NotebookOptimization_Solutions

August 28, 2025

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
import plotly.graph_objects as go
```

1 Finding minimum value of a function

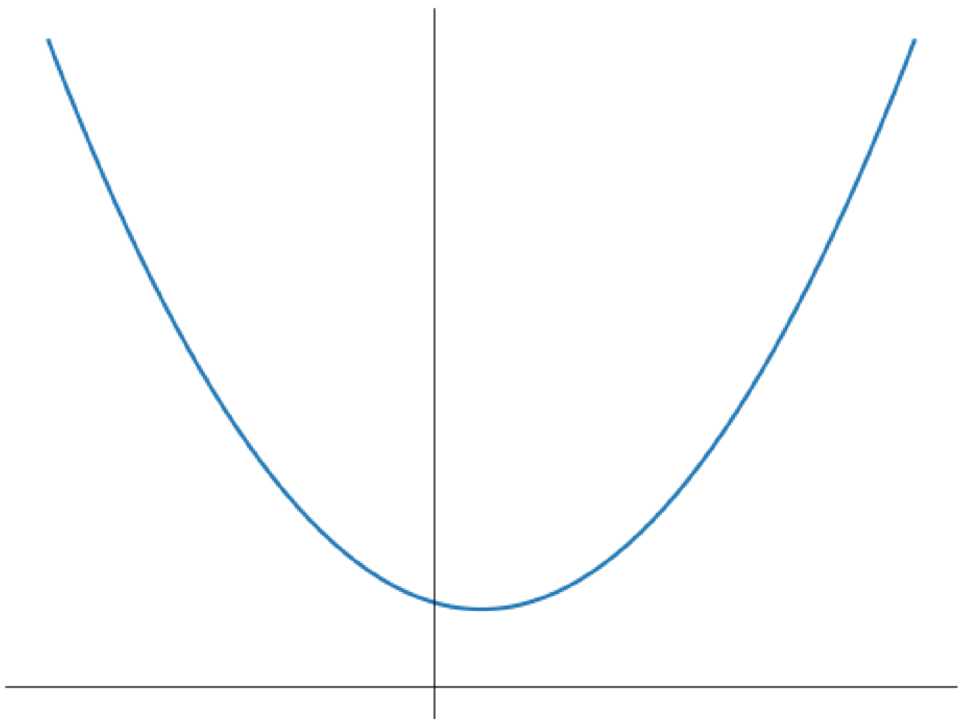
1.1 One dimensional function

$f(x) = x^2 - x + 3$

[2]:
```
def f(x):
    return x**2 - x + 3
```

Simple plot of the function. Note that if the fucntion had many (> 2 parameters, then plotting would not be possible).

[3]:
```
xs = np.linspace(-4, 5, 100)
ys = f(xs)
plt.plot(xs, ys)
plt.axis('off')
plt.axhline(0, color='black', lw=0.5)
plt.axvline(0, color='black', lw=0.5)
plt.show()
```



1.1.1 Method 1: Grid search

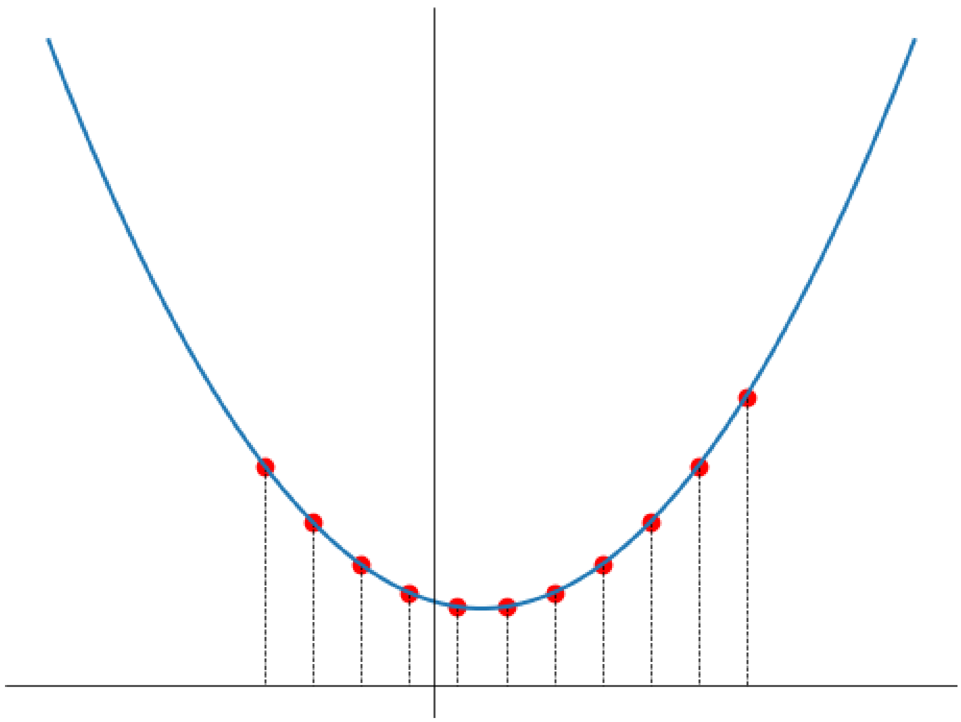
[4]:
```
x_vals = [-1.75, -1.25, -0.75, -0.25, 0.25, 0.75, 1.25, 1.75, 2.25, 2.75, 3.25]
for x in x_vals:
    print(f'f({x}) = {f(x)}')
```

```
f(-1.75) = 7.8125
f(-1.25) = 5.8125
f(-0.75) = 4.3125
f(-0.25) = 3.3125
f(0.25) = 2.8125
f(0.75) = 2.8125
f(1.25) = 3.3125
f(1.75) = 4.3125
f(2.25) = 5.8125
f(2.75) = 7.8125
f(3.25) = 10.3125
```

More visual version:

[5]:
```
x_vals = [-1.75, -1.25, -0.75, -0.25, 0.25, 0.75, 1.25, 1.75, 2.25, 2.75, 3.25]
for x in x_vals:
    plt.scatter(x, f(x), color='red')
    plt.plot([x, x], [f(x), 0], color='black', lw=0.5, linestyle='--')
```

