Crash Course:
Bases de Estadística y Econometría

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Informática par Economistas
Universidad de Piura
Ciclo 2024-II

**Nota basada en material del curso de Jonathan Roth y Peter Hull en Brown University.*

¿Así que quieres saber de machine learning? Primero las bases

¿Qué es Econometría?

$\rightarrow$ Es la caja de herramientas estadísticas que usamos los economistas para responder preguntas economicas usando data

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Por lo general, los economistas nos concentramos en las primeras dos, con un énfasis en preguntas causales

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- Worst case scenario: our sample is *not representative* of the population that we care about
  - E.g., workers with certain characteristics were more likely to respond to the survey

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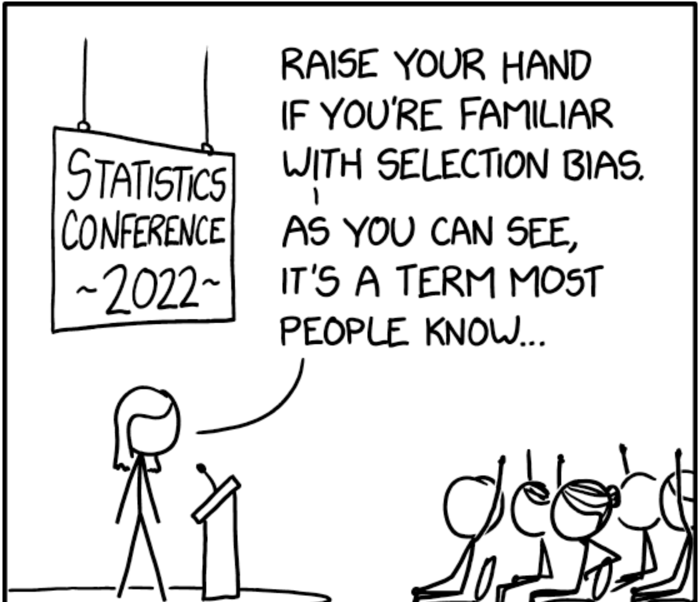
Selection bias referes to settings like Dewey-Truman where the sample is not drawn randomly from the population of interest



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- Counterfactual Qs can't ever be answered with data alone. Need additional assumptions to learn about them!

Splitting up the problem

- When thinking about causal Qs, it's often easier to split the problem in two
- **Identification:** what could we learn about the parameters we care about (causal effects) if we had the observable data for the entire population
  - Need to make assumptions about how observed outcomes relate to outcomes that would have been realized under different treatments
- **Statistics**: what can we learn about the full population that we care about from the finite sample that we have?
  - Need to understand the process by which our data is generated from the full population

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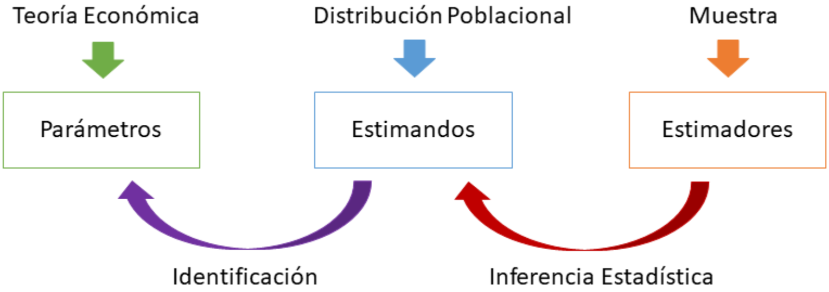
- The process of learning about the *parameter* from the *estimand* is called **identification**.



$\theta = E[X_i]/E[Y_i]$

$E[X_i]$

$E[Y_i]$

$\sum X_i/n$

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- We can write the observed outcome as $Y_i = D_i Y_i(1) + (1 - D_i) Y_i(0)$

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    - Difference in sample mean of earnings for people who went to Brown and people who went to URI:

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- Example estimand:
  - Difference in population mean of earnings for people went to Brown and people who went to URI:

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- Example target parameter:
  - Causal effect of Brown for Brown students:

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Why is causal identification hard?

- Thought experiment: suppose we had data on earnings for *every* Brown and URI graduate
- We can learn from the data:

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- The data doesn't tell us $\underbrace{E[Y_i(0)|D_i = 1]}_{\text{Earnings at URI for Brown Students}}$. Why not?
  - Because we never see Brown students going to URI!

- One idea to solve this problem would be to assume that:

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- Why might this give us the wrong answer?
- Because Brown students may be different from URI students in other ways that would affect their earnings (regardless of where they went to college)
  - Academic ability, family background, career goals, etc.
- These differences are referred to as *omitted variables* or *confounding factors*

What about experiments?

- The gold standard for learning about causal effects is a randomized controlled trial (RCT), aka experiment
- Suppose that the Brown and URI administration randomized who got into which college (assume these are the only 2 colleges for simplicity)
- Since college is randomly assigned, the only thing that differs between Brown and URI students is the college they went to
- Hence,

$$\underbrace{E[Y_i(0)|D_i = 1]}_{\text{Earnings at URI for Brown Students}} = \underbrace{E[Y_i(0)|D_i = 0]}_{\text{Earnings at URI for URI Students}}$$

since we've eliminated any confounding factors

But running experiments is often hard/impossible

- Unfortunately, Brown/URI have not let us randomize who gets into which college
  - At least not yet! If you could convince them to do this, it'd make for a cool senior thesis!
- Likewise, it is difficult to convince states to randomize their minimum wages, or other policies
- In some cases, randomization is not just difficult but would be immoral
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- In this course, we'll discuss tools economists try to use when running experiments is not possible.

Course Roadmap – Where we're going

- **Part I ($\sim$ 7 lectures): Review of probability/statistics**. This will give us a mathematical language to talk about:
  1. *Statistical estimation/inference:* how does the sample we observe relate to the population of interest
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- **Part III ($\sim$ 7 lectures:) Other "quasi-experimental" strategies**: We'll discuss other strategies for "mimicking" an experiment when it's not available, including instrumental variables (IV) and regression discontinuity (RD)

Outline

1. Deriving Multivariate Regression and OLS
2. Regression and Causality
3. Regression Odds and Ends

Moving Beyond One "Regressor"

- So far we've talked about regression as a way of approximating the CEF $E[Y_i|X_i = x] \approx \alpha + x\beta$ for a single scalar X_i
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2. We want a *nonlinear* CEF approx.: e.g. $E[Y_i \mid X_i] \approx \alpha + X_i\beta + X_i^2\gamma$
   - We can "trick" regression into doing this by setting $\mathbf{X_i} = (1, X_i, X_i^2)'$