Cauchy-Crofton Formula

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Theorem Statement

$$\iint_S d\rho\, d\theta = 2 * l$$

Explanations

test

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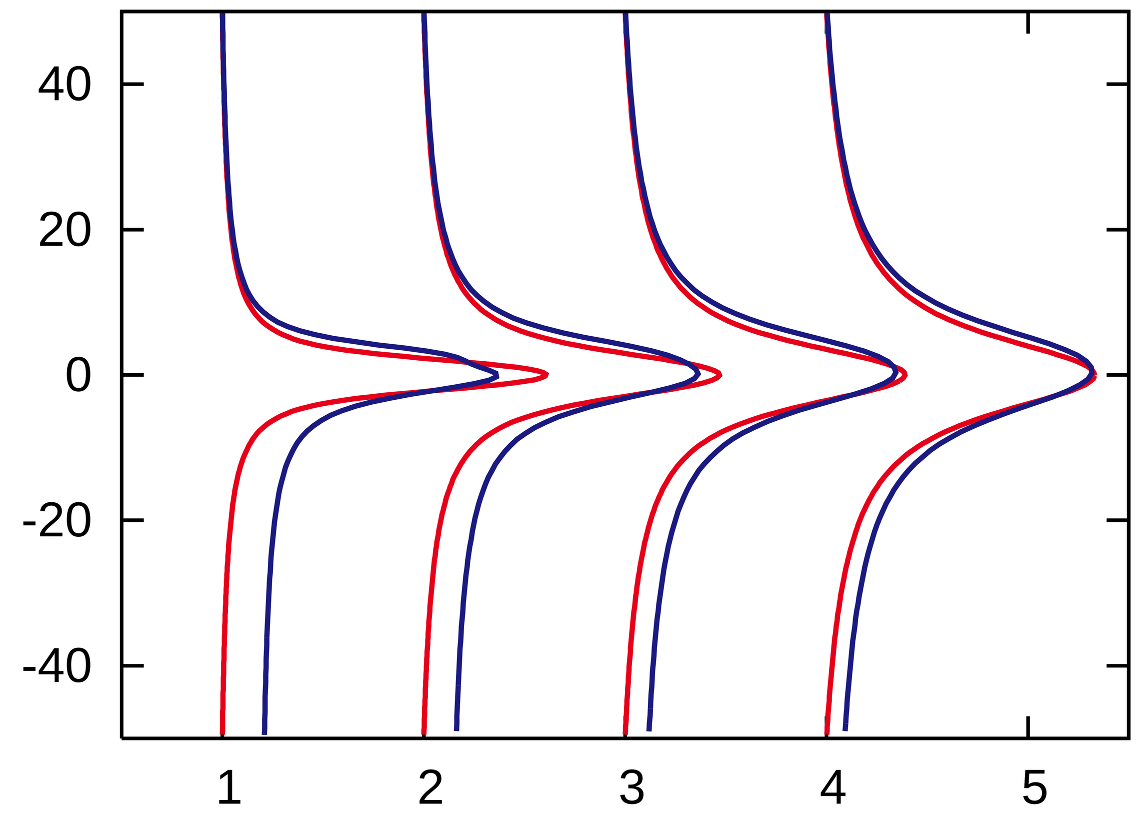
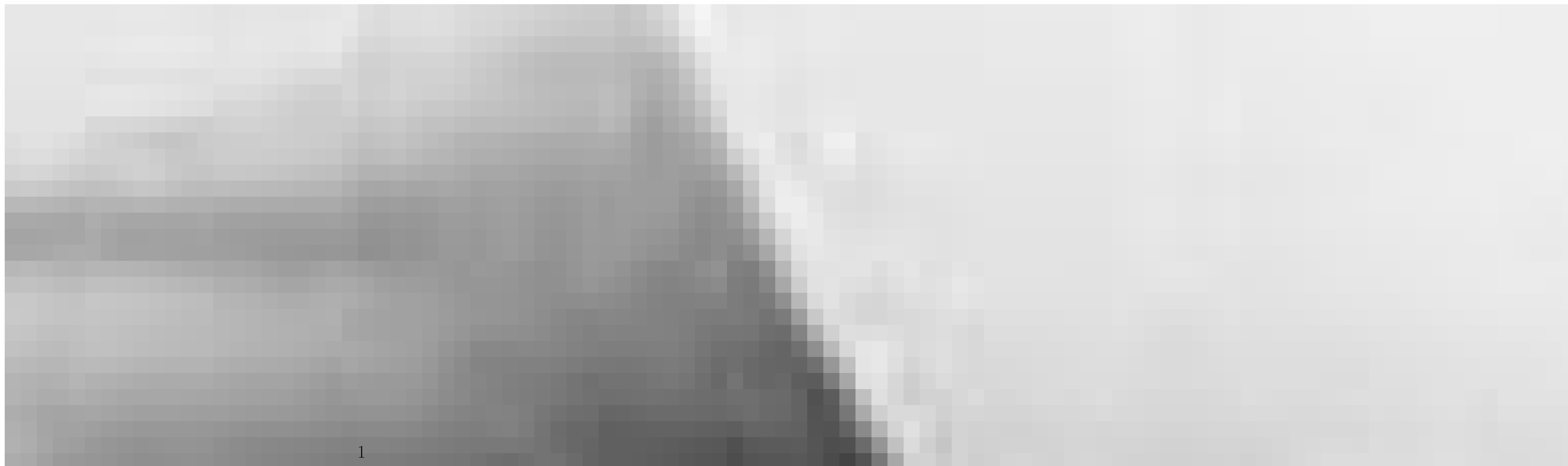
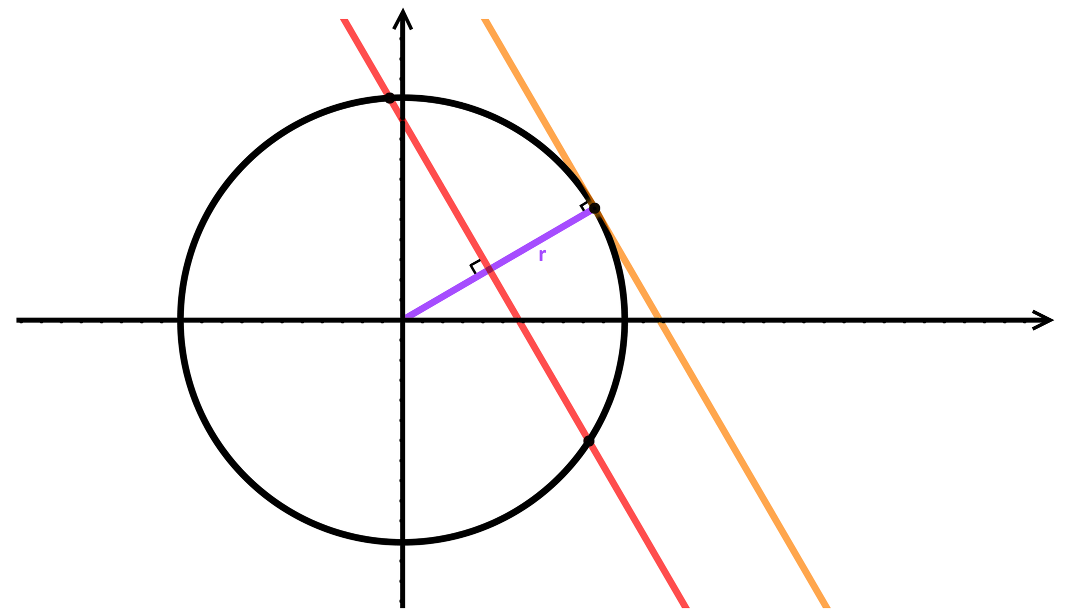




Applications

Length of a circle

Take a circle C of radius r. We need to measure the set of lines that crosses the circle, counted with multiplicity:



Here, we get that $\forall 0 \leq \theta \leq 2\pi$, the line represented by (ρ, θ) is crossing the circle once if $\rho = r$, twice if $\rho < r$ and none if $\rho > r$. So let $f : \mathbb{R} \to \mathbb{R}$ be such that:

$$f(x) = \begin{cases} 2 & if\ x < r \\ 1 & if\ x = r \\ 0 & if\ x > r \end{cases}$$

Thus, the measure of the set of lines crossing C counted with multiplicity is:

$$\begin{aligned} m(C) &= \int_0^{2\pi} \int_0^{+\infty} f(x) dx\, d\theta && (1) \\ &= \int_0^{2\pi} \int_0^{r} 2\ dx\, d\theta && (2) \\ &= \int_0^{2\pi} 2r\ d\theta && (3) \\ &= 4\pi r = 2 * 2\pi r && (4) \end{aligned}$$

And we already know that this is twice the length of the circle of radius r. Therefore, we checked the formula for circles.