



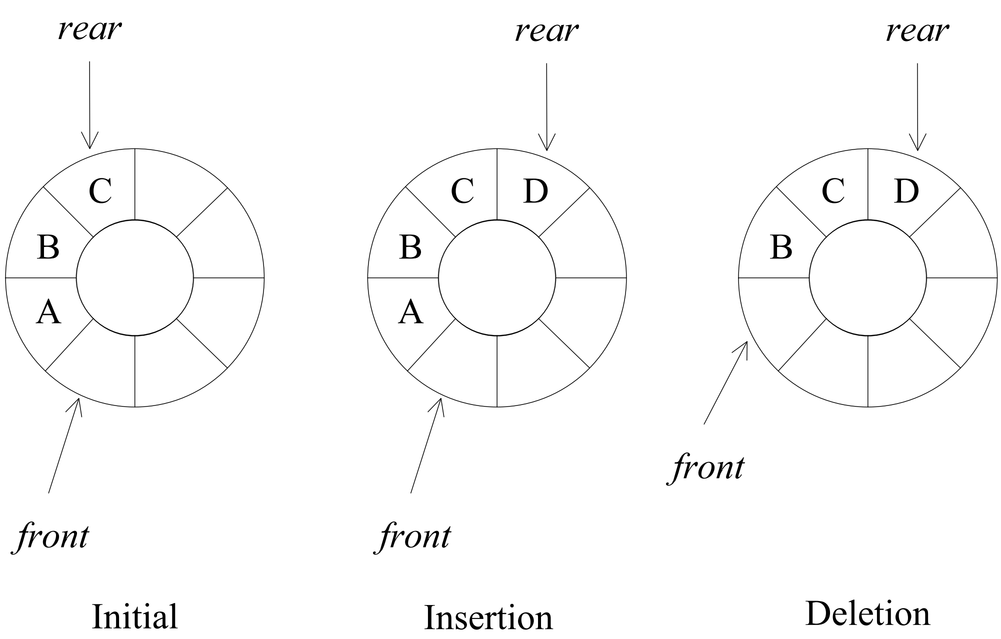
- Approach 1: Add more space



- Approach 2: Shift left



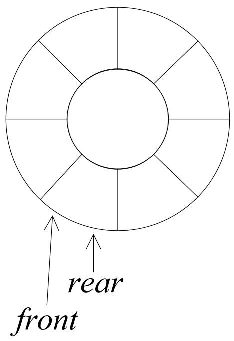
Circular Queue



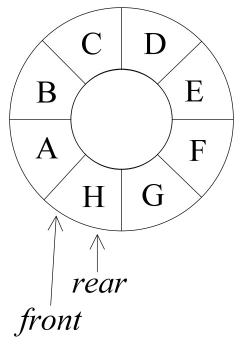

```
rear = (rear+1) % capacity;
```

Circular Queue: Empty

- rear == front ?

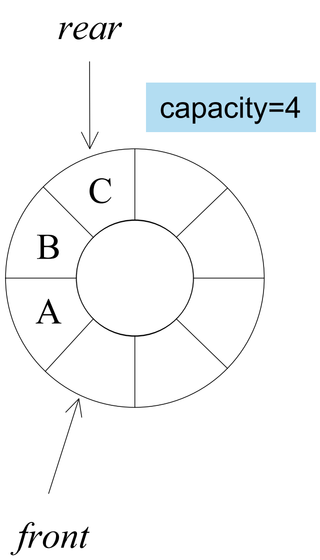


Queue is empty

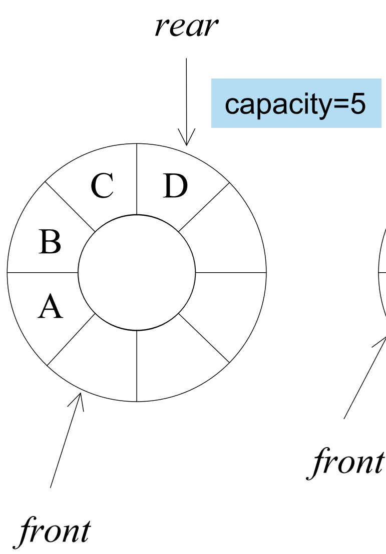


Queue is full

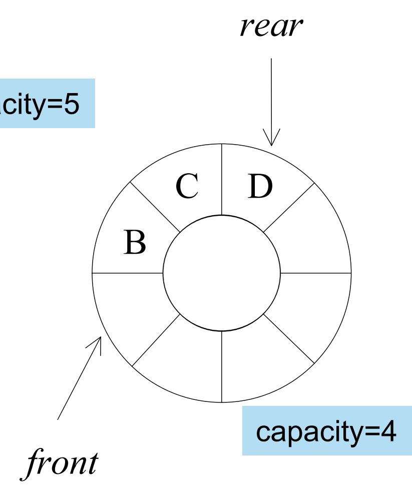
Circular Queue



Initial



Insertion



Deletion

rear = (rear+1) % capacity;

