Hierarchical Linear Models with PyMC3

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Metis - Chicago -2018

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Hierarchical Modeling

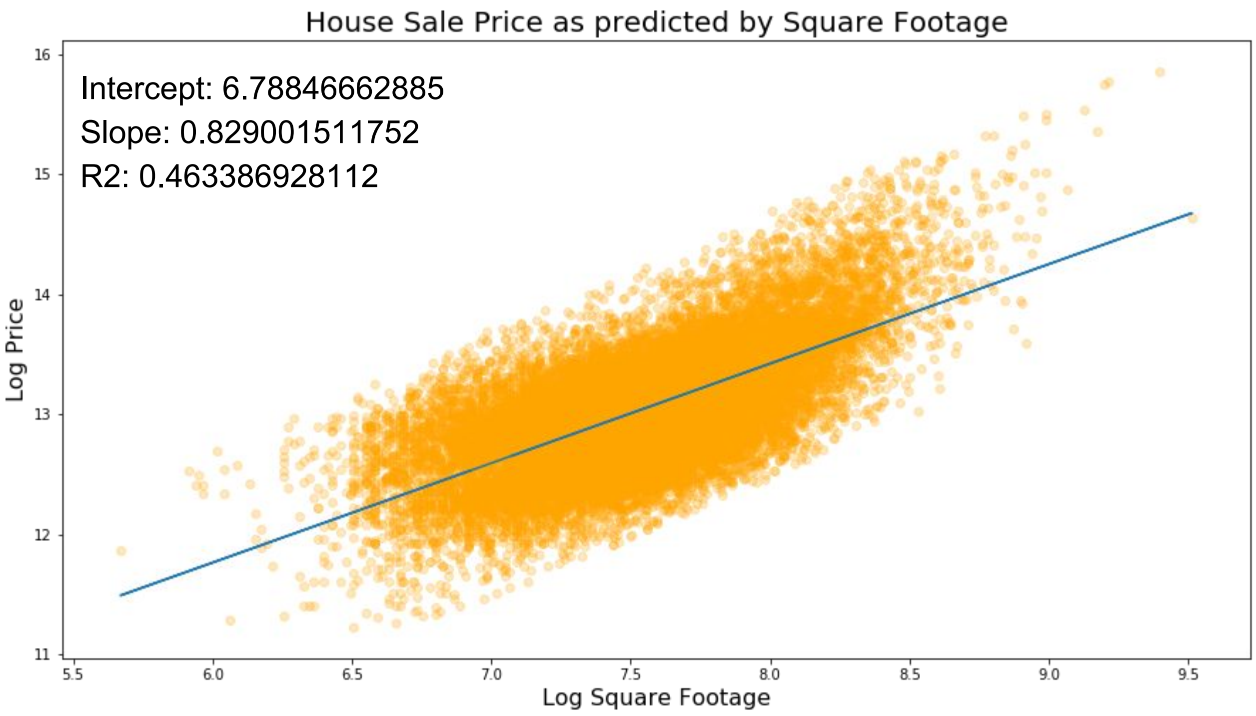
Hierarchical modeling is a tool of probabilistic programming and Bayesian statistics which predicts the parameters of a posterior distribution for data which can be observed on multiple levels.

It allows for between group varying slope and varying intercept bounding the parameters between complete and no pooled alternatives.

Case Study: Historical Housing Sale Data in Seattle

$$y = a + bx$$

	price	sqft_living	zipcode
0	221900	1180	98178
1	538000	2570	98125
2	180000	770	98028
3	604000	1960	98136
4	510000	1680	98074
5	1225000	5420	98053
6	257500	1715	98003
7	291850	1060	98198
8	229500	1780	98146
9	323000	1890	98038
10	662500	3560	98007
11	468000	1160	98115



Data source:
https://www.coursera.org/learn/ml-foundations/supplement/RP8te/reading-predicting-house-prices-assignment

Case Study: Historical Housing Sale Data in Seattle

What are the three most important things in real estate?

