Breadth Momentum and the Canary Universe: Defensive Asset Allocation (DAA)

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Abstract

We improve on our Vigilant Asset Allocation (VAA) by the introduction of a separate "canary" universe for signaling the need for crash protection, using the concept of breadth momentum. The amount of cash is now governed by the number of canary assets with bad (non-positive) momentum. The risky part is still based on relative momentum (or relative strength), just like VAA. We call this strategy Defensive Assets Allocation (DAA). The aim of DAA is to lower the average cash (or bond) fraction while keeping nearly the same degree of crash protection as with VAA. Using a very simple model from Dec 1926 to Dec 1970 with only the SP500 index as risky asset, we find an optimal canary universe of VWO and BND (aka EEM and AGG), which turns out to be rather effective also for nearly all our VAA universes, from Dec 1970 to Mar 2018. The average cash fraction of DAA is often less than half that of VAA's, while return and risk are similar and for recent years even better. The usage of a separate "canary" universe for signaling the need for crash protection also improves the tracking error with respect to the passive (buy-and-hold) benchmark and limits turnover.

1. Introducing DAA

In our Vigilant Asset Allocation (VAA, see Keller, 2017), we have shown that we could improve on the traditional crash protection (using trend following or absolute momentum per asset), by focusing on the so-called breadth momentum of the risky universe. By defining good (bad) assets as assets with (non) positive momentum, we define breadth momentum in VAA as the number of risky assets with good (positive) momentum. The strategy switches to cash (or low-risk bonds) with decreasing breadth momentum. So, the more bad assets, the higher the cash fraction.

Also, we use relative momentum in VAA by restricting the number of assets in our portfolio to the top performing assets, where relative performance is measured by the same momentum rule as for absolute or breadth momentum. In VAA we used a fast momentum filter (called 13612W momentum[3], see Keller 2017). We also used the same fast momentum filter for choosing the best bond in our "cash" universe when there is non-zero crash protection. The top T risky assets are selected long-only, equal weight (EW) for both VAA and DAA, as is traditionally done in relative strength strategies as in Faber, 2010. More literature on relative and absolute or intrinsic momentum (aka trendfollowing) can be found in Appendix B.

For VAA, parameters like T are determined in-sample (IS: eg. from Dec 1970 – Dec 1993) while results are evaluated out-of-sample (OS: eg. from Dec 1993 to today). This defines VAA (see also Keller, 2017).

One of the disadvantages of our VAA approach is the high average cash fraction (CF), often well above 50% for all four risky VAA universes under consideration (see Keller, 2017). The present paper (and its DAA strategy) aims at reducing this cash fraction substantially. The higher the cash fraction, the more often crash protection is active and the more our portfolio becomes bond-like with a heavy bond (or cash) portion. This is beneficial for eg. the seventies when the bond yields where high, but today that is not preferable in view

[1] We thank Denis Bergemann, Jos van den Berkmortel, Winfried Hallerbach, Steve LeCompte, and Bas Nagtzaam for useful comments on an earlier draft. All errors are ours.

[2] Besides correcting some small errors, we extended section 9 on DAA "aggressive" in the revised version v1.1.

[3] The 13612W momentum is the average 1, 3, 6, and 12 month lagged return, each weighted to annual return (so with weights 12x, 4x, 2x,and 1x, respectively) .

of the low bond yields. An effective lower cash fraction also lowers the risk on false positives[4] in our crash protection. In other words, our hit rate for crash prediction might improve.

Also, the tracking error with respect to the passive (buy-and-hold) risky portfolio is larger, the bigger the cash fraction becomes. This might lead to long periods (like 2003 – 2007) where VAA deviates negatively from its benchmark, which is not attractive for most (in particular short-term) investors. Therefore, we designed DAA to aim at (very) low cash fractions, while at the same time being nearly as defensive as VAA in terms of drawdown, using a separate "canary" (or protective) universe to quantify breadth momentum. We call this the canary universe since its role is similar to the canary in the coal mines (see Smithsonian, 2016).

The concept of a separate canary universe for quantifying breadth momentum also implies that we can arrive at nearly any cash fraction we want by just reducing or enlarging the number of assets in our canary (or protective) universe in combination with the breadth parameter B (see Keller, 2017), both independent of the risky and cash universes.

Our VAA strategy is then a special case of DAA with the protective or canary universe equal to the risky universe, along with the same cash universe. In other words, with VAA the level of crash protection was determined by the breadth momentum of the risky universe, while with DAA it is determined by the breadth momentum of the separate protective or "canary" universe. For both, momentum is measured by our fast 13612W momentum.

In Section 2 we focus on DAA in more detail. In Section 3, we search for the canary universe using data before 1971. We test the resulting canary universe in Sections 4, 5, 6 and 7 with the risky and cash universe of VAA-G12, -U6, U15, and -G4. In Section 8 we examine the additional effect of the crash protection of the (breadth of the) canary universe compared to EW, relative and absolute momentum for DAA-G12. In Section 9 we look at an aggressive variant with B=1 and therefore faster crash protection. We conclude in Section 10 with a summary of DAA, and some analysis of why the canary concept might work.

2. DAA in more detail

So, for DAA we distinguish three universes: Risky (denoted eg. R12 when there are N=NR=12 risky assets), Protective (eg. P2, with NP=2 canary assets), and Cash (eg. C3 for NC=3 cash/bonds assets).

Like VAA, there are two free parameters ie. the (risky) top T and the breadth parameter B (which determines the cash fraction, given the canary breadth). The cash fraction CF equals CF= b/B (max 100%), where b is related to the canary breadth: b is the number of bad canary assets (bad: with non-positive 13612W momentum). So with eg. B=2, we have CF=0, 50%, 100% for b=0,1,2+, respectively.

DAA (and VAA) without its crash protection is simply equal weight (EW) asset rotation based on relative strength (aka relative momentum) of the top T risky assets (short: EW-T). The selection of the top T risky and the single cash/bond asset (if CF>0) are also based on fast (13612W) momentum. We will use Easy Trading

[4] A false positive is an error in data reporting in which a test result improperly indicates presence of a condition, such as a disease (the test result is positive), when in reality it is not.

(ET, see Keller 2017), so when eg. CF=50% and B=2, we only select the top T/2 risky assets, so eg. Top 3 when T=6.[5]

Together with the new canary universe for DAA, we will also introduce a more defensive performance indicator with DAA than we used in our VAA strategy. In VAA we introduced the Return Adjusted for Drawdowns (RAD) indicator, here denoted as K50[6], defined as

K50 = R(1-D/(1-D)) when max drawdown D<50% and return R>0%, else K50=0%.

K50 corrects returns (CAGR) R by an amount depending upon the compensating rise in price D/(1-D) after a drawdown D. For example, after a 20% max drawdown one needs a 25% (=D/(1-D)) rise to get back at the max portfolio equity price before the drawdown, so we reduce return by 25% in K50. Notice that K50=0% when D>=50%, therefore we say that K50 has a max drawdown limit of 50%.

We will use a (more defensive) alternative to K50 called K25 for most in-sample optimizations (see also TrendXplorer, 2018). K25 has a (max drawdown) limit of 25%, and is defined as:

K25 = R(1-2D/(1-2D)) when max drawdown D<25% and return R>0%, else K25=0%.

Now a 20% drawdown implies a 67% reduction of return in K25. Since K25 only works when D<25% we are incidentally required (eg. from 1926) to use K50 in DAA for in-sample optimizations when D>25%.

A disadvantage of max drawdown D as risk measurement is that it is depending on extreme values (max drawdown), which makes it (and therefore K) a somewhat tricky statistic[7]. Therefore, we will also show UPI (the Ulcer Performance Index, see Martin, 1987) in our results, which takes the entire drawdown record into account. UPI is similar to the Sharpe Ratio but with the Ulcer index instead of volatility in the nominator and the excess return over riskfree in the denominator.

Besides the separate "canary" (or P) universe, and the more defensive performance indicator K25 as target for our in-sample optimizations, DAA is identical to VAA, including the fast 13612W momentum filter used in VAA, its one-way transactions costs of TC=0.1% and the same Easy Trading formula for CF when T is not a multiple of B.[8]

We will also use the same monthly total-return data (TR) as in VAA (see Keller 2017 and 2016, and TrendXplorer 2017 for details), one by Ibbotson/Morningstar and Fama French (Ibb/ FF for short) from (ultimate) Dec 1925 and one constructed by us from (ultimo) Dec 1969, now all extended until Mar 2018. The latter dataset includes ETF proxies including fees, while the Ibb/FF dataset contains unaltered total return indices. Only the recent years of our database has observed and tradable ETF prices, all the other historical prices are non-tradable proxies.

As always, backtesting will require the first year for initialization of our 13612W momentum, so all backtests start in Dec 1926 or in Dec 1970. Contrary to VAA we will do all DAA backtests from Dec 1970, with in-

[5] When T is even, we have eg. CF=0, 50%, 100% for b=0,1,2 when B=2. When T is uneven (and B=2), Easy Trading will introduce rounding for CF. The full ET formula is CF=(1/T)*rounddown(bT/B), with max(CF)=1 (see Keller 2017). When T is even and B=2 this boils down to CF= b/B (max 100%).

