EE 3070 Statistics

Homework #1

Due at 23:59, March 31, 2023
online submission to eeclass systems

Note: There are **15** questions, each worth **7** points, for a total of 105 points. The maximum score, however, will be capped at 100 points.

You may use computers, software packages, and online tools for this Homework.

1. At the beginning of a study of individuals, 15% were classified as heavy smokers, 30% were classified as light smokers, and 55% were classified as nonsmokers. In the five-year study, it was determined that the death rates of the heavy and light smokers were five and three times that of the nonsmokers, respectively. A randomly selected participant died over the five-year period: calculate the probability that the participant was a nonsmoker.

2. Let the space of the random variable X be $\mathcal{D} = \{x : 0 < x < 1\}$. If $D_1 = \{x : 0 < x < \frac{1}{2}\}$ and $D_2 = \{x : \frac{1}{2} \leq x < 1\}$, find $P_X(D_2)$ if $P_X(D_1) = \frac{1}{4}$.

3. Let X have the pmf $p(x) = \frac{1}{3}$, $x = -1, 0, 1$. Find the pmf of $Y = X^2$.

4. Let X have the pdf $f(x) = \frac{1}{9}x^2$, $0 < x < 3$, zero elsewhere. Find the pdf of $Y = X^3$.

5. Let X have the pdf $f(x) = 3x^2$, $0 < x < 1$, zero elsewhere.
   (a) Compute $E(X^3)$.
   (b) Show that $Y = X^3$ has a uniform $(0, 1)$ distribution.
   (c) Compute $E(Y)$ and compare this result with the answer obtained in part.

6. Suppose X_1 and X_2 have the joint pdf $f_{X_1,X_2}(x_1, x_2) = e^{-(x_1+x_2)}$, $0 < x_i < \infty, i = 1, 2$, zero elsewhere.
   (a) Use formula (2.2.2) to find the pdf of $Y_1 = X_1 + X_2$.
   (b) Find the mgf of Y_1.
   Note. formula (2.2.2) $f_{Y_1}(y_1) = \int_{-\infty}^{\infty} f_{X_1,X_2}(y_1 - y_2, y_2)dy_2$

7. Let the joint pdf of X and Y be given by

$$f(x,y) = \begin{cases} \dfrac{2}{(1+x+y)^3} & 0 < x < \infty, 0 < y < \infty \\ 0 & \text{elsewhere} \end{cases}$$

   (a) Compute the marginal pdf of X and the conditional pdf of Y, given $X = x$.
   (b) For a fixed $X = x$, compute $E(1 + x + Y|x)$ and use the result to compute $E(Y|x)$.

8. Let X and Y have the joint pmf $p(x, y) = \frac{1}{7}, (0, 0), (1, 0), (0, 1), (1, 1), (2, 1), (1, 2), (2, 2)$, zero elsewhere. Find the correlation coefficient ρ.

9. Let $f(x_1, x_2, x_3) = e^{-(x_1+x_2+x_3)}, 0 < x_1 < \infty, 0 < x_2 < \infty, 0 < x_3 < \infty$, zero elsewhere, be the joint pdf of X_1, X_2, X_3.
(a) Compute $P(X_1 < X_2 < X_3)$ and $P(X_1 = X_2 < X_3)$.
(b) Determine the joint mgf of X_1, X_2 and X_3. Are these random variables independent?

10. Let X_1, X_2 and X_3 be iid with common pdf $f(x) = e^{-x}, x > 0$, zero elsewhere. Find the joint pdf of $Y_1 = X_1$, $Y_2 = X_1 + X_2$ and $Y_3 = X_1 + X_2 + X_3$.

11. Find the mean and variance of the sum $Y = \sum_{i=1}^{5} X_i$, where $X_1, ..., X_5$ are iid, having pdf $f(x) = 6x(1-x), 0 < x < 1$, zero elsewhere.

12. Let the independent random variables X_1 and X_2 have binomial distribution with parameters $n_1 = 3$, $p = \frac{2}{3}$ and $n_2 = 4$, $p = \frac{1}{2}$, respectively. Compute $P(X_1 = X_2)$.

13. Let X have a Poisson distribution. If $P(X = 1) = P(X = 3)$, find the mode of the distribution.

14. If X is $N(1, 4)$, compute the probability $P(1 < X^2 < 9)$

15. Let the random variable X have a distribution that is $N(\mu, \sigma^2)$.
(a) Does the random variable $Y = X^2$ also have a normal distribution?
(b) Would the random variable $Y = aX + b$, a and b nonzero constants have a normal distribution?
Hint: In each case, first determine $P(Y \le y)$.