```
plt.plot(xs, ys)
plt.axis('off')
plt.axhline(0, color='black', lw=0.5)
plt.axvline(0, color='black', lw=0.5)
plt.show()
```



Minimum value of $f(x)$ found is 2.8125 at $x = 0.25$ and $x = 0.75$.

1.1.2 Method 2: Dichotomy

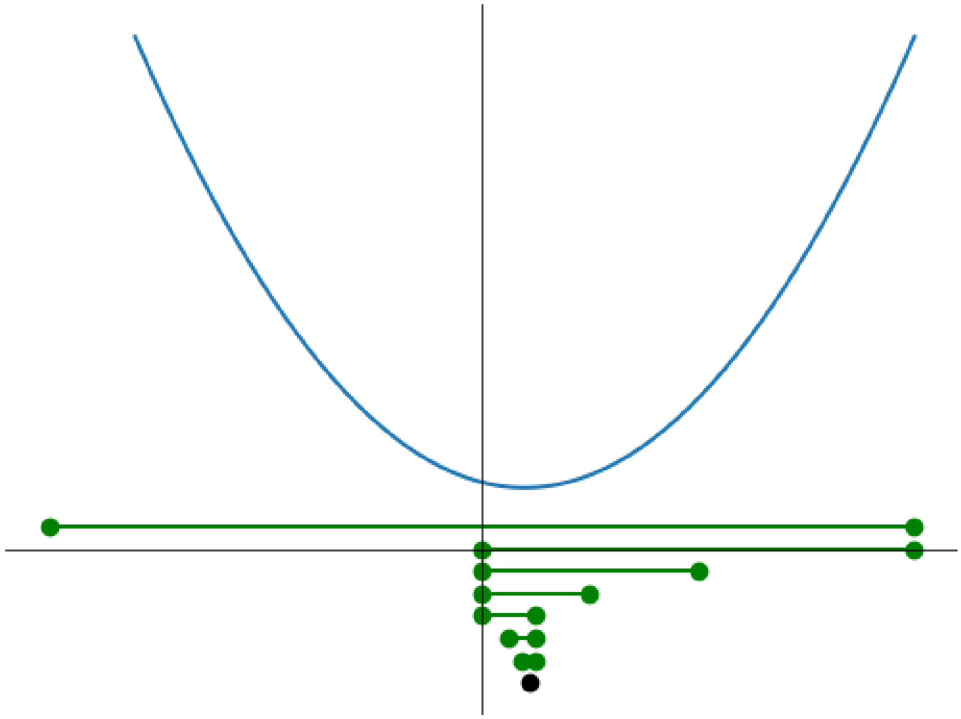
[6]:
```
a = -5
b = 5
for i in range(6):
    print(f"a: f({a}) = {f(a)} \t b: f({b}) = {f(b)}", end='\t\t')
    if f(a)>f(b):
        a = (a+b)/2
        print(f'set a = {a}')
    else:
        b = (a+b)/2
        print(f'set b = {b}')

x = ((a+b)/2)
print(f"best guess: x = {x}")
```

```
a: f(-5) = 33    b: f(5) = 23           set a = 0.0
a: f(0.0) = 3.0          b: f(5) = 23           set b = 2.5
a: f(0.0) = 3.0          b: f(2.5) = 6.75               set b = 1.25
a: f(0.0) = 3.0          b: f(1.25) = 3.3125            set b = 0.625
a: f(0.0) = 3.0          b: f(0.625) = 2.765625         set a = 0.3125
a: f(0.3125) = 2.78515625        b: f(0.625) = 2.765625         set a = 0.46875
best guess: x = 0.546875
```

More visual version

[7]:

```
a = -5
b = 5
plt.plot([a,b], [1, 1], color='green')
plt.scatter([a,b], [1, 1], color='green')
for i in range(6):
    if f(a)>f(b):
        a = (a+b)/2
    else:
        b = (a+b)/2
    plt.plot([a,b], [-i, -i], color='green')
    plt.scatter([a,b], [-i, -i], color='green')
x = ((a+b)/2)
plt.scatter([x], [-6], color='black')
plt.plot(xs, ys)
plt.axis('off')
plt.axhline(0, color='black', lw=0.5)
plt.axvline(0, color='black', lw=0.5)
plt.show()
```



1.1.3 Method 3: Gradient descent

We first calculate the derivative $f'(x) = 2x - 1$

[8]:
```
def df(x):
    return 2*x-1
```

[9]:
```
lr = 0.8
x = -0.5
l = [x]
for _ in range(7):
    print(f"f({x:.4f}) = {f(x):.4f}  \t f'({x:.4f}) = {df(x):.4f}")
    x = x - lr * df(x)
    l.append(x)
print(f'Final guess: x = {x:.4f}')

xs = np.linspace(min(l)-0.5, max(l+[2])+0.5, 100)
ys = f(xs)
```

```
f(-0.5000) = 3.7500      f'(-0.5000) = -2.0000
f(1.1000) = 3.1100       f'(1.1000) = 1.2000
f(0.1400) = 2.8796       f'(0.1400) = -0.7200
f(0.7160) = 2.7967       f'(0.7160) = 0.4320
f(0.3704) = 2.7668       f'(0.3704) = -0.2592
```

```
f(0.5778) = 2.7560       f'(0.5778) = 0.1555
f(0.4533) = 2.7522       f'(0.4533) = -0.0933
Final guess: x = 0.5280
```

[10]:
```
lr = 0.3
x = -0.5
l = [x]
for _ in range(7):
    print(f"f({x:.4f}) = {f(x):.4f}  \t f'({x:.4f}) = {df(x):.4f}")
    x = x - lr * df(x)
    l.append(x)
print(f'Final guess: x = {x:.4f}')

xs = np.linspace(min(l)-0.5, max(l+[2])+0.5, 100)
ys = f(xs)
```

```
f(-0.5000) = 3.7500      f'(-0.5000) = -2.0000
f(0.1000) = 2.9100       f'(0.1000) = -0.8000
f(0.3400) = 2.7756       f'(0.3400) = -0.3200
f(0.4360) = 2.7541       f'(0.4360) = -0.1280
f(0.4744) = 2.7507       f'(0.4744) = -0.0512
f(0.4898) = 2.7501       f'(0.4898) = -0.0205
f(0.4959) = 2.7500       f'(0.4959) = -0.0082
Final guess: x = 0.4984
```

Let's try other values for learning rate

[11]:
```
for lr in [0.3, 0.01, 1, 2]:
    print(f'\t learning rate: {lr}')
    x = -1
    l = [x]
    for _ in range(7):
        print(f"f({x:.2f}) = {f(x):.2f}  \t f'({x:.2f}) = {df(x):.2f}")
        x = x - lr * df(x)
        l.append(x)
    print(f'Final guess: x = {x:.2f} \n\n\n')

    xs = np.linspace(min(l)-0.5, max(l+[2])+0.5, 100)
    ys = f(xs)
```

```
         learning rate: 0.3
f(-1.00) = 5.00          f'(-1.00) = -3.00
f(-0.10) = 3.11          f'(-0.10) = -1.20
f(0.26) = 2.81           f'(0.26) = -0.48
f(0.40) = 2.76           f'(0.40) = -0.19
f(0.46) = 2.75           f'(0.46) = -0.08
f(0.48) = 2.75           f'(0.48) = -0.03
f(0.49) = 2.75           f'(0.49) = -0.01
Final guess: x = 0.50
```

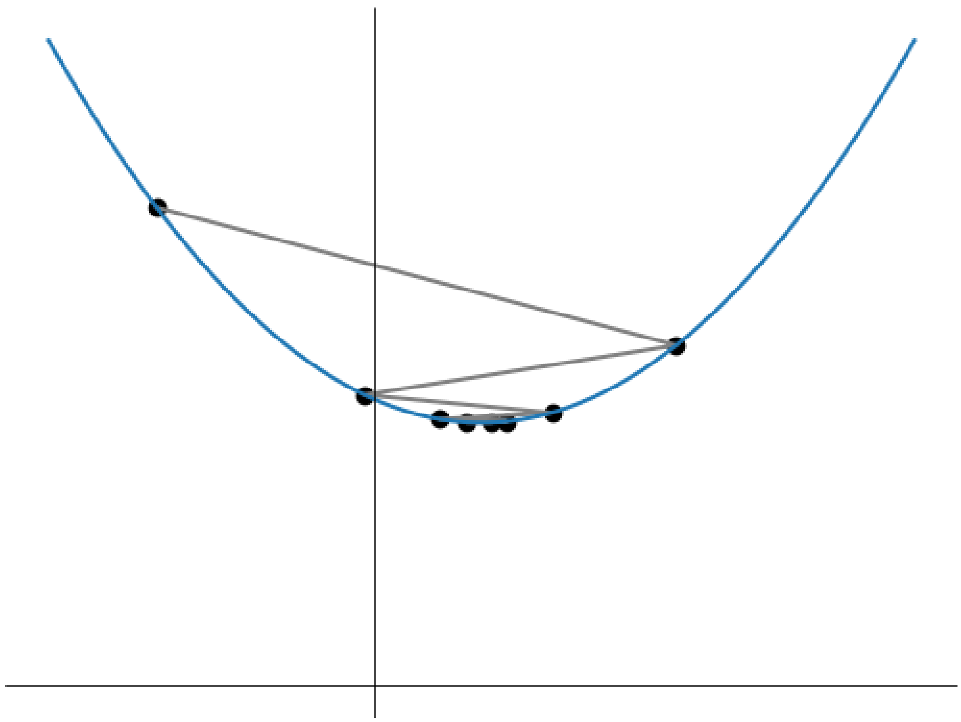
```
         learning rate: 0.01
f(-1.00) = 5.00          f'(-1.00) = -3.00
f(-0.97) = 4.91          f'(-0.97) = -2.94
f(-0.94) = 4.83          f'(-0.94) = -2.88
f(-0.91) = 4.74          f'(-0.91) = -2.82
f(-0.88) = 4.66          f'(-0.88) = -2.77
f(-0.86) = 4.59          f'(-0.86) = -2.71
f(-0.83) = 4.52          f'(-0.83) = -2.66
Final guess: x = -0.80