[6] See TrendXplorer (2018) for the explanation of the symbol K.

[7] See eg. Harding, 2003, Magdon-Ismail, 2004 and Goldberg, 2016

[8] See also note 5. We also have added the rule that with T=1, CF is simply b/B, in line with the ET idea, with b the number of bad assets.

sample (IS) optimization over Dec 1970 – Dec 1993, and out-of-sample (OS) tests from Dec 1993 onwards. This is done to avoid datasnooping, since the canary universe is determined from Dec 1926 – Dec 1970.

3. In search of the canary universe

We will use the same protective canary (or P) universe for all our DAA backtests. How do we find this special canary universe? Our search path is inspired by the recent performance of our VAA-G4 strategy (see Keller, 2017). This is shown in Fig. 1 below. As shown, VAA-G4 went 100% to cash at (ultimo) Jan 2018, just before the Feb/Mar 2018 crash of -7% of SPY.

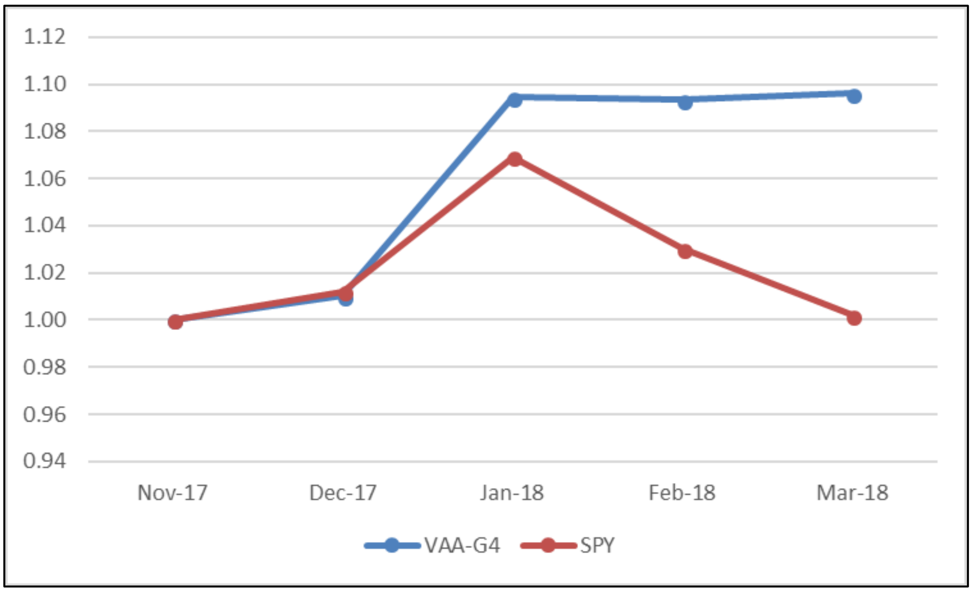

Fig. 1 VAA-G4 vs. SPY from Jan 1, 2018 – Mar 31, 2018

With VAA-G4, the canary universe is equal to the G4 Risky universe (R4= SPY, VEA, VWO, BND[9]) with parameters top T=1 and breadth B=1. In other words, this VAA scenario will always select the single (T=1) best asset (in terms of our 13612W momentum) while it switches from 100% invested to 100% cash when one or more (b>=1 with B=1) of these four assets turns bad (ie. 13612W mom<= 0). Notice that 100% cash is not really cash in our VAA model; the best bond (in terms of our 13612W momentum) from our three-asset Cash universe (C3= SHY, IEF, LQD) serves as cash proxy.

In Fig. 1, G4 goes to cash at ultimo Jan 2018 because BND turns bad. Over 1970-2017, the max drawdown with G4 is as low as 13% (see Keller, 2017). So G4 might be a good start to search for the best canary universe. Noteworthy, G4 also includes BND as a crash predictor, while BND is not very risky. The role of BND as crash predictor was also illustrated by Allocate Smartly (see AS, 2017) with Fig. 2.

As can be seen from Fig. 2, BND (aka AGG at AS) is an effective predictor for bad times in SPY, VEA, and VWO. Also, these three risky assets SPY, VEA, VWO themselves act successful as crash predictor (canary) in

[9] In contrast to VAA, we will use the Vanguard ETFs VEA, VWO, and BND here instead of the iShares ETFs (and symbols) EFA, EEM, and AGG used in VAA. Similar for Vanguard's VGK (Europe), VNQ (US REIT) and SPDR's SPY (SP500) used in VAA/DAA-G12, see Section 4. For the years before 1970 we used the best available index proxies, eg. IT Gov bonds for BND/AGG, while from 1970 we use our own ETF proxies (including fees).

G4, as is shown by the performance of VAA-G4 in terms of D and K since Dec 1970 until Mar 2018 for several subperiods, see Fig. 3.

Asset		Months following bad BND momentum			Months following good BND momentum	
		Annual Return	% Positive		Annual Return	% Positive
US Equites (SPY)		-5.40%	52.90%		13.30%	62.80%
Intl Equities (VEA)		-7.00%	51.90%		13.10%	61.50%
EM Equities (VWO)		-11.70%	46.20%		17.80%	63.90%

Fig. 2 Returns following BND momentum scores, Dec 1971 – Jun 2017 (AS, 2017)

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-70	Dec-93	21.9%	13.0%	18.6%	14.2%	4.13	54.2%
OS=	Dec-93	Mar-18	15.9%	10.5%	14.1%	11.7%	3.88	57.9%
RS=	Mar-08	Mar-18	10.5%	10.5%	9.3%	7.7%	2.37	51.2%
FS=	Dec-70	Mar-18	18.8%	13.0%	16.0%	12.2%	4.00	56.2%
Parms:	**T=**	**1**	**B=**	**1**	**NR=**	**4**	**NP=**	**4**

Fig. 3 VAA-G4, Dec 1970 – Mar 2018[10]

In Fig. 3, shown are In-Sample (IS), Out-of-Sample (OS), Recent (10y) Sample (RS), and Full Sample (FS) periods, and Return (R=CAGR), max (monthly) Drawdown (D), return/risk (K50, K25, UPI, see above), and the Cash (bond) Fraction (CF). In addition, we show the main parameters (Parms in the figure tables), i.e. the number of Top risky assets (T), the breadth parameter (B), Number of Risky (NR=N) and of Protective (NP) assets. Cash for DAA refers to the best bond of the three-asset cash universe (C3=SHY, IEF, LQD), just like we used for VAA (see Keller, 2017).

However, the disadvantage of the G4 combination (with B=1) as canary model is its high cash fraction CF (see Fig. 3), sometimes close to 60% (see OS from Dec 1993). CF decreases with a larger breadth parameter B, as is clear from CF=b/B. It is also easy to see that, for a given B, the CF increases with the number of assets NP in the canary or P-universe. Therefore, we search for a smaller P-universe than G4.

In order to generalize the concept of a canary universe beyond the risky G4 and for the longest period possible, we will now focus on the *smallest* (US) risky universe, i.e. the US large cap ETF SPY on the full sample (FS), from Dec 1926 to Mar 2018 (with the Ibbotson Large Cap total return index before 1970).

What would happen eg. if we now simply took as canary universe P1=BND, based on the analysis in Fig. 2 above? To test, we run it on FS (from Dec 1926 – Mar 2018) with risky R1=SPY, cash C3=SHY, IEF, LQD and P1= BND, with T=1 and B=1 as fixed parameters[11]. We also look at the passive (buy-and-hold) SPY strategy as benchmark, so without any crash protection. The results for SPY, with and without crash protection by this

[10] Legend: IS/OS/RS/FS: In-Sample/Out-of-Sample/Recent Sample/Full Sample period, R/D: return (CAGR)/ max (monthly) drawdown, K50/K25: Return adjusted for Drawdown (with threshold D=50/25%), UPI: Ulcer Performance Index, T/B: Top # of (risky) assets/Breadth parameter, NR/NP: # of Risky/Protection assets; CF= av. Cash Fraction b/B (0<=CF<=100%) with b # of bad assets in canary (protection) universe;

[11] Notice that for DAA/VAA, top T<=NR and breadth B<=NP, so that with NR=NP=1 here, we have T=1, B=1.

P1=BND canary universe, are shown in Fig. 4a and 4b, respectively.

