Analysis of brain data using graph theory techniques

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AI Lab
Friday 4-5

Friday, August 13, 2010

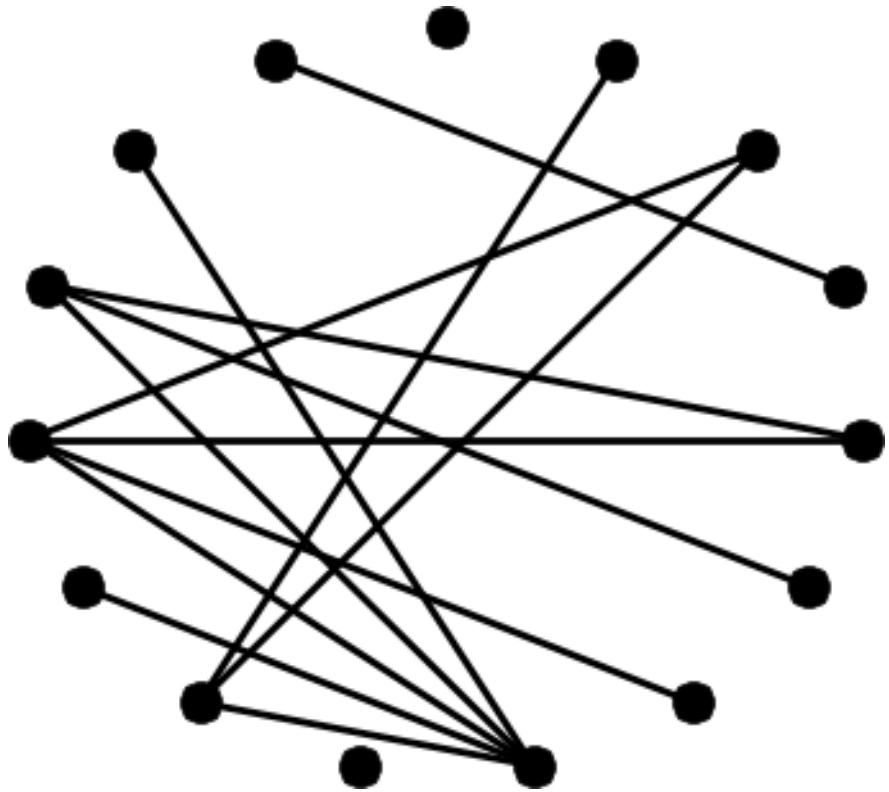
Outline

What is a graph?

"A *graph* G is an ordered pair of disjoint sets (V, E) such that E is a subset of the set $V^{(2)}$ of unordered pairs of V" [1].

What is a graph?

In other words:



Types of graphs

- Weighted/unweighted

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- Directed/undirected

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- Labelled/unlabelled

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- Path length: $L_G = \frac{\sum_{i \neq j \in V} l_{ij}}{|V|(|V|-1)}$
- Clustering coefficient: $C_i = \frac{2|\{(j,k): j \neq k \in N_i, (j,k) \in E\}|}{|N|(|N|-1)}$