         learning rate: 1
f(-1.00) = 5.00          f'(-1.00) = -3.00
f(2.00) = 5.00           f'(2.00) = 3.00
f(-1.00) = 5.00          f'(-1.00) = -3.00
f(2.00) = 5.00           f'(2.00) = 3.00
f(-1.00) = 5.00          f'(-1.00) = -3.00
f(2.00) = 5.00           f'(2.00) = 3.00
f(-1.00) = 5.00          f'(-1.00) = -3.00
Final guess: x = 2.00



         learning rate: 2
f(-1.00) = 5.00          f'(-1.00) = -3.00
f(5.00) = 23.00          f'(5.00) = 9.00
f(-13.00) = 185.00       f'(-13.00) = -27.00
f(41.00) = 1643.00       f'(41.00) = 81.00
f(-121.00) = 14765.00    f'(-121.00) = -243.00
f(365.00) = 132863.00    f'(365.00) = 729.00
f(-1093.00) = 1195745.00         f'(-1093.00) = -2187.00
Final guess: x = 3281.00
```

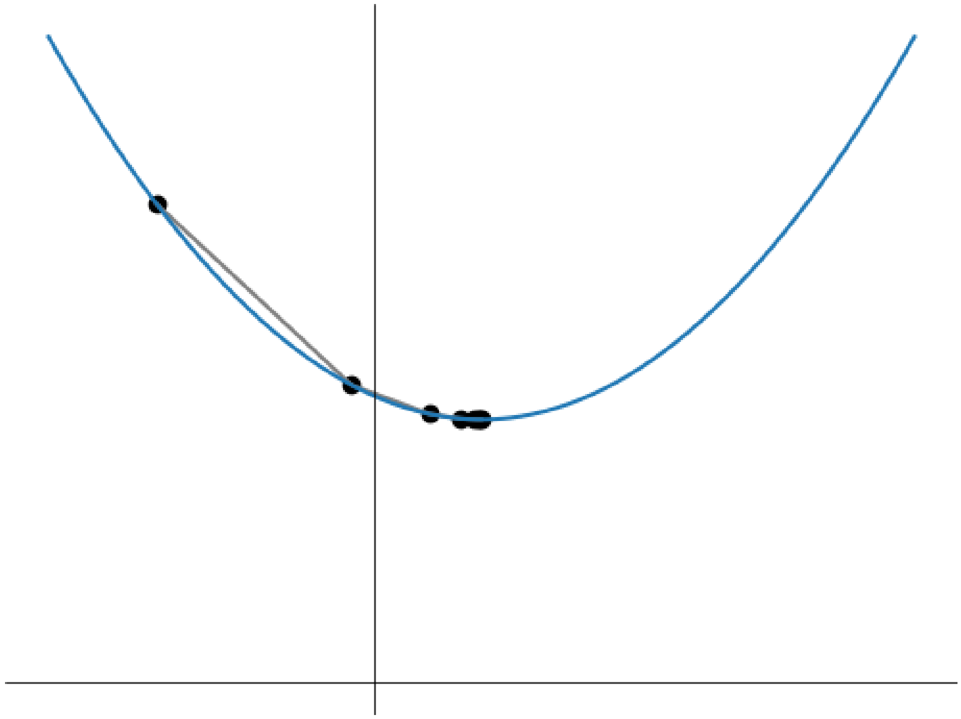
Visually:

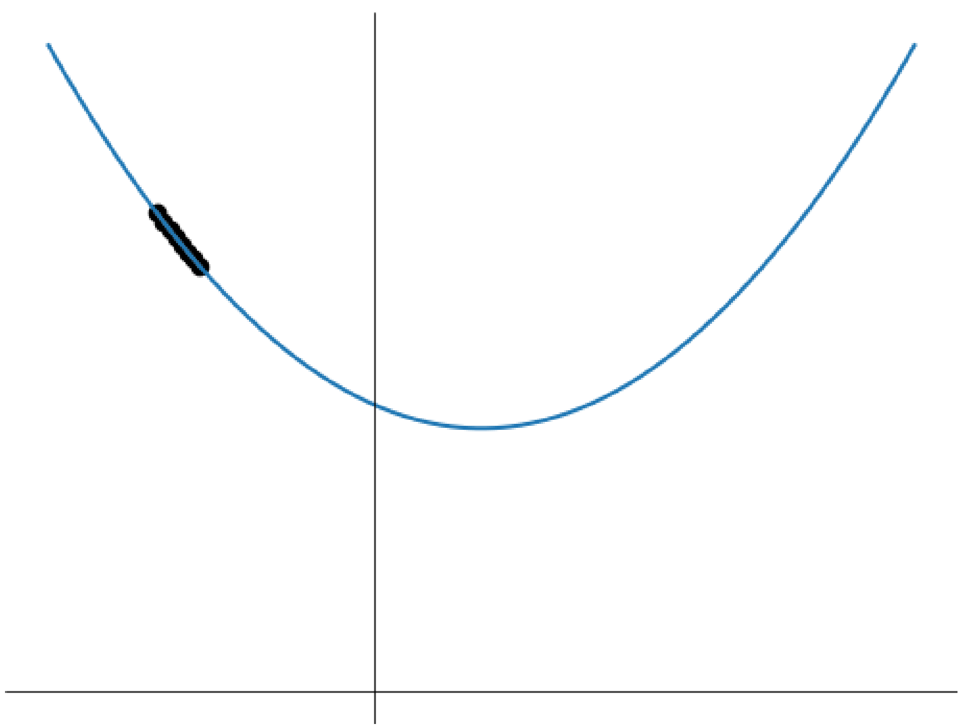
[12]:

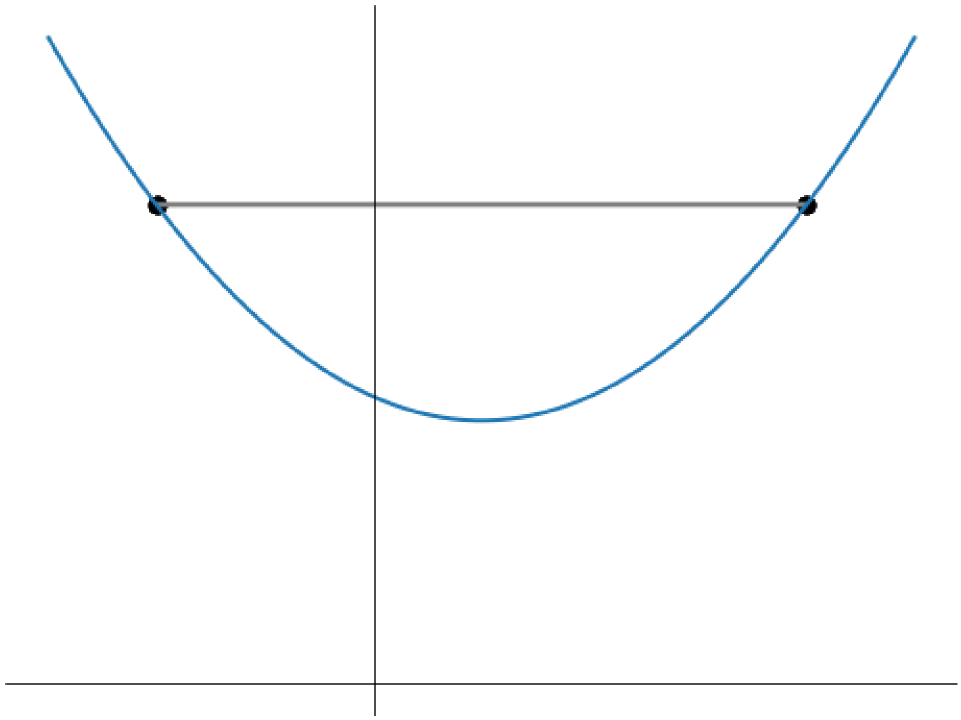
```python
for lr in [0.8, 0.3, 0.01, 1, 1.25]:
    x = -1
    l = [x]
    for _ in range(7):
        x = x - lr * df(x)
        l.append(x)
```

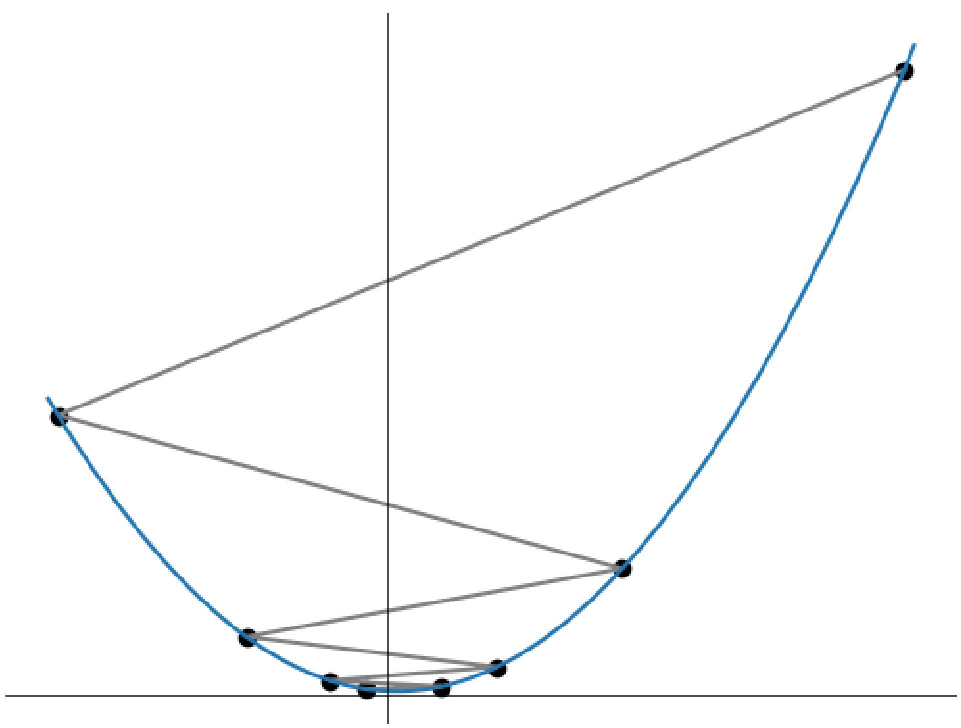
```
plt.plot(l, [f(x) for x in l], color='grey')
plt.scatter(l, [f(x) for x in l], color='black')
