## Modular Forms Modulo 2

# Governing Fields for the Hecke Algebra

### Oral Presentation Script

#### Title Slide

Hello, thank you for “coming”.

I will present my research project titled “Modular Forms Modulo 2: Governing Fields For the Hecke Algebra”.

First, we (quickly) go through reduction of modular forms modulo 2. We will also introduce Hecke operators modulo 2 and look at the structure of the Hecke algebra that they form.

The we will turn our interest to Dirichlet and Chebotarev density theorems. This will lead to Frobenian maps.

Finally, we will link the two theories by studying governing fields (a feature related to Frobenian property) for the maps “aij” (maps linked to the structure of the Hecke Algebra).

#### Modular Forms

Modular forms are analytic function on the upper half plane that satisfy a certain kind of equation with respect to the group action of the modular group.

The picture we usually see is the following *(show)*. Grey region is the fundamental cell, and other regions are “symmetric”.

We denote the complex vector space of modular forms modulo 2 my Mn.

We denote the coefficients of the q-series of a modular form by c(n).

What we well be interested in is modular forms MODULO 2.

#### Reduction Modulo 2

To reduce modulo 2, we need integers, so we will concentrate on modular forms having coefficients of q-series being integers (i.e. this set, *show*). It turns out to be generated by polynomials of the modular discriminant Delta, and Eisenstein series E2 and E3.

Once coefficients are reduced modulo 2, only Delta is not trivial. Thus, the reduction of modular forms are just polynomials of Delta over F2.

#### Modula Forms Modulo 2

Thus, we define F, the space of modular forms modulo 2 to be odd powers of Delta over F2. In this study, we ignore all even powers of Delta.

Note here that modular forms modulo 2 do not have “weights”, we lost this information while reducing mod 2.

Delta is defined as an infinite product. The coefficients that matches this are tau n (the Ramanujan function). Result from Kohlberg lead to Delta being only the sum of odd squares powers of q.

I would like to attract your attention here on the fact that a modular form modulo 2 has two definition: one as a (finite) polynomial of Delta over F2, one as an (infinite) q-series. This “duality” is very specific to MF mod 2, and we will use it.

#### Hecke Operators

One of the interesting things we can do with modular forms is calculate Hecke operators. They are define using the q-series of modular forms as follows *(show)*.

Again, we want to reduce this modulo 2. We also want a formula not too complicated to work with. We remark that this sum has only two summands if “m” is prime. Thus, we define Hecke operators on modular forms modulo 2 as follows:

#### Hecke Operators Modulo 2

Which corresponds to the sum from before. We ignore the case p=2 as T2 isn’t an operator on F.

#### Examples

#### The Hecke Algebra

#### Examples

#### Dirichlet Density Theorem

#### Chebotarev Density Theorem

#### Frobenian Maps

#### The maps aij

#### Known Governing Fields

#### Potential New Governing Fields

#### Governing Groups

#### Diagonal Governing Groups