Vector Spaces
(A003)

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1 Introduction

A vector space is a collection of objects called **vectors** that can be scaled and added together in a linear way. It is therefore a linear space. If $a \in \mathbb{R}$ then a real vector space is defined. If $a \in \mathbb{C}$ then a complex vector space exists. The scaling and addition rules for a vector space form a set of axioms. Roughly speaking, the number of independent directoins is the dimension of the vector space.

2 Operations

$$+ : V \times V \to V \tag{1}$$

Equation 1 defines the addition rule where $\times$ is the cartesian production [1] where V is a set. With a field F we have.

$$\cdot : F \times V \to V \tag{2}$$

Where $\cdot$ represents scalar multiplication. If F is a complex field then we have a complex vector space. If F is a real number field then we have a real vector space.

$$\text{Associativity of addition: } \hat{u} + (\hat{v} + \hat{w}) = (\hat{u} + \hat{v}) + \hat{w} \tag{3}$$

$$\text{Commutativity of addition: } \hat{u} + \hat{v} = \hat{v} + \hat{u} \tag{4}$$

3 Axioms

For the set V to form a vector space the operations $+$ and $\cdot$ must conform to the following axioms:

References

[1] P. Valentine, *Cartesian Products.* Paul Valentine, 2020.