xs = np.linspace(min(l)-0.5, max(l+[2])+0.5, 100)
ys = f(xs)
plt.plot(xs, ys)
plt.axis('off')
plt.axhline(0, color='black', lw=0.5)
plt.axvline(0, color='black', lw=0.5)
plt.show()
```











1.2 Two dimensional function

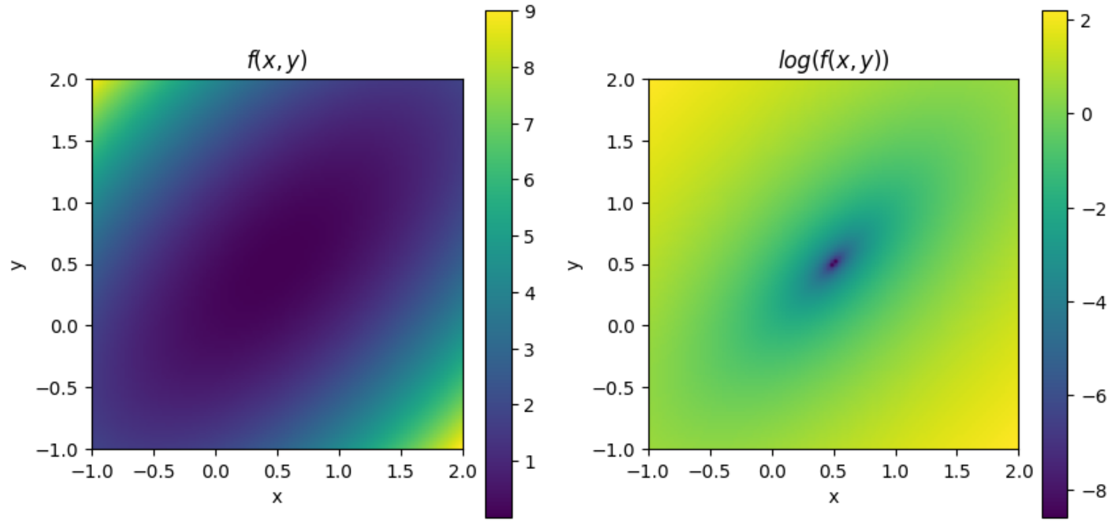
$f(x, y) = (x + y - 1)^2 + \frac{1}{5}(x - y)^2$

[13]:
```
def f(x, y):
    return (x+y-1)**2 + 0.2*(x-y)**2
```

Plotting the function as a 2D surface. Plotting log of the function helps to see the minimum more clearly.

[14]:
```
# interactive
x = np.linspace(-1, 2, 100)
y = np.linspace(-1, 2, 100)
X, Y = np.meshgrid(x, y)
Z = f(X, Y)
Zlog = np.log(Z)
go.Figure(data=[go.Surface(z=Z)])
```

[15]:
```
# not interactive
x = np.linspace(-1, 2, 100)
y = np.linspace(-1, 2, 100)
X, Y = np.meshgrid(x, y)
Z = f(X, Y)
Zlog = np.log(Z)

# plot a heatmap of the function
plt.figure(figsize=(10,5))
plt.subplot(1,2,1)
plt.imshow(Z, extent=(-1, 2, -1, 2))
plt.colorbar()
plt.title('$f(x,y)$')
plt.xlabel('x')
plt.ylabel('y')
plt.subplot(1,2,2)
plt.imshow(Zlog, extent=(-1, 2, -1, 2))
plt.colorbar()
plt.title('$log(f(x,y))$')
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```



