EE5630 DSP CLASSROOM MEETING

MAY 4-5, 2020

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OVERVIEW OF THIS UNPRECEDENTED SEMESTER

- 11 Video lectures from MIT
- 14 extra videos from YWL
- 7 HWs
- 3 streaming sessions

Below are the textbook sections you should already be familiar.

2.7-2.9 Discrete-time Fourier transform

4.2-4.4; 4.6 All about sampling

5.1-5.6: Linear time-invariant system analysis

5.7 (This week!)

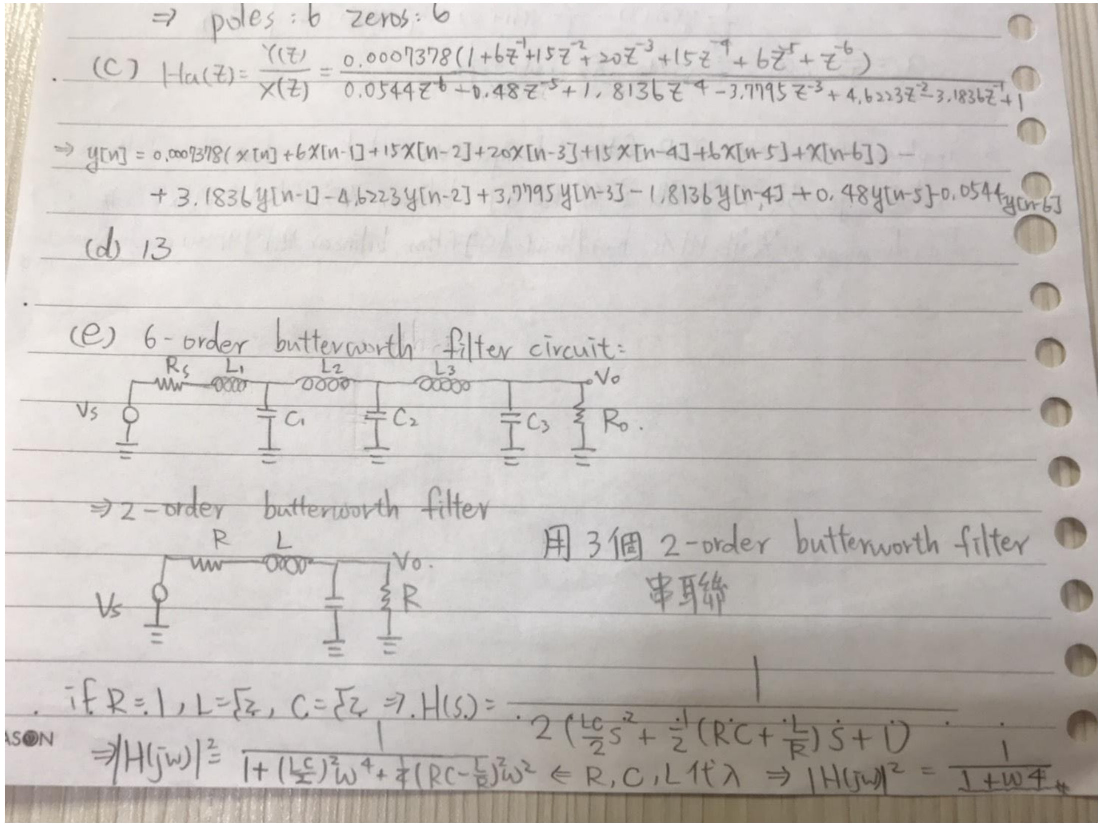
7.1-7.6 IIR and FIR Filter design

UPCOMING SCHEDULES

10	5/4-5/8	FIR types I to IV	Classroom 1		
11	5/11-5/15	Lec 11,12: Ntwk structures			HW8: Ch 6
12	5/18-5/22	Lec 13: Quantization effects	streaming 4		
13	5/25-29	**YWL supp (Optimal filter design)**			HW9: Optimal
14	6/1-6/5	**YWL supp (Lattice structure and Levinson-Durbin)**		Classroom 2	
15	6/8-6/12	**Advanced topic 1: Non LTI signal processing**			HW10: Lattice ladder
16	6/15-19	**Advanced topic 2: from Adaptive to NN**	streaming 5		
17	6/22-26	--	Final Exam (50%)		

SOME COMMENTS REGARDING HW6

My philosophy is to use HWs for harder and trickier questions, while keeping the exams simpler (not necessarily easier).



