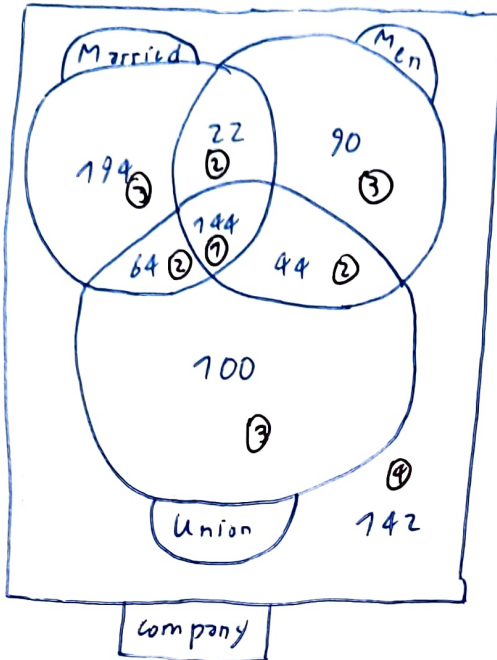


Exercise Set: Combinatorics

Solutions

1 - Company employees



$$208 - 144 = 64$$

$$166 - 144 = 22$$

$$188 - 144 = 44$$

$$424 - (22 + 64 + 144) = 194$$

$$352 - (44 + 64 + 144) = 100$$

$$300 - (22 + 44 + 144) = 90$$

$$800 - (22 + 44 + 64 + 90 + 100 + 144 + 194) = 142$$

There are 142 single women not in the union.

2 - The padlock

1) 10 possibilities for each digit $\Rightarrow 10^4 = 10\,000$.

2) 10 possibilities for 1st digit
9 _____ 2nd _____
8 _____ 3rd _____
7 _____ 4th _____
} $\frac{10!}{6!} = 10 \times 9 \times 8 \times 7 = 5040$

3) 10 possibilities for each digit 1, 2, 3
5 _____ last _____
} $5 \times 10^3 = 5\,000$

4) 5 possibilities for last digit
9 _____ 3rd _____
8 _____ 2nd _____
7 _____ 1st _____
} $5 \times 9 \times 8 \times 7 = 2520$

5) There are 9^4 not containing any "4", and 10^4 in total

$$\Rightarrow 10^4 - 9^4 = 3439$$

6) We put the "4" in one of the 4 positions. $\rightarrow 4$ possibilities

Then fill the rest with one of the 9 other digits $\rightarrow 9^3$ _____

$$\text{So } 4 \cdot 9^3 = 2916 \text{ possibilities}$$

3 - Airplanes

$4!$ ways to assign pilots

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$\binom{8}{2}$ ways to choose flight attendants for 1st airplane.

$\binom{6}{2}$ _____

2nd _____

$\binom{4}{2}$ _____

3rd _____

$\binom{2}{2}$ _____

4th _____

$$\text{So } 4! \binom{8}{2} \binom{6}{2} \binom{4}{2} \binom{2}{2} = 60\,480 \text{ ways in total}$$

4) Book shelf

• Grouped by subject:

$4!$ ways to order math.

$6!$ _____ physics.

$3!$ _____ chemistry.

and $3!$ ways to order subjects

$$\text{So } 4! 6! 3! 3! = 622\,080 \text{ in total}$$

• Only math grouped:

$4!$ ways to order math; then, consider math as a single book.

Then, there are $1+6+3 = 10$ books to order $\Rightarrow 10!$ ways

$$\text{So } 4! 10! = 87\,091\,200 \text{ in total}$$

5 - ESSEC's Stairs

for	0 steps:	1 ways	(0)
—	1 —	1 —	(1)
—	2 —	2 —	(1+1; 2)
—	3 —	3 —	(1+1+1; 2+1; 1+2)
—	4 —	5 —	(1+1+1+1; 2+1+1; 1+2+1; 1+1+2; 2+2)
—	5 —	8 —	(...)
—	6 —	13 —	(...)

\Rightarrow This is Fibonacci, but why?

Suppose $F(n)$ is the number of ways to climb n stairs.

Then, either we climb the first 2 together,
and there are $F(n-2)$ ways to climb the rest.

or we climb the first step alone, and there are
 $F(n-1)$ ways to climb the rest.

Thus, $F(n) = F(n-1) + F(n-2)$, and $F(0)=1$, $F(1)=1$

This is in fact, Fibonacci sequence.

The answer is $F(16) = 1597$.