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Intro to DSP HW3

1.

(a)

$$y[n] = \frac{1}{4}x[n] + \frac{1}{4}x[n-1] - \frac{1}{4}x[n-2] - \frac{1}{4}x[n-3]$$

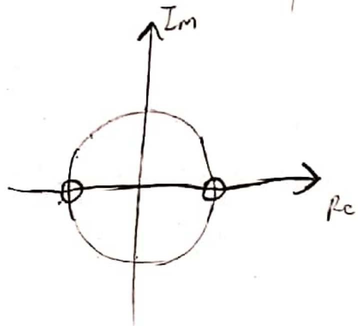
$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{1}{4}(1+z^{-1}-z^{-2}-z^{-3}) = \frac{-1}{4}(z^{-1}-1)(z^{-1}+1)^2$$

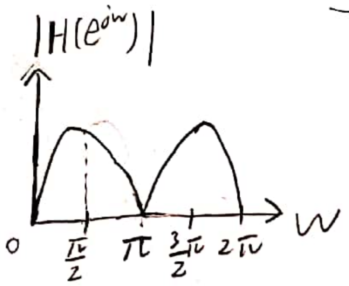
$$\Rightarrow H(e^{jw}) = \frac{1}{4}z^{-\frac{3}{2}}(z^{\frac{3}{2}}+z^{\frac{1}{2}}-z^{-\frac{1}{2}}-z^{-\frac{3}{2}})\Big|_{z=e^{jw}} = \frac{2j}{4}\left(\sin(\tfrac{3}{2}w)+\sin(\tfrac{1}{2}w)\right)e^{-j\frac{3}{2}w}$$

$$= \frac{1}{2}\left(\sin(\tfrac{3}{2}w)+\sin(\tfrac{1}{2}w)\right)e^{-j(\frac{3}{2}w-\frac{\pi}{2})}$$

$$\Rightarrow |H(e^{jw})| = \left|\frac{1}{2}\left(\sin(\tfrac{3}{2}w)+\sin(\tfrac{1}{2}w)\right)\right|$$

$$\measuredangle H(e^{jw}) = -\frac{3}{2}w+\frac{\pi}{2}$$





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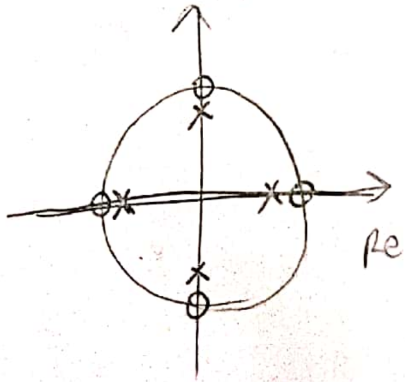
(b) $y[n] = x[n] - x[n-4] + 0.6561y[n-4]$

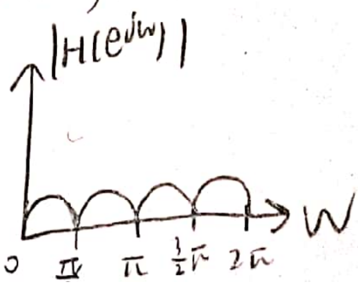
$$\Rightarrow H(z) = \frac{1-z^{-4}}{1-0.6561z^{-4}} = \frac{(1-z^{-2})(1+z^{-2})}{(1-0.81z^{-2})(1+0.81z^{-2})} = \frac{(1+z^{-1})(1-z^{-1})(z^{-1}+j)(z^{-1}-j)}{(0.9+z^{-1})(0.9-z^{-1})(z^{-1}+j0.9)(z^{-1}-j0.9)}$$

$$\Rightarrow H(e^{jw}) = \frac{e^{-j2w}(e^{j2w}-e^{-j2w})}{1-ae^{-j4w}} = \frac{2e^{-j2w}\cos(2w)}{1-ae^{-j4w}}, \text{ where } a = 0.6561$$

$$\Rightarrow |H(e^{jw})| = \left|\frac{2\cos(2w)}{\sqrt{1+a^2-2a\cos(4w)}}\right|$$

$$\measuredangle H(e^{jw}) = -2w - \tan^{-1}\left(\frac{-a\sin(4w)}{1-a\cos(4w)}\right)$$





