DSP HW1 Solution

Subject : ....

2.5 $y[n]-5y[n-1]+6y[n-2]=2x[n-1]$

(a) $y[n]-5y[n-1]+6y[n-2]=0 \rightleftharpoons 1-5z^{-1}+6z^{-2}=0 \Rightarrow (1-2z^{-1})(1-3z^{-1})=0$

$\Rightarrow$ homogeneous response $y_h[n]=A_1(2)^n+A_2(3)^n$

(b) Z-transform: $Y(z)\left[1-5z^{-1}+6z^{-2}\right]=X(z)\,(2z^{-1}) \Rightarrow \frac{Y(z)}{X(z)}=\frac{2z^{-1}}{1-5z^{-1}+6z^{-2}}=\frac{-2}{1-2z^{-1}}+\frac{2}{1-3z^{-1}}$

$H(z)=\frac{-2}{1-2z^{-1}}+\frac{2}{1-3z^{-1}} \rightleftharpoons h[n]=-2(2)^n u[n]+2(3)^n u[n]$

(c) $x[n]=u[n] \rightleftharpoons X(z)=\frac{1}{1-z^{-1}}$, $Y(z)=X(z)H(z)=\frac{1}{1-z^{-1}}\left(\frac{-2}{1-2z^{-1}}+\frac{2}{1-3z^{-1}}\right)$

$=\frac{2}{1-z^{-1}}+\frac{-4}{1-2z^{-1}}+\frac{-1}{1-z^{-1}}+\frac{3}{1-3z^{-1}}$

$=\frac{1}{1-z^{-1}}+\frac{-4}{1-2z^{-1}}+\frac{3}{1-3z^{-1}}$

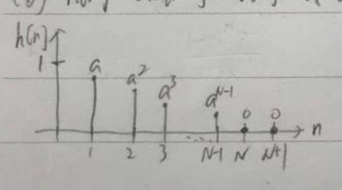
$\rightleftharpoons y[n]=u[n]-4\cdot(2)^n u[n]+3\cdot(3)^n u[n]$

2.30

(a) stable $\rightarrow \sum_{-\infty}^{\infty}|h[n]|<\infty \rightarrow \sum_{n=-\infty}^{\infty}|a^n u[n]|=\sum_{n=0}^{\infty}|a^n|<\infty \rightarrow |a|<1$

(b) $h[n]=ah[n-1]+\delta[n]-a^N\delta[n-N]$, causal $\rightarrow h[-1]=0$, $h[0]=1$, $h[1]=a$, $h[2]=a^2$

$h[N]=a^N-a^N=0$, $h[N+1]=0$



(c) the impulse response is finite length $\rightarrow$ FIR

(d) FIR is always stable $\rightarrow \sum_{n=-\infty}^{\infty}|h[n]|<\infty \rightarrow$ "a" can be any value

Double A

2.35

$H(e^{j\omega}) = e^{-j(\omega-\frac{\pi}{4})}\left(\text{—}\right) = e^{j\frac{\pi}{4}} \cdot e^{-j\omega}\left(\text{—}\right) = e^{j\frac{\pi}{4}} G(e^{j\omega})$

$x[n] \to \boxed{G} \xrightarrow{y_1[n]} \boxed{e^{j\frac{\pi}{4}}} \to y[n]$, $x[n] = \cos\left(\frac{\pi n}{2}\right) = \left(e^{j\frac{\pi n}{2}} + e^{-j\frac{\pi n}{2}}\right)/2$

$y_1[n] = x[n] * g[n] = \left(G(e^{j\frac{\pi}{2}}) e^{j\frac{\pi n}{2}} + G(e^{-j\frac{\pi}{2}}) e^{-j\frac{\pi n}{2}}\right)/2$

$G(e^{j\frac{\pi}{2}}) = e^{-j\frac{\pi}{2}}\left(\frac{1+e^{-j\pi}+4e^{-j2\pi}}{1+\frac{1}{2}e^{-j\pi}}\right) = e^{-j\frac{\pi}{2}}\left(\frac{1+(-1)+4}{1+\frac{1}{2}\times(-1)}\right) = 8e^{-j\frac{\pi}{2}}$, $G(e^{-j\frac{\pi}{2}}) = 8e^{j\frac{\pi}{2}}$

$y_1[n] = \left(8e^{j(\frac{\pi n}{2}-\frac{\pi}{2})} + 8e^{-j(\frac{\pi n}{2}-\frac{\pi}{2})}\right)/2 = 8\cos\left(\frac{\pi n}{2}-\frac{\pi}{2}\right)$

$y[n] = y_1[n] \times e^{j\frac{\pi}{4}} = 8e^{j\frac{\pi}{4}} \cos\left(\frac{\pi n}{2}-\frac{\pi}{2}\right)$

2.40

(a) $x[n] = e^{j(\frac{2\pi n}{5})}$, $x[n+N] = x[n] \to e^{j(\frac{2\pi(n+N)}{5})} = e^{j2\pi(\frac{n}{5})} \to e^{j\frac{2\pi N}{5}} = 1 \to \frac{N}{5} = 1$

(b) $x[n] = \sin\left(\frac{\pi n}{19}\right)$, $x[n+N] = \sin\left(\frac{\pi(n+N)}{19}\right) = \sin\left(\frac{\pi n}{19}+2\pi k\right)$, $\to \frac{N}{19} = 2k$, $\to N = 38$

(c) $x[n] = ne^{j\pi n}$, $x[n+N] = (n+N)e^{j\pi(n+N)} \neq x[n] = ne^{j\pi n}$ for all $N \neq 0$ $\to$ not periodic

(d) $x[n] = e^{jn}$, $x[n+N] = e^{j(n+N)} = e^{j(n+2\pi k)} \to N = 2\pi k$ $*$ $\to$ not periodic

2.47

(a) $x_1[n] = x_2[n] + x_3[n+4]$, but $y_1[n] \neq y_2[n] + y_3[n+4]$ $\to$ not linear

(b) $\delta[n] = x_3[n+4] \to T\{\delta[n]\} = y_3[n+4] = 3\delta[n+6] + 2\delta[n+5]$

(c) T is time-invariant but not linear $\to$ can only use shifted inputs x_1, x_2, x_3

$(x_1[n-k], x_2[n-k], x_3[n-k])$