Cartesian Products
(A002)

Paul Valentine

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Abstract

Cartesian products defin ordered sets of elements. Their relevance crops up in studies of topology and topological manifolds in physics. To describe motion a curve in the manifold M is paramaterised by $\lambda \in \mathbb{R} \mid \gamma : \mathbb{R} \to M, \exists x : M \to \mathbb{R}^n \implies x \circ \gamma : \mathbb{R} \to \mathbb{R}^n$. Where such coorindate maps exist on a topological space we can define mathematics by conidering only the mapping acros the Reals. In doing so the concept of space arises which is formed from the cartesian product of base space with, for example, a tangent space. This requires a basic understanding of the definition of a cartesian product.

1 Cartesian Products

1.1 General

Let A and B be two sets. The cartesian product is defined by:

$$A \times B = (a, b) \mid a \in A, b \in B \tag{1}$$

It is the multiplicatoin of two sets to form a set of **ordered** pairs. For example if:

$$\begin{aligned}
&A = \{jo, pip\} \\
&B = \{car, house\} \\
&A \times B = \{(jo, car), (jo, house), (pip, car), (pip, house)\}
\end{aligned}$$

The pair is ordered becase

$$\{(jo, car), (jo, house), (pip, car), (pip, house)\} \neq \{(car, jo), (house, jo), (car, pip), (house, pip)\}$$

A practical exmaple is to let X be the set of points on the x line and Y be the set of points on the y line. Then $X \times Y$ prepresents the points on the XY plane.

We can therefore say for n number of $\mathbb{R}$:

$$\underbrace{\mathbb{R} \times \mathbb{R} \times \cdots \times \mathbb{R}}_{\text{n times}} = \mathbb{R}^n \tag{2}$$

1.2 Empty Sets

The result of multiplying by the empty set is the empty set.

$$\mathbb{R} \times \varnothing = \varnothing \tag{3}$$

1.3 Non-commutativity and non-associativity

$$A \times B \neq B \times A \tag{4}$$

Unless $A = B$ or either A or B is the empty set.

$$(A \times B) \times C \neq A \times (B \times C) \tag{5}$$

Unless A, B or $C = \varnothing$.

□