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$$x[n] = \sin\left(\frac{1}{10}\pi n\right) + \frac{1}{3}\sin\left(\frac{3}{10}\pi n\right) + \frac{1}{5}\sin\left(\frac{1}{2}\pi n\right)$$

(a)

$$H(e^{jw}) = 1 - 2e^{-jw} + 3e^{-j2w} - 4e^{-j3w} + 4e^{-j5w} - 3e^{-j6w} + 2e^{-j7w} - e^{-j8w}$$

$$= e^{-j4w} \cdot 2j\left(\sin(4w) - 2\sin(3w) + 3\sin(2w) - 4\sin(w)\right)$$

=> The system has magnitude distortion since $|H(e^{jw})|$ is not constant but no phase distortion

(b) $y[n] = 10x[n-10]$

The system has no distortion, it only delays and amplifies the input by 10 samples and 10 times resp.

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3.

(a)

$$H_r(j\Omega) = \begin{cases} H_r(e^{j\Omega T}) & |\Omega| \le \pi/T \\ 0 & \text{else} \end{cases}$$

$$\Rightarrow H_r(e^{j\Omega T}) = \frac{\Omega T/2}{\sin(\Omega T/2)} e^{j\frac{\Omega T}{2}}, \quad |\Omega| \le \pi/T$$

let $w = \Omega T$

$$\Rightarrow H_r(e^{jw}) = \frac{w}{2\sin(\frac{w}{2})} e^{j\frac{w}{2}}, \quad |w| \le \pi$$

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(b) $H_{FIR}(z) = -\frac{1}{16} + \frac{9}{8} z^{-1} - \frac{1}{16} z^{-2}$

$$\Rightarrow H_{FIR}(e^{jw}) = \frac{-e^{-jw}}{16}(e^{jw} - 18 + e^{-jw}) = \frac{-1}{8}(\cos w - 9) e^{-jw}$$

$$\Rightarrow |H_{FIR}(e^{jw})| - |H_r(e^{jw})| = \frac{1}{8}(9 - \cos w) - \frac{w}{2\sin(\frac{w}{2})}, \quad |w| \le \pi$$

(c)

$$H_{IIR}(z) = \frac{9}{8 + z^{-1}}$$

$$\Rightarrow |H_{IIR}(e^{jw})| = \frac{9}{65 + 16\cos w}$$

$$\Rightarrow |H_{IIR}(e^{jw})| - |H_r(e^{jw})| = \frac{9}{65 + 16\cos w} - \frac{w}{2\sin(\frac{w}{2})}, \quad |w| \le \pi$$

4.

$$H(s) = \frac{s^4-6s^3+10s^2+2s-15}{s^5+15s^4+100s^3+390s^2+744s+720} = \frac{(s-3)(s+1)(s-2+i)(s-2-i)}{B(s)}$$

(a) $H(s)$ is a nonminimum phase system since it has 3 zero on $\{|s|>0\}$

(b) All poles of $H(s)$ are on $\{|s|<0\}$ since the coefficients of $B(s)$ are positive

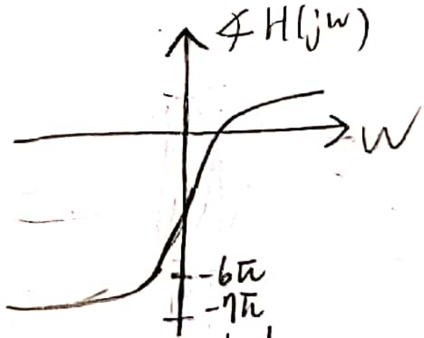
Thus, we only need to deal with the numerator of $H(s)$

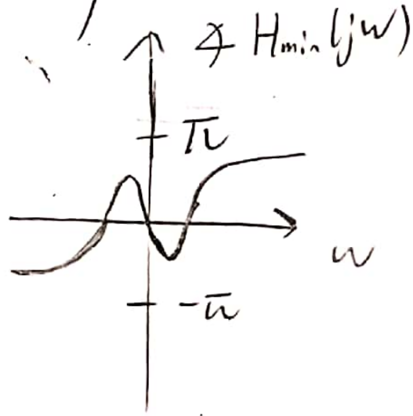
$$\Rightarrow H(s) = \frac{(s+1)(s-3)(s-2+i)(s-2-i)}{B(s)} = \underbrace{\frac{(s+1)(s+3)(s+2+i)(s+2-i)}{B(s)}}_{H_{min}(s)} \underbrace{\frac{(s-3)(s-2+i)(s-2-i)}{(s+3)(s+2+i)(s+2-i)}}_{H_{ap}(s)}$$