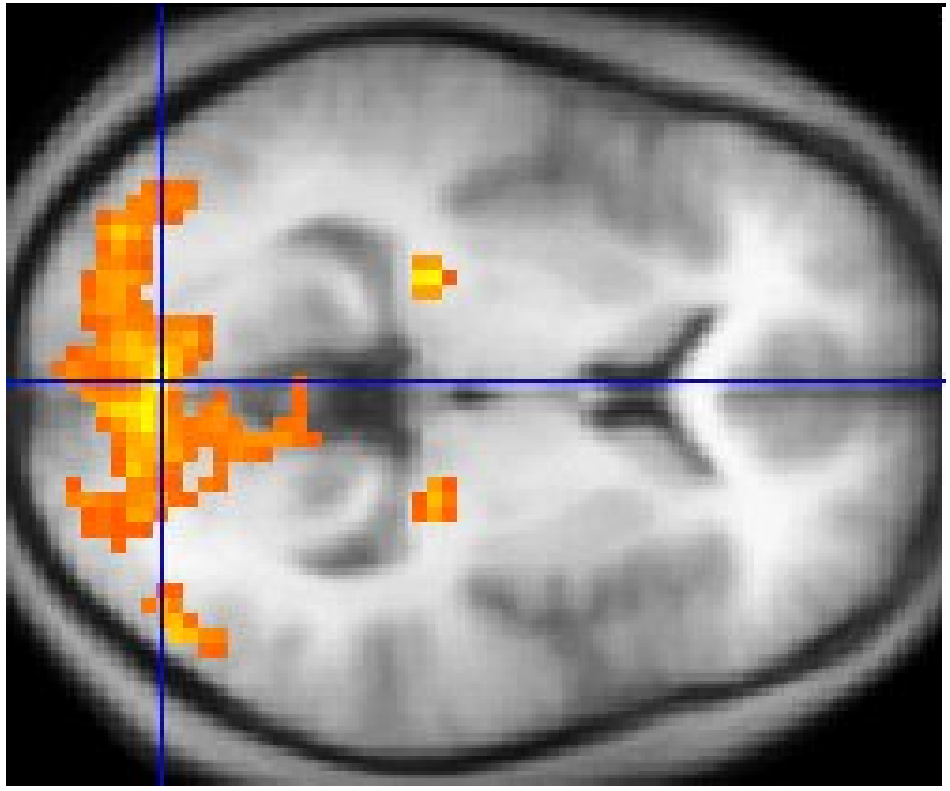
Outline

EEG/MEG


FP1-F3
FP2-F4
F3-C3
F4-C4
C3-P3
C4-P4
P3-O1
P4-O2
EP1-EZ
FP2-F8
F7-T3
F8-T4
T3-T5
T4-T6
T5-O1
T6-O2

[3]

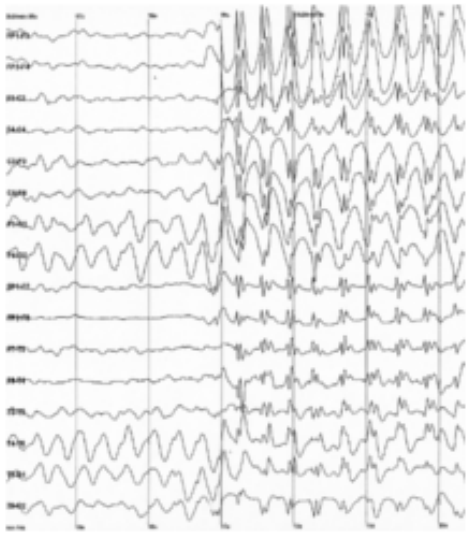
MRI/fMRI



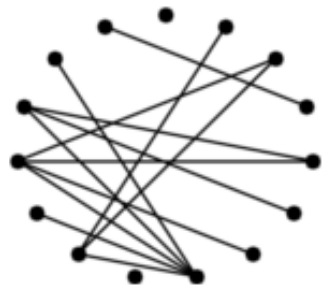
[2]

Outline

The Process



$$\rightarrow \quad \frac{\sum(x_i - \overline{x})\ (y_i - \overline{y})}{\sqrt{\sum(x_i - \overline{x})^2\ \sum(y_i - \overline{y})^2}} \quad \rightarrow$$



Outline

Correlation techinques

- Covariance/Correlation: $\rho(x, y) = \frac{\sum(x_i - \overline{x})\ (y_i - \overline{y})}{\sqrt{\sum(x_i - \overline{x})^2\ \sum(y_i - \overline{y})^2}}$

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- Synchronization Likelihood ([5])
- Wavelet techniques
- Multivariate techniques

Outline

Questions

- Can we find small-world/scale-free (or other) characteristics in graphs generated from brain data?
- Do paricular correlation techniques tend to generate graphs with specific characteristics?
- Can graph theory techniques be used in the diagnosis of neurological disorders?

