EE5630 Digital Signal Processing

HW#5

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Due Sunday April 12, 2020

1. (**Ambiguity in down-sampling, 20%**) In the video "Sampling rate conversion", around 37:45, we are on the topic of *decimation* for down-sampling by a factor of two. I said, after lowpass filtering, we are "skipping all the odd-number samples." How about skipping all the even-number samples instead? I.e., would there be a problem if we choose $\tilde{x}_d[n] = \tilde{x}[2n-1]$? If it is OK to do so, how come both $\tilde{x}_d[n] = \tilde{x}[2n-1]$ and $\tilde{x}_d[n] = \tilde{x}[2n]$ are both correct? Note that $\tilde{x}[2n-1] \neq \tilde{x}[2n]$ in general.

Remarks: The word *decimation*, I recently learned, has a cruel meaning in history. Check it out at Wikipedia. *Deci* means ten. The word meant "to remove one out of every ten", originally.

2. **(All-pass minimum-phase decomposition. 40%)**. Please do Problems 5.12, 5.18(a)(b).

5.12. A discrete-time causal LTI system has the system function

$$H(z) = \frac{(1+0.2z^{-1})(1-9z^{-2})}{(1+0.81z^{-2})}.$$

(a) Is the system stable?

(b) Determine expressions for a minimum-phase system $H_1(z)$ and an all-pass system $H_{\text{ap}}(z)$ such that

$$H(z) = H_1(z)H_{\text{ap}}(z).$$

5.18. For each of the following system functions $H_k(z)$, specify a minimum-phase system function $H_{\min}(z)$ such that the frequency-response magnitudes of the two systems are equal, i.e., $|H_k(e^{j\omega})| = |H_{\min}(e^{j\omega})|$.

(a)

$$H_1(z) = \frac{1-2z^{-1}}{1+\frac{1}{3}z^{-1}}$$

(b)

$$H_2(z) = \frac{(1+3z^{-1})\left(1-\frac{1}{2}z^{-1}\right)}{z^{-1}\left(1+\frac{1}{3}z^{-1}\right)}$$

3.(**Meaning of group delay, 40%**) Please work on Problem 5.63.

5.63. In the system shown in Figure P5.63-1, assume that the input can be expressed in the form

$$x[n] = s[n]\cos(\omega_0 n).$$

Assume also that $s[n]$ is lowpass and relatively narrowband; i.e., $S(e^{j\omega}) = 0$ for $|\omega| > \Delta$, with Δ very small and $\Delta \ll \omega_0$, so that $X(e^{j\omega})$ is narrowband around $\omega = \pm\omega_0$.

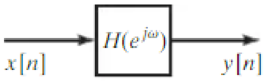

Figure P5.63-1

(a) If $|H(e^{j\omega})| = 1$ and $\angle H(e^{j\omega})$ is as illustrated in Figure P5.63-2, show that $y[n] = s[n]\cos(\omega_0 n - \phi_0)$.

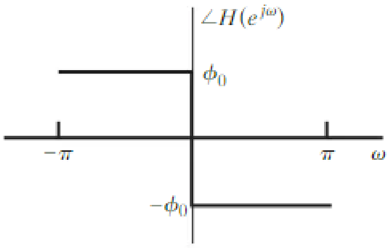

Figure P5.63-2

(b) If $|H(e^{j\omega})| = 1$ and $\angle H(e^{j\omega})$ is as illustrated in Figure P5.63-3, show that $y[n]$ can be expressed in the form

$$y[n] = s[n - n_d]\cos(\omega_0 n - \phi_0 - \omega_0 n_d).$$

Show also that $y[n]$ can be equivalently expressed as

$$y[n] = s[n - n_d]\cos(\omega_0 n - \phi_1),$$

where $-\phi_1$ is the phase of $H(e^{j\omega})$ at $\omega = \omega_0$.

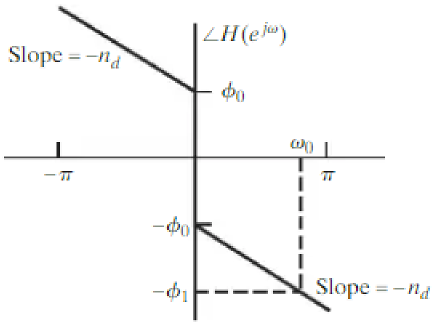

Figure P5.63-3

(c) The group delay associated with $H(e^{j\omega})$ is defined as

$$\tau_{gr}(\omega) = -\frac{d}{d\omega}\arg[H(e^{j\omega})],$$

and the phase delay is defined as $\tau_{ph}(\omega) = -(1/\omega)\angle H(e^{j\omega})$. Assume that $|H(e^{j\omega})|$ is unity over the bandwidth of $x[n]$. Based on your results in parts (a) and (b) and on the assumption that $x[n]$ is narrowband, show that if $\tau_{gr}(\omega_0)$ and $\tau_{ph}(\omega_0)$ are both integers, then

$$y[n] = s[n - \tau_{gr}(\omega_0)]\cos\{\omega_0[n - \tau_{ph}(\omega_0)]\}.$$

This equation shows that, for a narrowband signal $x[n]$, $\angle H(e^{j\omega})$ effectively applies a delay of $\tau_{gr}(\omega_0)$ to the envelope $s[n]$ of $x[n]$ and a delay of $\tau_{ph}(\omega_0)$ to the carrier $\cos\omega_0 n$.

(d) Referring to the discussion in Section 4.5 associated with noninteger delays of a sequence, how would you interpret the effect of group delay and phase delay if $\tau_{gr}(\omega_0)$ or $\tau_{ph}(\omega_0)$ (or both) is not an integer?