$$|H_{ap}(jw)| = \sqrt{\frac{(w^2+9)[(1+w)^2+4][(1-w)^2+4]}{(w^2+9)[(1+w)^2+4][(1-w)^2+4]}} = 1$$

and all poles and zeros of $H_{min}(s)$ are on $\{|s|<0\}$ clearly

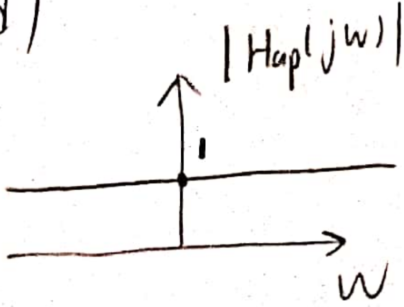
(c)

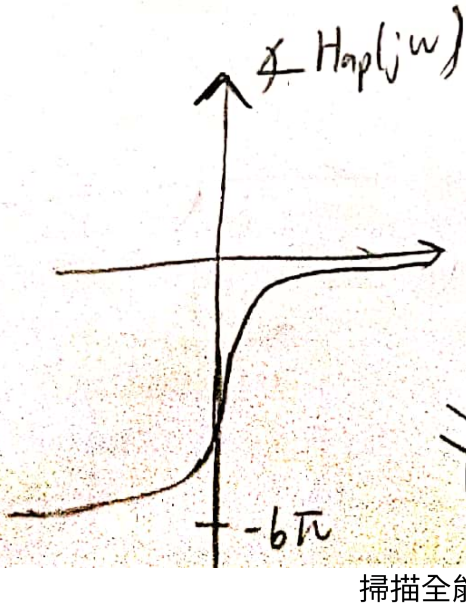




$H(s)$ and $H_{min}(s)$ has the same magnitude response

(d)





5.

(a) According to (1), $H(z)$ must has two zeros at $z=1,-1$ resp.

According to (2)(3), $H(z)$ has two poles with the phase $\pm\frac{\pi}{4}$ resp.

and the lengths of these poles are the same.

$$\Rightarrow H(z)=\frac{(z+1)(z-1)}{C(z-ae^{j\frac{\pi}{4}})(z-ae^{-j\frac{\pi}{4}})} \text{ where } C\in\mathbb{C},\ 0<a<1$$

Here C is used to control the gain of $H(z)$

Choose $a=0.95$, then choose $C=|H(e^{j\frac{\pi}{4}})|\simeq 20.5061$

$\Rightarrow |H(e^{jw})|$ has maximum 1 at $w_2=\frac{\pi}{4}$, $w_4=\frac{-\pi}{4}$

and $|H(e^{j(w_2+0.05)})|=|H(e^{j(w_4-0.05)})|\approx 0.7330$ $\quad\left(\frac{1}{\sqrt{2}}\simeq 0.7071\right)$