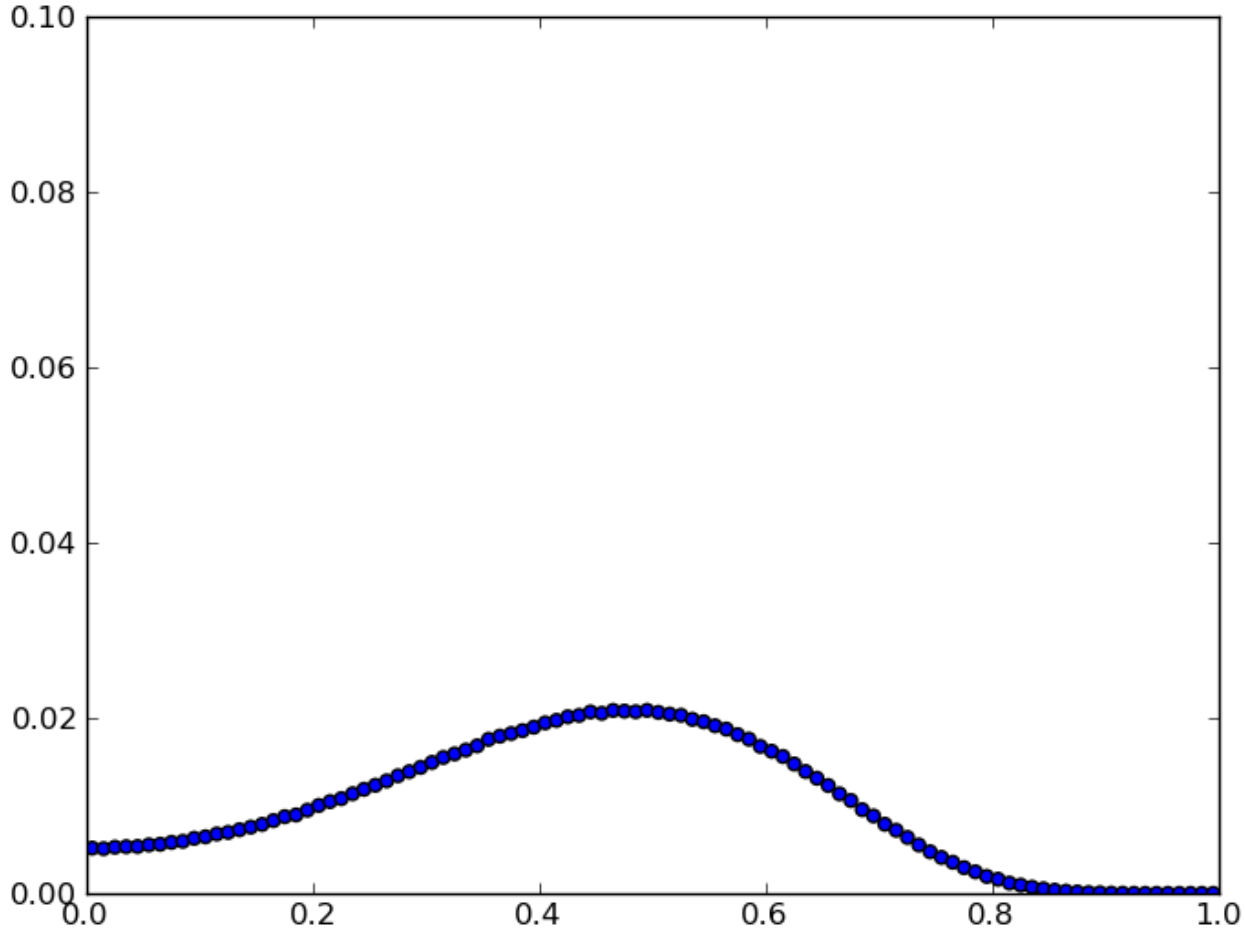
Outline

Correlation on randomly generated data

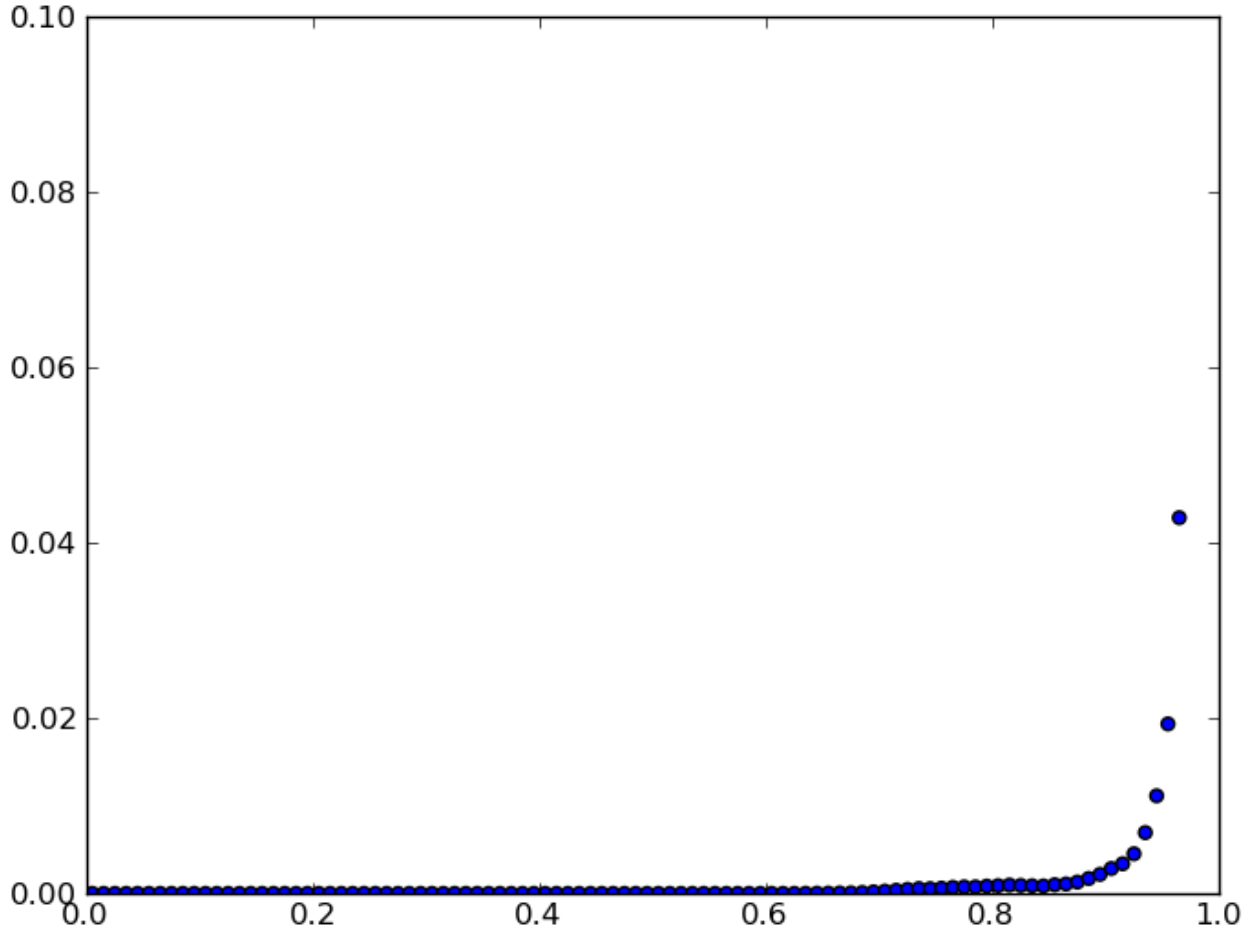
100 sets of 200 time series, each with 20 samples, were randomly generated via Cholesky factorisation, to have a desired correlation in the range $[0.4, 0.6]$.

