

Cartesian Products

(A002)

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1 Cartesian Products

1.1 General

Let A and B be two sets. The cartesian product is defined by:

$$A \times B = \{(a, b) \mid a \in A, b \in B\} \quad (1)$$

It is the multiplicatoin of two sets to form a set of **ordered** pairs. For example if:

$$\begin{aligned} A &= \{jo, pip\} \\ B &= \{car, house\} \\ A \times B &= \{(jo, car), (jo, house), (pip, car), (pip, house)\} \end{aligned}$$

A practical exmaple is to let X be the set of points on the x line and Y be the set of points on the y line. Then $X \times Y$ prepresents the points on the XY plane.

We can therefore say for n number of \mathbb{R} :

$$\underbrace{\mathbb{R} \times \mathbb{R} \times \cdots \times \mathbb{R}}_{n \text{ times}} = \mathbb{R}^n \quad (2)$$

1.2 Empty Sets

The result of multiplying by the empty set is the empty set.

$$\mathbb{R} \times \emptyset = \emptyset \quad (3)$$

1.3 Non-commutativity and non-associativity

$$A \times B \neq B \times A \quad (4)$$

Unless $A = B$ or either A or B is the empty set.

$$(A \times B) \times C \neq A \times (B \times C) \quad (5)$$

Unless A, B or $C = \emptyset$.