不確実かつ複雑な市場において適正な材料調達ポートフォリオを実現する  
マルチ期間ロバスト最適化技術

Risk-Aware Multiperiod Optimization for Procurement Portfolio Amid Extreme Complexity and Uncertainty

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**要　旨**

グローバルサプライチェーンにおける複雑性および不確実性の増加により、工場の材料調達は在庫日数、購入コストなどのキャッシュフロー関連KPIの最適化において大きな課題に直面している。我々は、これらの課題に対処するマルチ期間ロバスト最適化技術を開発した。これにより、材料購入先の選択、発注のタイミング、および注文量に関する最適化された調達計画を実現し、過剰在庫および欠品リスクを抑えることが可能となる。本アプローチをシミュレーションデータおよび実際の工場データを用いて評価を行った。従来の手法に比べて大幅な改善を示し、在庫コストと購買コストの合計で 43.3% 以上の削減を達成した。

**Abstract**

Due to the increasing complexity and uncertainties within global supply chains, factory procurement faces challenges in effectively optimizing KPIs such as inventory days, cash flow, and purchase costs. This paper introduces an innovative Multi-Period Robust Optimization technology to address these challenges, providing optimized recommendations on order timing, supplier selection, and order quantities with cost modelling of inventory and demand backlog risk. Utilizing synthetic and real factory data our approach demonstrates significant improvements over traditional methods. Our method achieves 43.3% reduction in combined inventory and purchase cost, thereby enhancing overall procurement decision-making and improving cost efficiency in the face of market uncertainties. We propose a metric to measure the risk and demonstrate that implementing the proposed strategy results in a dramatic reduction in risk. This proves the significant potential of the proposed technology to reduce operational costs while providing measured control over demand backlog risk.

1 Introduction

Procurement of raw materials for production involves key performance indicators such as inventory days, cash flow, and purchase costs. This issue of procurement management is a key to improve the cashflow. The optimization of procurement, encompassing two key decisions: a) determining optimal quantities and suppliers, and b) scheduling procurement periods effectively to minimize the operational cost including the ordering cost, storage cost and risk of demand backlog. A major issue faced by the production planning is the demand is unknown in advance. Even a small inaccuracy in forecasts can lead to very high inefficiencies in cash flow if inventory levels are too high or exposes to significant risks and financial losses if inventory levels are too low [2]. The uncertainty in demand makes procurement optimization inherently difficult, even though it is critical for financial performance.

A diagram of a factory

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Fig:1: Overview of supply chain process

Fig:1 presents the process flow of the supply chain. In this paper we propose a mathematical framework to optimize the procurement of semifinished silver paste whose cost varies as per the price of silver in the commodity market, when the factory is experiencing uncertain demand in a multiperiod setting. The framework considers multiple suppliers and outputs the optimal quantity to be procured across the planning horizon that minimizes the operational cost. In that sense we are optimizing the supplier portfolio. The reasons for choosing silver as the target use case is, silver is expensive, thereby significant effect due to price fluctuation in the commodity market. As per current practices the procurement managers tend to maintain inventory levels through manual gut feel that often leads to inefficient cashflow management and unquantified risk of backlog or demand miss.

We make three major contributions in this research. Firstly, we formulate a mathematical framework to optimize the procurement management in a multi supplier, multi period scenario to minimize the operational cost considering unknown demand with known price and lead time. Secondly, we consider the appropriate notion of risk to provide the optimal recommendation on *how much* to procure *when* (per month) *from which* supplier to strike an optimal tradeoff between overstocking (inefficient cashflow management) and understocking (demand backlog (risk)). Thirdly, to highlight the business value we perform extensive evaluation to quantify the performance of the proposed strategy in terms of cost reduction for a product that uses silver paste as the most expensive component. To have a representative performance we compare the proposed strategy with the existing benchmark with distribution of real demand, silver price from the commodity market etc.

2 Optimization for procurement management

2.1 Optimization with known demand (deterministic case)

Although mathematical optimization is a matured area in terms of academic research but due to the uncertainty in the real world for high impact decisions like procurement management the real implementation is not among the state of the art practices [1]. We start from the ideal case with hypothetical assumptions and eventually move on to the more realistic. The inputs to the problem are the demand for each period *t* in the planning horizon of length , set of suppliers *S*, where each supplier s can be characterized by a fixed ordering cost denoted by, variable ordering cost per unit in time *t* denoted by , capacity in time *t* denoted by and procurement lead time denoted by The problem is to decide the quantity to order from supplier *s* at time *t* that arrive at time *t′*, which consider the lead time *Ls* satisfying *t′* =*t* +*Ls*. The objective of this model is to minimize the total costs, which include fixed and variable ordering costs, as well as inventory and backlog costs, while subject to capacity and quality constraints. The unit inventory and backlog cost are denoted by *h* and *b*, respectively.

