

Csc 226 Assignment 1

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1. Comparison sorting like insertion sort has a running time $O(n)$ when on an already-sorted list. But in this case, it's not. Also Bubble sort and selection sort will have $\Omega(n^2)$ running time. Merge sort has to compare every $x_1, x_2; x_3, x_4$ and so on. So it has $\Omega(n \log n)$. Quicksort has $\Omega(n \log n)$ running time. So comparison sorting algorithm still can't be faster than $\Omega(n \log n)$ because there are $n!$ possible input permutations, so the best case we get $\log(n!)$, which is $\Omega(n \log n)$ (Stirling's approximation).

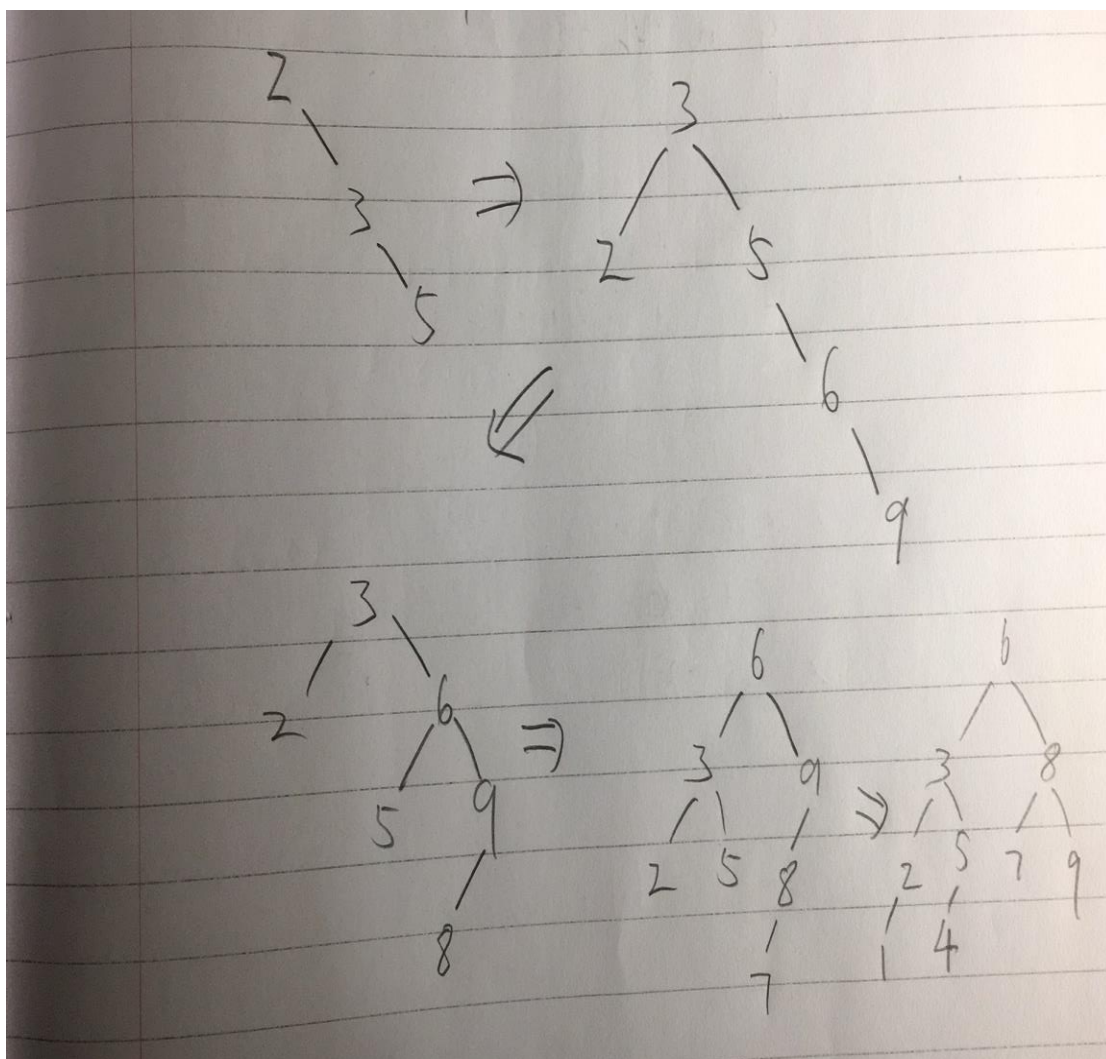
2. If we divide elements into groups of 3 then we will have

$$T(n) = O(n) + T(n/3) + T(2n/3)$$

The height of tree is $\log_3 2n$ and each level has n elements so

$$T(n) = O(n \log n)$$

3.



4.

Consider all nodes at levels $1, 2, 3, \dots, k-2$ have 2 children. (Otherwise, it's not balanced tree) So the tree has at least $2^{(k-1)}$ nodes. In an AVL tree of height h , the leaf closest to the root is at level $(h+1)/2$. On the first $(h-1)/2$ levels, the AVL tree is a complete binary tree thins out after $(h-1)/2$ level

Then $2^{(h-1)/2} \leq \text{number of nodes} \leq 2^h$.

Substitute $h = 2k-1$. Then the height is at most $2k-1$.