$|H(e^{j(w_2-0.05)})|=|H(e^{j(w_4+0.05)})|\approx 0.6971$

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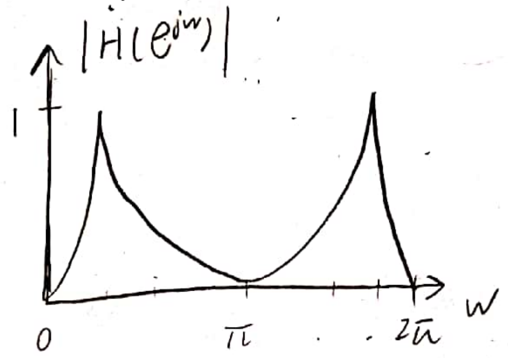
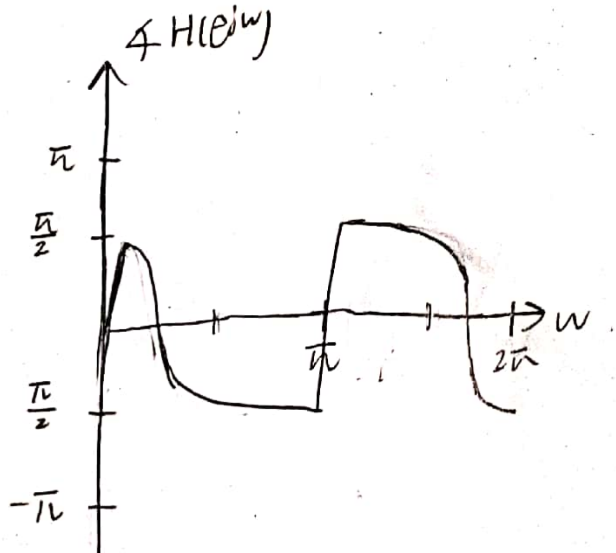
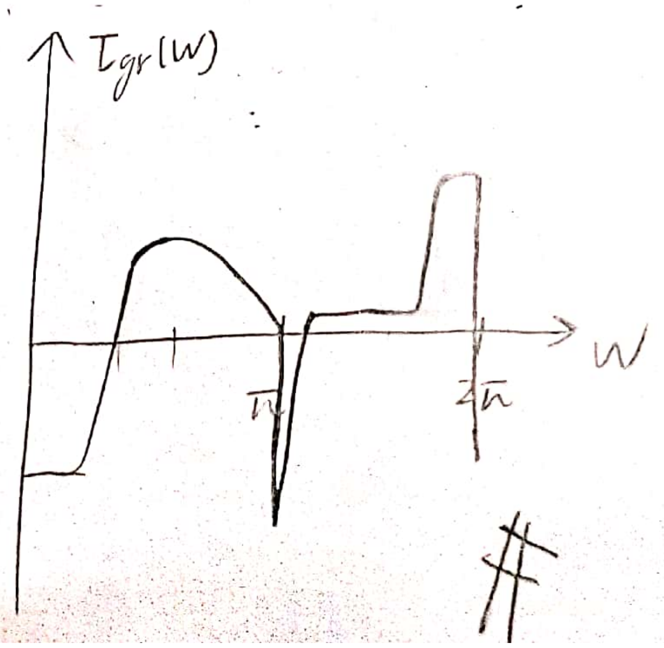
(b)



(c)





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6.

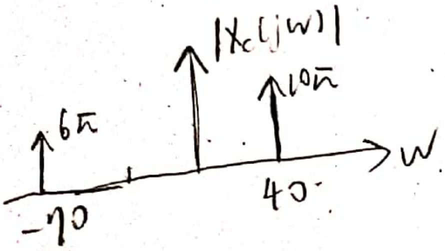
(a) (i) $x_c(t) = 5e^{j40t} + 3e^{-j70t}$

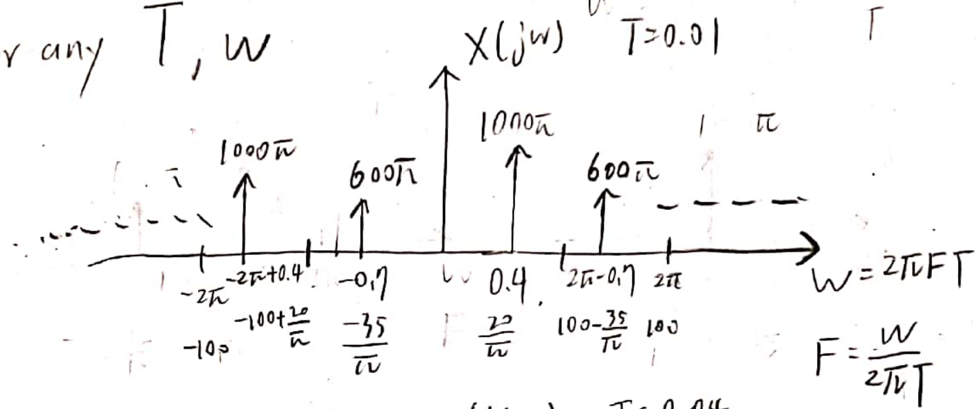
$X_c(jw) = 10\pi\delta(w-40) + 6\pi\delta(w+70)$