- Location
- Location
- Location

By creating zip code indicators we can move from a single featured line:

$$y = a + bx$$

to

$$y = a + b_1 x_1 + b_i x_i$$

	price	sqft_living	zipcode_98002	zipcode_98003	zipcode_98004	zipcode_98005	zipcode_98006
0	201000	900	1	0	0	0	0
1	300000	1984	1	0	0	0	0
2	142500	690	1	0	0	0	0
3	125000	920	1	0	0	0	0
4	213500	1220	1	0	0	0	0

Data source:
https://www.coursera.org/learn/ml-foundations/supplement/RP8te/reading-predicting-house-prices-assignment

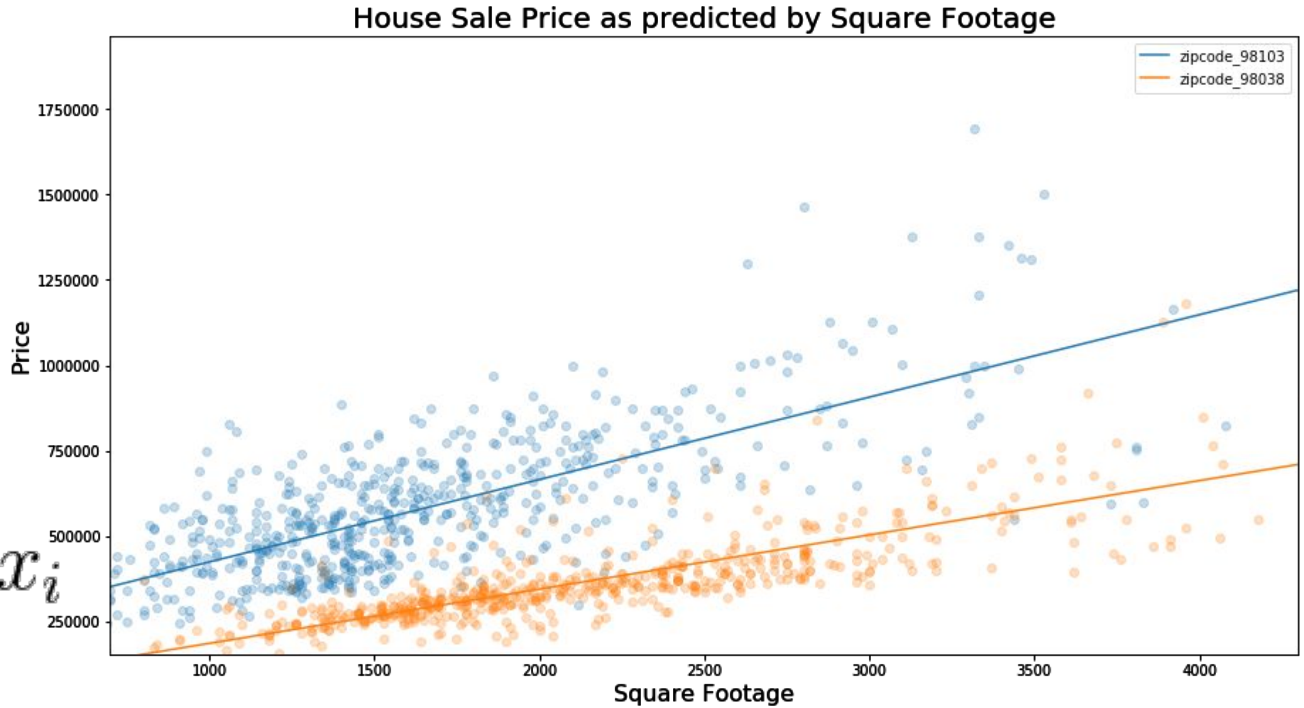
Case Study: Historical Housing Sale Data in Seattle

Varying Intercept

$$y = a + b_1x_1 + b_ix_i$$

Varying Intercept
Varying Slope

$$y = a + b_1x_1 + b_{i-1}x_1x_i + b_ix_i$$



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Pooling: Complete, no, and partial pooling

In the varying-slope varying-intercept, what would happen if we predicted on a new zip code?