⇒ poles : 6 zeros : 6

(c) $H_d(z)=\frac{Y(z)}{X(z)}=\frac{0.0007378(1+6z^{-1}+15z^{-2}+20z^{-3}+15z^{-4}+6z^{-5}+z^{-6})}{0.0544z^{-6}-0.48z^{-5}+1.8136z^{-4}-3.7795z^{-3}+4.6223z^{-2}-3.1836z^{-1}+1}$

⇒ $y[n]=0.0007378(x[n]+6x[n-1]+15x[n-2]+20x[n-3]+15x[n-4]+6x[n-5]+x[n-6])$ −

$+3.1836y[n-1]-4.6223y[n-2]+3.7795y[n-3]-1.8136y[n-4]+0.48y[n-5]-0.0544y[n-6]$

(d) 13

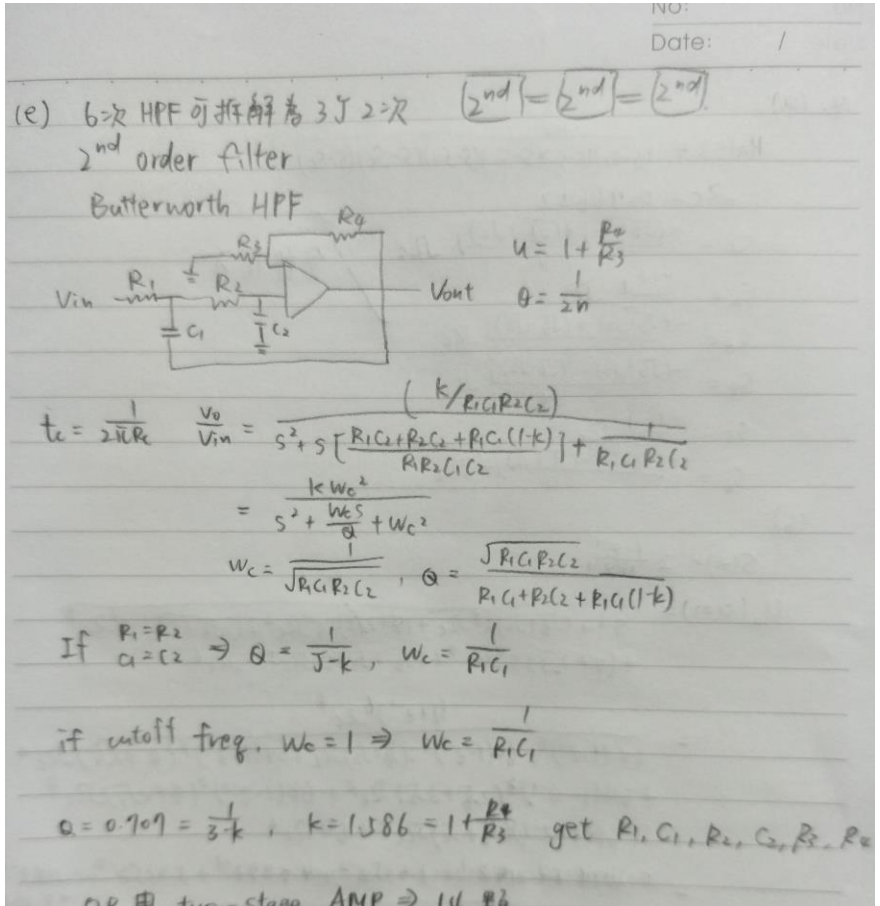
(e) 6-order butterworth filter circuit:

⇒ 2-order butterworth filter

用3個 2-order butterworth filter 串聯

if $R=1, L=\sqrt{2}, C=\sqrt{2}$ ⇒ $H(s)=\frac{1}{2(\frac{LC}{2}s^2+\frac{1}{2}(RC+\frac{L}{R})s+1)}$

⇒ $|H(jw)|^2=\frac{1}{1+(\frac{LC}{2})^2w^4+\frac{1}{4}(RC-\frac{L}{R})^2w^2}$ ⇐ R, C, L 代入 ⇒ $|H(jw)|^2=\frac{1}{1+w^4}$



No:

Date: /

(e) 6次 HPF 可拆解為 3 個 2次 $\boxed{2^{nd}}=\boxed{2^{nd}}=\boxed{2^{nd}}$

2^{nd} order filter

Butterworth HPF

$u=1+\frac{R_4}{R_3}$, $Q=\frac{1}{2n}$

$f_c=\frac{1}{2\pi RC}$ $\frac{V_o}{V_{in}}=\frac{(k/R_1C_1R_2C_2)}{s^2+s[\frac{R_1C_2+R_2C_2+R_1C_1(1-k)}{R_1R_2C_1C_2}]+\frac{1}{R_1C_1R_2C_2}}$

