EE5630 Digital Signal Processing

HW#2

Lecturer: Yi-Wen Liu

Due Sunday Mar 15, 2020

Q1. (20%) Briefly answer the following questions.

a) At 1:35 of Oppenheim's Lecture 5, he said, "z-transform is the discrete-time counterpart of the Laplace transform for continuous-time systems". Explain briefly what this means.

b) In this lecture, Oppenheim also mentioned that Fourier transform does not exist for all sequences. Except for those sequences that can be expressed in the form $x[n] = r^n u[n]$ where $|r| \geq 1$, and the linear combinations of them, can you think of a causal sequence of a different form that also does not have a discrete-time Fourier transform?

(80%) Please work on Problems 3.21, 3.24, 4.4, and 4.6 of the textbook.

Q2 (Problem 3.21)

Consider an LTI system with impulse response

$$h[n] = \begin{cases} a^n, n \geq 0, \\ 0, \;\; n < 0. \end{cases}$$

and input

$$x[n] = \begin{cases} 1, & 0 \leq n \leq (N-1), \\ 0, & \text{otherwise}. \end{cases}$$

(a) Determine the output y[n] by explicitly evaluating the discrete convolution of x[n] and h[n].

(b) Determine the output y[n] by computing the inverse z-transform of the product of the z-transforms of x[n] and h[n].

Q3 (Problem 3.24)

Sketch each of the following sequences and determine their z-transforms, including the ROC:

(a) $\sum_{k=-\infty}^{\infty} \delta[n-4k]$

(b) $\frac{1}{2}\left[e^{j\pi n} + \cos\left(\frac{\pi}{2}n\right) + \sin(\frac{\pi}{2} + 2\pi n)\right] \mathrm{u}[n]$

Q4 (Problem 4.4)

The continuous-time signal

$$x_c(t) = \sin(20\pi t) + \cos(40\pi t)$$

Is sampled with a sampling period *T* to obtain the discrete-time signal

$$x[n] = \sin\left(\frac{\pi n}{5}\right) + \cos(\frac{2\pi n}{5})$$

(a) Determine a choice for *T* consistent with this information.

(b) Is your choice for *T* in part (a) unique? If so, explain why. If not, specify another choice of *T* consistent with the information given.

Q5 (Problem 4.6)

Let $h_c(t)$ denote the impulse response of an LTI continuous-time filter and $h_d[n]$ the impulse response of an LTI discrete-time filter.

(a) If

$$h_c(t) = \begin{cases} e^{-at}, t \geq 0, \\ 0, \quad t < 0, \end{cases}$$

Where *a* is a positive real constant, determine the continuous-time filter frequency response and sketch its magnitude.

(b) If $h_d[n] = Th_c(nT)$ with $h_c(t)$ as in part (a), determine the discrete-time filter frequency response and sketch its magnitude.

(c) For a given value of *a*, determine, as a function of *T*, the minimum magnitude of the discrete-time filter frequency response.

Below are a few remarks that Yi-Wen would like to share with you.

- In this course I heavily focus on right-sided sequences. So please feel free to ignore left-side or double-sided signals such as $a^n u[-n-1]$. The reason is because all causal systems have, by definition, right-side impulse responses, and all physically possible systems are causal in reality.
- Near 32:00 of Lecture 5 professor Oppenheim had $|z| < \infty$ written on the board. I personally prefer not to use the symbol of ∞ as if it is a number. In my opinion, the statement $|z| < \infty$ gives no information because all real numbers are less than infinity.

Notice:

1. Each question should be in different file (.jpg .jpeg .png), and you should name those files Q1, Q2, … Q5.
2. Archive all the files into a zip file. (There would be 5 files inside the zip file.)
3. Name the zip file as “HW2_StudentID.zip”. (such as HW2_108061xxx.zip)
4. **We will close the submission system after due time.**