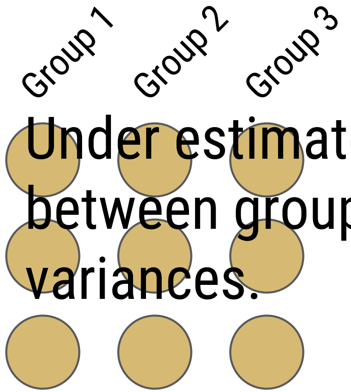
$$y = a + b_1x_1 + b_2x_2 + b_3x_3 \ldots b_ix_i$$

What if we had a zip code that lacked fully representative data? I.e. imagine we were predicting the effect of waterfront on price, but had coastal zip codes without waterfront property records

Pooling: Complete, no, and partial pooling

Complete Pooling

All groups are given the same slope and intercept.



Under estimates between group variances.

No Pooling

All groups are allowed to have unique slopes and intercepts.

Over estimates between group variances.

Partial Pooling

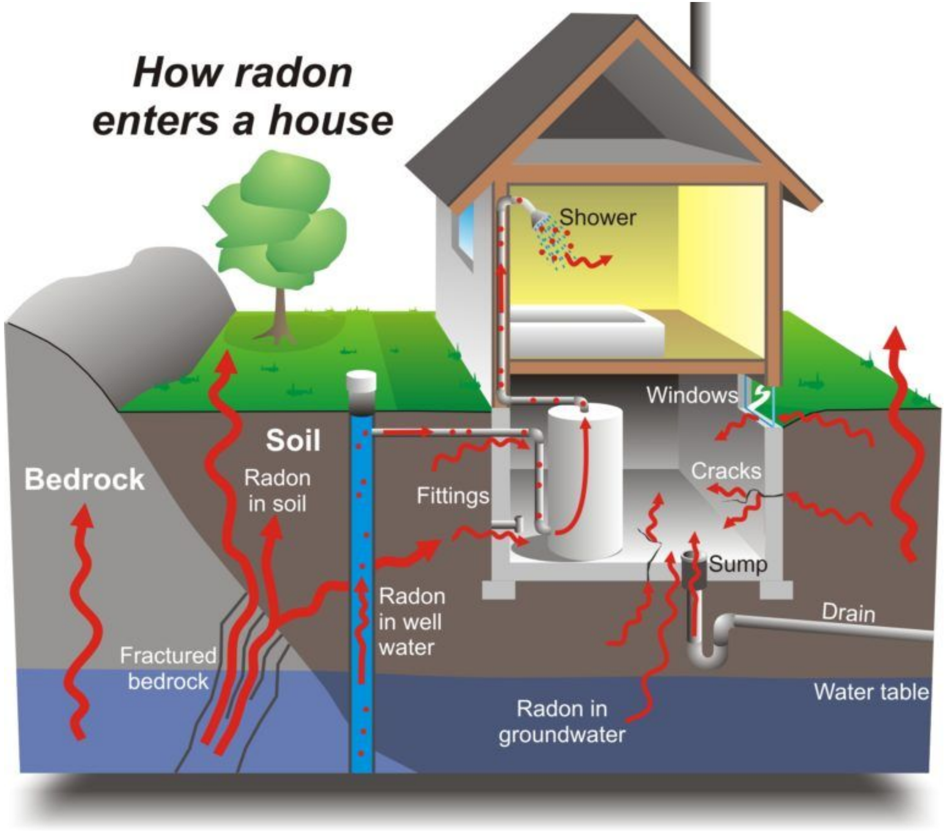
The slopes and intercepts are related, but allowed to vary. They are assumed to come from a distribution of betas.

Bound between the extremes of complete and no pooling, pulling betas towards the mean.