$=\frac{kW_c^2}{s^2+\frac{W_cs}{Q}+W_c^2}$

$W_c=\frac{1}{\sqrt{R_1C_1R_2C_2}}$, $Q=\frac{\sqrt{R_1C_1R_2C_2}}{R_1C_1+R_2C_2+R_1C_1(1-k)}$

If $R_1=R_2$, $C_1=C_2$ ⇒ $Q=\frac{1}{3-k}$, $W_c=\frac{1}{R_1C_1}$

if cutoff freq. $W_c=1$ ⇒ $W_c=\frac{1}{R_1C_1}$

$Q=0.707=\frac{1}{3-k}$, $k=1.586=1+\frac{R_4}{R_3}$ get $R_1, C_1, R_2, C_2, R_3, R_4$

LET'S BEGIN THE MOCK EXAM!

To warm up, the first few questions are about the discrete-time Fourier transform.

Let $x[n]$ and $y[n]$ be discrete-time signals. Define that $X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$, and $Y(\omega) = \sum_{n=-\infty}^{\infty} y[n]e^{-j\omega n}$. Some the following statements are false. Give a short proof to the one(s) you think is true, and point out what is wrong for those you think are false by correcting the statement or providing a counter example.

(2019, 1st MT)

Question 1 (30 sec):

If $x[n]$ is real for all n, then $X(\omega)$ is real for all ω.

Let $x[n]$ and $y[n]$ be discrete-time signals. Define that $X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$, and $Y(\omega) = \sum_{n=-\infty}^{\infty} y[n]e^{-j\omega n}$. Some the following statements are false. Give a short proof to the one(s) you think is true, and point out what is wrong for those you think are false by correcting the statement or providing a counter example.

(2019, 1st MT)

Question 2 (1 min):

If $y[n] = (x * x)[n]$, then $Y(\omega) = |X(\omega)|^2$.

Let $x[n]$ and $y[n]$ be discrete-time signals. Define that $X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$, and $Y(\omega) = \sum_{n=-\infty}^{\infty} y[n]e^{-j\omega n}$. Some the following statements are false. Give a short proof to the one(s) you think is true, and point out what is wrong for those you think are false by correcting the statement or providing a counter example.

Question 3 (1 min):

If $y[n] = x[2n]$, then $Y(\omega) = X(\omega/2)$.

Let $x[n]$ and $y[n]$ be discrete-time signals. Define that $X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$, and $Y(\omega) = \sum_{n=-\infty}^{\infty} y[n]e^{-j\omega n}$. Some the following statements are false. Give a short proof to the one(s) you think is true, and point out what is wrong for those you think are false by correcting the statement or providing a counter example.

(2019, 1st MT)

Question 4 (90 sec):
Assume that $y[n] = x[n/2]$ when n is even and $y[n] = 0$ when n is odd.
Then, $Y(\omega) = X(2\omega)$.

Suppose that you are sampling a signal $x_c(t) = A\,\mathrm{e}^{-\alpha t}\cos(2\pi f_0 t)\,u(t)$, where

- $\alpha = 600 \cdot \ln 2$ /sec,
- $f_0 = 200$ Hz,
- $A = 24.0$ mV, and
- the sampling rate f_s is 1200 Hz.

Question 5: Let $x[n] = x_c(nT)$, where $T = 1/f_s$.
Choose the correct expression of $x[n]$ at $n \geq 0$.

(a) $x[n](\mathrm{mV}) = 24(2^{-600n})\cos\frac{n}{6}$

(b) $x[n](\mathrm{mV}) = 24\left(\sqrt{2}^{-n}\right)\cos\frac{n}{6}$

(c) $x[n](\mathrm{mV}) = 24\left(\sqrt{2}^{-n}\right)\cos\frac{\pi n}{3}$

(d) $x[n](\mathrm{mV}) = 24(2^{-600n})\cos\frac{\pi n}{3}$

Continuing from the previous question, the correct answer should be $x[n] = 24\left(\sqrt{2}^{-n}\right)\cos\frac{\pi n}{3}u[n]$.

Question 6: Suppose that h[n] = x[n] is the impulse response of a LTI system, denote as H.

(a) H is causal and stable.
(b) H is non-causal but stable.
(c) H is causal but not stable.
(d) We cannot be sure.