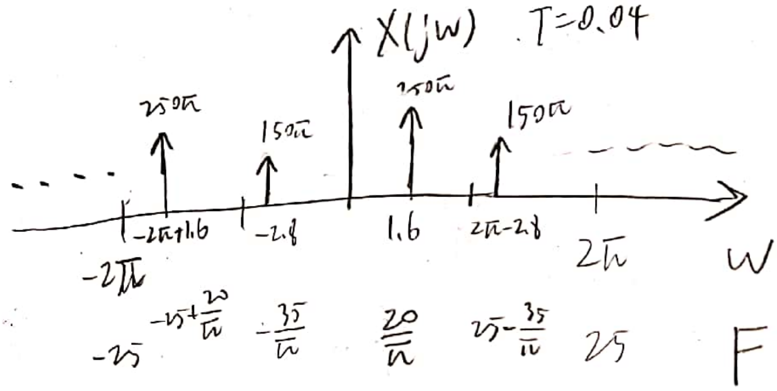
$X(e^{jw}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c(j(w-2\pi k)/T)$, $T = 0.01, 0.04, 0.1$

(ii) Since $X(e^{jw})$ is real and nonnegative,

$\angle X(e^{jw}) = 0$ for any T, w







(iii)

For $T = 0.01, 0.04$

$x_c(t)$ can be recovered from $x[n]$

But for $T = 0.1$

$x_c(t)$ can't be recovered from $x[n]$

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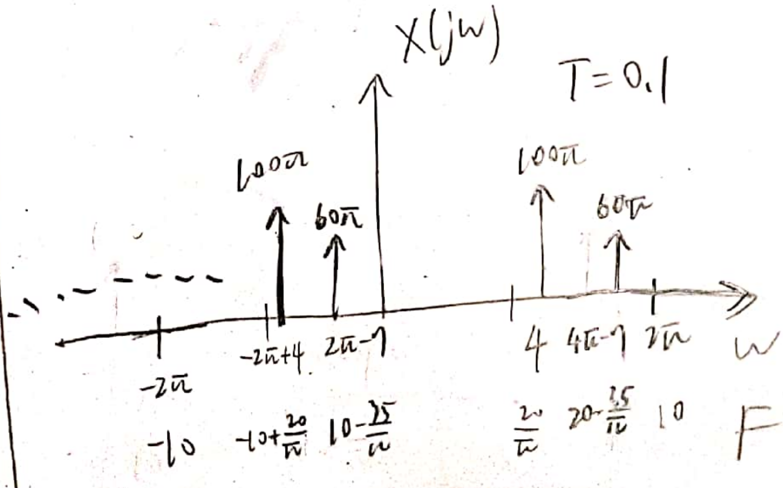


