

# Refresher Maths Course

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## Abstract

This course teaches basic mathematical methodologies for proofs. It is intended for students with a lack of mathematical background, or with a lack of confidence in mathematics. We will try to cover most of the prerequisites of the courses in the Master's, i.e. basic algebra/analysis and basic application.

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## 0 Introduction

Hello! welcome to this maths refresher course for DSBA 2022! This is the best course ever!

### Presentation

- Paul Dubois, PhD Student @ Centrale, end of 1st year
- Email: b00795695@essec.edu (for any question), answer within 1 working day

### Course Format

#### Lectures

- 8\*3h arranged as 1h20min lecture - 1/3h break - 1h20min lecture
- No pb class planned, but lectures will have integrated live exercises
- Interrupt if needed (but may also ask at the end of the lecture)

#### Examination

- Course is pass/fail
- Most (in fact hopefully all) of you will pass
- There will be sets of exercises (about one per lecture), it is advised to attempt it all (only the starred questions will be compulsory)
- As the goal is to learn, you will be able to resubmit exercise sets, but you will lose 10% every-time you re-submit (so that you have some incentive to try your best the 1st time)
- Best  $(n-1)/n$  count, need average  $\geq 70\%$  to pass
- In the unlikely event of not passing, you will be able to do some extra work to pass

### Questions?

# 1 Elementary Maths

## 1.1 Mathematical Objects & Notations

### Sets

**Definition** (Sets). *Unordered list of elements.*

**Notation** (Sets).  $\in$ ,  $\{True, False\}$ ,  $\{a \mid condition\}$ ,  $\{a, b, c \dots\}$ ,  $\emptyset$

**Remark** (Russell Paradox). *(digression)*

*Need to be careful when defining set: some definitions are pathological.*

*e.g.: Take  $Y = \{x \mid x \notin x\}$ :  $Y \in Y \iff Y \notin Y$*

**Notation** (Usual Sets).  $\mathbb{B}$ ,  $\mathbb{N}$ ,  $\mathbb{Z}$ ,  $\mathbb{Q}$ ,  $\mathbb{R}$ ,  $\mathbb{C}$ ,  $\mathbb{N}^*$ ,  $\mathbb{R}^+ \dots$

### Functions

**Definition** (Functions). *Assignment from a set to another.*

**Notation** (Function).  $f : X \rightarrow Y$ ,  $f(x) = blah$ ,  $f : x \mapsto blah$ .

**Question.** *Which ones of these function are well-defined ?*

- $f : k \in \{0, 1, 2, 3, 4\} \mapsto 24/k \in \mathbb{N}$
- $f : k \in \{1, 2, 3, 4\} \mapsto 24/k \in \mathbb{N}$
- $f : k \in \{1, 2, 3, 4, 5\} \mapsto 24/k \in \mathbb{N}$
- $f : k \in \{1, 2, 3, 4\} \mapsto k \in \{1, 2\}$
- $f : k \in \{1, 2, 3, 4\} \mapsto k \in \{1, 2, 3, 4, 5\}$

## 1.2 Objects & Notations

-  $\mathbb{N}$ ,  $\mathbb{Z}$ ,  $\mathbb{Q}$ ,  $\mathbb{R}$  - scalars vs vectors - logic & booleans

## 1.3 Proofs

Example of proofs and non-proofs - direct - splitting cases - induction - contradiction

## 1.4 Geometry

- equations of lines/planes, etc...  $\Rightarrow$  vectors / scalar & equation manipulations

## 1.5 Sets

- min/max & sup/inf  $\Rightarrow$  start using for all / there exists

## 1.6 Integers

- prime numbers (infinite nb by Euclide) - unique factorization - finding primes between 1 and 100  $\Rightarrow$  time complexity of algo?

## **2 Complex numbers**

argand diagram

## **3 Sizes of infinity**

[recycling house 6 pres']

## **4 Asymptotic analysis (limits)**

- def of sequence: recursive and general form - usual sequences (arithmetic/geometric) - convergence of sequences

## **5 Infinite & partial sums**

- sum of sequences - sum of usual (arithmetic/geometric) sequences - def of series - convergence of series

## **6 Functions & Inverses**

finding roots & inverses

## **7 Usual functions**

- plot & limit behaviour of: polynomials, exp, log, sin, cos, tan, sinh, cosh, tanh, arccos, arcsin, arctan

## **8 Differentiation**

- from scratch - derivatives of usual functions - chain-law & co

## **9 Integration**

- from scratch (area under curve, taking limit of rectangles) - antiderivative (do proof?) - integral of usual functions - integration by part? (if time!) - integration by substitution? (if time!)

## **10 Taylor series**

- theory & practice - usual Taylor expansions - example of convergence

## **11 Fourier series? (if not late!)**

## **12 Differential calculus? (if not late!)**

## **13 Vector spaces**

- def of vect sp - norm - basic propr

## **14 Matrices**

- def - linear mapping of vect sp - inverse: def, existence (det), finding inverse - rank & kernel - eigenvalues

## **15 Non-linear multi-dimensional functions**

- eg: cost func - partial derivatives - gradient - convexity? - optim: gradient descent

## **16 Regressions**

- by hand - theory - non linear

## **17 PCA? (if time)**

## **18 Basis of ML (perceptron)? (if time)**