THE NEXT FEW QUESTIONS ARE GETTING HARDER.

Let $h[n] = 24\left(\sqrt{2}^{-n}\right)\cos\frac{\pi n}{3}u[n]$ be the impulse response of H(z).

Question 7: (T or F)

$\exists \{a_1, a_2\}$ (constant coefficients) such that

$$h[n] + a_1 h[n-1] + a_2 h[n-2] = 0$$

for all $n \geq 2$.

Let $h[n] = 24\left(\sqrt{2}^{-n}\right)\cos\frac{\pi n}{3}u[n]$.

It is true that we can find constants $\{a_1, a_2\}$, such that

$h[n] + a_1 h[n-1] + a_2 h[n-2] = 0 \; n \geq 2$.

Question 8:

Is H(z) a minimum-phase filter?

Question 9: Let $h[n] = 24\left(\sqrt{2}^{-n}\right)\cos\frac{\pi n}{3}u[n]$.

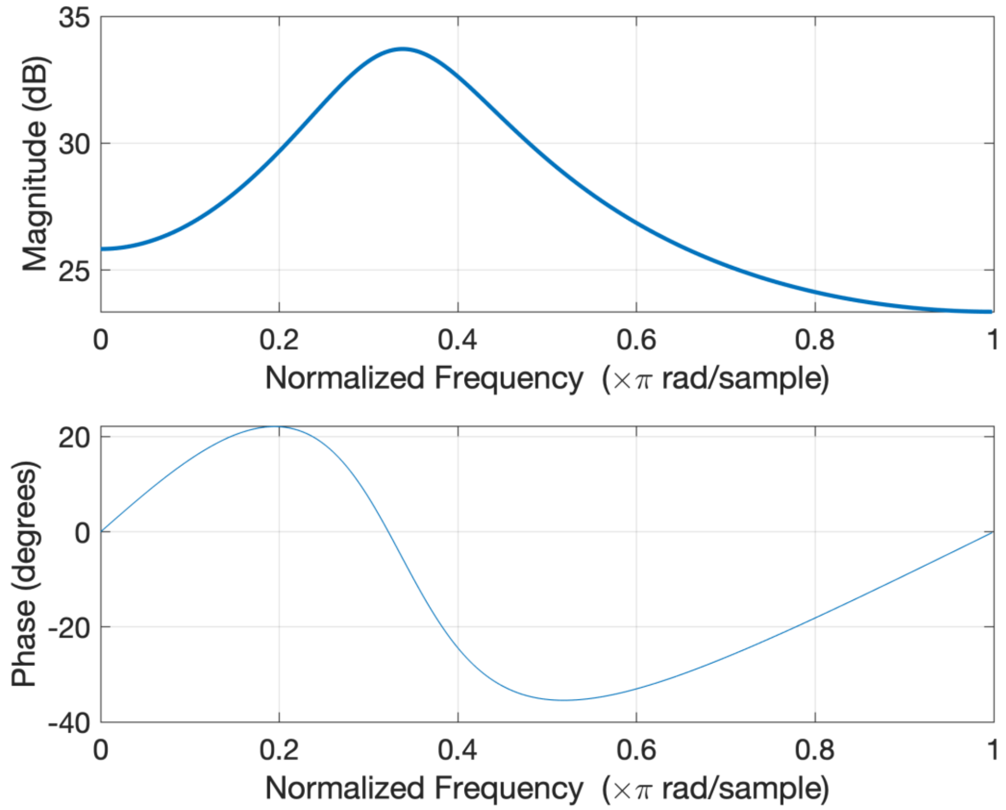
Calculate its z-transform H(z).

(Any volunteer?)