Queue: ADT

```
template < class T >
class Queue // A finite ordered list
{
public:
        // Constructor
        Queue (int queueCapacity = 10);

        // Check if the stack is empty
        bool IsEmpty ( ) const;

        // Return the front element
        T& Front ( ) const;

        // Return the rear element
        T& Rear ( ) const;

        // Insert a new element at rear
        void Push (const T& item);

        // Delete one element from front
        void Pop ( );
private:
    T* queue;
    int front, rear; // init. value = -1
    int capacity;
};
```


Queue Operations: Front & Rear

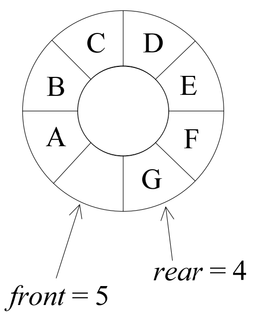
```
template < class T >
void Queue < T >::IsEmpty() const { return front==rear; }

template < class T >
T& Queue < T >::Front() const {
  if(IsEmpty()) throw "Queue is empty!";
  return queue[(front+1)%capacity];
}

template < class T >
T& Queue < T >::Rear() const {
  if(IsEmpty()) throw "Queue is empty!";
  return queue[rear];
}
```

Queue Operations: Push & Pop

```
template < class T >
void Queue< T >::Push (const T& x)
{   // Add x at rear of queue
    if((rear+1)%capacity == front)
    {
       // queue is going to full, double the capacity!
    }
    rear = (rear+1)%capacity;
    queue [rear] = x;
}
```

```
template < class T >
void Queue < T >::Pop ( )
{   // Delete front element from queue
    if(IsEmpty()) throw "Queue is empty. Cannot delete.";
    front = (front+1)%capacity;
    queue[front].~T(); // Delete the element
}
```


Doubling Queue Capacity



Full circular queue

capacity=8

queue

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
C	D	E	F	G		A	B
				r	f		

front = 5, rear = 4

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]
C	D	E	F	G		A	B								
				r	f										

Doubling the array

Anything wrong?

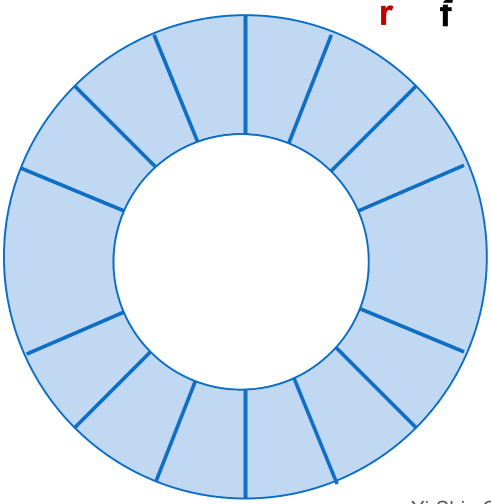
Doubling Queue Capacity: Scenario 1

capacity=16

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]
C	D	E	F	G		A	B								

r f

Doubling the array



Scenario 1: After shifting right segment

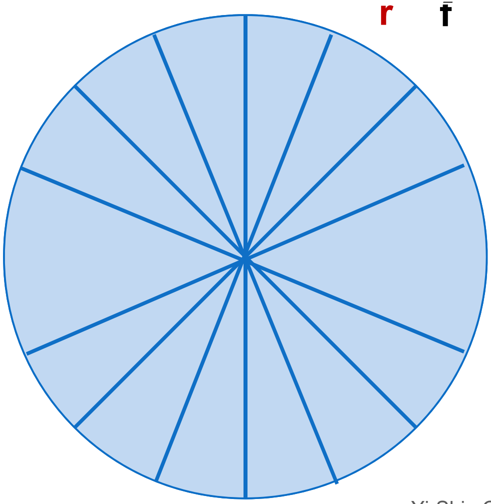
Doubling Queue Capacity: Scenario 2

capacity=16

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]
A	B	C	D	E	F	G									

r f

Doubling the array



Scenario 2: Alternative configuration

Subtyping and Inheritance

In C++

Subtype

- Inheritance is used to express subtype relationships
  - A Data object of Type B IS-A data object of Type A
  - Type B is more specialized than Type A
  - E.g., Chair IS-A Furniture

- Bag is a data structure, where
  - Elements can be inserted and deleted
- Stack is a data structure, where
  - Elements can be inserted and deleted
- Stack is more specialized
  - Stack IS-A Bag

Generic Bag ADT

```
Class Bag
{
public:
  Bag(int bagCapacity=10);
  virtual ~Bag();
  virtual int Size() const;
  virtual bool IsEmpty() const;
  virtual int Element() const;
  virtual void Push(const int);
  virtual void Pop();
protected:
  int *array;
  int capacity;
  int top;
};
```