1.2.1 Method 1: Grid search

[16]:
```
for x in np.arange(-1, 2.5, 0.5):
    for y in np.arange(-1, 2.5, 0.5):
        print(f'f({x}, {y}) = {f(x, y)}')
```

```
f(-1.0, -1.0) = 9.0
f(-1.0, -0.5) = 6.3
f(-1.0, 0.0) = 4.2
f(-1.0, 0.5) = 2.7
f(-1.0, 1.0) = 1.8
f(-1.0, 1.5) = 1.5
f(-1.0, 2.0) = 1.8
f(-0.5, -1.0) = 6.3
f(-0.5, -0.5) = 4.0
f(-0.5, 0.0) = 2.3
f(-0.5, 0.5) = 1.2
f(-0.5, 1.0) = 0.7
f(-0.5, 1.5) = 0.8
f(-0.5, 2.0) = 1.5
f(0.0, -1.0) = 4.2
f(0.0, -0.5) = 2.3
f(0.0, 0.0) = 1.0
f(0.0, 0.5) = 0.3
f(0.0, 1.0) = 0.2
f(0.0, 1.5) = 0.7
f(0.0, 2.0) = 1.8
f(0.5, -1.0) = 2.7
f(0.5, -0.5) = 1.2
f(0.5, 0.0) = 0.3
```

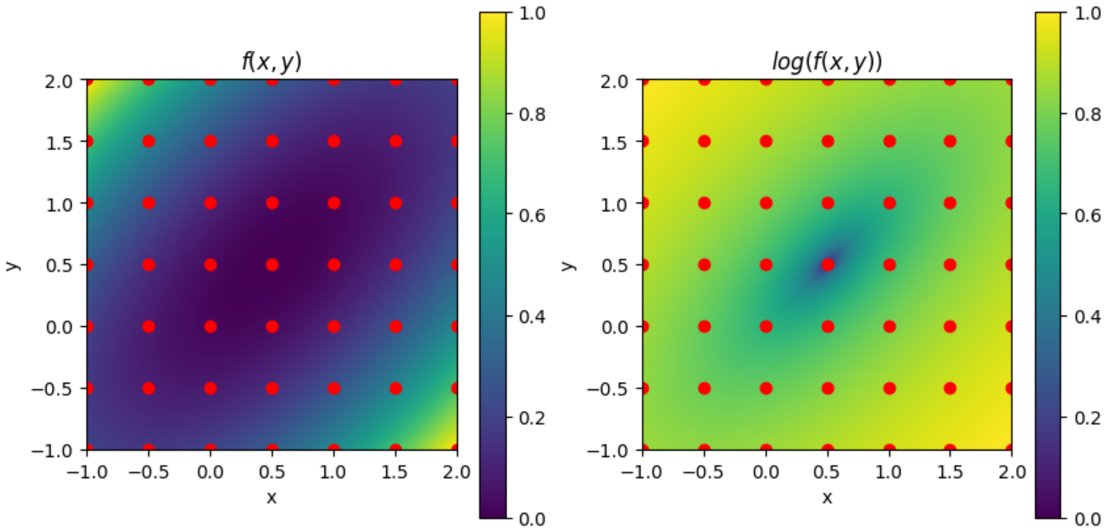
```
f(0.5, 0.5) = 0.0
f(0.5, 1.0) = 0.3
f(0.5, 1.5) = 1.2
f(0.5, 2.0) = 2.7
f(1.0, -1.0) = 1.8
f(1.0, -0.5) = 0.7
f(1.0, 0.0) = 0.2
f(1.0, 0.5) = 0.3
f(1.0, 1.0) = 1.0
f(1.0, 1.5) = 2.3
f(1.0, 2.0) = 4.2
f(1.5, -1.0) = 1.5
f(1.5, -0.5) = 0.8
f(1.5, 0.0) = 0.7
f(1.5, 0.5) = 1.2
f(1.5, 1.0) = 2.3
f(1.5, 1.5) = 4.0
f(1.5, 2.0) = 6.3
f(2.0, -1.0) = 1.8
f(2.0, -0.5) = 1.5
f(2.0, 0.0) = 1.8
f(2.0, 0.5) = 2.7
f(2.0, 1.0) = 4.2
f(2.0, 1.5) = 6.3
f(2.0, 2.0) = 9.0
```

Note that it becomes computationally expensive to search for the minimum in a 2D grid. For higher dimensions, this method is not feasible.

More visually:

[17]:
```
# plot a heatmap of the function
plt.figure(figsize=(10,5))
plt.subplot(1,2,1)
plt.imshow(Z, extent=(-1, 2, -1, 2))
for x in np.arange(-1, 2.5, 0.5):
    for y in np.arange(-1, 2.5, 0.5):
        plt.scatter(x, y, color='red')
plt.colorbar()
plt.title('$f(x,y)$')
plt.xlabel('x')
plt.ylabel('y')
plt.subplot(1,2,2)
plt.imshow(Zlog, extent=(-1, 2, -1, 2))
for x in np.arange(-1, 2.5, 0.5):
    for y in np.arange(-1, 2.5, 0.5):
        plt.scatter(x, y, color='red')
plt.colorbar()
```

```
plt.title('$log(f(x,y))$')
plt.xlabel('x')
plt.ylabel('y')
plt.show()
```



1.2.2 Method 2: Dichotomy

[18]:
```
a, b = -10, 10
c, d = -10, 10
for _ in range(10):
    f1 = f(a, c)
    f2 = f(a, d)
    f3 = f(b, c)
    f4 = f(b, d)
    if f1<=f2 and f1<=f3 and f1<=f4:
        b = (a+b)/2
        d = (c+d)/2
    if f2<=f1 and f2<=f3 and f2<=f4:
        b = (a+b)/2
        c = (c+d)/2
    if f3<=f2 and f3<=f1 and f3<=f4:
        a = (a+b)/2
        d = (c+d)/2
    if f4<=f2 and f4<=f3 and f4<=f1:
        a = (a+b)/2
        c = (c+d)/2
    print(f'f({(a+b)/2:.2f}, {(c+d)/2:.2f}) = {f((a+b)/2, (c+d)/2):.4f} \t {a:.
 ↪2f} < x < {b:.2f} \t {c:.2f} < y < {d:.2f} \t ')
```

