EE5630 Digital Signal Processing

HW#3

Lecturer: Yi-Wen Liu

Due Sunday Mar 22, 2020

0. This week's videos (**Lec. 6 and 7**) correspond to textbook Sections **3.3**-**3.4** and **5.2**-**5.3**, so please read along and questions are welcome.

Q1. (20%) Briefly answer the following questions.

a) At about 14:05 of Oppenheim's Lecture 6, he assumed that P(x)/Q(x) has two properties: first, the order of P is smaller than that of Q. Secondly, Q(x) has no multiple roots. What if Q(x) has a multiple root? [Hint: Please check textbook Eq. (3.47).]

b) At about 42:00 in Lecture 6, there was a tiny error on the board. What is it?

c) At about 15:10 of Lecture 7, what do "zero vectors" and "pole vectors" mean?

(80%) Please work on Problems 3.23, 5.1, 5.8, and 5.23 of the text book.

Q2. (3.23)

An LTI system is characterized by the system function

$$H(z) = \frac{\left(1 - \frac{1}{2}z^{-2}\right)}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)}, \qquad |z| > \frac{1}{2}.$$

(a) Determine the impulse response of the system.

(b) Determine the difference equation relating the system input $x[n]$ and the system output $y[n]$.

Q3. (5.1)

In the system shown in Figure P5.1-1, $H(e^{j\omega})$ is an ideal lowpass filter. Determine whether for some choice of input $x[n]$ and cutoff frequency ω_c, the output can be the pulse

$$y[n] = \begin{cases} 1, & 0 \le n \le 10, \\ 0, & \text{otherwise}, \end{cases}$$

shown in Figure P5.1-2.

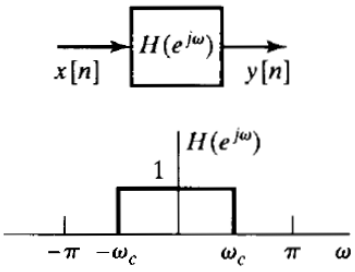

Figure P5.1-1

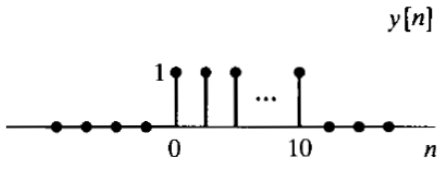

Figure P5.1-2

Q4. (5.8)

A causal linear time-invariant system is described by the difference equation

$$y[n] = \tfrac{3}{2}y[n-1] + y[n-2] + x[n-1].$$

(a) Find the system function $H(z) = Y(z)/X(z)$ for this system. Plot the poles and zeros of $H(z)$, and indicate the region of convergence.

(b) Find the impulse response of the system.

(c) You should have found the system to be unstable. Find a stable (noncausal) impulse response that satisfies the difference equation.

Q5. (5.23)

Consider a causal linear time-invariant system with system function

$$H(z) = \frac{1 - a^{-1}z^{-1}}{1 - az^{-1}},$$

where a is real.

(a) Write the difference equation that relates the input and the output of this system.

(b) For what range of values of a is the system stable?

(c) For $a = \frac{1}{2}$, plot the pole–zero diagram and shade the region of convergence.

(d) Find the impulse response $h[n]$ for the system.

(e) Show that the system is an all-pass system, i.e., that the magnitude of the frequency response is a constant. Also, specify the value of the constant.

Below are a few remarks that Yi-Wen would like to share with you.

- In Lecture 6 Oppenheim spent some time to introduce the residue theorem for calculation of inverse z-transform in terms of a contour integral. From my own teaching experience (I've also taught Functions of a Complex Variable in recent years), the way the residue theorem and series expansion are introduced here might be not so rigorous. In my opinion, rigor cannot be established without introducing the concept of analyticity and Cauchy's integral formula first. Anyway, we won't emphasize on the residue theorem and contour integral in this class.
- Again, we focus on right-sided sequences since we will mostly be interested in analyzing and designing causal systems than can be specified by linear difference equations.

Preview for next week: We will switch to Yi-Wen's lectures to cover a few topics from Textbook Chapter 5.

Notice:

1. Each question should be in different file (.jpg .jpeg .png), and you should name those files Q1, Q2, … Q5.
2. Archive all the files into a zip file. (There would be 5 files inside the zip file.)
3. Name the zip file as “HW3_StudentID.zip”. (such as HW3_108061xxx.zip)
4. **We will close the submission system after due time.**