COM 599200 Statistical Learning
Homework #3

(Due May 28, 2018 at the beginning of class.)

Note: Detailed derivations are required to obtain a full score for each problem. (Total 15% + 80%)

0. (15%) Please turn in the remaining parts of Problem 4 of HW #2.

1. (12%) Let us consider the basis representation for cubic splines with K interior knots, where

$$f(X) = \sum_{j=0}^{3} \beta_j X^j + \sum_{k=1}^{K} \beta_{k+3}(X - \xi_k)_+^3.$$

Prove that the linear boundary conditions for natural cubic splines imply that $\beta_2 = \beta_3 = 0$ and $\sum_{k=1}^{K} \beta_{k+3} = 0$, $\sum_{k=1}^{K} \xi_k \beta_{k+3} = 0$.

2. (8%+6%) Consider 3 data points in the 2-d space: $(-2, -1)$, $(0, 0)$, $(1, 2)$.

(a) What are the first and second principal components (write down the actual vector)?

(b) If we project the original data points into the 1-d subspace by the principal component you choose, what are their coordinates in the 1-d subspace? And what is the variance of the projected data?

3. (8%) Let us consider the generalized additive model

$$y_i = \beta_0 + f_1(x_{i1}) + f_2(x_{i2}) + \epsilon_i,$$

for $i = 1, \ldots, n$. Here, we use the backfitting algorithm to fit a ridge regression model with tuning parameters λ_1 and λ_2 for f_1 and f_2, respectively. Find the fits for f_1 and f_2 in the first iteration of the backfitting algorithm.

4. (4%+4%+2%+2%+2%) Problem 1 of Chapter 7.

5. (8%+6%+6%) Suppose we fit a curve with basis functions $b_1(X) = X$, $b_2(X) = (X-1)^2 I(X \geq 1)$. (Note that $I(X \geq 1)$ equals 1 for $X \geq 1$ and 0 otherwise.) We fit the linear regression model

$$Y = \beta_0 + \beta_1 b_1(X) + \beta_2 b_2(X) + \epsilon.$$

(a) Suppose that we obtain coefficient estimates $\hat{\beta}_0 = 1$, $\hat{\beta}_1 = 1$, $\hat{\beta}_2 = 2$. Sketch the estimated curve between $X = -2$ and $X = 2$. Note the intercepts, slopes, and other relevant information.

(b) By fitting the model to the 3 data points $(X, Y) = (-1, -1), (1, 2), (2, 8)$, find the resulting coefficients $\hat{\beta}_1$ and $\hat{\beta}_2$.

(c) Using the data points in (b), sketch the result of local linear regression with weight K_{i0} equal to constant K for the 2 closest training points and 0, otherwise.

6. (6%+6%) Problem 4 of Chapter 8 in the textbook.