Case Study: Radon Prediction

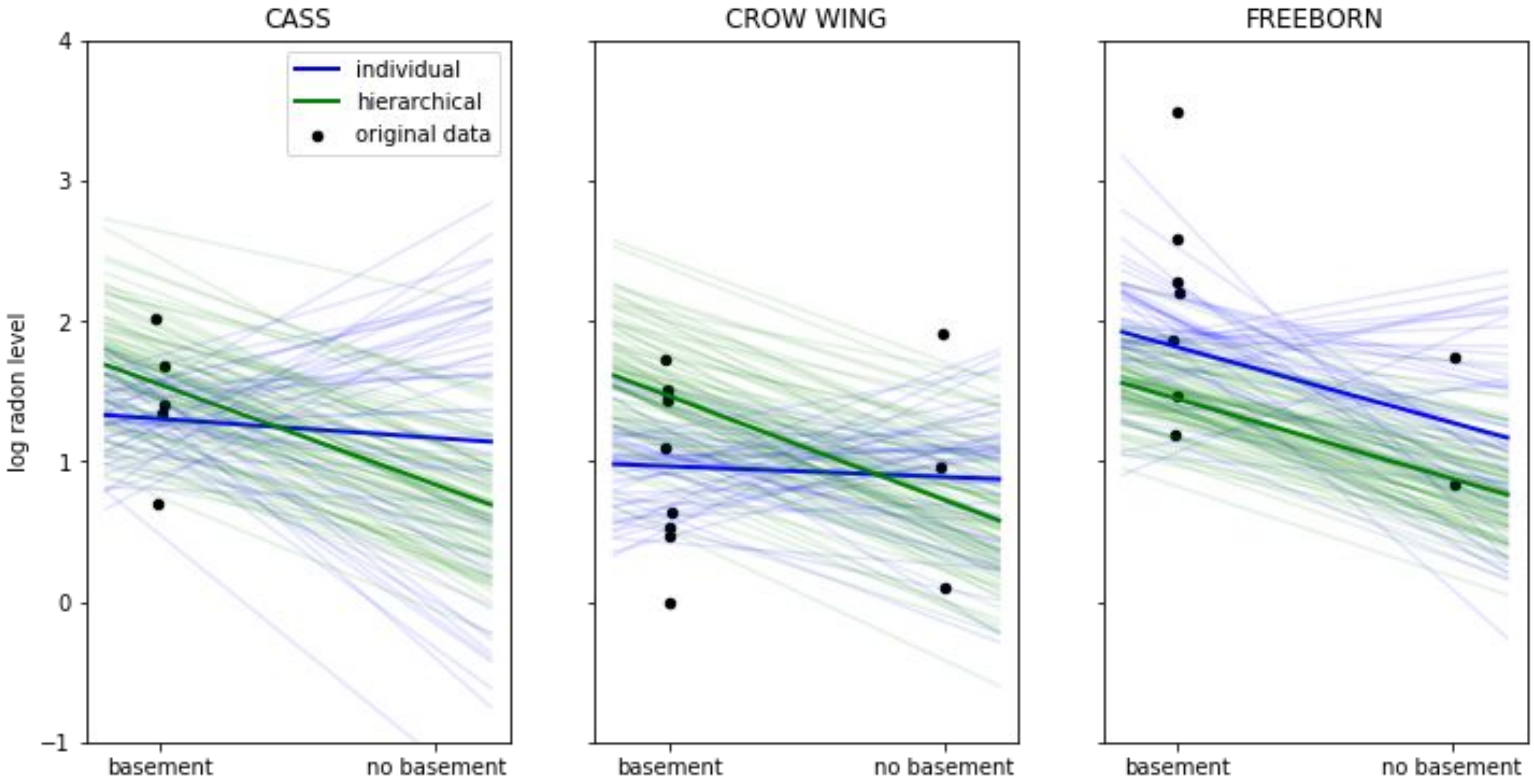
Radon is a toxic gas that seeps into homes from the ground and is the number one cause of lung cancer outside of smoking.

