

Solutions

1 - Change of Basis

$$\underline{v}_B = \begin{pmatrix} -1 & 0,5 & + & 2,2 & + & 2 & 0,25 \\ -1 & (-1) & + & 3,0 & + & 2 & 0,5 \\ -1 & 1 & + & 3(-1) & + & 2 & 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ -4 \end{pmatrix}$$

$$P = \begin{pmatrix} 1/2 & 2 & -1/4 \\ -1 & 0 & 1/2 \\ 1 & -1 & 0 \end{pmatrix}$$

$$\underline{w}_B = P \underline{w}_{B'} \Rightarrow P^{-1} \underline{w}_B = \underline{w}_{B'}$$

$$P^{-1} = \begin{pmatrix} 1/2 & 1/4 & 1 \\ 1/2 & 1/4 & 0 \\ 1 & 5/2 & 2 \end{pmatrix}$$

$$\text{so } \underline{w}_{B'} = \begin{pmatrix} 9/4 \\ 1/4 \\ 21/2 \end{pmatrix}$$

2 - Variance & Covariance

$$S_1 \text{ mean : } 17/3$$

$$S_1 \text{ std. : } \sim 2.99$$

$$S_1 \text{ var. : } 8.9$$

$$S_2 \text{ mean : } 5$$

$$S_2 \text{ std. : } \sim 3.74$$

$$S_2 \text{ var. : } 12.7$$

$$\text{cov}(S_1, S_2) : 11$$

$$\hat{S}_1 = \{-1.39 ; -0.89 ; -0.22 ; 0.61 ; 0.78 ; 1.11\}$$

$$\hat{S}_2 = \{-1.34 ; -0.80 ; -0.27 ; 0.27 ; 0.80 ; 1.34\}$$

$$\text{cov}(\hat{S}_1, \hat{S}_2) \simeq 0.98$$

3 - Principal Component Analysis

$$1) \hat{S} = \{A', B', C', D', E', F'\} \text{ with :}$$

$$A' = \begin{pmatrix} -1.414 \\ 1.414 \\ 0.304 \end{pmatrix} \quad B' = \begin{pmatrix} -1.061 \\ 1.061 \\ -0.304 \end{pmatrix} \quad C' = \begin{pmatrix} 0.354 \\ -0.354 \\ -1.521 \end{pmatrix} \quad D' = \begin{pmatrix} 0.354 \\ -0.354 \\ 1.521 \end{pmatrix} \quad E' = \begin{pmatrix} 0.707 \\ -0.707 \\ -0.304 \end{pmatrix} \quad F' = \begin{pmatrix} 1.061 \\ -1.061 \\ 0.304 \end{pmatrix}$$

2)

$$Cov = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(2)

$$3) \quad ch_{Cov}(\lambda) = \det(C - \lambda I) = (1-\lambda)^3 - (-1)(-1)(1-\lambda) = \lambda(\lambda-1)(\lambda-2)$$

Eigenvalues  $0, 1, 2$   
 $\nearrow 00\% \quad \uparrow 33.3\% \quad \nwarrow 66.6\%$

$$4) \quad [\lambda=2] \quad C\underline{u} = 2\underline{u} \quad \underline{u} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$(\Rightarrow) \begin{cases} x-y = 2x \\ y-x = 2y \\ z = 2z \end{cases} \rightarrow \begin{cases} x = -y \\ z = 0 \end{cases} \quad \text{set } x=1 \quad \text{gives } \underline{u}_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

$$[\lambda=1] \quad C\underline{u} = \underline{u}$$

$$(\Rightarrow) \begin{cases} x-y = x \\ y-x = y \\ z = z \end{cases} \rightarrow \begin{cases} x=0 \\ y=0 \end{cases} \quad \text{set } z=1 \quad \text{gives } \underline{u}_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$[\lambda=0] \quad \text{No root! (It has 0\% variance)} \quad \underline{u}_0 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$5) \quad \underline{u} = (\underline{u}_2, \underline{u}_1, \underline{u}_0)$$

$$A_u = \begin{pmatrix} -2.791 \\ 0.333 \\ 0 \end{pmatrix}$$

$$B_u = \begin{pmatrix} -1.643 \\ -0.333 \\ 0 \end{pmatrix}$$

$$C_u = \begin{pmatrix} 0.548 \\ -1.667 \\ 0 \end{pmatrix}$$

$$D_u = \begin{pmatrix} 0.548 \\ 1.667 \\ 0 \end{pmatrix}$$

$$E_u = \begin{pmatrix} 1.095 \\ -0.333 \\ 0 \end{pmatrix}$$

$$F_u = \begin{pmatrix} 1.643 \\ 0.333 \\ 0 \end{pmatrix}$$