Cartesian Products
(A002)

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1 Cartesian Products

1.1 General

Let A and B be two sets. The cartesian product is defined by:

$$AxB = (a, b) \mid a \in A, b \in B \tag{1}$$

It is the multiplicatoin of two sets to form a set of **ordered** pairs. For example if:

$$A = \{jo, pip\}$$
$$B = \{car, house\}$$
$$A \times B = \{(jo, car), (jo, house), (pip, car), (pip, house)\}$$

A practical exmaple is to let X be the set of points on the x line and Y be the set of points on the y line. Then $X \times Y$ prepresents the points on the XY plane.

We can therefore say for n number of $\mathbb{R}$:

$$\underbrace{\mathbb{R} \times \mathbb{R} \times \cdots \times \mathbb{R}}_{\text{n times}} = \mathbb{R}^n \tag{2}$$

1.2 Empty Sets

The result of multiplying by the empty set is the empty set.

$$\mathbb{R} \times \varnothing = \varnothing \tag{3}$$

1.3 Non-commutativity and non-associativity

$$A \times B \neq B \times A \tag{4}$$

Unless $A = B$ or either A or B is the empty set.

$$(A \times B) \times C \neq A \times (B \times C) \tag{5}$$

Unless A, B or $C = \varnothing$.