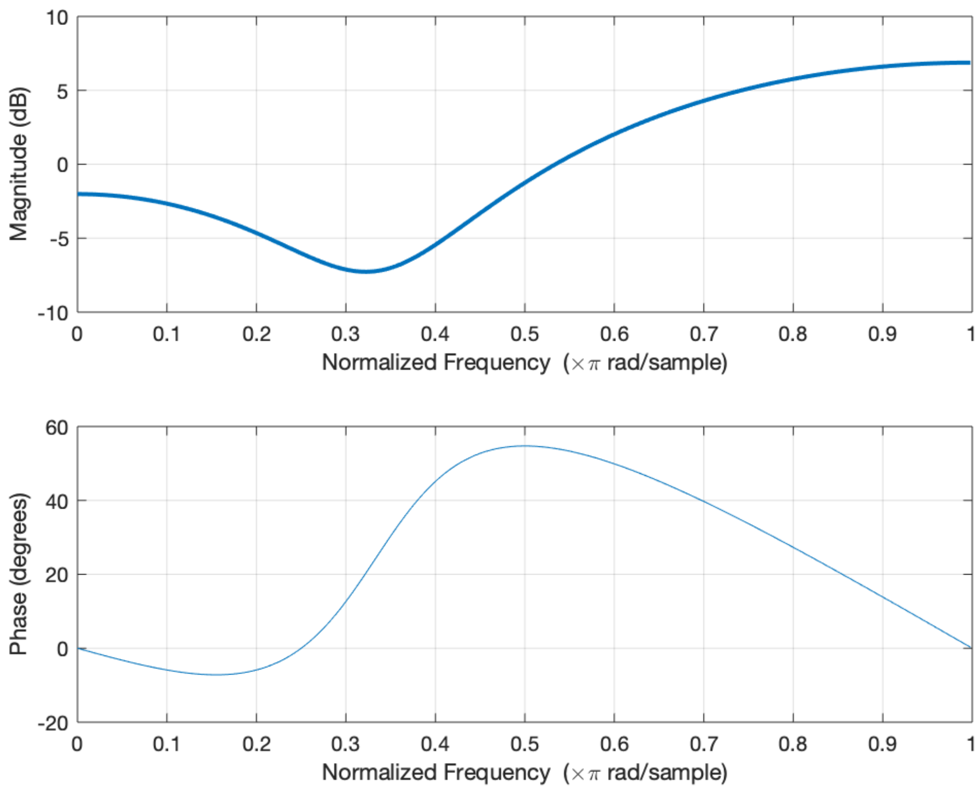
Question 10: Let $h[n] = 24\left(\sqrt{2}^{-n}\right)\cos\frac{\pi n}{3}u[n]$.

Which one of the below shows its magnitude and phase spectrum?

(a)



(b)



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Assume that $H(z) = \frac{1-3z^{-1}}{(1+0.5z^{-2})^2}$, $|z| > 1/\sqrt{2}$ is an LTI system function.
Denote its all-pass minimum-phase decomposition as

$$H(z) = H_{\mathrm{ap}}(z)H_m(z).$$

Question 11:
How many poles and zeros does the minimum-phase part $H_m(z)$ have?

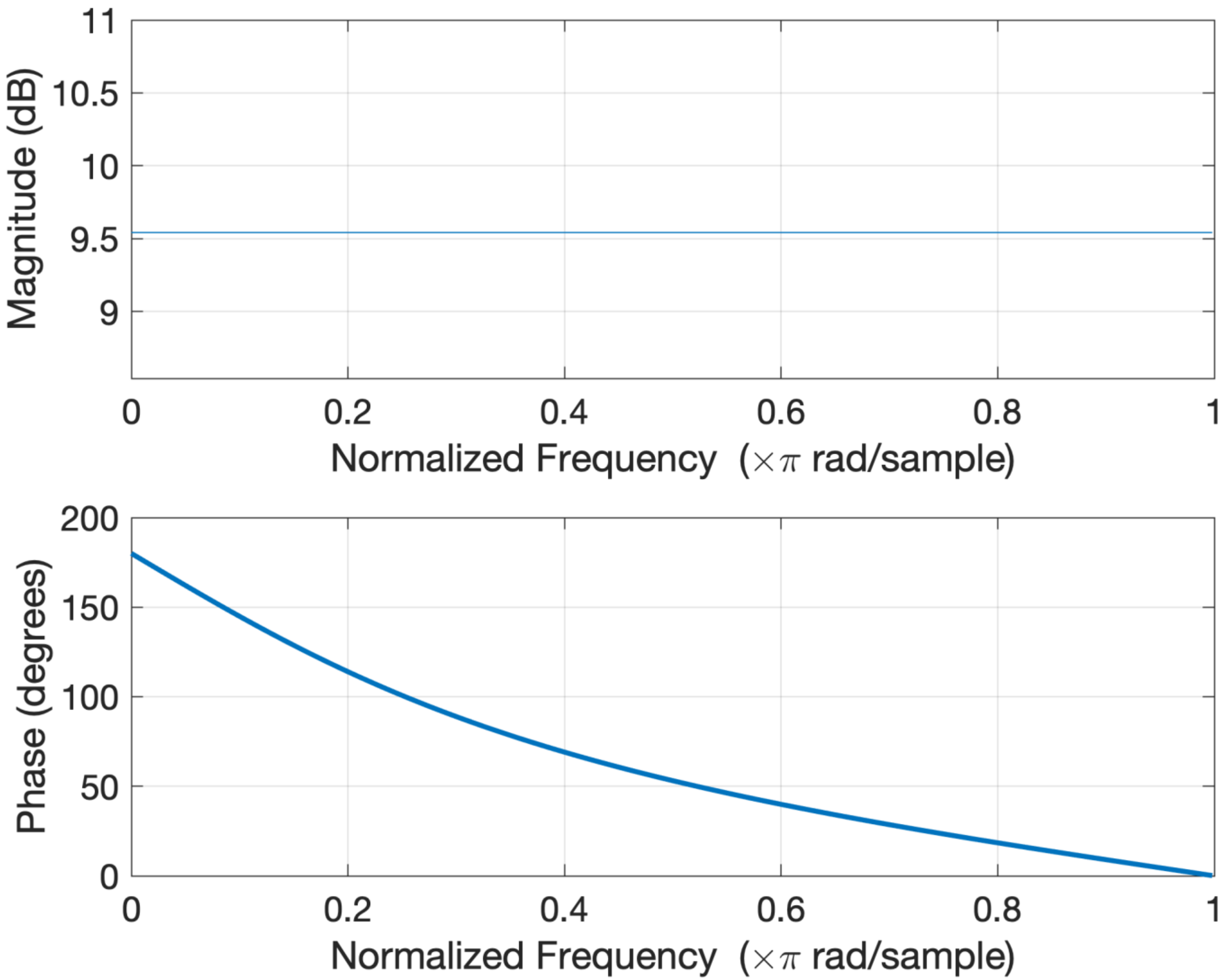
(a) 4 poles, 1 zero.
(b) 2 poles, 1 zero.
(c) 3 poles.
(d) 1 pole, 1 zero.