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-26	Dec-70	9.9%	72.9%	0.0%	0.0%	0.41	17.6%
OS=	Dec-70	Mar-18	12.8%	42.6%	3.3%	0.0%	0.88	18.0%
RS=	Mar-08	Mar-18	13.8%	29.7%	7.9%	0.0%	2.08	19.8%
FS=	Dec-26	Mar-18	11.4%	72.9%	0.0%	0.0%	0.53	17.8%
Parms:	**T=**	**1**	**B=**	**1**	**NR=**	**1**	**NP=**	**1**

Fig. 4a R1=SPY and P1=BND, Dec 1926 – Mar 2018 [10]

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-26	Dec-70	9.4%	83.7%	0.0%	0.0%	0.26	0.0%
OS=	Dec-70	Mar-18	10.4%	50.8%	0.0%	0.0%	0.42	0.0%
RS=	Mar-08	Mar-18	9.3%	50.8%	0.0%	0.0%	0.57	0.0%
FS=	Dec-26	Mar-18	9.9%	83.7%	0.0%	0.0%	0.30	0.0%
Parms:	**T=**	**1**	**B=**	**9999**	**NR=**	**1**	**NP=**	**1**

Fig. 4b R1=SPY passive (CF=0), Dec 1926 – Mar 2018 [10]

Although the strategy with P=BND (see Fig. 4a) improves over SPY without cash protection (see Fig. 4b), the effect of the canary universe P1=BND on eg. R, D, K and UPI is limited. Return R on FS (1926-2018) improved from 9.9% to 11.4%, drawdown D from 83.7% to 72.9% and UPI from 0.30 to 0.53. Notice the low CF/FS of 18% for our strategy (and CF=0 for passive SPY) and the improved recent sample (RS) period in Fig. 4a with regards to return R (13.8% vs. 9.3% for passive) and D (29.7% vs. 50.8%).

As a next step, we search for the best canary universe with 1,2,3, or 4 assets (NP=1..4) from G4 (= SPY, VEA, VWO, BND) with risky R1=SPY (so T=1) and cash C3=SHY, IEF, LQD . To prevent datasnooping, we split the Dec 1926 – Mar 2018 full-sample (FS) period into in-sample (IS) Dec 1927 - Dec 1970 and out-of-sample (OS) Dec 1970 – Mar 2018. Now, we search for the best protection assets from G4 by optimizing the return/risk indicator K50[12] on IS. This in-sample optimization will allow us to use the resulting "optimal" canary universe with DAA while avoiding datasnooping risk at the out-of-sample period (OS) from Dec 1970.

So, what is the best (in terms of K50/IS) canary universe from G4= SPY, VEA, VWO, BND? This turns out to be the combination of VWO (aka EEM)[13] and BND (aka AGG). This P2=VWO/BND universe with B=1 gives the best K50 on IS out of all asset combinations out of G4 over Dec 1926-Dec 1970, with K50= 7.1%[14]. The results of this canary universe for various periods (including OS from Dec 1970) is shown in Fig. 4c.

[12] We use K50 since K25=0% for most of the scenarios at IS.

[13] The best single asset canary universe on K50/IS was VWO. We used Meb Faber's backtesting tool (see Faber, 2018) for these tests from Dec 1926.

[14] This combination VWO/BND also gives the best Sharpe Ratio (rf=5%) on IS.

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-26	Dec-70	11.7%	28.0%	7.1%	0.0%	1.45	47.3%
OS=	Dec-70	Mar-18	12.7%	17.5%	10.0%	5.9%	1.76	44.9%
RS=	Mar-08	Mar-18	14.7%	13.3%	12.5%	9.4%	3.86	46.3%
FS=	Dec-26	Mar-18	12.2%	28.0%	7.5%	0.0%	1.55	46.0%
Parms:	**T=**	**1**	**B=**	**1**	**NR=**	**1**	**NP=**	**2**

Fig. 4c. R1=SPY, with P2=VWO, BND, Dec 1926 – Mar 2018 [10]

If we compare Fig. 4c (with P2= VWO, BND) with Fig. 4b (passive SPY, so without crash protection), we see significant improvements on OS for all performance indicators, like R (12.7% vs. 10.4% for passive), D (17.5% vs. 50.8%), K50 (10.0% vs. 0%), K25 (5.9% vs. 0%) and UPI (1.76 vs. 0.42). For IS the results are even better (eg. D= 28.0% vs. 83.7% for passive, see Fig. 4b), but this includes datasnooping for P2 (since we optimized K50/IS). Also note (again) the improvements on recent years (RS: Mar 2008 – Mar 2018): return R (14.7% vs. 9.3% for passive) and max drawdown D (13.3% vs. 50.8%), resulting in much better K50 (12.5% vs. 0%).

Similar improvements on OS (and IS), although smaller, are to be found compared to P1=BND (Fig. 4a), eg. see D (17.5% vs. 42.6% for Fig. 4a), K50 (10.0% vs. 3.3%) and UPI (1.76 vs. 0.88). Although CF is increased from less than 20% in Fig. 4a to 45% at OS in Fig. 4c, CF is still less than that of most VAA strategies in the period after 1970 (see eg. CF/FS= 56% in Fig. 3). Below, we will address this further.

We also look in a more statistical way at this canary universe of VWO/BND. What about the analysis of Fig. 2, now applied to both VWO and BND (aka EEM and AGG) over nearly 100 years? Fig. 5 gives the answer.

	Next month SPY			
	Following bad Canary momentum		**Otherwise**	
Canary Assets	Annual Return	% Positive	Annual Return	% Positive
VWO	3.36%	57.1%	15.36%	64.9%
BND	-4.00%	50.3%	14.56%	64.8%
VWO and BND	-14.17%	47.0%	12.89%	63.2%
VWO or BND	2.81%	55.8%	18.46%	67.7%

Fig. 5 Returns of SPY following bad VWO, BND momentum, Dec 1926 – Mar 2018

As shown in Fig. 5, the number of months where next month SPY returns are positive after bad (13612W) momentum of VWO and/or BND are smaller than when momentum of our canary assets are not bad. For example, when both VWO and BND have bad momentum (b=2), 47% of the months have positive next-month SPY performance and 63% otherwise (b<2). Also, the average next month SPY return after bad VWO and/or BND momentum is also lower than otherwise, in particular when bóth VWO and BND have bad momentum. In this case (b=2), the spread with otherwise (b<2) equals 12.89--14.17= 27.06% on an annual basis.

We conclude that the combination of VWO and BND forms an effective canary universe of merely two assets at both IS and OS and a substantial improvement over the simple BND universe. Does this canary universe of VWO, BND also hold for other and larger risky universes? We will examine this in detail in the next sections, where we revisit the four risky universes (G12, U6, U15, and G4) from our VAA testbed.

Finally, given the optimal canary universe consisting of P2=VWO/BND, we consider CF as a function of the breadth parameter B=1, 2 (<=NP=2). Importantly, CF is independent of the risky and cash universe and of the top T; it only depends on the canary universe and B, since CF=b/B with b equal to the number of bad canary assets[15]. So, CF is smaller when B is larger. For example, Fig. 6 shows CF for the period Dec 1970 – Dec 1993.

	CF/IS
B=1	39.40%
B=2	21.80%

Fig. 6. CF/IS for G12 as function of B with P2=VWO/BND, Dec 1970 – Dec 1993

From Fig. 6, it is clear that for this period the cash fraction CF for B=2 is nearly half (CF=21.8%) of B=1 (39.40%), given our canary universe of VWO/BND. This breadth parameter B=2 is also preferable since it allows for a more granular three steps in crash protection: depending on the number of bad canary assets b (=0,1,2), CF equals 0%, 50%, 100%, respectively, when B=2, in contrast to the all-or-nothing response of B=1 (CF= 0%, 100% for b=0, b>0, respectively).

Therefore, we will choose B=2 as default for DAA as our breadth momentum parameter in combination with P2 (VWO/BND) as our canary universe (but see Section 9 for B=1). The choice of B=2 guarantees a very small CF (see Fig. 6), given P2= VWO/BND and independent of the value of T and of the population of the chosen risky and cash universes. Only when we use Easy Trading with rounding (see note 5 and Keller 2017, p. 6), the resulting CF for uneven T (like T=5) will be smaller (as can be shown using the ET formula for CF, see note 5). For even T (like T=6), the CF values for a given period are always equal for any DAA (see eg. Fig. 6), given our fixed canary protection universe P2=VWO, BND and B=2.

From here, we will refer to our new strategy with P2=VWO, BND and B=2 as the *Defensive Asset Allocation* (or “DAA” for short) strategy. Notice that the protective universe is completely separate from the risky and cash universe, so VWO and/or BND does not have to be part of both universes.

4. The DAA-G12 strategy

The DAA-G12 model is inspired by the VAA-G12 model. The VAA-G12 model (see Keller, 2017) consists of twelve Global risky assets: R12=SPY, IWM, QQQ, VGK, EWJ, VWO, VNQ, GSG, GLD, TLT, HYG, LQD and our default cash universe C3= SHY, IEF, LQD, with T=2 and B=4. The backtest results for VAA-G12 are shown in Fig. 7a. Notice that in contrast to Section 3 (where we searched for the canary universe), in-sample/out-of-sample (IS/OS) now refers to Dec 1970 – Dec 1993 / Dec 1993 – Mar 2018, respectively, since VAA-G12 was optimized for T and B over this IS period.