Radon levels were measured across counties in Minnesota with a field denoting which floor the measurement was taken.
Not all counties had records with basement measurements



http://twiecki.github.io/blog/2014/03/17/bayesian-glms-3/

Case Study: Radon Prediction



Making your model hierarchical

$$y = \alpha + \beta\chi$$

Simple Model

$$y = \alpha_j + \beta\chi$$

Varying intercept

$$y = \alpha_j + \beta_j\chi$$

Varying Slope - Varying Intercept

$$y \sim N\left(\alpha_j + \chi\beta_j, \sigma_y^2\right) \begin{pmatrix} \alpha_j \\ \beta_j \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_\alpha \\ \mu_\beta \end{pmatrix}, \begin{pmatrix} \sigma_\alpha^2 \\ \sigma_\beta^2 \end{pmatrix}\right)$$

Probabilistic Models in three steps

1. Posit priors and declare likelihood estimator

2. Infer values for latent variables

3. Check your model

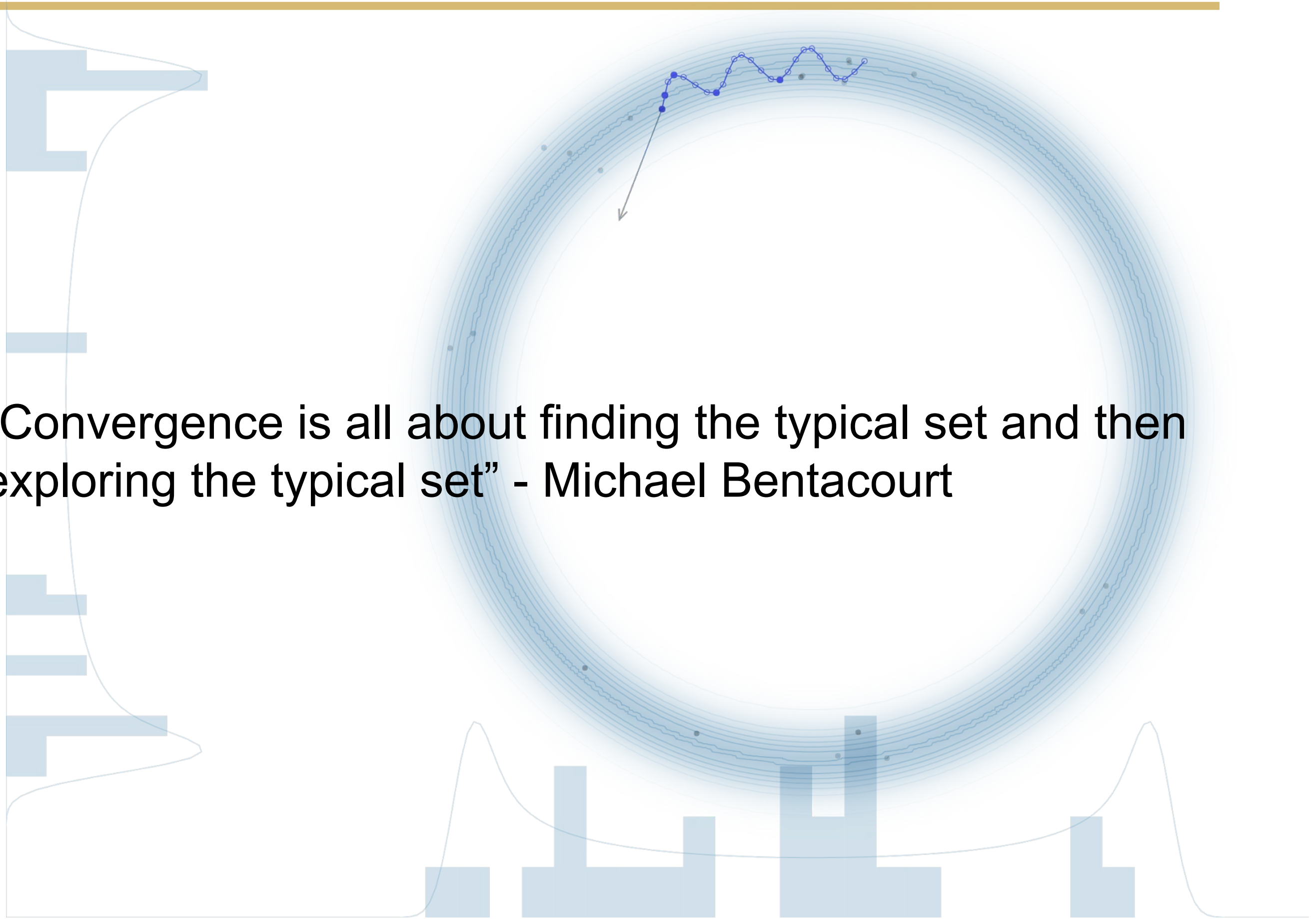
Posit priors and declare likelihood estimator

```
lr = pm.Model()

with lr:

    alpha = pm.Normal('alpha', mu=0, sd=10e4, shape=(1))
    betas = pm.Normal('betas', mu=0, sd=10e4, shape=(1,len(X_cols)))
    sigma = pm.HalfNormal('sigma', sd=10e4)

    temp = alpha + T.dot(model_input, betas.T)

    y = pm.Lognormal('y', mu=temp , sd=sigma, observed=model_output)
```

