

EE 3070 Statistics

Homework #2

Due at 23:59, May 23, 2023
online submission to eclass systems

Note: There are **8** questions, each worth **13** points, for a total of 104 points. The maximum score, however, will be capped at 100 points.

You may use computers, software packages, and online tools for this Homework.

1. **(Exercise 4.1.3)** Suppose the number of customers X that enter a store between the hours 9:00 a.m. and 10:00 a.m. follows a Poisson distribution with parameter θ . Suppose a random sample of the number of customers that enter the store between 9:00 a.m. and 10:00 a.m. for 10 days results in the values: 9, 7, 9, 15, 10, 13, 11, 7, 2, 12.

Note: You may feel that this question seems familiar. Yup, it's the same one from the midterm.

(a) Determine the maximum likelihood estimate of θ . Show that it is an unbiased estimator.

(b) Based on these data, obtain the realization of your estimator in part **(a)**. Explain the meaning of this estimate in terms of the number of customers.

2. **(Exercise 4.2.1)** Let the observed value of the mean \bar{X} and of the sample variance of a random sample of size 20 from a distribution that is $N(\mu, \sigma^2)$ be 81.2 and 26.5, respectively. Find respectively 90%, 95% and 99% confidence intervals for μ .

Note: how the lengths of the confidence intervals increase as the confidence increases.

3. **(Exercise 4.2.9)** Let \bar{X} denote the mean of a random sample of size n from a distribution that has mean μ and variance $\sigma^2 = 10$. Find n so that the probability is approximately 0.954 that the random interval $(\bar{X} - \frac{1}{2}, \bar{X} + \frac{1}{2})$ includes μ .

4. **(Exercise 4.4.7)** Let $f(x) = \frac{1}{6}, x = 1, 2, 3, 4, 5, 6$, zero elsewhere, be the pmf of a distribution of the discrete type. Show that the pmf of the smallest observation of a random sample of size 5 from this distribution is

$$g_1(y_1) = \left(\frac{7-y_1}{6}\right)^5 - \left(\frac{6-y_1}{6}\right)^5, y_1 = 1, 2, \dots, 6, \text{ zero elsewhere.}$$

Note that in this exercise the random sample is from a distribution of the discrete type. All formulas in the text were derived under the assumption that the random sample is from a distribution of the continuous type and are not applicable. Why?

5. **(Exercise 4.4.20)** Let the joint pdf of X and Y be $f(x, y) = \frac{12}{7} \cdot x(x+y), 0 < x < 1, 0 < y < 1$, zero elsewhere. Let $U = \min(X, Y)$ and $V = \max(X, Y)$. Find the joint pdf of U and V .

6. **(Exercise 4.5.3)** Let X have a pdf of the form $f(x; \theta) = \theta \cdot x^{(\theta-1)}$, $0 < x < 1$, zero elsewhere, where $\theta \in \{\theta : \theta = 1, 2\}$. To test the simple hypothesis $H_0 : \theta = 1$ against the alternative simple hypothesis $H_1 : \theta = 2$, use a random sample X_1, X_2 of size $n = 2$ and define the critical region to be $C = \{(x_1, x_2) : \frac{3}{4} \leq x_1 \cdot x_2\}$. Find the power function of the test.
7. **(Exercise 4.6.2)** Consider the power function $\gamma(\mu)$ and its derivative $\gamma'(\mu)$ given by equations (4.6.5) and (4.6.6), on page 249. Show that $\gamma'(\mu)$ is strictly negative for $\mu < \mu_0$ and strictly positive for $\mu > \mu_0$.
8. **(Exercise 6.1.1)** Let X_1, X_2, \dots, X_n be a random sample from a $\Gamma(\alpha = 3, \beta = \theta)$ distribution, $0 < \theta < \infty$. Determine the mle of θ .
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