4. **6.5.4.** Let $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_m$ be independent random samples from the two normal distributions $N(0, \theta_1)$ and $N(0, \theta_2)$.

(a) Find the likelihood ratio Λ for testing the composite hypothesis $H_0 : \theta_1 = \theta_2$ against the composite alternative $H_1 : \theta_1 \neq \theta_2$.

(b) This Λ is a function of what F-statistic that would actually be used in this test?

$X \sim N(0, \theta_1) \qquad Y \sim N(0, \theta_2)$

(a)

$$\Lambda = \frac{L(\hat{\omega})}{L(\hat{\Omega})} = \frac{f(X, Y; \theta_1 = \theta_2)}{f(X, Y; \theta_1, \theta_2)}$$

ω：假設空間

Ω：原空間

$$\Lambda = \frac{\left[\dfrac{1}{2\pi\left(\frac{1}{n+m}\left(\sum X_i^2 + \sum Y_j^2\right)\right)}\right]^{\frac{m+n}{2}}}{\left[\dfrac{1}{2\pi\left(\frac{1}{n}\sum X_i^2\right)}\right]^{n/2}\left[\dfrac{1}{(2\pi)\left(\frac{1}{m}\sum Y_j^2\right)}\right]^{m/2}}$$

$$= \frac{n^{\frac{n}{2}} m^{\frac{m}{2}}}{(m+n)^{\frac{m+n}{2}}} \frac{\left(\sum_{i=1}^{n} X_i^2 + \sum_{j=1}^{m} Y_j^2\right)^{\frac{m+n}{2}}}{\left(\sum_{i=1}^{n} X_i^2\right)^{\frac{n}{2}}\left(\sum_{j=1}^{m} Y_j^2\right)^{\frac{m}{2}}}$$

$$= \frac{n^{\frac{n}{2}} m^{\frac{m}{2}}}{(m+n)^{\frac{m+n}{2}}} \frac{\left(\dfrac{\sum_{i=1}^{n} X_i^2}{\sum_{j=1}^{m} Y_j^2} + 1\right)^{\frac{m+n}{2}}}{\left(\dfrac{\sum_{i=1}^{n} X_i^2}{\sum_{j=1}^{m} Y_j^2}\right)^{\frac{n}{2}}}$$

(b)

$$F = \frac{\frac{1}{n}\sum_{i=1}^{n} X_i^2}{\frac{1}{m}\sum_{j=1}^{m} Y_j^2}$$

The relevant F statistic $F = \dfrac{\frac{1}{n}\sum_{i=1}^{n} X_i^2}{\frac{1}{m}\sum_{j=1}^{m} Y_j^2}$ #