Problem Set 5

Due 29nd September 2021

/11 -> /10

(BONUS pt for free)

Abstract

Only the questions with a star (*) are compulsory for submission;
It is however *strongly* advised to attempt all question.

1 Sequences

Question 1. *(*) Show that* $S_n = \sum_{k=1}^{n} \frac{1}{k}$ *is such that* $S_{2^n} \geq \frac{n}{2}$.
Is $\sum_{n=1}^{\infty} \frac{1}{n}$ *convergent?*

Question 2. *(Sum of Arithmetic Sequence) a*) Rewrite* $S = \sum_{k=0}^{n} k$ *with* k *running from* n *to* 0 *(instead of from* 0 *to* n*).*
b) Using both expressions for* S*, express* $2S$ *as a product of two terms, and deduce an expression for* S*.*
c) Let* (x_n) *be an arithmetic sequence:* $x_0 = b$, $x_{n+1} = a + x_n$ *(we have shown before that the explicit formula for* x_n *is* $x_n = b + a * n$*).*
Find the sum of the first n *terms:* $\sum_{k=0}^{n} x_k$
d) When is the series* $\sum_{n=0}^{\infty} x_n$ *convergent/divergent?*

Question 3. *(Sum of Geometric Sequence) a*) Multiply* $S = \sum_{k=0}^{n} \alpha^k$ *by* $(1-\alpha)$ *and simplify the expression*
b) Deduce an expression for* S*.*
c) Let* (x_n) *be a geometric sequence:* $x_0 = b$, $x_{n+1} = a * x_n$ *(we have shown before that the explicit formula for* x_n *is* $x_n = b * a^n$*).*
Find the sum of the first n *terms:* $\sum_{k=0}^{n} x_k$
d) When is the series* $\sum_{n=0}^{\infty} x_n$ *convergent/divergent?*
(BONUS) e) When convergent (if ever), what values does the series* $\sum_{n=0}^{\infty} x_n$ *converge to?*

Question 4. *Are these convergent series?*

- $\sum_{n=1}^{+\infty} \frac{(-17)^n}{4^{2n+1}(n+1)}$
- $\sum_{n=1}^{+\infty} \frac{3^{2n+1}}{n^n}$
- $\sum_{n=1}^{+\infty} \frac{n!}{n^n}$
- $\sum_{n=1}^{+\infty} \frac{n^n}{(2n-1)!}$
- $\sum_{n=1}^{+\infty} \left(\frac{5n-3n^3}{0.5n^3+2}\right)^n$
- $\sum_{n=1}^{+\infty} \frac{(-12)^n}{n}$
- $\sum_{n=1}^{+\infty} \frac{(-2)^{n+1} n}{9^n}$
- $\sum_{n=1}^{+\infty} \frac{1}{n^3}$

Fact: $\lim_{n \to +\infty} n^{\frac{1}{n}} = 1$

Question 5. *Calculate the derivative of the following functions:*
$f(x) = \frac{5x^3-2x+1}{2x-7}$ $g(x) = ax^2 + bx + c$

Question 6. *Find the anti-derivative of* x^n.