We also present in Fig. 7b and 7c respectively two passive Equal Weight benchmarks (monthly rebalanced), one (EW-G12) with only the 12 risky assets and one (EW-C3) with only the three cash assets. The EW-G12 portfolio has no crash protection (B=9999, so CF=0%) and no relative momentum (T=12=N) while the passive EW-C3 cash portfolio has B=0 so CF=100% and also no relative momentum (EW of all three bonds).

[15] To be more precise, CF=b/B when b<B, and CF=1 when b>=B (and b, B <= N).

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-70	Dec-93	20.9%	5.8%	19.6%	18.2%	9.23	55.6%
OS=	Dec-93	Mar-18	10.1%	13.1%	8.6%	6.5%	1.93	60.8%
RS=	Mar-08	Mar-18	8.6%	7.1%	7.9%	7.2%	2.98	57.4%
FS=	Dec-70	Mar-18	15.3%	13.1%	13.0%	9.8%	3.44	58.3%
Parms:	**T=**	**2**	**B=**	**4**	**NR=**	**12**	**NP=**	**12**

Fig. 7a. VAA-G12, Dec 1970 – Mar 2017 [10]

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-70	Dec-93	13.2%	24.6%	8.9%	0.4%	1.20	0.0%
OS=	Dec-93	Mar-18	7.7%	39.2%	2.7%	0.0%	0.62	0.0%
RS=	Mar-08	Mar-18	5.9%	39.2%	2.1%	0.0%	0.56	0.0%
FS=	Dec-70	Mar-18	10.4%	39.2%	3.7%	0.0%	0.79	0.0%
Parms:	**T=**	**12**	**B=**	**9999**	**NR=**	**12**	**NP=**	**2**

Fig. 7b. EW-G12, passive risky EW (CF=0%), Dec 1970 – Mar 2017 [10]

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-70	Dec-93	8.6%	9.1%	7.7%	6.7%	0.62	100.0%
OS=	Dec-93	Mar-18	5.0%	4.7%	4.7%	4.4%	1.45	100.0%
RS=	Mar-08	Mar-18	3.4%	4.7%	3.2%	3.0%	1.76	100.0%
FS=	Dec-70	Mar-18	6.7%	9.1%	6.0%	5.2%	1.05	100.0%
Parms:	**T=**	**2**	**B=**	**0**	**NR=**	**12**	**NP=**	**12**

Fig. 7c. EW-C3, passive cash EW (CF=100%), Dec 1970 – Mar 2017 [10]

The results for VAA-G12 in Fig. 7a clearly show outperformance with regard to the passive EW-G12, however with the side-effect of a high cash fraction CF of around 60%. Now, for DAA-G12 we will use the same G12 and C3 risky and cash universe, together with the canary universe VWO/BND and breadth parameter B=2 from Section 3. The only unknown parameter is now the Top T for relative momentum of the risky assets. For this, we will simply optimize K25 with a one- dimensional sweep over T=1,2,3,4,5,6 on IS (like we did for T and B with VAA). Notice that by using K25 here we are more defensive than eg. with VAA-G12, where K50 was used as optimization target. The optimization of K25/IS results into best T=6 for DAA-G12, given P2=VWO/BND and B=2, see Fig. 8.

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-70	Dec-93	19.3%	10.6%	17.0%	14.1%	5.44	21.8%
OS=	Dec-93	Mar-18	12.9%	9.0%	11.7%	10.1%	3.73	28.8%
RS=	Mar-08	Mar-18	9.9%	9.0%	8.9%	7.7%	2.90	28.5%
FS=	Dec-70	Mar-18	16.0%	10.6%	14.1%	11.7%	4.42	25.4%
Parms:	**T=**	**6**	**B=**	**2**	**NR=**	**12**	**NP=**	**2**

Fig. 8. DAA-G12 with P2=VWO/BND, Dec 1970 – Mar 2018 [10]

The DAA results for R, D, K50, K25 and UPI on OS are all better than VAA-G12 (see fig. 7a), while CF/OS is less than half (28.8% vs. 60.8% for VAA). In particular, K25 on OS rises from 6.5% to 10.1% while UPI goes up from 1.93 to 3.73. Also return R for RS is slightly improved (9.9% vs. 8.6% for VAA).

Compared to both passive (risky and cash) benchmarks on OS in Fig. 7b and 7c, K25 on OS also shows substantial improvement (10.1% vs. 0% for EW-G12 and 4.4% for EW-C3), while max drawdown D is reduced by three-quarters (9.0% vs. 39.2% for EW-G12) and return R is also improved (12.9% vs. 7.7% for EW-G12). Similar results hold for RS. Finally, notice also that CF/IS= 21.8% equals CF in Fig. 6, demonstrating CF's independence of the (size of the) risky universe and of the parameter T. The same holds for CF in other periods.

Fig. 9a and 9b show the (log) equity lines and drawdowns, respectively, for DAA-G12 and its benchmark EW-G12. We also give the relative price line DAA/EW in Fig. 9a. When this price goes up over a period, DAA wins over the benchmark for that period and vice versa. As shown, the relative price nearly never goes down substantially (except to some extent in 1978/79) and is stable in the years from 2008 onwards. This is probably the result of the low CF, which is indicative for a low tracking error during uptrending periods for the global market. Also, from Fig. 9b it is clear that the drawdowns of DAA-G12 are very limited as compared to EW-G12: the max (monthly) drawdown of 10.6% for DAA-G12 occurs at Mar 1980. In Appendix A we show more detailed graphs of VAA-G12.