Three correlation techniques (pearson, coherence, functional distance) were applied to the data sets, and the probability distributions of the generated correlation values were plotted.

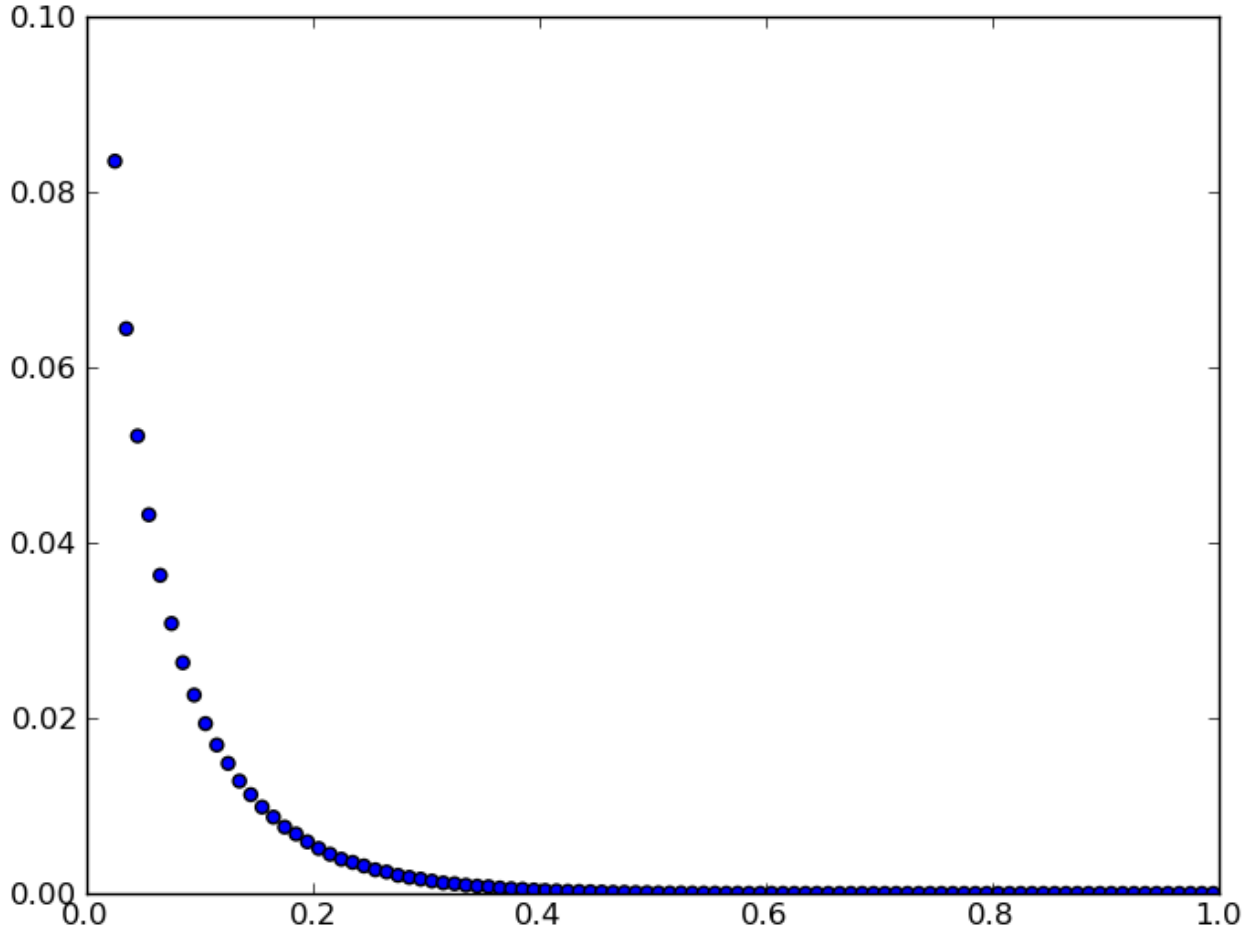