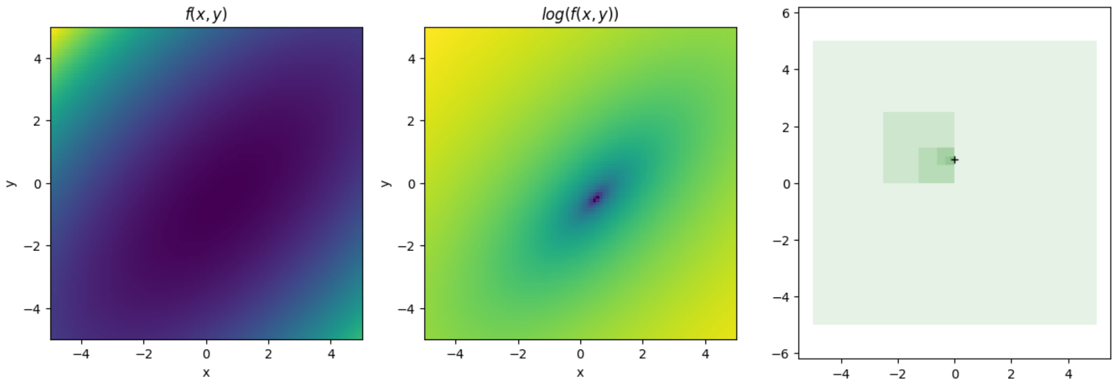
```
print(f'Best guess: x = {(a+b)/2:.2f}, y = {(c+d)/2:.2f}')
```

```
f(-2.50, 2.50) = 6.0000          -5.00 < x < 0.00        0.00 < y < 5.00
f(-1.25, 1.25) = 2.2500          -2.50 < x < 0.00        0.00 < y < 2.50
f(-0.62, 0.62) = 1.3125          -1.25 < x < 0.00        0.00 < y < 1.25
f(-0.31, 0.94) = 0.4531          -0.62 < x < 0.00        0.62 < y < 1.25
f(-0.16, 0.78) = 0.3164          -0.31 < x < 0.00        0.62 < y < 0.94
f(-0.08, 0.86) = 0.2236          -0.16 < x < 0.00        0.78 < y < 0.94
f(-0.04, 0.82) = 0.1956          -0.08 < x < 0.00        0.78 < y < 0.86
f(-0.02, 0.84) = 0.1800          -0.04 < x < 0.00        0.82 < y < 0.86
f(-0.01, 0.83) = 0.1734          -0.02 < x < 0.00        0.82 < y < 0.84
f(-0.00, 0.83) = 0.1699          -0.01 < x < 0.00        0.83 < y < 0.84
Best guess: x = -0.00, y = 0.83
```

More visual version

[19]:
```
a, b = -5, 5
c, d = -5, 5

fig = plt.figure(figsize=(15,5))
x = np.linspace(a,b, 100)
y = np.linspace(c,d, 100)
X, Y = np.meshgrid(x, y)
Z = f(X, Y)
Zlog = np.log(Z)
plt.subplot(1,3,1)
plt.imshow(Z, extent=(a, b, c, d))
plt.title('$f(x,y)$')
plt.xlabel('x')
plt.ylabel('y')
plt.subplot(1,3,2)
plt.imshow(Zlog, extent=(a, b, c, d))
plt.title('$log(f(x,y))$')
plt.xlabel('x')
plt.ylabel('y')

ax = plt.subplot(1,3,3)
rectangles = []
for _ in range(10):
    rectangle = plt.Rectangle((a, c), b-a, d-c, facecolor='green', alpha=0.1)
    ax.add_patch(rectangle)
    f1 = f(a, c)
    f2 = f(a, d)
    f3 = f(b, c)
    f4 = f(b, d)
    if f1<=f2 and f1<=f3 and f1<=f4:
        b = (a+b)/2
```

```
        d = (c+d)/2
    if f2<=f1 and f2<=f3 and f2<=f4:
        b = (a+b)/2
        c = (c+d)/2
    if f3<=f2 and f3<=f1 and f3<=f4:
        a = (a+b)/2
        d = (c+d)/2
    if f4<=f2 and f4<=f3 and f4<=f1:
        a = (a+b)/2
        c = (c+d)/2
plt.plot((a+b)/2, (c+d)/2, '+', color='black')
plt.axis('equal')
plt.show()
```



1.2.3 Method 3: Gradient descent

We start by calculating the partial derivatives of the function with respect to x and y.

$\frac{\partial f}{\partial x}(x, y) = 2(x + y - 1) + \frac{2}{5}(x - y)$

$\frac{\partial f}{\partial y}(x, y) = 2(x + y - 1) - \frac{2}{5}(x - y)$

[20]:
```
def dfx(x, y):
    return 2*(x+y-1) + 0.4*(x-y)
def dfy(x, y):
    return 2*(x+y-1) - 0.4*(x-y)
```

[21]:
```
lr = 0.6
x = 2.5
y = 3
lx = [x]
ly = [y]
for _ in range(7):
    x = x - lr * dfx(x,y)
```

```
    y = y - lr * dfy(x,y)
    lx.append(x)
    ly.append(y)
    print(f'f(x,y) = {f(x,y):.2f} \t (x,y) = ({x:.2f}, {y:.2f})')
```

```
f(x,y) = 7.20    (x,y) = (-2.78, 2.55)
f(x,y) = 0.85    (x,y) = (-0.02, 0.10)
f(x,y) = 0.25    (x,y) = (1.11, 0.09)
f(x,y) = 0.04    (x,y) = (0.63, 0.56)
f(x,y) = 0.01    (x,y) = (0.39, 0.58)
f(x,y) = 0.00    (x,y) = (0.47, 0.49)
f(x,y) = 0.00    (x,y) = (0.52, 0.48)
```

[22]:

```
lr = 0.6
for lr in [0.3, 0.1, 0.01, 1.25]:
    print(f'learning rate: {lr}')
    x = 2.5
    y = 3
    lx = [x]
    ly = [y]
    for _ in range(7):
        x = x - lr * dfx(x,y)
        y = y - lr * dfy(x,y)
        lx.append(x)
        ly.append(y)
        print(f'f(x,y) = {f(x,y):.2f} \t (x,y) = ({x:.2f}, {y:.2f})')
    print('\n\n\n')
```

```
learning rate: 0.3
f(x,y) = 0.68    (x,y) = (-0.14, 1.51)
f(x,y) = 0.32    (x,y) = (-0.16, 1.10)
f(x,y) = 0.16    (x,y) = (0.03, 0.90)
f(x,y) = 0.07    (x,y) = (0.18, 0.77)
f(x,y) = 0.03    (x,y) = (0.28, 0.68)
f(x,y) = 0.01    (x,y) = (0.35, 0.62)
f(x,y) = 0.01    (x,y) = (0.40, 0.58)




learning rate: 0.1
f(x,y) = 8.14    (x,y) = (1.62, 2.22)
f(x,y) = 3.29    (x,y) = (1.08, 1.72)
f(x,y) = 1.36    (x,y) = (0.74, 1.39)
f(x,y) = 0.58    (x,y) = (0.54, 1.17)
f(x,y) = 0.27    (x,y) = (0.43, 1.02)
f(x,y) = 0.14    (x,y) = (0.36, 0.92)
f(x,y) = 0.08    (x,y) = (0.33, 0.84)
```

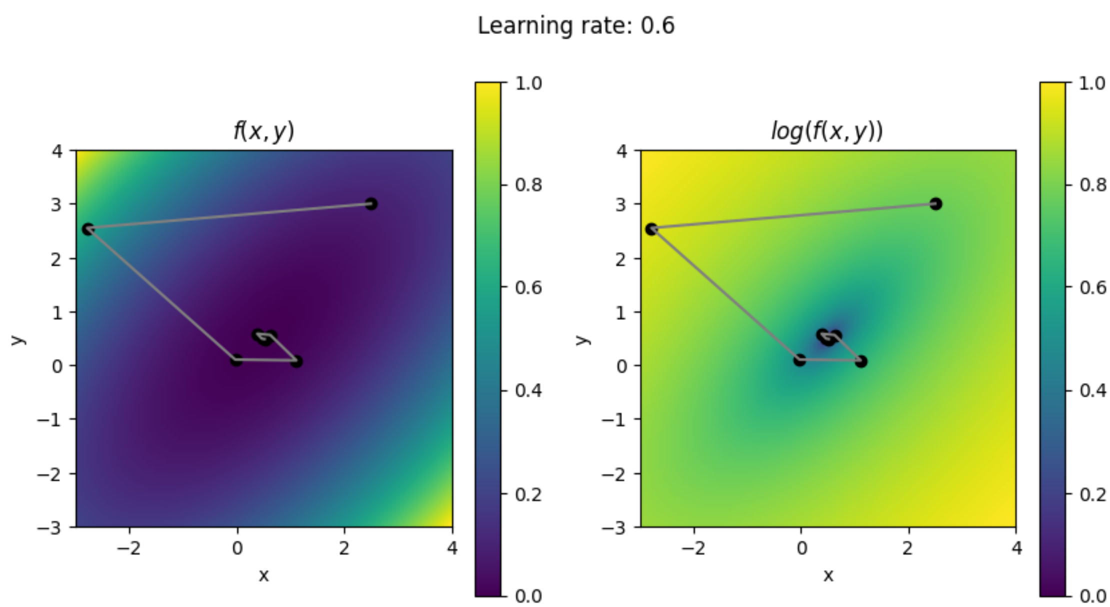
```
learning rate: 0.01
f(x,y) = 18.72   (x,y) = (2.41, 2.91)
f(x,y) = 17.27   (x,y) = (2.33, 2.82)
f(x,y) = 15.93   (x,y) = (2.25, 2.74)
f(x,y) = 14.69   (x,y) = (2.17, 2.66)
f(x,y) = 13.55   (x,y) = (2.09, 2.58)
f(x,y) = 12.50   (x,y) = (2.02, 2.51)
f(x,y) = 11.53   (x,y) = (1.95, 2.44)
```