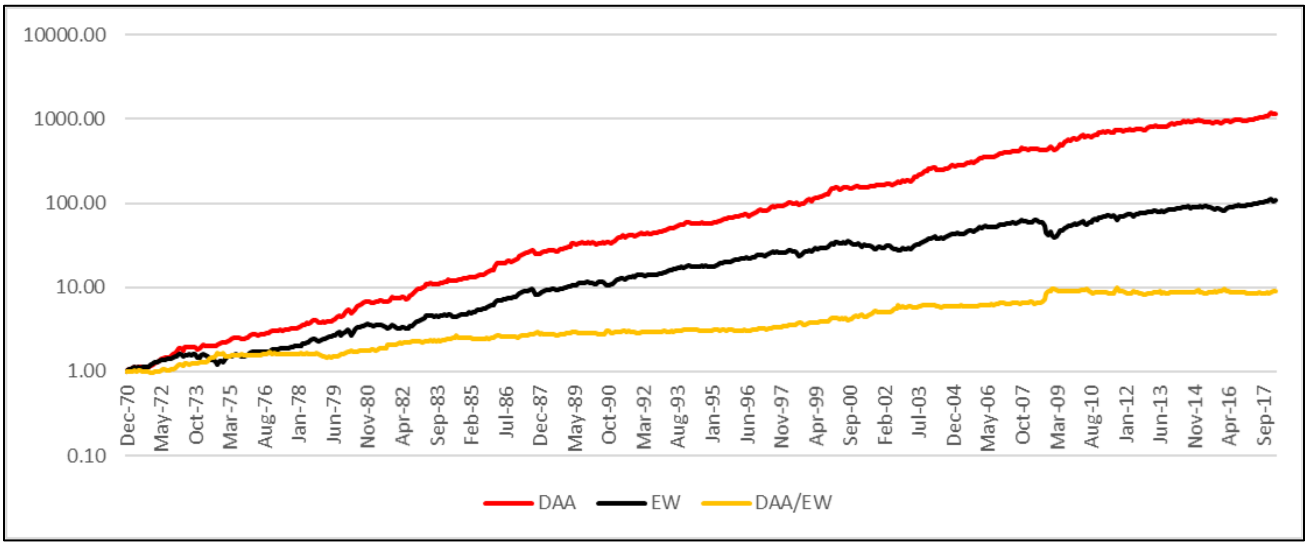

Fig. 9a Log equity DAA-G12 vs. EW-G12 (and relative DAA/EW), Dec 1970 – Mar 2018

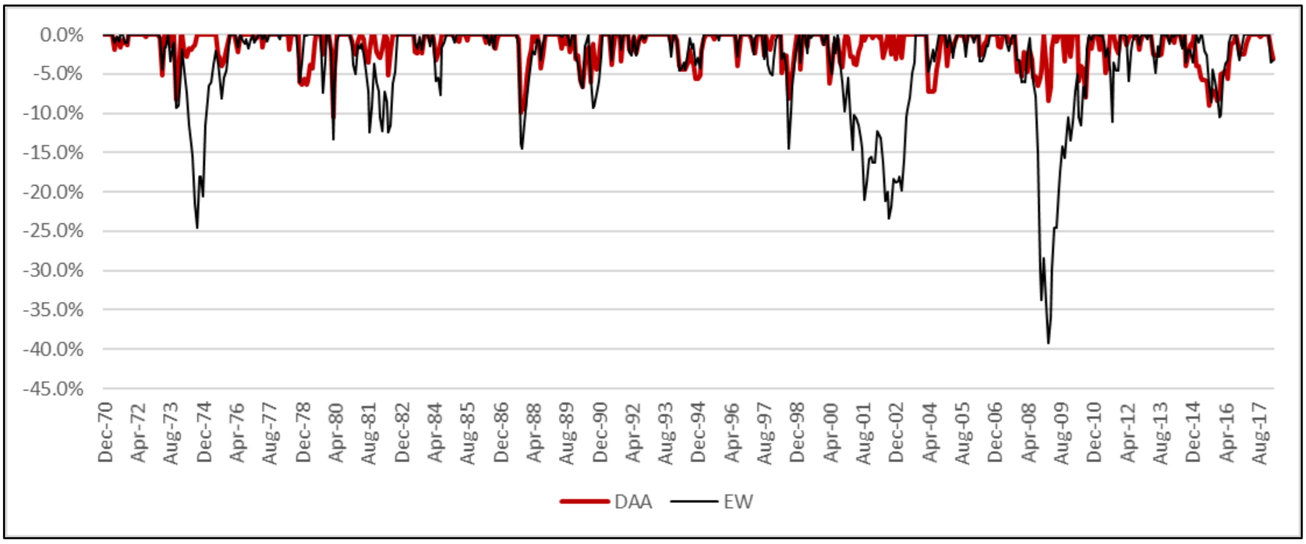

Fig. 9b Drawdowns of DAA-G12 vs. EW-G12, Dec 1970 – Mar 2018

5. The DAA-U6 strategy

With the same risky and cash universe for DAA-U6 as in our VAA-U6 strategy, we again use P2= VWO/BND and B=2 (see sections 3) for crash protection. We used only six US risky assets (which explains U6) and the default C3 cash universe (SHY, IEF, LQD). Like VAA-U6, our diversified R6 "risky" universe has four equity assets (US Large/Small Cap x Growth/Value), from Fama French (FF) and two bonds, BIL and LQD (see also Keller, 2017).

In contrast to our VAA-U6 backtest, we will restrict ourselves to the period Dec 1970 – Mar 2018 in order to avoid datasnooping induced results for our canary universe choice (optimized on Dec 1926 – Dec 1970, see section 3). Below, we give the VAA-U6 result for the period from Dec 1970, see Fig. 10a, with parameters T=6, B=1 taken from Keller, 2017. We also give the results for the passive equal weight EW-U6 portfolio (so with T=6 and B=9999) as benchmark in Fig. 10b. The passive EW-C3 benchmark is the same as in Fig. 7c.

Using again (like DAA-G12) P2=VWO/BND as canary universe for DAA-U6 with B=2, we find T by optimizing K25 with a one dimensional sweep over T=1,2,3,4,5,6 on IS, which gives best T=5 with K25=8.0%. See Fig. 11. Notice that both P2 assets (VWO and BND) are not part of the risky universe.

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-70	Dec-93	13.3%	11.6%	11.5%	9.2%	2.39	53.1%
OS=	Dec-93	Mar-18	8.2%	7.9%	7.5%	6.7%	2.46	65.4%
RS=	Mar-08	Mar-18	6.8%	7.1%	6.3%	5.7%	2.58	73.6%
FS=	Dec-70	Mar-18	10.7%	11.6%	9.3%	7.4%	2.43	59.5%
Parms:	T=	6	B=	1	NR=	6	NP=	6

Fig. 10a VAA-U6, Dec 1970 – Mar 2018 [10]

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-70	Dec-93	12.1%	32.1%	6.4%	0.0%	0.68	0.0%
OS=	Dec-93	Mar-18	8.6%	39.5%	3.0%	0.0%	0.80	0.0%
RS=	Mar-08	Mar-18	8.0%	39.5%	2.8%	0.0%	0.77	0.0%
FS=	Dec-70	Mar-18	10.3%	39.5%	3.6%	0.0%	0.75	0.0%
Parms:	**T=**	**6**	**B=**	**9999**	**NR=**	**6**	**NP=**	**2**

Fig. 10b EW-U6, passive risky EW (CF=0), Dec 1970 – Mar 2017 [10]

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-70	Dec-93	14.3%	15.3%	11.7%	8.0%	1.57	18.3%
OS=	Dec-93	Mar-18	11.5%	12.6%	9.8%	7.6%	2.67	24.5%
RS=	Mar-08	Mar-18	11.6%	12.6%	9.9%	7.7%	3.13	25.0%
FS=	Dec-70	Mar-18	12.9%	15.3%	10.6%	7.2%	2.06	21.5%
Parms:	**T=**	**5**	**B=**	**2**	**NR=**	**6**	**NP=**	**2**

Fig. 11. DAA-U6 with P2=VWO/BND, Dec 1970 – Mar 2018 [10]

Again, the DAA-U6 result in Fig. 11 is better on R on OS than VAA-U6 (11.5% vs. 8.2% for VAA-U6 in Fig. 10a) and much better for RS (11.6% vs. 6.8%). Only "risk" D on OS is larger than VAA-U6 (12.6 vs. 7.9%) but still below 15%, while UPI/OS is slightly better (2.67 vs. 2.46). Interestingly, the return/risk indicators K50 at OS (9.8% vs. 7.5% for VAA) and K25 (7.6% vs. 6.7%) are both better than VAA. The cash fraction CF on OS is less than half (24.5% vs. 65.4% for VAA), while CF/IS=18.3% is even smaller than 21.8% (see Fig. 11 and 7) since ET rounding occurs when T=5 (being no multiple of B=2).

As before, low CF values holds for all periods for DAA, being only dependent on P, B (and with ET also sometimes on T, as is shown here for the uneven T=5). Compared to the passive benchmark EW-U6, the drawdown D on OS is reduced by more than two-third (12.6% vs. 39.5% for EW-U6), while R is also improved by the DAA approach (11.5% vs. 8.6% for EW-U6).

Fig. 12a and 12b gives the (log) equity lines and drawdowns, respectively, for DAA-U6 and its benchmark EW-U6. We also give the relative price line DAA/EW in Fig. 9a which is again hardly ever decreasing. Also, from Fig. 9b it is clear that the drawdowns of DAA-U6 are limited as compared to EW-U6: the max drawdown of 15.3% for DAA-U6 occurs in Oct 1987.