Implement operations not exist in the Bag class

```
class Stack : public Bag
{
public:
  Stack(int stackCapacity=10);
  virtual ~Stack();
  int Top()const;
  virtual void Pop();
};
```


Evaluation of Expressions

Expression

$$X = \boxed{A}/\boxed{B} - \boxed{C} + \boxed{D} * \boxed{E} - \boxed{A} * \boxed{C}$$

- Operators
  - +,-,*,/,…,etc
- Operands
  - A,B,C,D,E,F
- Execution order might affect the final result

Expression Evaluation

- For $X = A/B - C + D * E - A * C$
- If A = 4, B=C=2, D=E=3
- X = ((4/2)-2)+(3*3)+(4*2)=1

- For $X = (A/(B - C + D)) * (E - A) * C$
- If A = 4, B=C=2, D=E=3
- X = (4/(2-2+3))*(3-4)*2 = -2.6666666

Evaluation Rules

- Operators have priority
- Operator with higher priority is evaluated first
- Operators of equal priority are evaluated from left to right
- Unary operators are evaluated from right to left

Priority of Operators in CPP

Priority	Operators
1	Minus, !
2	*, /, %
3	+, -
4	<, <=, >=, >
5	= =, !=
6	&&
7	||

Infix and Postfix Notation

- Infix notation
  - Operator comes in–between the operands
  - E.g., A+B*C
  - Hard to evaluate using codes…
- Postfix notation
  - Each operator appears after its operands
  - E.g., ABC*+

Advantages of Postfix Notation

- You don't need parentheses
- Priority of operators is no longer relevant!
- Expression can be efficiently evaluated by
  - Making a left to right scan
  - Stacking operands
  - Evaluating operators
  - Push the result into stack

Example 1

- Infix : A+B – C => Postfix : A B + C –
- Suppose A = 4, B = 3, C = 2

$\boxed{A}$ B + C –

4

Operand Stack

Operation
See operand A, put it into stack

Example 1

- Infix : A+B – C => Postfix : A B + C –
- Suppose A = 4, B = 3, C = 2

A $\boxed{B}$ + C –

3
4

Operand Stack

Operation
See operand B, put it into stack

Example 1

- Infix : A+B – C => Postfix : A B + C –
- Suppose A = 4, B = 3, C = 2

A B $\boxed{+}$ C –

3
4

Operand Stack

Operation
See operator ‘+’ (binary operator) 1. Pop two elements from stack 2. Perform evaluation (3+4) 3. Push result into stack (7)

Example 1

- Infix : A+B – C => Postfix : A B + C –
- Suppose A = 4, B = 3, C = 2

A B + $\boxed{C}$ –

2
7

Operand Stack

Operation
See operand C, put it into stack

Example 1

- Infix : A+B – C => Postfix : A B + C –
- Suppose A = 4, B = 3, C = 2

A B + C $\boxed{–}$

Operand
Stack

Operation
See operator '-' (binary operator) 1. Pop two elements from stack 2. Perform evaluation (7-2) 3. Push result into stack (5)

Example 2

- Infix : $X = A/B - C + D * E - A * C$
- Postfix : $X = \boxed{A}B/C - DE * +AC * -$

Operand
Stack

Operation
See operand A, put it into stack

Evaluation Pseudo Codes

```
void Eval(Expression e)
{  // Assume the last token of e is '#'
   // A function NextToken is used to get next token in e
   Stack<Token> stack; // initialize stack
   for (Token x = NextToken(e); x != '#'; x = NextToken(e)){
     if(x is an operand) stack.Push(x);
     else{
         // Remove the correct number of operands from stack
         // Perform the evaluation
         // Push the result back to stack
         // ***Try to fill up the codes by your own***
     }
   }
};
```


Infix to Postfix

- Fully parenthesize algorithm:
  - Fully parenthesize the expression
  - Move all operators so the they replace the corresponding right parentheses
  - Delete all parentheses

$$((((A / B) – C) + (D * E)) - (A * C))$$

Infix to Postfix

- Smarter algorithm
  - Scan the expression only once
  - Utilize stack
- The order of operands dose not change
- Output every visiting operand directly
- Use stack to store visited operators
  - Pop them out at the right moment
    - The ***priority*** of operator on top of stack is *higher or equal to* that of the incoming operator
    - left-to-right associativity

Example 1

- Infix : A + B * C

Next token	Stack	Output
None	Empty	None
A	Empty	A
+	+	A
B	+	AB
*	+*	AB
C	+*	ABC
	+	ABC*
	Empty	ABC*+

Example 2

- Infix : A * (B + C) * D

Next token	Stack	Output

Notes

- Expression with ( )
  - ‘(‘ has the highest priority, always push to stack.
  - Once pushed, ‘(’ get lowest priority.
  - Pop the operators until you see the matched ‘)’

Self-Study Topics

- Prefix representation

infix	prefix
A*B/C	/*ABC
A/B-C+D*E-A*C	-+-/ABC*DE*AC
A*(B+C)/D-G	-/*A+BCDG