Cauchy Crofton Formula

Theorem:

$$\iint_S d\rho, d\theta = 2\,\ell$$

Description of the set S + Example?

Usual Integration

Length of the curve

Rigid motions: def + image

One may prove that the integral above remains unchanged after applying rigid motions

Line case…

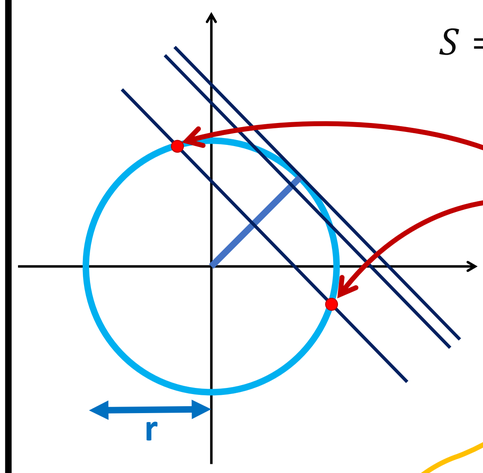
Polygon case…

Regular curve case…

We finally proved the statement for any regular curve

Check for circle:

$$S = \{(\rho, \theta) \mid \rho \leq r \quad \theta \in (0, 2\pi)\}$$



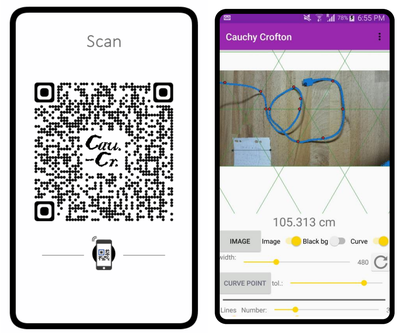
2 intersections, so count twice in the integral

$$\iint_S d\rho, d\theta = \int_0^{2\pi} \int_0^{r} 2\, d\rho d\theta = 2\pi r * 2 = 2 * \ell$$

Finally: $\boldsymbol{\ell = 2\pi r}$

Did you expect this result? ☺

Note: This is not that useful for the case of a circle, but remember that you may use the formula on any curve in a plane, which make it very powerful.
Also, as the formula involves an integral, we can use all the tools we already know about integrals. In particular, it is quite easy to make approximations.



Try this formula in real life !

- Only available on Play Store – (sorry)

This poster summarizes a second-year project undertaken in June 2018 ; By Aadil Khan, Caroline Bouat, Chris Bishop and Paul Dubois