Assume that $H(z) = \frac{1-3z^{-1}}{(1+0.5z^{-2})^2}, |\mathrm{z}| > 1/\sqrt{2}$ is an LTI system function.
Denote its all-pass minimum-phase decomposition as

$$H(z) = H_{\mathrm{ap}}(z)H_m(z).$$

Question 12:
For the all-pass part $H_{\mathrm{ap}}(z)$, which of the following statement is true?

(a) Its group delay is positive for all ω because it is a causal system.
(b) Its group delay is **not** always positive but monotonically increases at $\omega \in (0, \pi)$.
(c) Its group delay is always positive and monotonically decreases at $\omega \in (0, \pi)$.
(d) Its group delay is always positive and monotonically increases at $\omega \in (0, \pi)$.



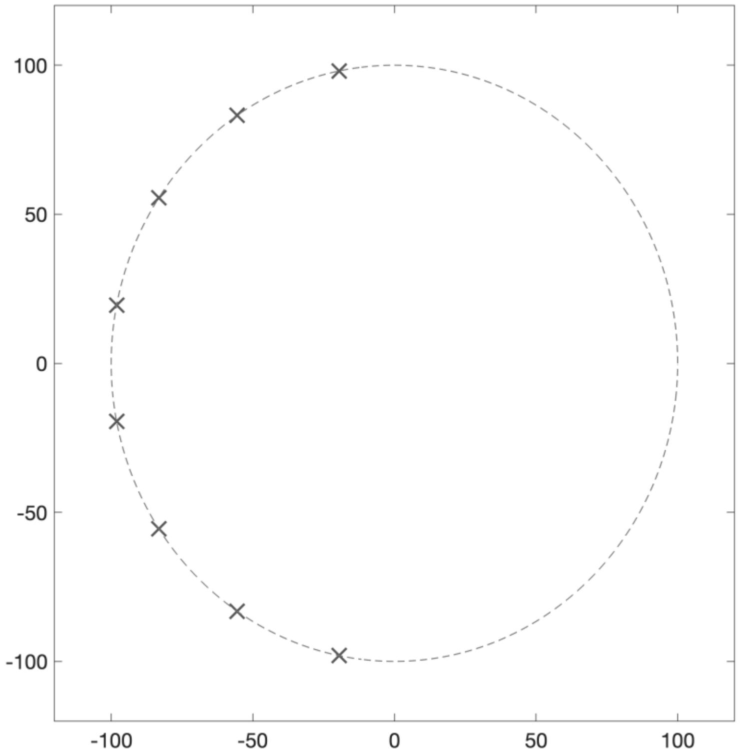
Magnitude (dB)
11
10.5
10
9.5
9
0
0.2
0.4
0.6
0.8
1
Normalized Frequency ($\times\pi$ rad/sample)
Phase (degrees)
200
150
100
50
0
0
0.2
0.4
0.6
0.8
1
Normalized Frequency ($\times\pi$ rad/sample)

Let us define bilinear transformation as $s = \frac{2}{T_d}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$.

