Notes on Combinatorics

DSBA Mathematics Refresher 2024

Abstract

Combinatorics is the branch of mathematics dealing with counting, arrangement, and combination of objects. It provides the foundation for various concepts in probability, statistics, computer science, and more.

Choosing n Times: Power n

When we choose an item from a set n times ***with replacement***, each choice is independent of the previous ones. If the set has k distinct elements, the total number of ways to make these choices is given by: k^n

Example:

Suppose you have a set of 3 elements $\{a, b, c\}$. The number of different sequences of length 4 that can be formed by choosing from this set with replacement is: $3^4 = 81$

Ordering n Different Items: Factorial

When selecting n different items from a set, the number of ways to arrange these n items in order is given by $n!$ (read as "n factorial"), where:

$$n! = n \times (n-1) \times \cdots \times 2 \times 1$$

Example:

For a set of 5 distinct items $\{a, b, c, d, e\}$, the number of ways to arrange all 5 items is:

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Choosing k from n *(in Order)*

When choosing k elements from a set of n elements (without replacement) where the order of selection matters (permutations), the number of possible arrangements is given by:

$$P(n, k) = \frac{n!}{(n-k)!}$$

Example:

Consider a set of 6 elements $\{a, b, c, d, e, f\}$, and you want to select and arrange 3 elements. The number of different arrangements is:

$$P(6, 3) = \frac{6!}{(6-3)!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} = 120$$

Choosing k from n *(Without Order)*
When choosing k elements from a set of n elements where the order of selection does not matter (combinations), the number of possible combinations is given by the binomial coefficient:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Example:
Given a set of 7 elements $\{a, b, c, d, e, f, g\}$, the number of ways to choose 3 elements without regard to order is:

$$\binom{7}{3} = \frac{7!}{3!(7-3)!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{(3 \times 2 \times 1)(4 \times 3 \times 2 \times 1)} = 35$$