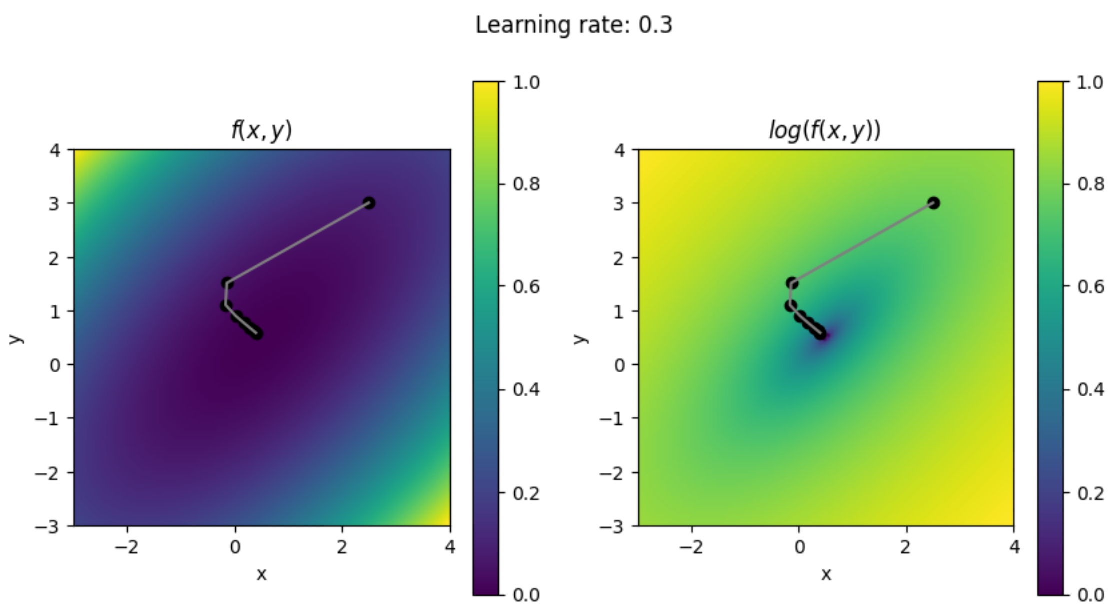
```
learning rate: 1.25
f(x,y) = 112.80          (x,y) = (-8.50, 13.50)
f(x,y) = 324.80          (x,y) = (-7.50, -9.50)
f(x,y) = 1804.80         (x,y) = (36.50, -51.50)
f(x,y) = 5196.80         (x,y) = (32.50, 40.50)
f(x,y) = 28876.80        (x,y) = (-143.50, 208.50)
f(x,y) = 83148.80        (x,y) = (-127.50, -159.50)
f(x,y) = 462028.80       (x,y) = (576.50, -831.50)
```

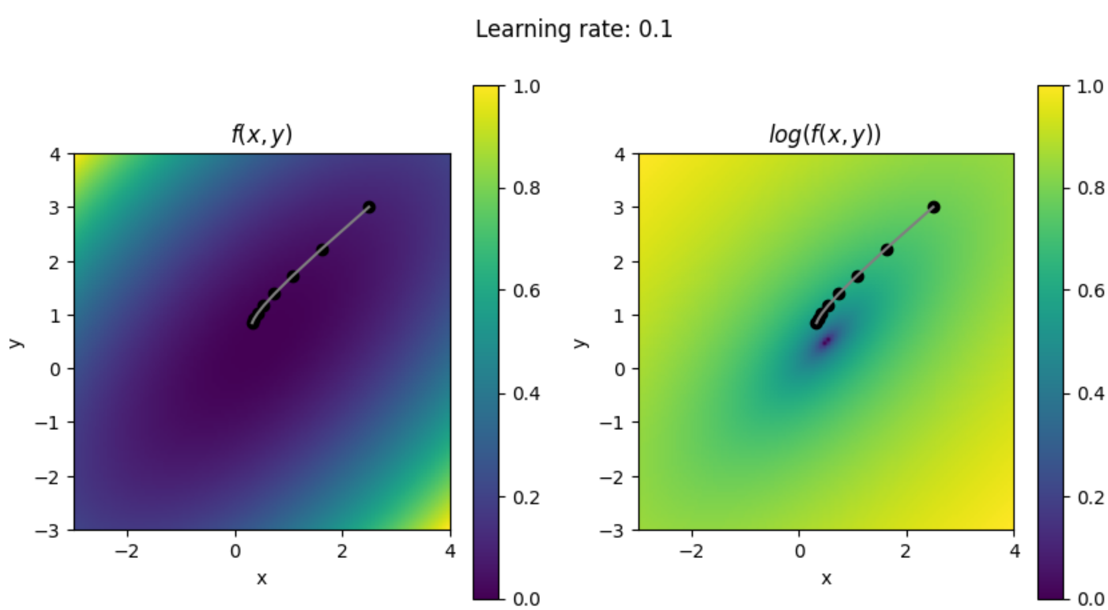
A more visual version of the gradient descent method