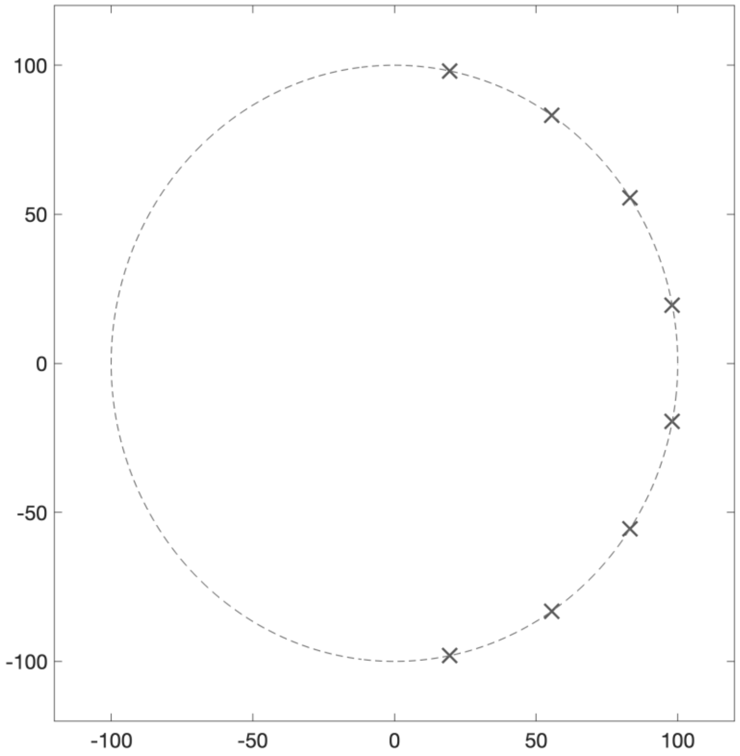
If $G_{\text{des}}(\Omega) = \frac{1}{1+\left({\Omega}/{\Omega_c}\right)^{16}}$ is the desired magnitude response of a prototype continuous-time filter, briefly explain how to construct a rational function $H_c(s)$ such that $|H_c(j\Omega)|^2 = G_{\text{des}}(\Omega)$.

Question 13: Please draw a sketch to show where the poles of $H_c(s)$ should be located in the Laplace s-plane.

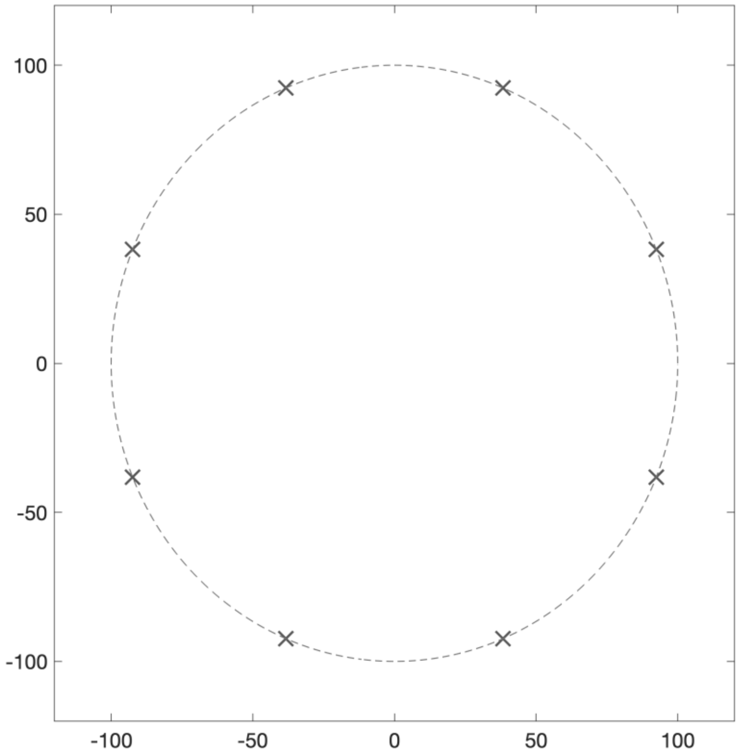
Question 13 (continued). Suppose that $\Omega_c = 100$ rad/s. Which one of the plots below shows the locations of the poles of $H_c(s)$ most accurately?