Pearson correlation



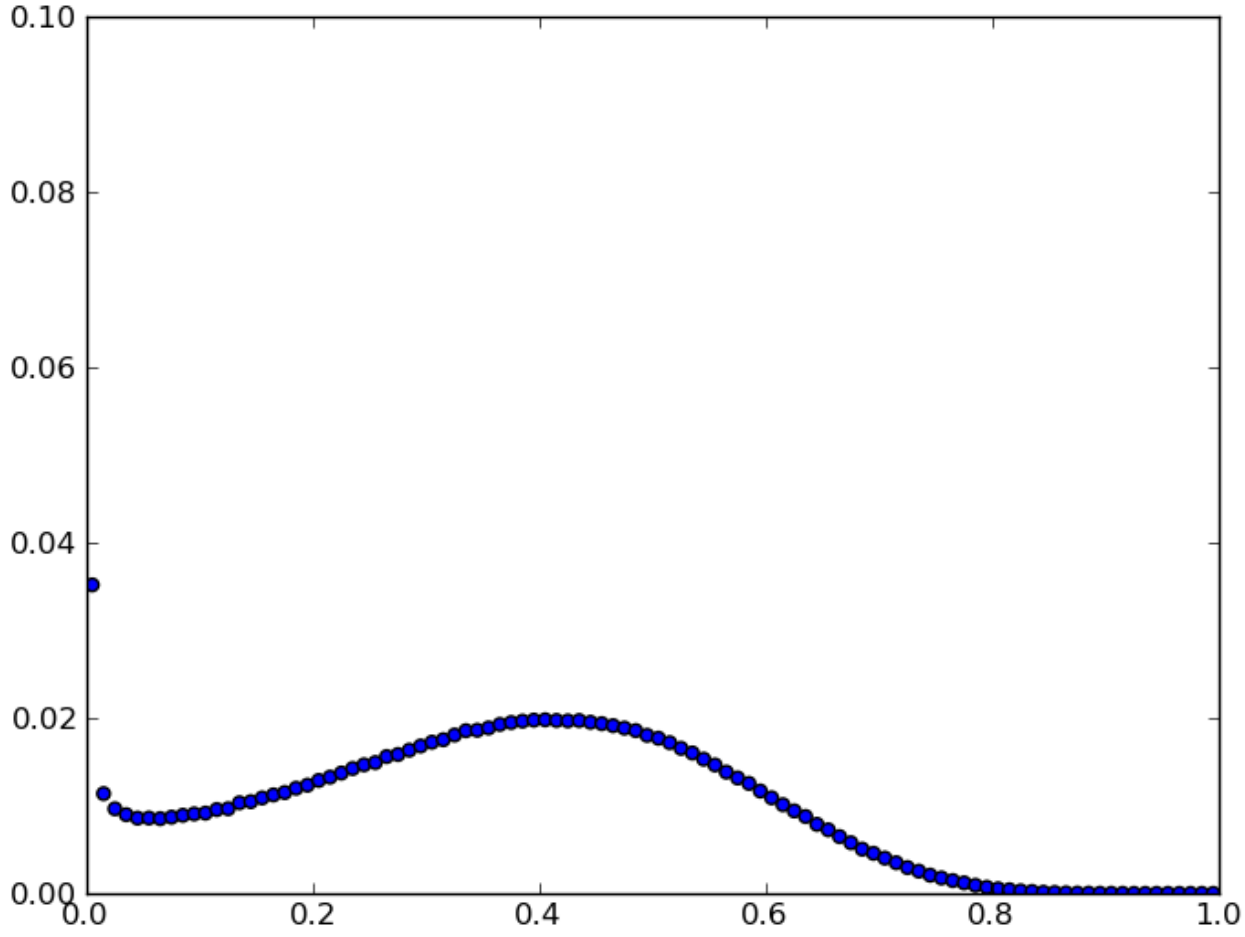
Coherence



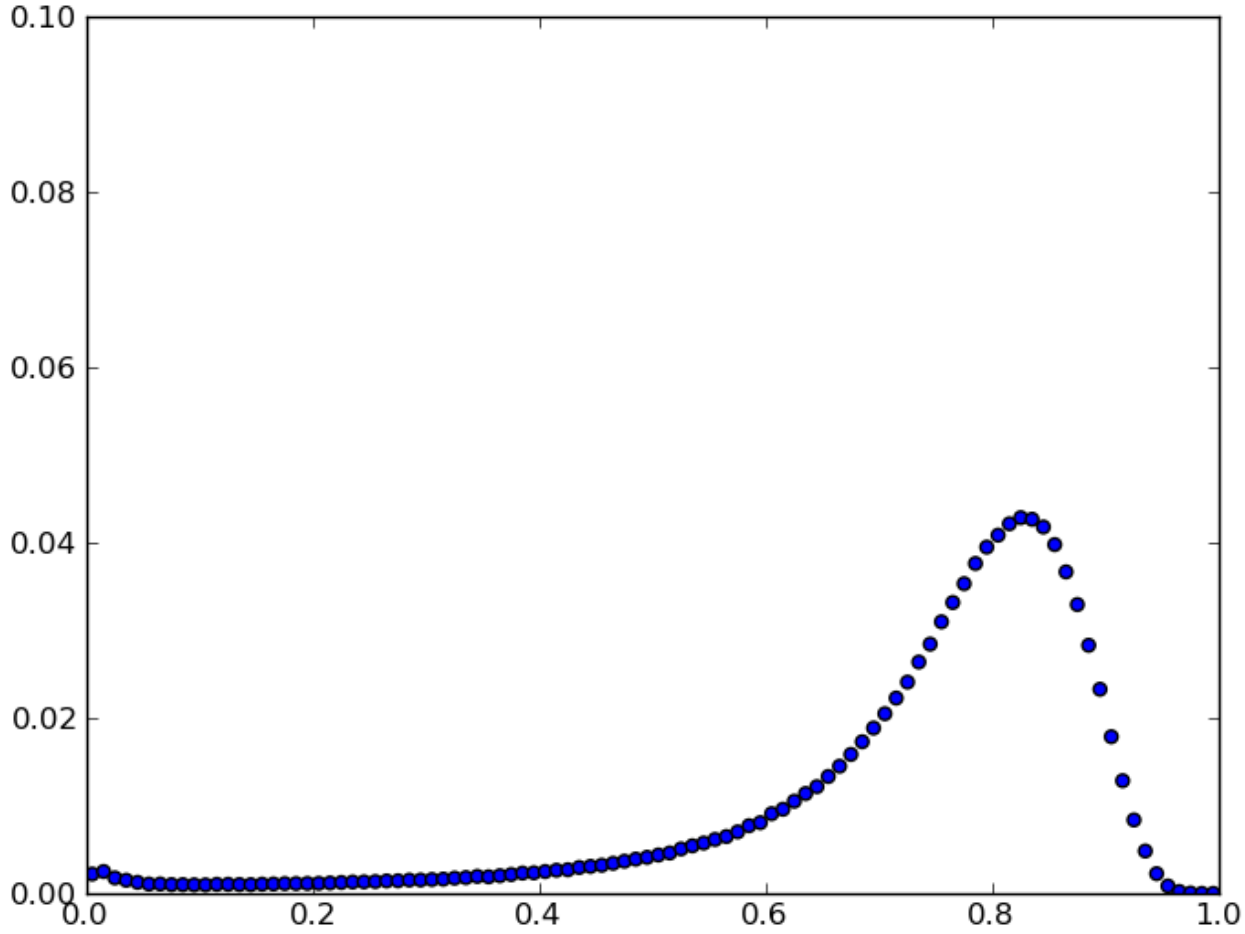
Functional distance ($\sigma = 0.5$)



Functional distance ($\sigma = 1.0$)



Functional distance ($\sigma = 2.0$)



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Thanks!

Questions?

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References I


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