EE 3070 Statistics

Final Exam

May 30, 2023
10:10 $\sim$ 12:00

Note: There are **6** problems with total 100 points within **2** pages, please write your answer with detail in the answer sheet.

No credit without detail. Closed books. You may use scientific calculator.

1. (13%) Show that the mean $\bar{X}$ of a random sample of size n from a distribution having pdf $f(x;\theta) = (1/\theta) \cdot e^{-(x/\theta)},\ 0 < x < \infty,\ 0 < \theta < \infty$, zero elsewhere, is an unbiased estimator of θ and has variance θ^2/n.

2. (13%) Let the joint pdf of X and Y be $f(x,y) = \frac{12}{7} \cdot x(x+y), 0 < x < 1, 0 < y < 1$, zero elsewhere. Let $U = \min(X,Y)$ and $V = \max(X,Y)$. Find the joint pdf of U and V.

3. (14%) Let $X_1, X_2, \cdots, X_n$ be a random sample from a $\Gamma(\alpha = 3, \beta = \theta)$ distribution, $0 < \theta < \infty$. Determine the mle of θ.

4. (20%) Let $X_1, X_2, \cdots, X_n$ and $Y_1, Y_2, \cdots, Y_m$ be independent random samples from the two normal distributions $N(0, \theta_1)$ and $N(0, \theta_2)$.
   (a) Find the likelihood ratio Λ for testing the composite hypothesis $H_0 : \theta_1 = \theta_2$ against the composite alternative $H_1 : \theta_1 \neq \theta_2$.
   (b) This Λ is a function of what F-statistic that would actually be used in this test?

5. (20%) Let $Y_1 < Y_2 < Y_3$ be the order statistics of a random sample of size 3 from the uniform distribution having pdf $f(x; \theta) = 1/\theta,\ 0 < x < \theta,\ 0 < \theta < \infty$, zero elsewhere. Show that $4Y_1$ is unbiased estimators of θ.

6. (20%)
