UBC, ECONOMICS 627 2019 MIDTERM EXAMINATION

You have 80 minutes. Answer all questions.

1. (30 points) Consider the linear IV model

$$Y = X\beta + U,$$
$$X = Z\Pi + V,$$

where Y is the n-vector of observations on some dependent variable, X is the $n \times k$ matrix of observations on potentially endogenous regressors, and Z is the $n \times l$ matrix of observations on IVs. Suppose that there are no common variables between X and Z, and that the usual exogeneity conditions hold:

$$EZ_iU_i = 0$$
 and $EZ_iV_i' = 0$,

where Z_i' is the *i*-th row of the matrix Z (the *i*-th observation), and U_i and V_i are defined similarly with Z replaced by U and V respectively. Assume also that data are iid. Write

$$Y = X\beta + V\lambda + \epsilon, \text{ where}$$

$$\lambda = (EV_iV_i')^{-1} EV_iU_i,$$

$$\epsilon = U - V\lambda.$$

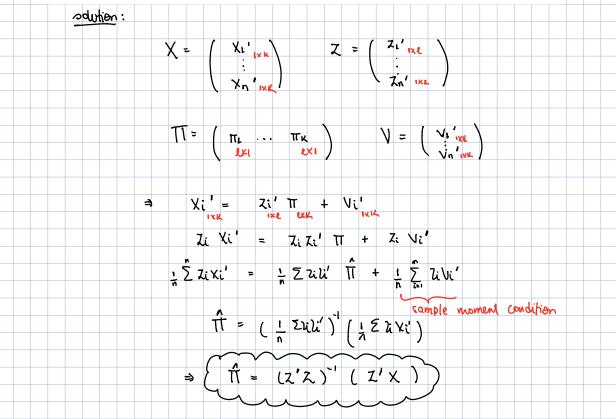
(a) Let \hat{V} denote the residuals from the OLS regression of X against Z. Let $\hat{\lambda}$ denote the estimated coefficient on \hat{V} from the OLS regression of Y against X and \hat{V} :

$$Y = X\hat{\beta} + \hat{V}\hat{\lambda} + \hat{\epsilon},$$

Show that

$$\hat{\lambda} = (V'M_ZM_XM_ZV)^{-1}V'M_ZM_X(V\lambda + \epsilon),$$

where $M_X = I - P_X$, $P_X = X(X'X)^{-1}X'$, and M_Z is defined similarly with X replaced by Z.



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\hat{V} = X - Z \hat{\Pi} = X - Z(Z'Z)^{-1}Z'X = MZX
             \hat{\lambda} = (\hat{V} M_{X} \hat{V})^{-1} \hat{V} M_{X} Y
                  = ( X ' Mz Mx Mz X) - X' Mz Mx Y
                   = (x, Mz Mx Me X) - X, Mz Mx (XB+ V)+E)
                                (*) Mx X = 0 by construction.
                     = (x' Mz Mx Mz x) X Mz Mx (Vx+E)
                       (x) Max = Mz 2 TT + nz V = MzV.
                          ( N' MZ MX MZV) V' MZ MX(VX +E).
   ( X and V should be driven by V inhide of X ! (X= ITT +V)
      (b) Find the probability limit of \hat{\lambda}. Specify additional assumptions you have to make.
      solution:
       λ = (V'MZMXMZV)-1 V'MZMX (Vλ+E) ( Our hope if that www
                                                                        can help or simplify this!
        V' \xrightarrow{Mz \ Mx \ Mz} V = \underbrace{V' (I-Pz)(I-Px)(I-Pz)} V
                               = 1 V'( I - Px -Pz + Pz Px - Pz + Px Pz - Pz Px Pz) V
                                 (x) notice that Z'V \rightarrow EZiVi' = 0 to the terms that involve PZV or (P2V) will be op(1)
                                = L V' ( I - Px ) V + op(()
                             = \frac{\sqrt{\sqrt{N}} - \sqrt{\sqrt{\sqrt{X}} \times \sqrt{X}} + op(1)}{\sqrt{\sqrt{X} \times \sqrt{X}} + op(1)}
 \frac{1}{\sqrt{N}} = \sqrt{\frac{N}{1}} + \sqrt{\frac{N}{1}} 
                                = \frac{\sqrt{1}\sqrt{1-\sqrt{1}\sqrt{1-1}}}{\sqrt{1-1}\sqrt{1-1}} \frac{\sqrt{1}\sqrt{1-1-1}}{\sqrt{1-1-1}} + o_p(1)
     EliVil=0.
                                 = EV(\);' - EV(\);' (EX(X(')-'EV(\)' + op(1)
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