(a)



(b)



(c)

Let $G_{\text{des}}(\Omega) = \frac{1}{1+\left(\Omega/\Omega_c\right)^{16}} = |H_c(j\Omega)|^2$, and define $H(z) = H_c\left(\frac{2}{T_d}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)$.

Question 14: If we want the cut-off frequency* to be $\pi/4$ in the digital domain, which value should we choose for the parameter T_d ?

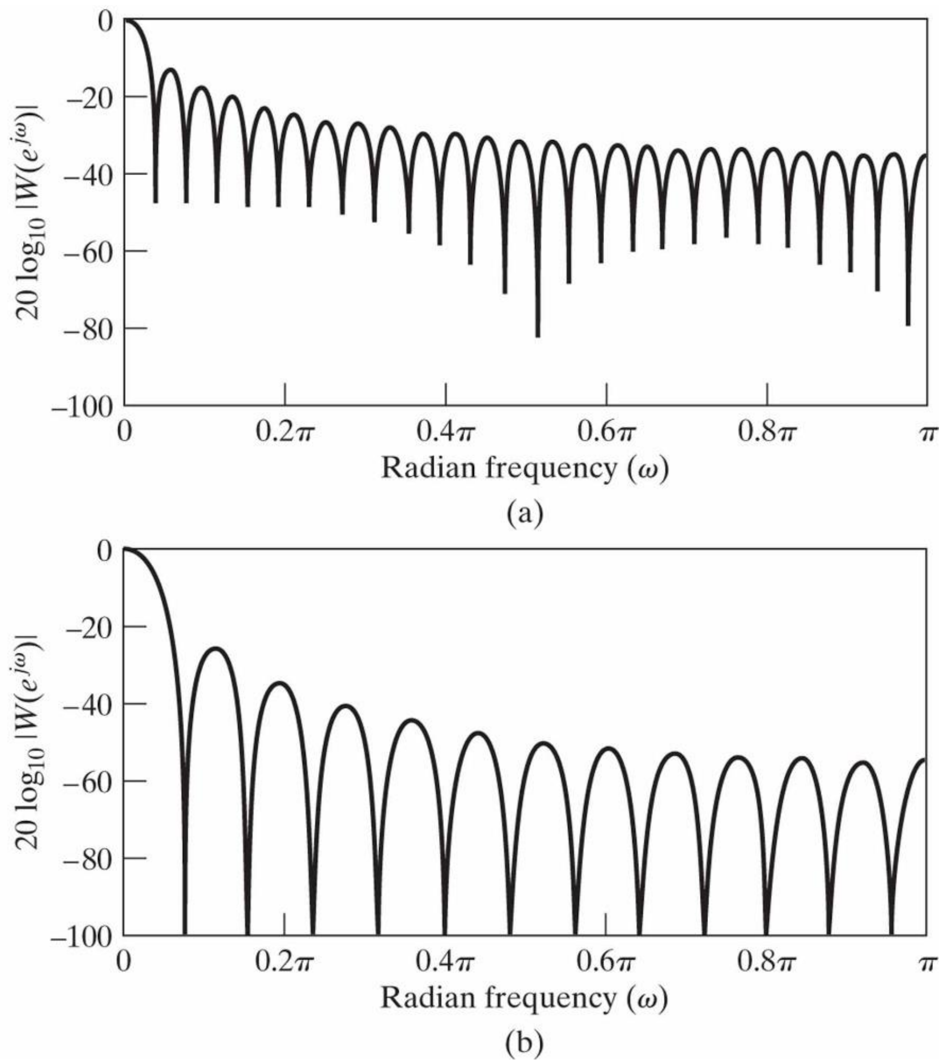
*Remark: Here, cut-off frequency means the frequency at which the gain drops by 3 dB.

(a) $T_d = \frac{1}{\Omega_c}$

(b) $T_d = 2/\Omega_c$

(c) $T_d = \frac{2}{\Omega_c}\tan\frac{\pi}{8}$

(d) $T_d = \frac{1}{\Omega_c}\tan\frac{\pi}{8}$



Question 15: On the left, we have the magnitude spectrum of a rectangular window and a Barlett (triangular) window, respectively.

How many non-zero samples does the rectangular window have?

(a) 25
(b) 49
(c) 26
(d) 51