Chapter 01: Introduction

Lesson 01
Evolution Computers Part 1: Mechanical Systems, Babbage method of Finite Difference Engine and Turing Hypothesis

Objective

- Understand how mechanical computation systems evolved
- Babbage Difference Engine
- Turing Hypothesis

Mechanical Systems

- 16th Century
- Gears, Handles and levers based systems
- Based on Concept pioneered by Pascal
- Addition of decimal numbers
- Subtraction of decimal numbers
- Carry to Left Concept

Mechanical Systems

- Based on concept pioneered by Leibniz—Added, subtracted, multiplied, and divided decimal numbers

Gear Box and Carry left concept

- 100 teeth/360° with a decimal number mark (0, 1, …9) at each 3.6° at each tooth
- 10 teeth/360° with a decimal number mark (0, 1, …9) at each 36° at each tooth
- When first rotates by 10 teeth (3.6° each) the second rotates by 1 tooth (36°)
- Each tooth has a decimal number mark number at 36°

Mechanical system based on Gear Box

- The rotation either clockwise or anticlockwise and a gear ratio of 10
- Allows the mechanical system to either carry to left during a decimal addition
- Borrow from left during subtraction

Babbage's Multi-step Programmable Computer

- Concept pioneered by Babbage
- 19th Century (Difference Engine)
- Multiple steps of adding to arrive at a result
- Application─ A machine generated and printed tables of equally spaced numbers by a method of finite differences
- Easy to implement with mechanical gears and levers

Method of finite Difference

- Finding table in which each number $x = i \times x_0$; $i = 1, 2, \ldots$
- Difference between each number is finite and difference $\Delta = i.\ x_0 - (i - 1).\ x_0 = x_0$ when $i = 1, 2, 3, \ldots$
- Store initial value x_0 in first register
- Store difference x_0 in second register

Method of finite Difference

- Step 1: Add first with second. Answer in first register = x + Δ
- Repeat Step 1: Add first with second Answer = x + 2 Δ
- Repeat Step 1: Answer = x + 3 Δ
- Repeat Step 1: Answer = x + 4 Δ
- Repeat Step 1: Answer = x + 5 Δ

Cycles with one step	1^{st} Initial Value	2^{nd} Initial Value	1^{st} Initial Value	2^{nd} Initial Value	$x = i \times 3$
First Register	3	6	9	12	
Second Register	3	3	3	3	

Finite Difference

Method of finite Difference

- Finding table in which each number = $n_0 + i \times n$; $i = 1, 2, \ldots$
- Difference between each number is finite and difference $\Delta = i.\ n - (i - 1).\ n = n$ when $i = 1, 2, 3, \ldots$
- Store initial value n_0 in first register
- Store difference n in second register

Method of finite Difference

- Step 1: Add first with second. Answer in first register = $n_0 + n$
- Repeat Step 1: Add first with second Answer = $n_0 + 2\ n$
- Repeat Step 1: Answer = $n_0 + 3\ n$
- Repeat Step 1: Answer = $n_0 + 4\ n$
- Repeat Step 1: Answer = $n_0 + 5\ n$

Cycles with one step	1st Initial Value	2nd Initial Value	1st Initial Value	2nd Initial Value
R1	2000	2200	2400	2600
R2	200	200	200	200
R3	0	0	0	0
R3	0	0	0	0

Finite Differences →

Finite Differences →

Finite Differences →

$x = n0 + i \times n$

Method of finite Difference

- Finding table in which each number = i^2; i = 1, 2, …
- Differentiate i^2 with respect to i. $\partial(i^2)/\partial i$ Answer is 2. i
- Differentiate 2i with respect to i. $\partial(2i)/\partial i$ Answer is 2
- Difference between third and second registers is finite and difference $\Delta_{23} = 2.\ i - 2.\ (i - 1) = 2$ in each successive step when i= 1, 2, 3, …