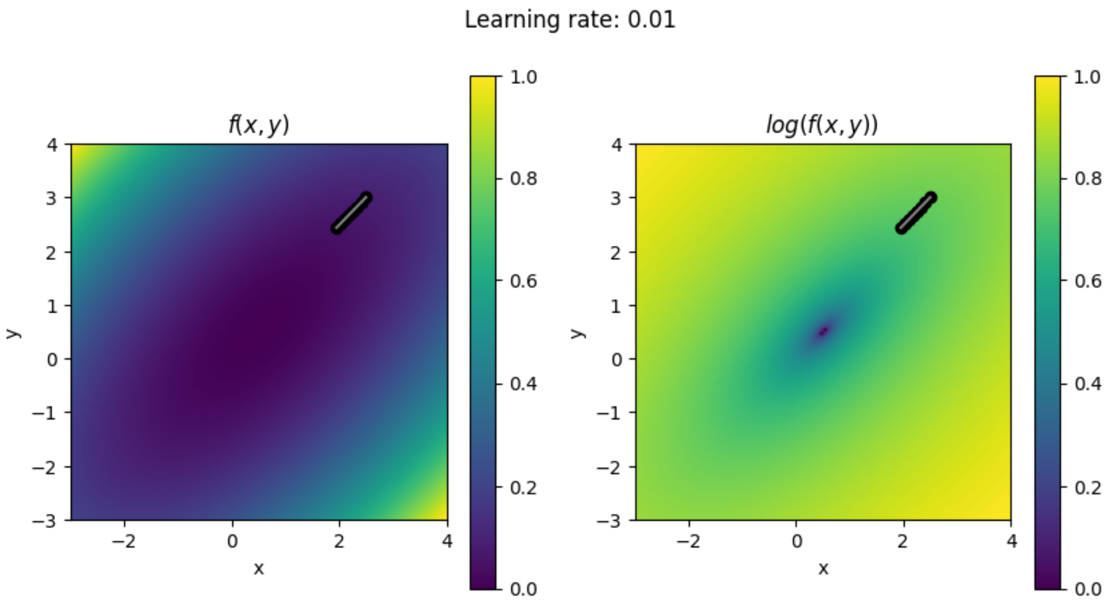
[23]:
```
for lr in [0.6, 0.3, 0.1, 0.01, 1.25]:
    x = 2.5
    y = 3
    lx = [x]
    ly = [y]
    for _ in range(7):
        x = x - lr * dfx(x,y)
        y = y - lr * dfy(x,y)
        lx.append(x)
        ly.append(y)

    # plot a heatmap of the function with the line search
    x = np.linspace(-3, 4, 100)
    y = np.linspace(-3, 4, 100)
    X, Y = np.meshgrid(x, y)
    Z = f(X, Y)
    Zlog = np.log(Z)
```

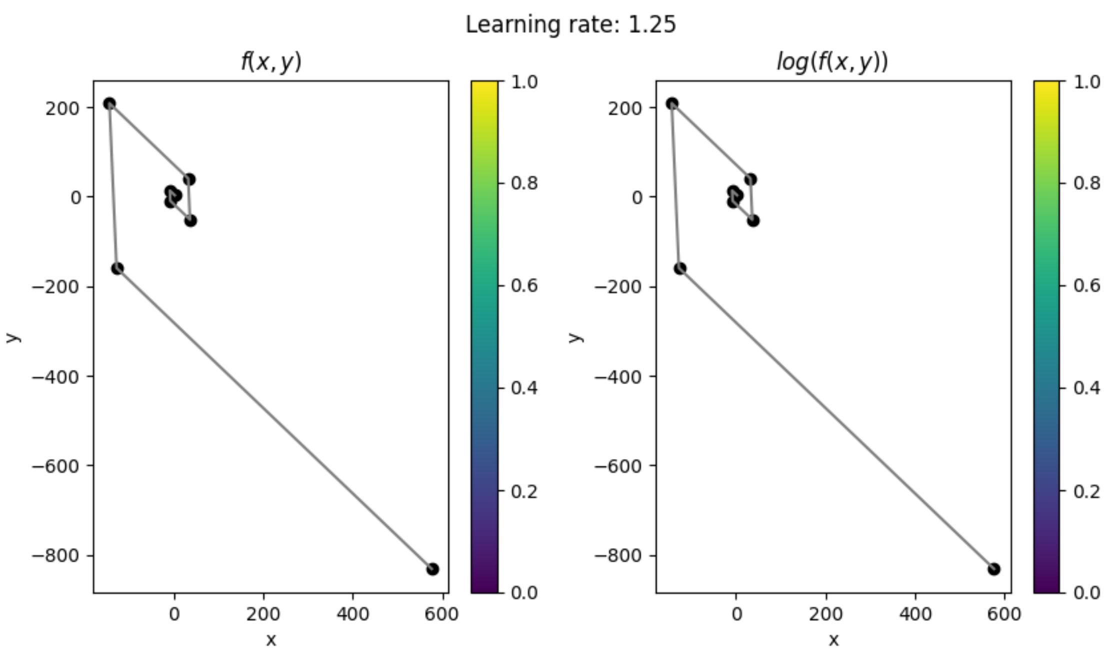
```
plt.figure(figsize=(10,5))
plt.subplot(1,2,1)
plt.imshow(Z, extent=(-3, 4, -3, 4))
plt.plot(lx, ly, color='grey')
plt.scatter(lx, ly, color='black')
plt.colorbar()
plt.title('$f(x,y)$')
plt.xlabel('x')
plt.ylabel('y')
plt.subplot(1,2,2)
plt.imshow(Zlog, extent=(-3, 4, -3, 4))
plt.plot(lx, ly, color='grey')
plt.scatter(lx, ly, color='black')
plt.colorbar()
plt.title('$log(f(x,y))$')
plt.xlabel('x')
plt.ylabel('y')
plt.suptitle(f'Learning rate: {lr}')
plt.show()
```




Learning rate: 0.3
$f(x,y)$
$log(f(x,y))$
x
y
1.0
0.8
0.6
0.4
0.2
0.0


Learning rate: 0.1
$f(x,y)$
$log(f(x,y))$
x
y
1.0
0.8
0.6
0.4
0.2
0.0





---

2 Linear regression

We want to find the line of best fit for the following points: - $P_1 = (0, 1)$ - $P_2 = (3, 8)$ - $P_4 = (5, 12)$ - $P_5 = (8, 17)$

The line of best fit is given by the equation $y = ax + b$ where a is the slope and b is the y-intercept.

Let's define the loss function as the sum of squared errors:

$$\mathcal{L}(a, b) = \sum_{i=1}^{n} (y_i - (ax_i + b))^2$$

so

$$\mathcal{L}(a, b) = (1 - (0a + b))^2 + (8 - (3a + b))^2 + (12 - (5a + b))^2 + (17 - (8a + b))^2$$

We can find the minimum of the loss function by taking the partial derivatives with respect to a and b and setting them to zero.

With respect to a:

$\frac{\partial \mathcal{L}}{\partial a}(a, b) = -2(1 - (0a + b))0 + -2(8 - (3a + b))3 + -2(12 - (5a + b))5 + -2(17 - (8a + b))8$

$\frac{\partial \mathcal{L}}{\partial a}(a, b) = -6(-3a - b + 8) + -10(-5a - b + 12) + -16(-8a - b + 17)$

$\frac{\partial \mathcal{L}}{\partial a}(a, b) = 18a + 6b - 48 + 50a + 10b - 120 + 128a + 16b - 272$

$\frac{\partial \mathcal{L}}{\partial a}(a, b) = 196a + 32b - 440$

With respect to b:

$\frac{\partial \mathcal{L}}{\partial b}(a, b) = -2(1 - (0a + b)) + -2(8 - (3a + b)) + -2(12 - (5a + b)) + -2(17 - (8a + b))$

$\frac{\partial \mathcal{L}}{\partial b}(a, b) = -2(1 - b) - 2(-3a - b + 8) - 2(-5a - b + 12) - 2(-8a - b + 17)$

$\frac{\partial \mathcal{L}}{\partial b}(a, b) = 2b - 2 + 6a + 2b - 16 + 10a + 2b - 24 + 16a + 2b - 34$

$\frac{\partial \mathcal{L}}{\partial b}(a, b) = 8b - 76 + 32a$

Setting the partial derivatives to zero:

$196a + 32b - 440 = 0 \quad \text{and} \quad 8b - 76 + 32a = 0$

So $b = 9.5 - 4a$

Hence, $196a + 32(9.5 - 4a) - 440 = 0 \implies 196a + 304 - 128a - 440 = 0 \implies 68a = 136 \implies a = 2$

Finally, $b = 9.5 - 4(2) = 1.5$

Hence, the line of best fit is $y = 2x + 1.5$