EE 3070 Statistics

Practice #1

No need to turn it in, this is for practice purpose only.
Practice and get familiar with the questions are encouraged.
Some types of the questions might appear on the test.

1. Find the complement $\mathcal{C}^c$ of the set $\mathcal{C}$ with respect to the space $\mathcal{C}$ if
(a) $\mathcal{C} = \{x : 0 < x < 1\}$, $\mathcal{C} = \{x : \frac{5}{8} < x < 1\}$
(b) $\mathcal{C} = \{(x,\ y,\ z) : x^2 + y^2 + z^2 \leq 1\}$, $\mathcal{C} = \{(x,\ y,\ z) : x^2 + y^2 + z^2 = 1\}$
(c) $\mathcal{C} = \{(x,\ y) : |x| + |y| \leq 2\}$, $\mathcal{C} = \{(x,\ y) : x^2 + y^2 < 2\}$

Solution:
(a) $\mathcal{C}^c = \{x : 0 < x \leq \frac{5}{8}\}$
(b) $\mathcal{C}^c = \{(x,\ y,\ z) : x^2 + y^2 + z^2 \leq 1\ \cap\ x^2 + y^2 + z^2 \neq 1\}$
(c) $\mathcal{C}^c = \{(x,\ y) : |x| + |y| \leq 2\ \cap\ x^2 + y^2 \geq 2\}$

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2. A coin is to be tossed as many times as necessary to turn up one head.Thus the elements c of the sample space $\mathcal{C}$ are H, TH, TTH, $TTTH$, and so forth. Let the probability set function P assign to these elements the respective probabilities $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$, and so forth.
(a) Show that $P(\mathcal{C}) = 1$.
(b) Let $C_1 = \{c : c \text{ is } H, TH, TTH, TTTH, \text{ or } TTTTH\}$. Compute $P(C_1)$.
(c) Next, let $C_2 = \{c : c \text{ is } TTTTH \text{ or } TTTTTH\}$. Compute $P(C_2)$, $P(C_1 \cap C_2)$, $P(C_1 \cup C_2)$.

Solution:
(a)

$$\mathcal{C} = \{c : c \text{ is } H, TH, TTH, ..., T\cdots TH\}$$
$$P(\mathcal{C}) = P(H) + P(TH) + \cdots = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots = \frac{\frac{1}{2}}{1 - \frac{1}{2}} = 1$$

(b)

$$P(C_1) = P(H) + P(TH) + \cdots + P(TTTTH) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} = \frac{31}{32}$$

(c)

$$P(C_2) = P(TTTTH) + P(TTTTTH) = \frac{1}{32} + \frac{1}{64} = \frac{3}{64}$$
$$P(C_1 \cap C_2) = P(TTTTH) = \frac{1}{32}$$
$$\begin{aligned} P(C_1 \cup C_2) &= P(H) + P(TH) + \cdots + P(TTTTTH) \\ &= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} = \frac{63}{64} \end{aligned}$$

$\blacksquare$

3. In a certain factory, machines I, II, and III are all producing springs of the same length. Machines I, II, and III produce 1%, 4%, and 2% defective springs, respectively. Of the total production of springs in the factory, Machine I produces 30%, Machine II produces 25%, and Machine III produces 45%.
(a) If one spring is selected at random from the total springs produced in a given day, determine the probability that it is defective.

(b) Given that the selected spring is defective, find the conditional probability that it was produced by Machine II

Solution:
(a)
Let $\mathcal{S}$ be the set of all springs produced in the factory. Machines I, II, and III produce springs such that:
- Machine I produces 30% of all springs in $\mathcal{S}$ and 1% of springs in $\mathcal{S}$ are defective.
- Machine II produces 25% of all springs in $\mathcal{S}$ and 4% of springs in $\mathcal{S}$ are defective.
- Machine III produces 45% of all springs in $\mathcal{S}$ and 2% of springs in $\mathcal{S}$ are defective.

$$\begin{aligned} P(\text{defective}) &= (1\% \text{ of Machine I}) + (4\% \text{ of Machine II}) + (2\% \text{ of Machine III}) \\ &= 0.01 \times 0.3 + 0.04 \times 0.25 + 0.02 \times 0.45 = 0.022 \end{aligned}$$

(b)

$$P(\text{Machine II production} \mid \text{defective}) = \frac{P(\text{Machine II production } \cap \text{ defective})}{P(\text{defective})}$$
$$= \frac{0.25 \times 0.04}{0.3 \times 0.01 + 0.25 \times 0.04 + 0.45 \times 0.02} = \frac{0.01}{0.022} = \frac{5}{11} = 0.\overline{45}$$

$\blacksquare$
