EE 3070 Statistics

Practice #3

No need to turn it in. Strongly recommend to practice these questions.

1. **(Exercise 6.2.3)** Given the pdf

$$f(x;\theta) = \frac{1}{\pi \cdot (1 + (x-\theta)^2)}, \quad -\infty < x < \infty, \quad -\infty < \theta < \infty,$$

   show that the Rao–Cramer lower bound is $2/n$, where n is the size of a random sample from this Cauchy distribution. What is the asymptotic distribution of $\sqrt{n}(\hat{\theta} - \theta)$ if $\hat{\theta}$ is the mle of θ.

2. **(Exercise 6.3.8)** Let $X_1, X_2, \cdots, X_n$ be a random sample from a Poisson distribution with mean $\theta > 0$.

(a) Show that the likelihood ratio test of $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$ is based upon the statistic $Y = \sum_{i=1}^{n} X_i$. Obtain the null distribution of Y.

(b) For $\theta_0 = 2$ and $n = 5$, find the significance level of the test that rejects H_0 if $Y \leq 4$ or $Y \geq 17$.

3. **(Exercise 6.4.4)** The *Pareto distribution* is a frequently used model in the study of incomes and has the distribution function

$$F(x; \theta_1, \theta_2) = \begin{cases} 1 - (\theta_1/x)^{\theta_2} & \theta_1 \le x \\ 0 & \text{elsewhere,} \end{cases}$$

where $\theta_1 > 0$ and $\theta_2 > 0$. If $X_1, X_2, \cdots, X_n$ is a random sample from this distribution, find the maximum likelihood estimators of θ_1 and θ_2. (*Hint*: This exercise deals with a non-regular case.)

4. **(Exercise 6.5.1)** In Example 6.5.1 let $n = 10$, and let the experimental value of the random variables yield $\bar{x} = 0.6$ and $\sum_{i=1}^{10}(x_i - \bar{x})^2 = 3.6$. If the test derived in that example is used, do we accept or reject $H_0 = \theta_1 = 0$ at the 5% significance level?

5. **(Exercise 6.5.5)** Let X and Y be two independent random variables with respective pdfs

$$f(x;\theta_i) = \begin{cases} (1/\theta_i) \cdot e^{-(x/\theta_i)} & -\infty < x < \infty, \quad -\infty < \theta_i < \infty \\ 0 & \text{elsewhere,} \end{cases}$$

for $i = 1, 2$. To test $H_0 : \theta_1 = \theta_2$ against $H_1 : \theta_1 \neq \theta_2$, two independent samples of sizes n_1 and n_2, respectively, were taken from these distributions. Find the likelihood ratio Λ and show that Λ can be written as a function of a statistic having an F-distribution, under H_0.

6. **(Exercise 7.1.1)** Show that the mean $\bar{X}$ of a random sample of size n from a distribution having pdf $f(x;\theta) = (1/\theta) \cdot e^{-(x/\theta)},\ 0 < x < \infty,\ 0 < \theta < \infty$, zero elsewhere, is an unbiased estimator of θ and has variance θ^2/n.

7. **(Exercise 7.2.1)** Let $X_1, X_2, \cdots, X_n$ be iid $N(0, \theta)$, $0 < \theta < \infty$. Show that $\sum_{i=1}^n X_i^2$ is a sufficient statistic for θ.

8. **(Exercise 7.2.6)** Let $X_1, X_2, \cdots, X_n$ be a random sample of size n from a beta distribution with parameters $\alpha = \theta$ and $\beta = 5$. Show that the product $X_1, X_2, \cdots, X_n$ is a sufficient statistics for θ.