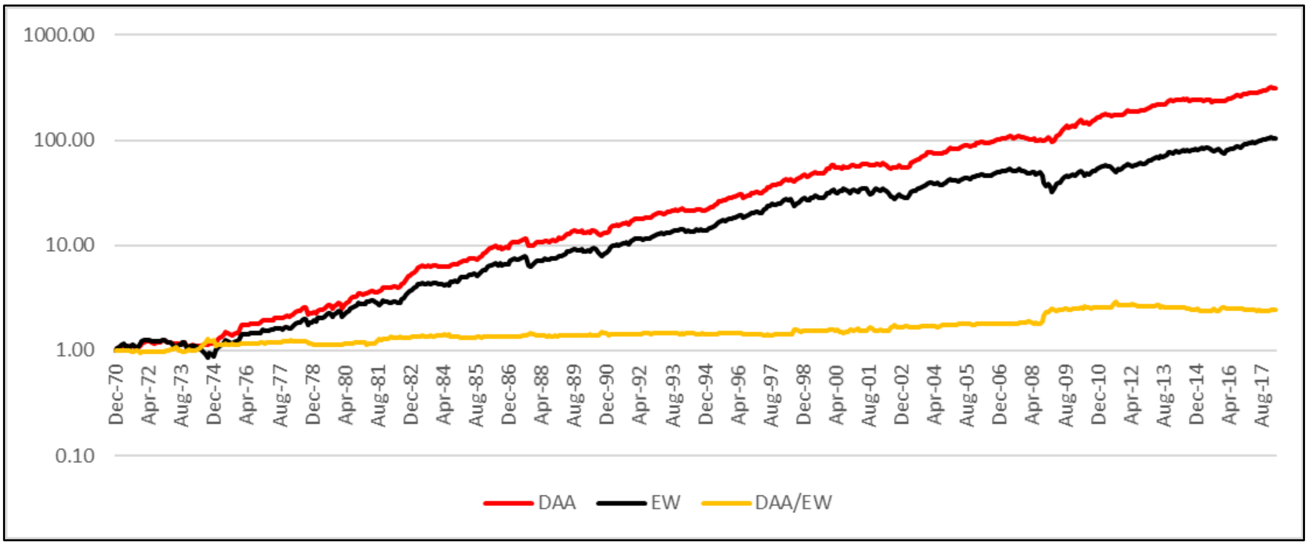

Fig. 12a Log equity DAA-U6 vs. EW-U6 (and relative DAA/EW), Dec 1970 – Mar 2018

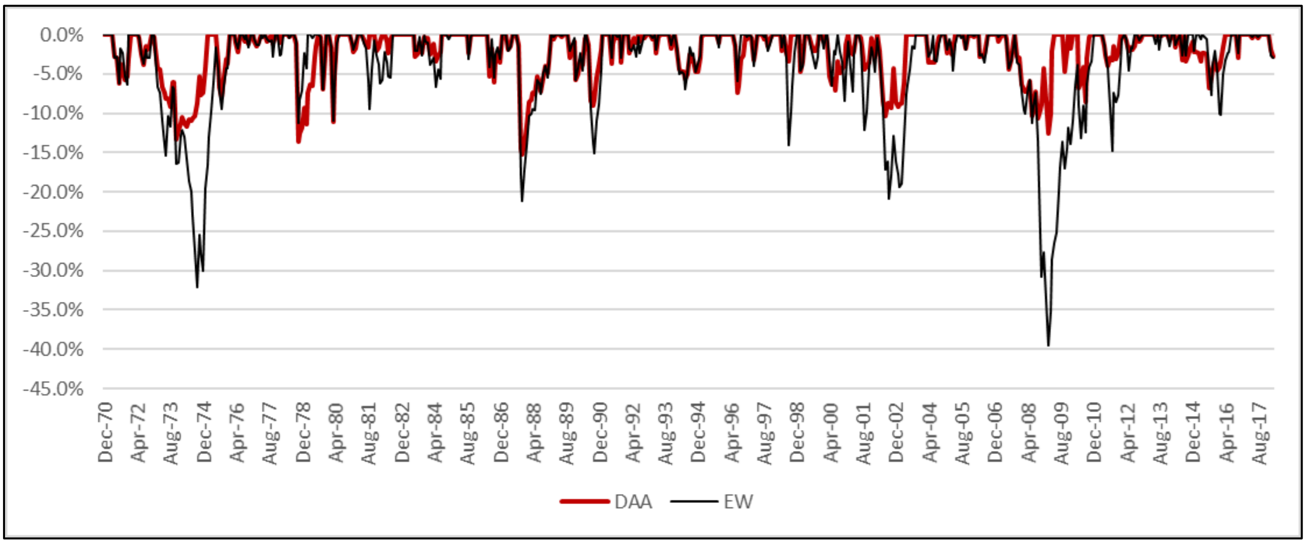

Fig. 12b Drawdowns of DAA-U6 vs. EW-U6, Dec 1970 – Mar 2018

6. The DAA-U15 strategy

The DAA-U15 strategy is based on the VAA-U15 strategy, but now from Dec 1970, see Fig. 13. There are 15 diversified “risky” US assets (10 FF sectors plus BIL, IEF, LQD, TLT, HYG) with the default cash universe (C3= SHY, IEF, LQD). The results for VAA-U15 (see also Keller 2017 for T/B) are shown in Fig. 13a, while the passive EW-U15 benchmark (with CF=0 and T=15) is shown in Fig. 13b.

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-70	Dec-93	13.1%	15.1%	10.8%	7.4%	1.86	59.2%
OS=	Dec-93	Mar-18	9.2%	7.8%	8.4%	7.5%	3.31	69.8%
RS=	Mar-08	Mar-18	6.1%	7.1%	5.6%	5.1%	2.46	72.9%
FS=	Dec-70	Mar-18	11.1%	15.1%	9.1%	6.3%	2.42	64.7%
Parms:	T=	4	B=	3	NR=	15	NP=	15

Fig. 13a VAA-U15, Dec 1970 – Mar 2018 [10]

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-70	Dec-93	11.6%	33.5%	5.7%	0.0%	0.63	0.0%
OS=	Dec-93	Mar-18	8.8%	34.7%	4.1%	0.0%	0.96	0.0%
RS=	Mar-08	Mar-18	8.2%	34.7%	3.8%	0.0%	0.92	0.0%
FS=	Dec-70	Mar-18	10.1%	34.7%	4.7%	0.0%	0.80	0.0%
Parms:	T=	15	B=	9999	NR=	15	NP=	2

Fig. 13b EW-U15, passive risky EW (CF=0), Dec 1970 – Mar 2017 [10]

For DAA-U15, we find best T=6 by optimizing K25 on IS, given P2=VWO/BND and B=2, see Fig. 14. Again, VWO and BND are not part of the risky universe.

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-70	Dec-93	13.8%	14.0%	11.5%	8.4%	1.45	21.8%
OS=	Dec-93	Mar-18	11.2%	12.8%	9.5%	7.3%	2.46	28.8%
RS=	Mar-08	Mar-18	11.1%	12.8%	9.4%	7.3%	2.52	28.5%
FS=	Dec-70	Mar-18	12.4%	14.0%	10.4%	7.6%	1.91	25.4%
Parms:	T=	6	B=	2	NR=	15	NP=	2

Fig. 14 DAA-U15 with P2=VWO/BND, Dec 1970 – Mar 2018 [10]

From Fig. 14 it is clear that DAA-U15 outperforms VAA-U15 on OS on R (11.2% vs. 9.2%), K50 (9.5% vs. 8.4%), and CF (28.8% vs. 69.8%), but not on K25 (7.3% vs. 7.5% for VAA), D (12.8% vs. 7.8%) and UPI (2.46 vs. 3.31). So DAA-U15 has a higher return but is slightly more risky than VAA-U15. As before, return R on RS is improved substantially (11.1% vs. 6.1% for VAA), while max drawdown D is worse (12.8% vs. 7.1%), but still limited.

DAA has a CF/OS less than half that of VAA-U15 (28.8% vs. 69.8%), which implies a much smaller tracking error to the risky portfolio and a lower cash/bond dependency. Since T=6 (even), again CF/IS=21.8% cf. Fig. 6. The comparison to the EW benchmark on OS is very similar to the previous strategies: D is reduced by more than half (12.8% vs. 34.7% for EW) while R is also improved (11.2% vs. 8.8% for EW).

Fig. 15a and 6 again shows the (log) equity curve and drawdown, respectively, compared to the benchmark EW. The relative price DAA/EW is hardly ever decreasing. The max drawdown of DAA-U15 is 14.0% in Oct 1978.

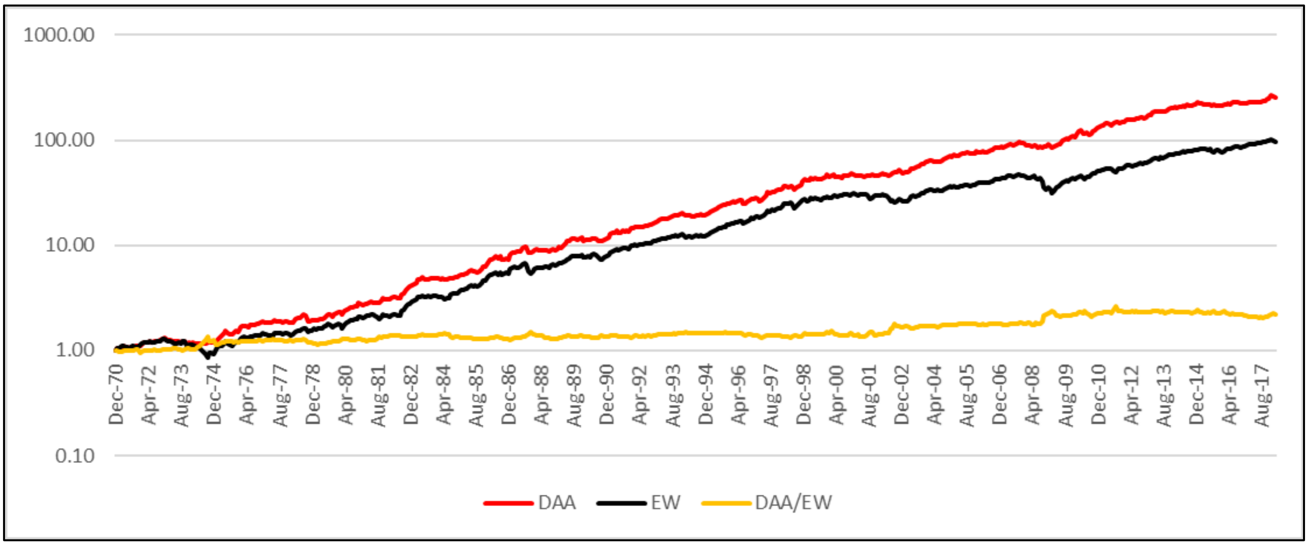

Fig. 15a Log equity DAA-U15 vs. EW-U15 (and relative DAA/EW), Dec 1970 – Mar 2018