Non-informative priors can be used when no prior information is available, but all possibilities must be represented.

PyMC3

Infer Values for Latent Variables



"Convergence is all about finding the typical set and then exploring the typical set" - Michael Bentacourt

[No U-Turn Sampler]

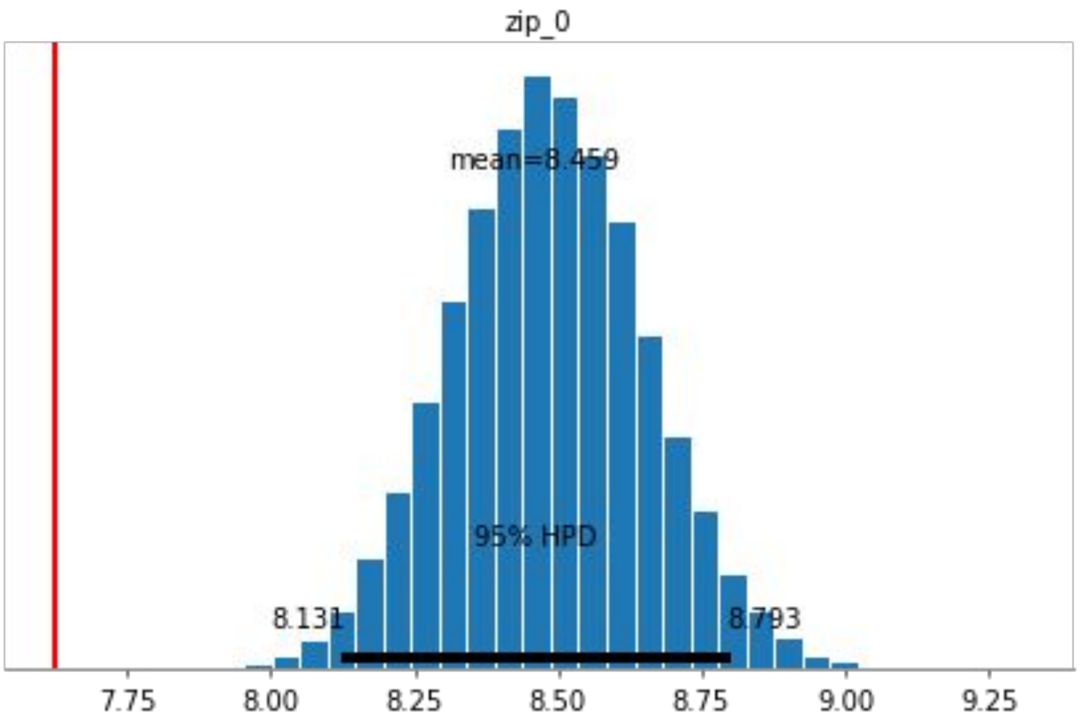
A Hamiltonian Monte Carlo Markov Chain sampler for efficient sampling

Check your Model

Is your model sensitive to initialization of new priors?

Has your model converged?

Could your actual data have come from your posterior?



Recap

1. A hierarchical model is a probabilistic model which allows for varying slope and intercepts that come from a beta distribution based on group indicators.
2. Probabilistic can be generated using priors and a likelihood estimator.

3. When there is no prior knowledge a non-informative prior can be used.

4. Model health can be assessed through comparing the posterior distribution to actual data and from the sampling traceplot.

Putting it all Together

PyMC3 and Hierarchical models in practice