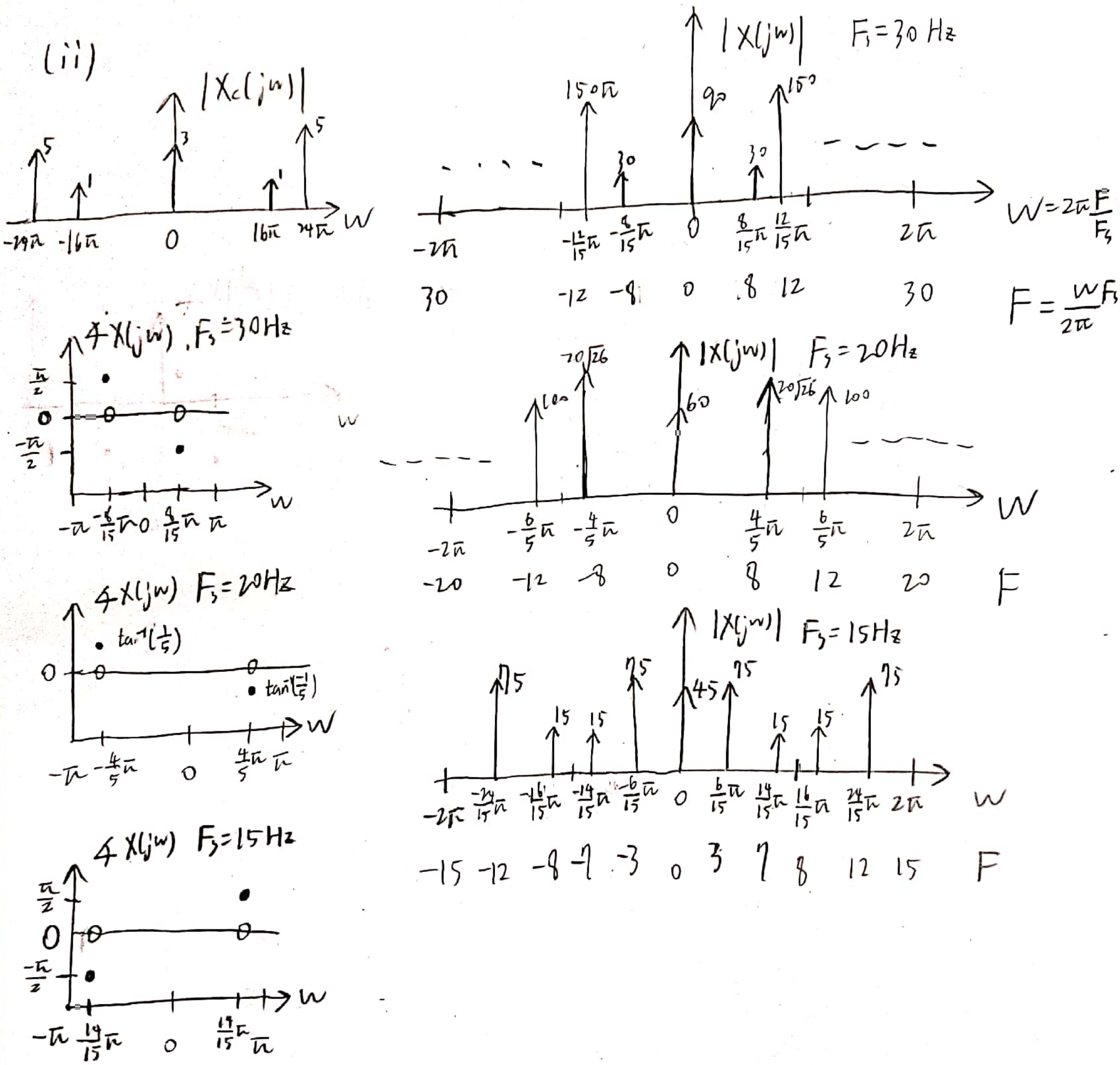
(b)

(i) $x_c(t) = 3 + 2\sin(16\pi t) + 10\cos(24\pi t)$

$$\Rightarrow X_c(jw) = 3\delta[w] - j\left(\delta[w-16\pi] - \delta[w+16\pi]\right) + 5\left(\delta[w-24\pi] + \delta[w+24\pi]\right)$$

$$X(e^{jw}) = F_s \sum_{k=-\infty}^{\infty} X_c\left(jF_s(w-2\pi k)\right), \quad F_s = 30, 20, 15\ \text{Hz}$$

(ii)



(iii) For $F_s = 30$ Hz, $x_c(t)$ can be recovered from $x[n]$

But For $F_s = 20, 15$ Hz $x_c(t)$ can't be recovered from $x[n]$ #

7.

(a)

Quantizer step $\Delta = 1$

(b)

$e(t) = \frac{\Delta}{2} t \qquad |t| < \frac{\Delta}{2}$

$\Rightarrow P_Q = \frac{1}{\Delta}\int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} e^2(t)\,dt = \frac{\Delta^2}{12} = \frac{1}{12}$

$X_c(t) = 2\cos(200\pi t) + 3\sin(500\pi t)$

$\Rightarrow P_s = \frac{1}{T}\int_0^T X_c^2(t)\,dt = \frac{1}{T}\int_0^T 4\cos^2(200\pi t)\,dt + \frac{1}{T}\int_0^T 9\sin^2(500\pi t)\,dt$

$= 2 + \frac{9}{2} = \frac{13}{2}$ (since sin and cos are orthogonal at any common period)

$\Rightarrow SQNR = \frac{P_s}{P_Q} = 78$

$\Rightarrow SQNR(dB) = 10\log_{10} 78 \approx 18.92$

(c.)

Nyquist rate: 1000π

folding frequency: $128\ Hz$ (2048 bits/s $\Rightarrow$ 256 symbol/s $\Rightarrow F_s = 256 \Rightarrow \frac{F_s}{2} = 128$)