University of Oxford

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Random Fractals: [subtitle]

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Random Fractals

Abstract

[blah]

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0 Introduction

1 Background Theory

1.1 Dimensions

The concept of dimension is quite intuitive from a day-life perceptive. However, the mathematical concept is more involved. From the non-mathematical world, this can be used to have better understanding of biology, as DNA segments are crooked enough be considered as object with dimension greater than one.

1.1.1 Intuition

Some object that we are used to work with have a very commonly admitted dimension:

- **Empty set / Point**: dimension 0
- **Curve** (e.g.: *line*): dimension 1
- **Surface** (e.g.: *square*): dimension 2
- **Volume** (e.g.: *cube*): dimension 3
- N**-dim space** (or N*-dimensional cuboid*): dimension N

All of these usual objects have integral dimensions, making it (relatively) easy to understand.

The rule of thumb to calculate the dimension is to double (or, in general, multiply by n) the size of the object, and count the number of copies of the original object obtained. If there is N original objects, the dimension is $\frac{ln(N)}{ln(n)}$.

Some objects have a more complicated dimension (in fact, a non-integral one):

- **Cantor Set**: dimension $log_3(2) = \frac{ln(2)}{ln(3)}$
- **Koch Snowflake**: dimension $log_3(4) = \frac{ln(4)}{ln(3)}$
- **Sierpiski Triangle**: dimension $log_2(3) = \frac{ln(3)}{ln(2)}$

The dimension is much less intuitive for these objects, and it justifies creating a formal mathematical definition.

After this quick overview, 3 properties seem desirable for a definition of dimension Pol. For a set X ($\subset \mathbb{R}^n$, in general):

1. If X is a manifold, dimension coincide with the natural preconception.
2. In some cases, X may have a fractional (i.e. non-integral) dimension.
3. If X is countable, then X has dimension 0.

There are several definition for dimension, satisfying different properties.

1.1.2 Topological Dimension

Definition

Properties

1.1.3 Box Dimension

Definition

Properties

1.1.4 Hausdorff Dimension

Definition

Properties

1.1.5 Relation Between Dimensions

1.2 Fractals

1.2.1 Definitions

Fractals are mathematical objects that have been studied since the 17th century. Generally having a recursive self-similarity pattern, a fractal is defined as a subset of an Euclidean space with non-integral Hausdorff dimension. exceeding its topological dimension.

1.2.2 Famous Examples

2 Fractal Percolation

This is the main object we intend to study.

2.1 Definition

We will begin with the 2D case, to as it is the most intuitive.

3 Numerics

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Appendix

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A Plots

B Codes

References

Lectures on fractals and dimension theory.