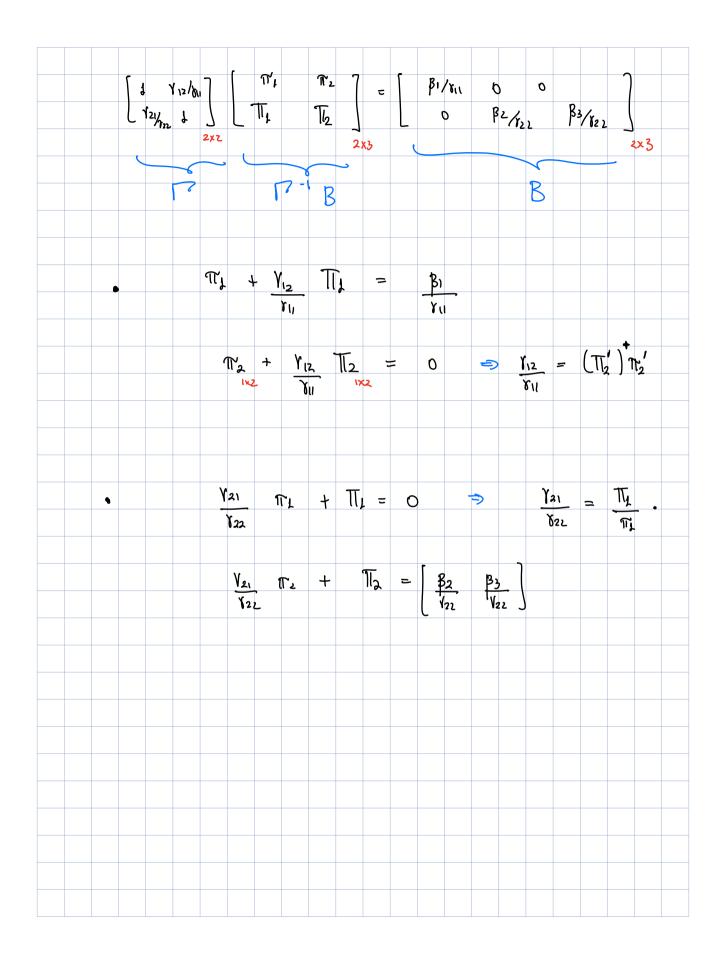
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Question 2

2011 nuidterm

Consider the following structural equation:

$$y_i = Y_i'\gamma + u_i, (8)$$

where y_i is the "dependent" endogenous variable, Y_i is the $m \times 1$ vector of endogenous regressors, and γ is the $m \times 1$ vector of unknown structural parameters. Let Z_i denote the $l \times 1$ vector of excluded exogenous variables (there are no included exogenous regressors). The reduced form equations are

$$y_i = \pi Z_i + v_i,$$

$$Y_i = \Pi Z_i + V_i,$$
(9)

where π is the $1 \times l$ vector of reduced form coefficients, and Π is the $m \times l$ matrix of the reduced form coefficients. Assume that m < l. Assume also that the structural equation is identified and therefore

$$\gamma = \left(\Pi'\right)^{+} \pi',\tag{10}$$

where A^+ denotes the Moore-Penrose (MP) inverse of a matrix A.

(a) (10 points) Derive the asymptotic distribution of the MP estimator of γ :

$$\hat{\gamma}^+ = \left(\hat{\Pi}'\right)^+ \hat{\pi}',\tag{11}$$

where $\hat{\pi}$ and $\hat{\Pi}$ denote the OLS estimators of π and Π respectively. Justify any additional assumptions you have to make.

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(b) (10 points) Is $\hat{\gamma}^+$ an efficient estimator under heteroskedastic or homoskedastic errors? Explain.

1. Consider the following system of equations: Midtern 2017

$$y_{1i} = X'_{1i}\delta_1 + u_{1i},$$

 $y_{2i} = X'_{2i}\delta_2 + u_{2i},$

where X_{1i} and X_{2i} are the k_1 - and k_2 -vectors of endogenous regressors respectively, and $\delta_j \in \mathbb{R}^{k_j}$, j = 1, 2. Let Z_i be the l-vector of instruments:

$$EZ_i u_{ji} = 0 \text{ for } j = 1, 2.$$

Assume that $l \geq k_j$, j = 1, 2, and that data are iid. Consider the following GMM estimator of $(\delta'_1, \delta'_2)'$,