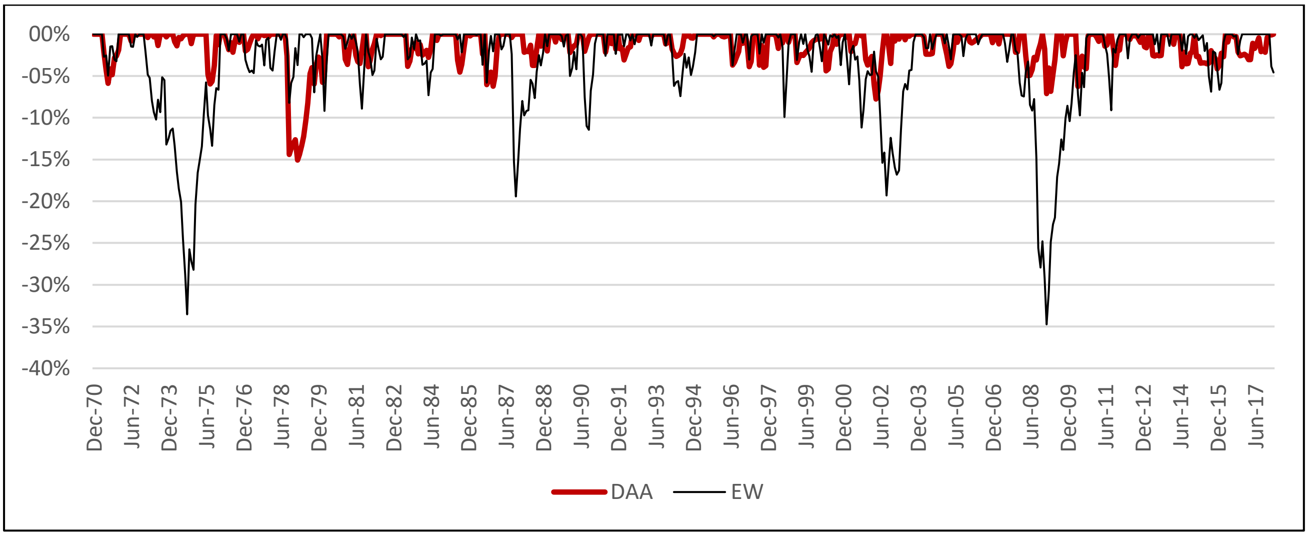

Fig. 15b Drawdowns of DAA-U15 vs. EW-U15, Dec 1970 – Mar 2018

7. What about G4?

In section 3 we started with the outperformance of VAA-G4 (with four risky Global assets, ie. SPY, VEA, VWO, and BND) in recent months (and over the longer period from Dec 1970). Does our canary P-universe also work for G4? We first show VAA-G4 in Fig. 16a together with the passive EW-G4 benchmark strategy in Fig. 16b.

As is clear from Fig. 17, our DAA canary approach seems less successful for R4=G4 compared to VAA-G4, although max drawdown D is less than half of the passive benchmark EW-G4 on OS (20.3 vs. 44.9% for EW). Albeit the cash fraction CF (approximately half of VAA-G4's for all periods) is very low, the other parameters like R, K50, K25, and UPI, are underperforming the results of VAA-G4.

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-70	Dec-93	21.9%	13.0%	18.6%	14.2%	4.13	54.2%
OS=	Dec-93	Mar-18	15.9%	10.5%	14.1%	11.7%	3.88	57.9%
RS=	Mar-08	Mar-18	10.5%	10.5%	9.3%	7.7%	2.37	51.2%
FS=	Dec-70	Mar-18	18.8%	13.0%	16.0%	12.2%	4.00	56.2%
Parms:	**T=**	**1**	**B=**	**1**	**NR=**	**4**	**NP=**	**4**

Fig. 16a VAA-G4, Dec 1970 – Mar 2018 [10]

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-70	Dec-93	13.8%	42.4%	3.6%	0.0%	0.68	0.0%
OS=	Dec-93	Mar-18	6.9%	44.9%	1.3%	0.0%	0.38	0.0%
RS=	Mar-08	Mar-18	5.1%	44.9%	0.9%	0.0%	0.37	0.0%
FS=	Dec-70	Mar-18	10.2%	44.9%	1.9%	0.0%	0.51	0.0%
Parms:	**T=**	**4**	**B=**	**9999**	**NR=**	**4**	**NP=**	**4**

Fig. 16b EW-G4 Passive risky EW (CF=0%), Dec 1970 – Mar 2017 [10]

To test DAA-G4, we again used P2=VWO/BND and B=2 and optimize K25/IS for T, to get best T=4, see Fig. 17.

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-70	Dec-93	15.2%	21.5%	11.0%	3.7%	1.37	21.8%
OS=	Dec-93	Mar-18	10.1%	20.3%	7.5%	3.2%	1.82	28.8%
RS=	Mar-08	Mar-18	9.4%	20.3%	7.0%	3.0%	1.79	28.5%
FS=	Dec-70	Mar-18	12.6%	21.5%	9.1%	3.1%	1.55	25.4%
Parms:	**T=**	**4**	**B=**	**2**	**NR=**	**4**	**NP=**	**2**

Fig. 17 DAA-G4 with P2=VWO/BND, Dec 1970 – Mar 2018 [10]

So, our canary approach with P2= VWO, BND and B=2 does not seem to work as well for all universes as compared to VAA (but see Section 9 for B=1). Also notice that in Jan 2018 only one of the assets in the canary universe (BND) was bad, so DAA-G4 with B=2 only goes halfway to cash (while VAA-G4 with B=1 fully turns to cash, see Fig. 1).

What could explain the subpar risk/return performance of DAA-G4? We suspect it is the G4 universe itself: too risky and without enough diversification. To test our supposition, we replace BND in G4 by LQD, TLT, HYG resulting in a G6 risky universe and optimize K25/OS to arrive at T=6 (so no relative momentum selection) with DAA-G6, see Fig. 18a (and 18b for the passive EW benchmark).

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-70	Dec-93	13.1%	16.8%	10.4%	6.5%	1.32	21.8%
OS=	Dec-93	Mar-18	9.4%	13.7%	7.9%	5.9%	2.25	28.8%
RS=	Mar-08	Mar-18	8.8%	13.7%	7.4%	5.5%	2.18	28.5%
FS=	Dec-70	Mar-18	11.2%	16.8%	8.9%	5.5%	1.71	25.4%
Parms:	**T=**	**6**	**B=**	**2**	**NR=**	**6**	**NP=**	**2**

Fig. 18a DAA-G6 with P2=VWO/BND, Dec 1970 – Mar 2018 [10]

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-70	Dec-93	12.0%	33.8%	5.9%	0.0%	0.69	0.0%
OS=	Dec-93	Mar-18	7.0%	35.0%	3.2%	0.0%	0.63	0.0%
RS=	Mar-08	Mar-18	6.0%	35.0%	2.8%	0.0%	0.64	0.0%
FS=	Dec-70	Mar-18	9.4%	35.0%	4.3%	0.0%	0.66	0.0%
Parms:	**T=**	**6**	**B=**	**9999**	**NR=**	**6**	**NP=**	**2**

Fig. 18b EW-G6 passive risky (CF=0%), Dec 1970 – Mar 2017 [10]

From Fig. 18a we see we are now closer to VAA-G4 in terms of risk D on OS (13.7% vs. 10.5% for VAA-G4) and UPI (2.25 vs. 3.88 for VAA-G4), although return R/OS is much less (9.4% vs. 15.9% for VAA-G4), but with a cash fraction CF at half of that of VAA-G4 (28.8% vs. 57.9% on OS). Also notice the risk D on OS (Fig. 18a) which is less than half of EW (13.7% vs. 35.0%, see Fig. 18b), while return is also better than EW (9.4% vs. 7.0% for EW).

Comparing DAA-G6 (Fig. 18a) with DAA-G12 (Fig. 8) yields the same conclusion: DAA-G12 with the larger, more diversified G12 universe results in better performance metrics than DAA-G6 when B=2 (see section 9 for B=1).

So we conclude that (at least for B=2) DAA's canary effect appears to benefit and improve with larger and more diversified (risky) universes. This raises the question how the effect of crash protection through P2 compares to the effect of relative momentum (top T<N). This is addressed in the next section.

8. Types of momentum

With DAA, we distinguish two types of momentum: relative momentum (choosing the best Top T assets) and breadth momentum (crash protection based on P2= VWO, BND and B=2). Can we get an impression of both effects separately? And what about absolute momentum?

To test the effects of relative and breadth momentum separately, we consider DAA-G12. In Fig. 19a, b and c we show EW-G12 (without crash protection as well as without relative momentum: T=12, B=9999), DAA-G12 without crash protection (but with relative momentum: T=6, B=9999), and full DAA-G12 (combining relative momentum and crash protection: T=6, B=2). We also show VAA with T=6 (for comparison with DAA-G12 with T=6), with B4 optimized on K25/IS (see Fig. 19d).

