DSP HW3 solution

Q1

(a) Partial fraction, if $Q(x) = (x-a)^2(x-b)$, then $\frac{P(x)}{Q(x)} \Rightarrow \frac{C_1}{x-a} + \frac{C_2}{(x-a)^2} + \frac{C_3}{x-b}$

(b) $|P| < \frac{1}{2} \rightarrow |P| < 2$

(c) The vector from the pole/zero to a certain point on the unit circle.

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Q2 (3.23)

(a) $H(z) = \frac{(1-\frac{1}{2}z^{-2})}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})} = -4 + \frac{5-3z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})} = -4 + \frac{-2}{1-\frac{1}{2}z^{-1}} + \frac{7}{1-\frac{1}{4}z^{-1}}$

$\overset{Z^{-1}}{\rightleftharpoons} h[n] = -4\delta[n] + (-2)(\frac{1}{2})^n u[n] + 7(\frac{1}{4})^n u[n]$

(b) $H(z) = \frac{1-\frac{1}{2}z^{-2}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})} = \frac{1-\frac{1}{2}z^{-2}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}} = \frac{Y(z)}{X(z)} \Rightarrow Y(z)(1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}) = X(z)(1-\frac{1}{2}z^{-2})$

$\overset{Z^{-1}}{\rightleftharpoons} y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = x[n] - \frac{1}{2}x[n-2]$

Q3 (5.1)

$y[n] = \begin{cases} 1, & 0 \le n \le 10 \\ 0, & \text{otherwise} \end{cases} \overset{F}{\rightleftharpoons} Y(e^{jw}) = \sum_{n=0}^{10} e^{-jwn} = \frac{1-e^{-jw \cdot 11}}{1-e^{-jw}}, \quad -\pi < w \le \pi$

Since the frequency band of $Y(e^{jw})$ is $-\pi < w \le \pi$, the cutoff frequency of LPF $H(e^{jw})$ should be $w_c = \pi$. and then $X(e^{jw}) = Y(e^{jw}) \overset{F^{-1}}{\rightleftharpoons} x[n] = y[n]$

$y[n] = \begin{cases} 1, & 0 \le n \le 10 \\ 0, & \text{otherwise} \end{cases} \overset{\mathcal{F}}{\rightleftharpoons} Y(e^{j\omega}) = \sum_{n=0}^{10} e^{-j\omega n} = \frac{1-e^{-j\omega 11}}{1-e^{-j\omega}}, \quad -\pi < \omega \le \pi$

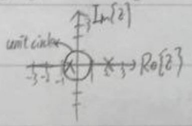
Since the frequency band of $Y(e^{j\omega})$ is $-\pi < \omega \le \pi$, the cutoff frequency of LPF $H(e^{j\omega})$ should be $\omega_c = \pi$, and then $X(e^{j\omega}) = Y(e^{j\omega}) \overset{\mathcal{F}^{-1}}{\rightleftharpoons} x[n] = y[n]$

Q4 (5.8)

(a) $y[n] - \frac{3}{2}y[n-1] - y[n-2] = x[n-1] \overset{Z}{\rightleftharpoons} Y(z)(1-\frac{3}{2}z^{-1}-z^{-2}) = X(z)z^{-1} \Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1-\frac{3}{2}z^{-1}-z^{-2}} = \frac{z^{-1}}{(1-2z^{-1})(1+\frac{1}{2}z^{-1})}$, ROC $|z|>2$

(b) $H(z) = \frac{z^{-1}}{1-\frac{3}{2}z^{-1}-z^{-2}} = \frac{2/5}{1-2z^{-1}} + \frac{-2/5}{1+\frac{1}{2}z^{-1}} \overset{Z^{-1}}{\rightleftharpoons} h[n] = \frac{2}{5}(2^n)u[n] + (-\frac{2}{5})(\frac{-1}{2})^n u[n]$

(a) plot



x: pole, poles at $z = 2, -\frac{1}{2}$

(c) The ROC of a stable system must include $|z|=1$, and since the system $H(z)$ has two poles at $z=2$ and $z=\frac{1}{2}$, so the one possible ROC is $\frac{1}{2}<|z|<2$, then the impulse response would be $h[n] = \frac{-2}{5}(2^n)u[-n-1] + (\frac{-2}{5})(\frac{-1}{2})^n u[n]$

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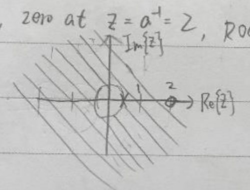
Q5 (5.23)

(a) $H(z) = \frac{Y(z)}{X(z)} = \frac{1-a^{-1}z^{-1}}{1-az^{-1}} \Rightarrow Y(z)(1-az^{-1}) = X(z)(1-a^{-1}z^{-1}) \rightleftharpoons y[n] - ay[n-1] = x[n] - a^{-1}x[n-1]$

(b) The ROC of a stable system must include $|z|=1$, $H(z)$ has a pole at $z=a$, so the ROC of $H(z)$ is $|z|>|a|$ since it is causal, so the range of "a" should be $|a|<1$ for stability.

(c) $a = \frac{1}{2} \rightarrow$ pole at $z=\frac{1}{2}$, zero at $z = a^{-1} = 2$, ROC: $|z|>\frac{1}{2}$



(d) $H(z) = \frac{1-a^{-1}z^{-1}}{1-az^{-1}} \begin{cases} = a^{-2} + \frac{1-a^{-2}}{1-az^{-1}} \overset{Z^{-1}}{\rightleftharpoons} h[n] = a^{-2}\delta[n] + (1-a^{-2})a^n u[n] \\ = \frac{1}{1-az^{-1}} - \frac{a^{-1}z^{-1}}{1-az^{-1}} \overset{Z^{-1}}{\rightleftharpoons} h[n] = a^n u[n] - a^{-1}(a^{n-1})u[n-1] = a^n u[n] - a^{n-2}u[n-1] \end{cases}$

(e) $H(e^{j\omega}) = H(z)|_{z=e^{j\omega}} = \frac{1-a^{-1}e^{-j\omega}}{1-ae^{-j\omega}}$ ($h[n]$ is stable)

$|H(e^{j\omega})| = (|H(e^{j\omega})|^2)^{1/2} = (H(e^{j\omega})H^*(e^{j\omega}))^{1/2} = \left(\frac{1-a^{-1}e^{-j\omega}}{1-ae^{-j\omega}} \times \frac{1-a^{-1}e^{j\omega}}{1-ae^{j\omega}}\right)^{1/2} = \left(\frac{1+a^{-2}-2a^{-1}\cos(\omega)}{1+a^2-2a\cos(\omega)}\right)^{1/2} = \frac{1}{a}$

$\rightarrow$ The magnitude is a constant $\frac{1}{a} \rightarrow$ All-pass system