A mixed integer linear programming (MILP) is developed for the deterministic model presented below. In addition to the key decision variables , other decision variables are also included as auxiliary variables. Specifically, we introduce a set of binary variables which is equal to 1 if an order is made from supplier s at time *t* and arrives at time *t′* and 0 otherwise. We use to denote the arrive quantity from supplier *s* at time *t*. Finally, are introduced to denote the inventory and backlog quantity at the end of each time *t*.

(1)

where : planning horizon, *S*: set of all suppliers. The constraints are

(2)

(3)

(4)

(5)

(6)

(7)

(8)

(9)

(10)

(11)

The objective function (1) minimizes the total cost, including fixed and variable order cost, inventory holding and backlog cost. The constraint (2) describes the inventory balance for different planning period. Constraint (3), (4), (5) together defines the relationship between ordering time, lead time and the arrival time. Constraint (6) ensure order quantity equals to the arrival quantity. Constraint (7) links the decision of whether make order with the decision of order quantity. Constraint (8) and (9) limits the quality requirement and supplier capacity. (10) and (11) are the integer and non-negativity constraints. The optimization problem presented from (1) to (11) serves as a framework for optimal multiperiod procurement management when all the parameters such as demand, silver price etc. are perfectly known in advance. It is to be noted that this is an ideal scenario and serves only as a starting point for building up more sophisticated formulations. This gives an upper bound of the performance of the optimization framework in this context. In the following section we translate this deterministic optimization problem to a stochastic optimization problem where the demand is not perfectly known in advance.

2.2 Optimization with unknown demand (stochastic case)

Under this setting observing an instance of historical demand we make some distributional assumptions to address the uncertainty to consider the practical scenario of stochasticity. In our case, demand is the most critical parameter and often exhibits high fluctuation over time [2]. To address the realistic scenario of uncertain nature of demand we leverage sample average approximation (SAA) that translate the deterministic optimization problem presented in (1) to (11) to its stochastic equivalent.

SAA is a method used in stochastic optimization to convert problems involving randomness into deterministic ones. In stochastic optimization, the objective function and constraints often contain random variables, making them complex to solve. SAA addresses this by drawing a set of independent and identically distributed (i.i.d.) samples from the random variables’ distribution. These samples are then used to approximate the expectation in the objective function and constraints with their sample averages. For details on SAA we direct the reader to [3].

By substituting the expectation with the sample average, the original stochastic problem is transformed into a deterministic optimization problem. This new problem can be solved using standard deterministic optimization techniques. The solution obtained from this deterministic problem approximates the solution of the original stochastic problem.

To address the stochastic case the there are two modifications over optimization problem proposed in (1) to (11). Firstly, the objective function in (1) gets transformed into:

(13)

(14)

Under the SAA framework equation (2) and (7) get transformed into equation (13) and (14), respectively. As the sample size increases, the approximated objective function and constraints become more accurate representations of their expected values. Consequently, the solution derived from the SAA method converges to the true optimal solution of the stochastic problem with high probability. This method simplifies the problem and makes it more manageable, while it can be computationally expensive if supplying large sample sizes. We call the above formulation as SAA based procurement portfolio optimization (SAA-PPO). Despite this, SAA is widely applicable in fields like supply chain management, financial planning, and energy systems, where it helps in making informed decisions under uncertainty by leveraging the power of random sampling.

2.3 Baseline strategy

We compare the two strategies namely, i) optimization with known demand in (1) to (11), and ii) SAA-PPO in (3) to (13) with a greedy heuristic considered as the baseline strategy for comparison. Under this baseline strategy firstly, select a primary supplier: choose the supplier with the lowest price and establish a long-term contract for regular orders set at 70% of the average historical demand, unchanged throughout the planning period. Next, engage a secondary supplier: for residual demand in each period, order from the next cheapest supplier, up to a limit of 200 units and their capacity constraint. Finally, fulfil remaining demand: if demand exceeds these sources, procure additional supplies from other suppliers follow the same 200 units upper limit and capacity constraint, until all the order is fulfilled. One week’s safety inventory is considered equivalent to the average demand during the average lead time.

2.4 Performance evaluation: Experiments and results

In this section we describe the setting under which the performance evaluation is conducted and discuss the results.

2.4.1 Experimental setup and results

To appreciate the business contribution of this research initiative we conduct the performance evaluation in a realistic setting with real demand experienced in the factory of a business-to-business industrial electronic product. Each timeslot corresponds to a month.

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Fig.2: Total cost incurred by Baseline method (existing)

We consider three suppliers with cost varying as per the silver price in the commodity price of silver that is known in advance. We consider lead time corresponding to each suppliers that is inversely proportional to the ratio of the mean price charged by the respective suppliers. In the experiement we generate 100 instances of demand from 2020 to 2023 following the same mean and variance of the actual historical demand experienced by the factory. There are two important performance attributes, namely, i) average performance and, ii) risk. Average performance corresponds to the notion of cost incurred under the respective strategy in a nominal case where risk is associated with the worst case performance and closely related to the robustness of the objective function with respect to the decision variables. We evaluate the strategies with, i) insample and ii) outsample inputs to effectiveness on similar inputs and test the model's generalizability to new data. Insample evaluation measures the model's performance on the same dataset used to train or develop the optimization model, i.e, measures the objective function in the same instance.

