

Refresher Maths Course

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Abstract

This course teaches basic mathematical methodologies for proofs. It is intended for students with a lack of mathematical background, or with a lack of confidence in mathematics. We will try to cover most of the prerequisites of the courses in the Master's, i.e. basic algebra/analysis and basic application.

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0.0 Introduction

Hello! welcome to this maths refresher course for DSBA 2022! This is the best course ever!

Presentation

- Paul Dubois, PhD Student @ Centrale, end of 1st year
- Email: b00795695@essec.edu (for any question), answer within 1 working day

Course Format

Lectures

- 8*3h arranged as 1h20min lecture - 1/3h break - 1h20min lecture
- No pb class planned, but lectures will have integrated live exercises
- Interrupt if needed (but may also ask at the end of the lecture)

Examination

- Course is pass/fail
- Most (in fact hopefully all) of you will pass
- There will be sets of exercises (about one per lecture), it is advised to attempt it all (only the starred questions will be compulsory)
- As the goal is to learn, you will be able to resubmit exercise sets, but you will lose 10% every-time you re-submit (so that you have some incentive to try your best the 1st time)
- Best $(n-1)/n$ count, need average $\geq 70\%$ to pass
- In the unlikely event of not passing, you will be able to do some extra work to pass

Questions?

0.1 Elementary Maths

0.1.1 Objects & Notations

- set notation - function notations - \mathbb{N} , \mathbb{Z} , \mathbb{Q} , \mathbb{R} - scalars vs vectors - logic & booleans

0.1.2 Proofs

Example of proofs and non-proofs - direct - splitting cases - induction - contradiction

0.1.3 Geometry

- equations of lines/planes, etc... \Rightarrow vectors / scalar & equation manipulations

0.1.4 Sets

- min/max & sup/inf \Rightarrow start using for all / there exists

0.1.5 Integers

- prime numbers (infinite nb by Euclide) - unique factorization - finding primes between 1 and 100 \Rightarrow time complexity of algo?

0.2 Complex numbers

argand diagram

0.3 Sizes of infinity

[recycling house 6 pres']

0.4 Asymptotic analysis (limits)

- def of sequence: recursive and general form - usual sequences (arithmetic/geometric) - convergence of sequences

0.5 Infinite & partial sums

- sum of sequences - sum of usual (arithmetic/geometric) sequences - def of series - convergence of series

0.6 Functions & Inverses

finding roots & inverses

0.7 Usual functions

- plot & limit behaviour of: polynomials, exp, log, sin, cos, tan, sinh, cosh, tanh, arccos, arcsin, arctan

0.8 Differentiation

- from scratch - derivatives of usual functions - chain-law & co

0.9 Integration

- from scratch (area under curve, taking limit of rectangles) - antiderivative (do proof?) - integral of usual functions - integration by part? (if time!) - integration by substitution? (if time!)

0.10 Taylor series

- theory & practice - usual Taylor expansions - example of convergence

0.11 Fourier series? (if not late!)

0.12 Differential calculus? (if not late!)

0.13 Vector spaces

- def of vect sp - norm - basic propr

0.14 Matrices

- def - linear mapping of vect sp - inverse: def, existence (det), finding inverse - rank & kernel - eigenvalues

0.15 Non-linear multi-dimensional functions

- eg: cost func - partial derivatives - gradient - convexity? - optim: gradient descent

0.16 Regressions

- by hand - theory - non linear

0.17 PCA? (if time)

0.18 Basis of ML (perceptron)? (if time)