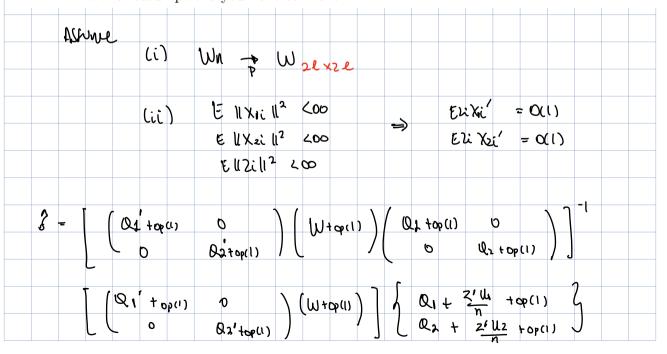
$$\begin{pmatrix}
\widehat{\delta}_{1}(W_{n}) \\
\widehat{\delta}_{2}(W_{n})
\end{pmatrix} = \begin{bmatrix}
\begin{pmatrix}
\sum_{i=1}^{n} X_{1i} Z'_{i} & 0 \\
0 & \sum_{i=1}^{n} X_{2i} Z'_{i}
\end{pmatrix}
\begin{pmatrix}
W_{n,11} & W_{n,12} \\
W_{n,21} & W_{n,22}
\end{pmatrix}
\begin{pmatrix}
\sum_{i=1}^{n} Z_{i} X'_{1i} & 0 \\
0 & \sum_{i=1}^{n} Z_{i} X'_{2i}
\end{pmatrix}
\end{bmatrix}^{-1} \times \begin{pmatrix}
\sum_{i=1}^{n} X_{1i} Z'_{i} & 0 \\
0 & \sum_{i=1}^{n} X_{2i} Z'_{i}
\end{pmatrix}
\begin{pmatrix}
W_{n,11} & W_{n,12} \\
W_{n,21} & W_{n,22}
\end{pmatrix}
\begin{pmatrix}
\sum_{i=1}^{n} Z_{i} y_{1i} \\
\sum_{i=1}^{n} Z_{i} y_{2i}
\end{pmatrix},$$

where

$$W_n = \left(\begin{array}{cc} W_{n,11} & W_{n,12} \\ W_{n,21} & W_{n,22} \end{array} \right)$$

is some positive definite symmetric matrix (possibly random and data-dependent).

(a) (10 points) Show consistency of $\hat{\delta}_1(W_n)$ and $\hat{\delta}_2(W_n)$. Carefully justify all additional assumptions you have to make.



$$= \begin{cases} f + \left(Q_{1}' | W_{11} | Q_{1} | W_{12} | Q_{2} \right) \\ Q_{2}' | W_{21} | Q_{1} | Q_{2}' | W_{22} | Q_{22} \end{cases} \begin{pmatrix} Q_{1}' | V \\ Q_{2}' | W_{21} | Q_{1} | Q_{2}' | W_{22} | Q_{22} \end{pmatrix} \begin{pmatrix} Q_{1}' | V \\ Q_{2}' | W_{21} | Q_{1} | Q_{2}' | W_{22} | Q_{22} \end{pmatrix} \begin{pmatrix} Q_{1}' | V \\ Q_{2}' | W_{21} | Q_{1} | Q_{2}' | W_{22} | Q_{22} \end{pmatrix} \begin{pmatrix} Q_{1}' | V \\ Q_{2}' | W_{21} | Q_{1} | Q_{2}' | W_{22} | Q_{22} \end{pmatrix} \begin{pmatrix} Q_{1}' | V \\ Q_{2}' | W_{21} | Q_{1} | Q_{2}' | W_{22} | Q_{22} \end{pmatrix} \begin{pmatrix} Q_{1}' | V \\ Q_{2}' | W_{21} | Q_{1} | Q_{2}' | W_{22} | Q_{22} \end{pmatrix} \begin{pmatrix} Q_{1}' | V \\ Q_{2}' | W_{21} | Q_{1} | Q_{2}' | W_{21} | Q_{2}' | Q_{22} \end{pmatrix} \begin{pmatrix} Q_{1}' | V \\ Q_{2}' | W_{21} | Q_{1} | Q_{2}' | W_{22} | Q_{22} \end{pmatrix} \begin{pmatrix} Q_{1}' | V \\ Q_{2}' | W_{21} | Q_{2}' | Q_{2}' | W_{21} | Q_{2}' | Q_{2}'$$

(b) (15 points) Show joint asymptotic normality of $\hat{\delta}_1(W_n)$ and $\hat{\delta}_2(W_n)$. Find their asymptotic variance-covariance matrix. Carefully justify all additional assumptions you have to make.

Change the rate of convergence to get influence function

$$\begin{array}{lll}
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