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Fig. 3: Total cost incurred by SAA-PPO

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Fig. 4: Total cost incurred by Detrministic optimization

A diagram of a graph

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Fig. 5: Box plot of distribution of cost for all strategies

More specifically in this, the optimization problem is solved with one of inputs and optimal order is computed. Now this solution is applied on 100 other instances of input data and the objective function is computed. The results obtained thus far makes some assumptions about the cost of storage and backlog. On the other hand, outsample evaluation measures the model's performance on a separate, unseen dataset not used during the model's training phase, i.e measures the objective function with another set of input data.

Table: 1: Summary statistics

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Some more experiments are under progress with the business users. Table 1 given above provides the summary statistics of the evaluations we perform. Fig.2, 3 and 4 shows the cost incurred under baseline, SAA-PPO and deterministic strategies, respectively. From the above, we observe that with baseline in general the cost incurred is much higher, however there is less performance degradation with generalization. On the other extreme with deterministic strategy in general the cost incurred is much lower however the performance degradation with generalization is much worse. However, the outsample performance of deterministic strategy is still better than insample performance of baseline. This highlights the great potential of mathematical optimization-based procurement management. As observed in demand forecast project we observe for some product the forecast error is very high while for some products it is very low [2]. The insample cost is more representative in the low forecast error arena whereas outsample cost is more representative in the high demand forecast error arena.

To standardize the variance around the mean we report the coefficient of variance (CoV), which is defined as:

{"mathml":"<math style=\"font-family:stix;font-size:14px;\" xmlns=\"http://www.w3.org/1998/Math/MathML\"><mstyle mathsize=\"14px\"><mi>C</mi><mi>o</mi><mi>V</mi><mo>=</mo><mfrac><mi>&#x3C3;</mi><mi>&#x3BC;</mi></mfrac><mo>&#xD7;</mo><mn>100</mn></mstyle></math>","origin":"MathType for Microsoft Add-in"} (15)

*CoV* provides a standardized measure of variability regardless of the unit of measurement, making it useful for comparing the relative variability of different distributions. A higher *CoV* indicates greater dispersion around the mean, while a lower *CoV* indicates less dispersion. Fig.(5) provides the distributions of cost under different strategies. The nominal value of cost represents the effectiveness on an average case, whereas the spread represents the risk associated with the respective strategy. Experiments show that compared between SAA-PPO with baseline (both outsample) there is 43.3% decrease in average cost computed across 100 instances of synthetic demand emulating the essential statistical properties of the actual demand. In the context of *CoV* there is a decrease of 79.6% in SAA-PPO over the Baseline strategy for procurement management. This empirically demonstrates the value of leveraging mathematical optimization in reducing the cost and risk associated with the procurement management.

2.5 Technology roadmap

The theme of data driven risk averse multiperiod robust optimization is a new theme in PID company and the results presented in this paper is just middle fruit. With ongoing digital transformation initiatives, we are likely to have transition to AI powered inventory management of which procurement management is an instrumental component. The SAA-PPO model presented in this paper is a starting point for stochastic optimization and we wish to develop advanced robust optimization-based techniques to address this issue. We found some advanced and customized variant of distributionally robust optimization as the probability distributions of the input parameters are not known in advance [4], [5] can be useful for our problem. Additionally, we only observe one realization of the historical parameters that serve as inputs to the optimization problem, so distributional assumptions with parametric family of models may be restrictive.

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Fig. 5: Total cost incurred by detrministic optimization with in-sample and out-sample evaluation

3 Conclusions

This paper presented a mathematical optimization framework for multiperiod procurement optimization with uncertain demand. We performed a real case study with data from a capacitor factory with silver paste and demonstrated that the proposed strategy achieves 43.3% improvement over the existing strategy. We choose to quantify the risk through the coefficient of variance (CoV) and observe the CoV under baseline and SAA-PPO strategy to be 6.3 and 1.3, respectively. This demonstrate that there is a substantial reduction in risk after implementation of SAA-PPO. To accurately calculate cost savings, it is essential to consider the actual storage costs and the risks associated with backlogs. For a thorough evaluation, we analyzed 100 instances of demand samples. We are now exploring a more equitable method of performance evaluation in comparison to the existing strategy, with results to be reported soon. This project is a steppingstone towards the broader theme of supply chain digitalization initiative, and we discuss the technology roadmap that leverages advanced variants of distributionally robust optimization. We hope this will serve as an incredibly valuable strategy for procurement management augmenting in terms of cost reduction with adequate control of operational risk.

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