**Response to the comments and suggestions from the reviewers**

We would like to take this opportunity to express our sincere gratitude to the reviewers for their extremely valuable comments and suggestions to improve the quality of our work.

We have revised the manuscript in accordance with the comments/suggestions. All the changes made in the revised paper are presented in Red. We provide a discussion of modifications with respect to the comments in this letter.

Comments

* This paper proposes a robust optimization method for procurement portfolios. The proposal is based on mixed integer linear programming (MILP) and considers the lead time and the costs associated with each factory. To address the uncertainty of future forecasts, some deterministic variables are replaced with sampled data, which is a common practice in robust optimization. Experimental results show that the proposed method outperforms both the baseline method and the deterministic method.

Reply: Thanks a lot for the comment.

* I have some comments and questions regarding the abstract. In my opinion, the abstract should provide the overview of your proposal to clarify your contributions. The abstract should clearly state the meanings of “Risk-Aware,” “Multiperiod,” and “Uncertainty” as used in the title. “Risk-aware” and “Uncertainty” seem to have the duplicated meanings, both indicating the uncertainty of future forecasts. Does “Risk-aware” in the title refer to robust optimization as used in the abstract?

Reply: In this context risk aware corresponds to optimization under uncertainty.

Multiperiod corresponding to the fact that we consider a discrete time setting where ordering and other decisions are updated in every timeslot. The planning is conducted over a planning horizon through this optimization problem.

Uncertainty refers to uncertain parameters. This results in risk of missing demand that is explicitly addressed through the optimization problem.

To clarify this we modify the abstract as follows:

“Cash flow optimization in terms of inventory management to strike a balance between risk of demand miss and having extra inventory is incrementally challenging because of uncertain demand, price, lead time etc. This paper introduces an innovative stochastic technology to address these challenges, providing optimized recommendations on order timing, supplier selection, and order quantities with cost modelling of inventory and demand backlog risk considering multiple time periods, dynamically.”

* Although the title contains the word of “Complexity,” I cannot identify your contribution to addressing the “Complexity” after reading the main document. “Complexity” seems to refer to the situation where the optimization problem should consider the lead time and the costs associated with each factory. Is it particularly complex for the MILP framework? If so, the main document should clarify the differences of the “Complexity” between your problem setting and a typical MILP problem setting, as well as how your proposal overcomes this complexity.

Reply: In this paper complexity is an attribute of the supply chain. We do not make any contribution to analyze/reduce the algorithmic complexity to solve the MILP problem.

This is clarified with the following modification in the modified draft:

“Even when all the parameters associated with the inventory management is known the planning is challenging. The uncertainties associated with various parameters such as demand makes procurement optimization inherently complex, even though it is critical for financial performance. Additionally, at times the uncertainties can even be very dramatic which further aggravates the issue [2]. In this paper we address the issue of this complexity of the supply chain.”

We have also added the following to reiterate this point in the revised draft:

“In this paper we do not discuss about the complexity of the above optimization problem, the complexity appearing in the title corresponds to the supply chain with uncertain parameters.”

* The abstract should clearly state the meaning of “Extreme.” Because this is a technical report, such an adjective should be mathematically and quantitatively defined. Can you distinguish “EXTREME Complexity and Uncertainty” from merely “Complexity and Uncertainty”? To solve such an “Extreme” problem, what contributions does your research make?

Reply: Our thought behind this is that at times the fluctuation of the input parameters such as demand is dramatic.

Complexity refers to complexity of the physical supply chain.

Uncertainty refers to the fact that some parameters are uncertain.

Extreme refers to the fact that the demand uncertainty is dramatic, that makes the inventory planning challenging. Additionally the presence of lead time in the physical process introduces subtlety in the corresponding optimization problem.

Nevertheless we realize that there are more advanced method especially in the domain of robust optimization to address this scenario of uncertainty with heavy tailed distributions. We are in the journey of developing an advanced method to address all the three aspects and this paper reports early results. Therefore we chose to omit the word *extreme* from the title of the present paper.

* I have one question about your experimental results. What causes the difference in the performance between the in-sample and out-sample experiments of your proposal, as described in Fig. 5? Equations (1) and (12) are optimization methods and seem to have no trainable parameters that can be used in the out-of-sample experiment.

Reply: In our context of stochastic optimization, in-sample and out-sample evaluations still refer to the assessment of the performance of the solution of the optimization problem (can be thought of a model in his case), but in relation to random variables and uncertainties. We address the issue of decision-making under uncertainty, where future conditions or outcomes are uncertain. The evaluation is carried out after realization of the uncertain parameters that were not available at the time of solving the optimization problem.

In-sample evaluation: In-sample evaluation measures the performance in terms of actual cost incurred using the same set of scenarios under which the optimization problem is solved. Basically it is the value of the objective function. As there is no trainable parameter in the optimization problem this is not same as training (typically used in mainstream machine learning), but in some sense it involves similar concept of seen data.

Out-sample evaluation: Out-sample evaluation, on the other hand, measures the performance of the decision variables optimized with in-sample scenario but evaluated on a new, independent set of random scenarios that were not used in solving the optimization problem. Although there is no trainable parameter but this is an estimation of the performance of the method in unseen scenarios.

In the light of the above the following is added in the modified draft:

“We evaluate the strategies with, i) in-sample and ii) out-sample inputs to quantify the effectiveness on similar inputs and test the model's generalizability to new data. In-sample evaluation measures the model's performance on the same dataset used to solve the optimization problem, i.e, measures the objective function in the same instance. More specifically in this, the optimization problem is solved given the inputs, optimal order is computed and the objective function in (1) is evaluated. This value becomes the in-sample performance.

On the other hand, out-sample evaluation measures the model's performance on a separate, unseen dataset not used when the optimization problem is solved. This measures the objective function on a set of input parameter where the solution comes from another set of input parameters. We perform this for 100 such instances of input data and report the average objective value. The name comes from the fact that this input data is totally unseen by the optimization problem where the optimal decision variables are computed. This is an estimation of the generalization capability and robustness.”

The following are small comments regarding the writing.

* The lower left of the first page should include the affiliations of the authors and the email address of the corresponding author. Please refer to the template.

Reply: Thanks for pointing this out. We have amended the draft accordingly.

* In my opinion, Sections 2.4 and 2.5 should be included to a new section, such as “3. Results & Discussion.”

Reply: Thanks for pointing this out. We have amended the draft accordingly.

* The terms “insample” and “outsample” should be replaced with “in-sample” and “out-sample,” respectively.

Reply: Thanks for pointing this out. We have amended the draft accordingly.

* Some sentences are not easy to read due to a lack of commas; for example, the first sentence of Section 2.4.1.

Reply: Thanks for pointing this out. We have made efforts to improve the readability.