INDENG 290: Problem Set 1

Due: September 8, 2023 at 3 pm PST

Homework Collaboration Policy.
Please feel free to discuss the homework problems in groups if you prefer, however, you must write/code you solutions independently.

1. **Problem 1 (10 pts).** Consider the following limit order book for FinTrade stock at time t. The last trade in the stock occurred at a price of $50.

Limit Buy Orders		Limit Sell Orders	
Price	Volume	Price	Volume
$49.75	500	$50.25	100
$49.50	800	$51.50	100
$49.25	500	$54.75	300
$49.00	200	$58.25	100
$48.50	600		

Write down what limit order book for Fintrade stock will look after each of the following events:

   (a) At time $t + 1$, market order to buy for 100 shares comes in (2 pts).

   (b) At time $t + 2$, limit order to buy 50 shares at $49.25 comes in (2 pts).

   (c) At time $t + 3$, limit order to buy 200 shares at $50 comes in (2 pts).

   (d) At time $t + 4$, market order to sell 100 shares comes in (2 pts).

   (e) At time $t + 5$, limit order to sell 100 shares at $55 comes in (2 pts).

2. **Problem 2 (20 pts).** Suppose I play a game where I toss a fair coin over and over and record the values in a list ('H' for head, and 'T' for tail). I am interested in finding specific sequences in the list, such as 'HT', which would occur on the nth roll if the nth roll is 'T', and the $(n-1)$st roll is 'H'. Find the expected number of coin tosses until the first occurrence of sequence 'HHH'.

3. **Problem 3 (20 pts).** Suppose I play a game where I roll a six-sided die over and over and record the values in a list. I am interested in finding specific pairs of numbers in the list, such as 23, which would occur on the nth roll if the nth roll is 3, and the $(n-1)$st roll is 2. Let X be the number of rolls required to observe 23 and let Y be the number of rolls required to observe 22. That is, X is the smallest n for which 23 occurs on roll n, and Y is the smallest n for which 22 occurs on roll m.

   (a) Find the expected value $E(X)$ (8 pts).

   (b) Find the expected value $E(Y)$ (8 pts).

   (c) Compare you answers in (a) and (b) and explain the result (4 pts).

4. **Problem 4 (50 pts).** Download sample orderbook data for order book level 1 for AMZN, AAPL, GOOG, INTC, MSFT from LOBSTER. Once you unzip the downloaded files, details on how to interpret this data can be located in LOBSTER_SampleFiles_ReadMe.txt. By definition hidden order placements and cancellations are not communicated to other market participants. Their execution is however reported to give market participants accurate knowledge of volumes traded. For the purpose of this exercise you can assume that hidden limit limit orders are executed via market orders with the opposite interest (i.e. hidden orders to buy are matched

with market orders to sell). A lot of useful information about processing this dataset can be found on LOBSTER website – please explore.

Tick time moves every time a limit order book event occurs, whereas *physical time* moves every fixed time interval (e.g, the distance between two order book ticks can be greater or less than one second).

Please complete all coding in Python.

(a) Plot midprices for AMZN, APPL, GOOGL, INTC, MSFT as a function of physical time. Now repeat in tick time (10 pts).

(b) Plot cumulative executed volumes for these stocks as a function of physical time. Now repeat in tick time. On these plots mark occurrence of individual trades (10 pts).

(c) Plot observed spreads for these stocks as a function of physical time. Now repeat in tick time. On these plots mark occurrence of individual trades (10 pts).

(d) Plot 10-sec and 1-min midprice returns and their distribution histograms. For these quantities, plot distribution of linear autocorrelations of lag 1 and 5 (5 pts).

(e) Plot 5-tick and 10-tick midprice returns and their distribution histograms. For these quantities, plot distribution of autocorrelations of lag 1 and 5 (5 pts).

(f) Comment on the observed stylized facts - please use the list of stylized facts from the "Get Real" paper that we discussed in class (5 pts).

(g) Describe your observations for working in physical time vs. tick time. When, in your opinion, microstructure models in physical time are preferred to tick time, and other way around (5 pts).