Method of finite Difference

- Hence Store finite difference $\Delta_{23} = 2$ in third register
- Difference between first and second registers is finite and difference $\Delta_{12} = i^2 - (i - 1)^2 = i^2 - (i^2 - 2\,i + 1) = 2\,i - 1$ in each successive step when i= 1, 2, 3, …
- Store initial value R1 = 0 in first register
- Store initial difference value $\Delta_{12} = 2 \times 1 - 1 = 1$ in second register for $i = 1$

Method of finite Difference

- Step 1: Add third with second. Answer in second register = $\Delta_{23} + \Delta_{12.} = 2 + 1 = 3$
- Step 2: Add second with first Answer in first register = $0 + \Delta_{12} = 0 + 1 = 1 = 1^2$
- Repeat Steps 1 and 2 : Add second with first Answer in first register = $\Delta_{23} + \Delta_{12} + 1^2 = 2 + 1 + 1 = 4 = 2^2$
- Repeat Steps 1 and 2: Answer in first register = $2 + 3 + 4 = 9 = 3^2$

Method of finite Difference

- Repeat Steps 1 and 2: Answer in first register = $2 + 5 + 9 = 4^2$
- Repeat Steps 1 and 2: Answer in first register = $2 + 7 + 16 = 5^2$
- Repeat Steps 1 and 2: Answer in first register = $2 + 9 + 25 = 6^2$

Cycles with two steps	1st Initial Value	2nd Initial Value	1st Initial Value	2nd Initial Value
R1	0	1	4	9
R2	1	3	5	7
R3	2	2	2	2
R3	0	0	0	0

Finite Differences →

Finite Differences →

Finite Differences →

$x = i^2$

Method of finite Difference

- Finding table in which each number = i^3; i = 1, 2, …
- Differentiate i^3 with respect to i. $\partial(i^3)/\partial i$ Answer is $3.\ i^2$
- Differentiate $3.\ i^2$ with respect to i. $\partial(3.\ i^2)/\partial i$ Answer is 6 i
- Differentiate 6 i with respect to i. $\partial(6\ i)/\partial i$ Answer = 6

Method of finite Difference

- Difference between fourth and third registers is finite and difference $\Delta_{34} = 6.\ i - 6.\ (i - 1) = 6$ in each successive step when i= 1, 2, 3, …
- Difference between third and second registers is finite and difference $\Delta_{12} = 6.\ (i\ )^2 - 6.(i - 1)^2 = 6.i^2 - 6.(i^2 - 2\ i + 1) = 6\ i - 6$ in each successive step when i= 1, 2, 3, …

Cycles with three steps	1st Initial Value	2nd Initial Value	1st Initial Value	2nd Initial Value
R1	1	8	27	64
R2	1	7	19	37
	Finite Differences →			
R3	0	6	12	18
	Finite Differences →			
R3	6	6	6	6
	Finite Differences →			

$x = i^3$

Turing's Hypothesis

- **Alan Turing (1937)**
- Every computation can be performed by some Turing machine
- Turing Machine that adds
- $T_{add}(a, b) = a + b$
- Turing Machine that multiplies
- $T_{mul}(a, b) = a \times b$

What is a Turing Machine?

- A mathematical model of a device that can perform any computation
- Which Writes/Reads symbols on an infinite "tape"
- Performs state transitions, based on current state and symbol

What is universal Turing machine?

- A Turing machine that could implement all other Turing machines
- Which takes in inputs the data, plus a description of computation

Universal Turing Machine

- Programmable — Instructions are part of the input data
- A Universal Turing Machine can emulate a computer
- A computer can emulate a Universal Turing Machine

Universal Turing Machine

- Do any computations
- Therefore, a computer is also a universal computing device

A Universal Computing Device

- All computers, given enough memory and time are capable of computing exactly the same things

Summary

We learnt

- Mechanical Systems — 16th Century
  Gears, Handles, and lever based systems
- Concept pioneered by Babbage —
- Multiple steps of Addition to arrive at the result
- Every computation can be performed by some Turing machine.
- A universal Turing machine can perform any computation provided given enough memory and time

End of Lesson 01

Evolution Computers Part 1: Mechanical Systems, Babbage method of Finite Difference Engine and Turing Hypothesis