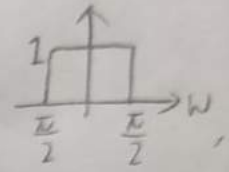
HW5 Q1

Since $\tilde{x}[n]$ is the signal from $x(n)$ passed by an ideal LPF 

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so both $\tilde{x}_{d1}[n] = \tilde{x}[2n]$ and $\tilde{x}_{d2}[n] = \tilde{x}[2n-1]$ satisfy the sampling theorem. (They have same period of 2.)

It means that if we use $\tilde{x}_{d_1}(n)$, $\tilde{x}_{d_2}[n]$ to perfectly reconstruct continuous time signal with sampling rate $\frac{f_s}{2}$, respetively, both results would be the same as reconstruct $\tilde{x}[n]$ with sampling rate f_s but <u>with different time delay.</u>

5.12 (a) $H(z) = \dfrac{(1+0.2z^{-1})(1-9z^{-2})}{(1+0.81z^{-2})} \rightarrow$ poles at $z = \pm 0.9i \rightarrow \because |z_p| < 1 \therefore$ stable

(b) $H(z) = \dfrac{(1+0.2z^{-1})(1-9z^{-2})}{(1+0.81z^{-2})} = \dfrac{(1+0.2z^{-1})}{(1+0.81z^{-2})}\left(\dfrac{(1-9z^{-2})}{(1-\frac{1}{9}z^{-2})}(1-\frac{1}{9}z^{-2})\right)$

inside circle

$$= \frac{(1+0.2z^{-1})(1-\frac{1}{9}z^{-2})}{(1+0.81z^{-2})} \cdot \frac{1-9z^{-2}}{1-\frac{1}{9}z^{-2}} = H_1 \cdot H_{ap}$$

5.18 (a) $H_1(z) = \dfrac{1-2z^{-1}}{1+\frac{1}{3}z^{-1}} = \dfrac{1-\frac{1}{2}z^{-1}}{1+\frac{1}{3}z^{-1}} \cdot \dfrac{1-2z^{-1}}{1-\frac{1}{2}z^{-1}} = \underbrace{\left(\dfrac{1-\frac{1}{2}z^{-1}}{1+\frac{1}{3}z^{-1}}\right)(2)}_{H_{min}} \underbrace{\left(\frac{1}{2}\right)\left(\dfrac{1-2z^{-1}}{1-\frac{1}{2}z^{-1}}\right)}_{H_{ap}}$

$$H_{min} = \frac{2-z^{-1}}{1+\frac{1}{3}z^{-1}}$$

(b) $H_2(z) = \dfrac{(1+3z^{-1})(1-\frac{1}{2}z^{-1})}{z^{-1}(1+\frac{1}{3}z^{-1})} = \dfrac{(1-\frac{1}{2}z^{-1})}{z^{-1}}(3)\left(\frac{1}{3}\right)\left(\dfrac{1+3z^{-1}}{1+\frac{1}{3}z^{-1}}\right)$

Since $\frac{1}{z^{-1}}$ term does not affect the freq. response magnitude $\rightarrow H_{min} = 3 - \frac{3}{2}z^{-1}$

and this term makes the system noncausal.

5.63

(a) $x[n] = s[n]\cos(w_0 n) \rightleftharpoons X(e^{jw}) = \dfrac{S(e^{j(w-w_0)}) + S(e^{j(w+w_0)})}{2}$

$H(e^{jw}) = |H(e^{jw})|\, e^{j\measuredangle H(e^{jw})} = \begin{cases} e^{-j\phi_0}, & w \text{ is positive} \\ e^{j\phi_0}, & w \text{ is negative} \end{cases}$

$Y(e^{jw}) = X(e^{jw})H(e^{jw}) = \frac{1}{2}S(e^{j(w-w_0)})e^{-j\phi_0} + \frac{1}{2}S(e^{j(w+w_0)})e^{j\phi_0}$

$\rightleftharpoons y[n] = \frac{1}{2}s[n]e^{j(w_0 n-\phi_0)} + \frac{1}{2}s[n]e^{-j(w_0 n-\phi_0)} = \frac{1}{2}s[n]\cos(w_0 n-\phi_0)$

(b) $H(e^{jw}) = \begin{cases} e^{j(-\phi_0 - n_d w)}, & w \text{ is positive} \\ e^{j(\phi_0 - n_d w)}, & w \text{ is negative} \end{cases}$

$Y(e^{jw}) = X(e^{jw})H(e^{jw}) = \frac{1}{2}S(e^{j(w-w_0)})e^{j(-\phi_0-n_d w)} + \frac{1}{2}S(e^{j(w+w_0)})e^{j(\phi_0-n_d w)}$

$\rightleftharpoons y[n] = \delta[n-n_d] * (s[n]\cos(w_0 n-\phi_0)) = s[n-n_d]\cos(w_0 n-\phi_0-w_0 n_d)$

$-\phi_1 = -\phi_0 - w_0 n_d \rightarrow y[n] = s[n-n_d]\cos(w_0 n-\phi_1)$

(c) $\tau_{gr}(w) = -\frac{d}{dw}\measuredangle H(e^{jw}) = n_d$

$\tau_{ph}(w) = \frac{-1}{w}\measuredangle H(e^{jw}) = \begin{cases} \frac{\phi_0}{w} + n_d, & w \text{ is positive} \\ \frac{-\phi_0}{w} + n_d, & w \text{ is negative} \end{cases}$

for $w_0 > 0$

$y[n] = s[n-n_d]\cos(w_0 n - \phi_0 - w_0 n_d)$

$= s[n-n_d]\cos\left(w_0\left(n - \frac{\phi_0}{w} - n_d\right)\right)$

$= s[n-\tau_{gr}(w_0)]\cos(w_0(n-\tau_{ph}(w_0)))$

(d) The effect is the same as the following steps:

Reconstruct the continuous-time signal with sampling rate f_s.

Then delay the envelope by $\frac{\tau_{gr}}{f_s}$, delay the carrier by $\frac{\tau_{ph}}{f_s}$.

Then sample the signal to discrete-time with sampling rate f_s.

$$\underbrace{s[n]}_{\text{envelope}}\underbrace{\cos(\omega_0 n)}_{\text{carrier}}$$