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-70	Dec-93	13.2%	24.6%	8.9%	0.4%	1.20	0.0%
OS=	Dec-93	Mar-18	7.7%	39.2%	2.7%	0.0%	0.62	0.0%
RS=	Mar-08	Mar-18	5.9%	39.2%	2.1%	0.0%	0.56	0.0%
FS=	Dec-70	Mar-18	10.4%	39.2%	3.7%	0.0%	0.79	0.0%
Parms:	**T=**	**12**	**B=**	**9999**	**NR=**	**12**	**NP=**	**2**

Fig. 19a EW-G12, Dec 1970 -Mar 2018 [10]

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-70	Dec-93	17.4%	18.9%	13.4%	6.9%	2.11	0.0%
OS=	Dec-93	Mar-18	10.9%	30.1%	6.2%	0.0%	1.39	0.0%
RS=	Mar-08	Mar-18	6.9%	30.1%	3.9%	0.0%	0.80	0.0%
FS=	Dec-70	Mar-18	14.1%	30.1%	8.0%	0.0%	1.68	0.0%
Parms:	T=	6	B=	9999	NR=	12	NP=	2

Fig. 19b T6-G12 (CF=0), Dec 1970 – Mar 2018 [10]

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-70	Dec-93	19.3%	10.6%	17.0%	14.1%	5.44	21.8%
OS=	Dec-93	Mar-18	12.9%	9.0%	11.7%	10.1%	3.73	28.8%
RS=	Mar-08	Mar-18	9.9%	9.0%	8.9%	7.7%	2.90	28.5%
FS=	Dec-70	Mar-18	16.0%	10.6%	14.1%	11.7%	4.42	25.4%
Parms:	T=	6	B=	2	NR=	12	NP=	2

Fig. 19c DAA-G12, Dec 1970 – Mar 2018 [10]

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS=	Dec-70	Dec-93	17.5%	4.1%	16.8%	16.0%	9.70	61.1%
OS=	Dec-93	Mar-18	8.2%	8.0%	7.5%	6.7%	2.39	65.6%
RS=	Mar-08	Mar-18	5.7%	7.1%	5.3%	4.8%	2.36	62.5%
FS=	Dec-70	Mar-18	12.7%	8.0%	11.6%	10.3%	4.17	63.4%
Parms:	T=	6	B=	4	NR=	12	NP=	12

Fig. 19d VAA-G12 with T6, Dec 1970 – Mar 2018 [10]

From Figs. 19a, b, and c, we conclude that both effects (relative momentum and breadth momentum) are substantial on R and K. However, for risk (see D) and therefore return/risk (see eg. K25, and UPI), breadth momentum really shines. Risk D on OS goes from 39.2% (EW), 30.1% (T6), to 9.0% (for DAA), while return/risk K50[16] on OS goes from 2.7% (EW), 6.2% (T6) to 11.7% (DAA). And remember, the optimal canary protection universe of P2= VWO, BND was determined in-sample on Dec 1926 – Dec 1970, so no P2-datasnooping from 1970.

From Fig. 19c and d (DAA-G12 vs. VAA-G12 both with the same T6) we conclude that the advantage of DAA to VAA in terms of return R is probably not related to T but to the use of a different protective universe. While max drawdown D on OS is only slightly worse for DAA (9.0% vs. 8.0% for VAA), its return R on OS is much better than that of VAA (12.9% vs.8.2% for VAA). Also note an even larger discrepancy for the recent sample period (RS), where the return spread on RS is even better (9.9% vs. 5.7% for VAA). This might also demonstrate the advantage of a low CF and tracking error in low cash yield and market uptrend environments (like 2009).

Finally, notice that we did not use absolute momentum (ie. eliminating bad assets in favor of cash), except for the number of bad canary assets (ie. breadth momentum) in relation to the breadth parameter B (=2). So for DAA, we did not eliminate bad assets in the top T selection of risky assets, although we reduced the top T

[16] We use K50 instead of K25 since D>25% in Fig. 18a and b.

eg. by half when CF=50% (based on P2 with B=2), given our Easy Trading (ET) strategy (see note 5).[17] Easy Trading also limits turnover and trading.

9. DAA "Aggressive"

Until now, we have defined DAA as an allocation strategy with canary universe P2= VWO, BND and breadth parameter B=2. The B=2 choice implies 50% to cash when VWO ór BND becomes bad, and 100% when both are bad. As a result (see Fig. 6), the average cash fraction for our in-sample period (IS: Dec 1970 – Dec 1993) equals CF=21.8%, while for the full sample (FS: Dec 1970 – Mar 2018, see eg. Fig. 8) equals CF= 25.4%.

Compared to eg. the average cash fraction on FS for VAA-G12 of CF=58.3% (see Fig. 7a), this is a reduction of CF by more than half for DAA compared to VAA-G12. Notice that this holds for all DAA universes with B=2 (and P2=VWO/BND) when T is even, while the average cash fraction is even lower in case of odd tops like T=5 (see Fig. 11: CF/FS= 18.3%) because of the Easy Trading rounding (see note 5 and the text below Fig. 6).

However, we could also consider a more aggressive crash protection, using the breadth parameter B=1. With B=1, crash protection is more aggressive (ie. faster) since the allocation goes fully to cash (CF=100%) when VWO ór BND (or both) are bad, and stays fully invested in risky (so CF=0%) otherwise.

To see the effect of this aggressive crash protection, we show DAA-G12 with B=1. As always, we will optimize K25 by parameter T in-sample (IS: Dec 1970 – Dec 1993), which results in T=5, see Fig. 20:

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS	Dec-70	Dec-93	19.9%	8.0%	18.1%	16.1%	7.03	39.4%
OS	Dec-93	Mar-18	12.6%	7.7%	11.5%	10.3%	4.34	50.0%
RS	Mar-08	Mar-18	10.7%	7.7%	9.8%	8.7%	4.16	46.3%
FS	Dec-70	Mar-18	16.1%	8.0%	14.7%	13.0%	5.40	44.9%
Parms:	**T=**	**5**	**B=**	**1**	**NR=**	**12**	**NP=**	**2**

Fig. 20. DAA-G12 with B=1 and T=5, Dec 1970 – Mar 2018 [10]

From Fig. 20, we see that K25 with B=1 is better than DAA-G12 (with B=2, see Fig. 19c) on IS (16.1 vs. 14.1%), OS (10.3 vs. 10.1%), RS (8.7 vs. 7.7%) and FS (13.0 vs. 11.7%). Of course, CF is larger (around 45% on FS), but still much less than VAA-G12 (around 58% on FS, see Fig. 7a). But because of the (much) lower CF (and therefore lower tracking error and lower dependencies on bond rates), we will often prefer DAA with B=2. Notice also that B=1 implies all-in/all-out with regards to crash protection (CF=0/100% for b=0/>0), while our default B=2 results in more granular cash allocations (CF=0/50/100% for b=0/1/2+).

Besides B=1, there is another method to make DAA more "aggressive", that is by adapting the cash/bond universe C3. By default (for VAA and DAA), we used as the cash universe C3=SHY, IEF, LQD. First, we replaced LQD (Investment grade Corporate Bonds) by UST (2x leveraged 7-10y Treasuries). Compared to LQD, UST is as T-Bond ETF less equity-like and as such offers more hedging protection against the general stock market than LQD, while it has a better in-sample (IS) Sharpe ratio than TLT (20y+ Treasuries) . And in view of the

[17] We also tried using traditional absolute momentum in the risky top T instead of ET and found for DAA-G12 with absolute momentum and the same T=6/B=2, slightly better returns with slightly worse drawdowns and a little higher return/risk (K25/OS= 10.2 vs. 10.1%) than DAA-G12 with ET. As expected, turnover was also slightly higher, while CF was the same as DAA-G12 with ET.

volatility of SHY these days of low rates, we will also replace SHY (1-3y T-Bond) by the shorter maturity SHV (1-12m T-Bond) which is close to the volatility of BIL (3m T-Bills) but with slightly better returns due to SHV's longer maturity. So, C3 becomes SHV, IEF, UST.

We will call the combination of DAA with B=1 and the new C3 (= SHV, IEF, UST) the DAA "Aggressive" or "DAA1" for short (with the added "1" referring to B=1). We have for DAA1-G12 (after optimizing T on K25/IS at T=2), see Fig. 21:

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS	Dec-70	Dec-93	28.6%	12.9%	24.4%	18.6%	6.89	39.4%
OS	Dec-93	Mar-18	16.5%	12.7%	14.1%	10.9%	3.30	50.0%
RS	Mar-08	Mar-18	12.3%	12.2%	10.6%	8.3%	2.51	46.3%
FS	Dec-70	Mar-18	22.3%	12.9%	19.0%	14.5%	4.67	44.9%
Parms:	**T=**	**2**	**B=**	**1**	**NR=**	**12**	**NP=**	**2**

Fig. 21. DAA1-G12 (B=1, C3=SHV, IEF, UST) with T=2, Dec 1970 – Mar 2018 [10]

This DAA1-G12 is on FS (Dec 1970 – Mar 2018) even better than Fig. 20 (B1, old C3) on K25 (14.5 vs. 13.0%), with an impressive 6% higher R (22.3 vs. 16.1%) and a slightly worse D (12.9 vs. 8.0%), but still less than 15%. It is a fortiori better than DAA-G12 with B=2 and T=6 (see Fig. 19c). The (log) equity curve and drawdown curves for DAA1-G12 compared to G12-EW are shown in Fig. 22 and 23.

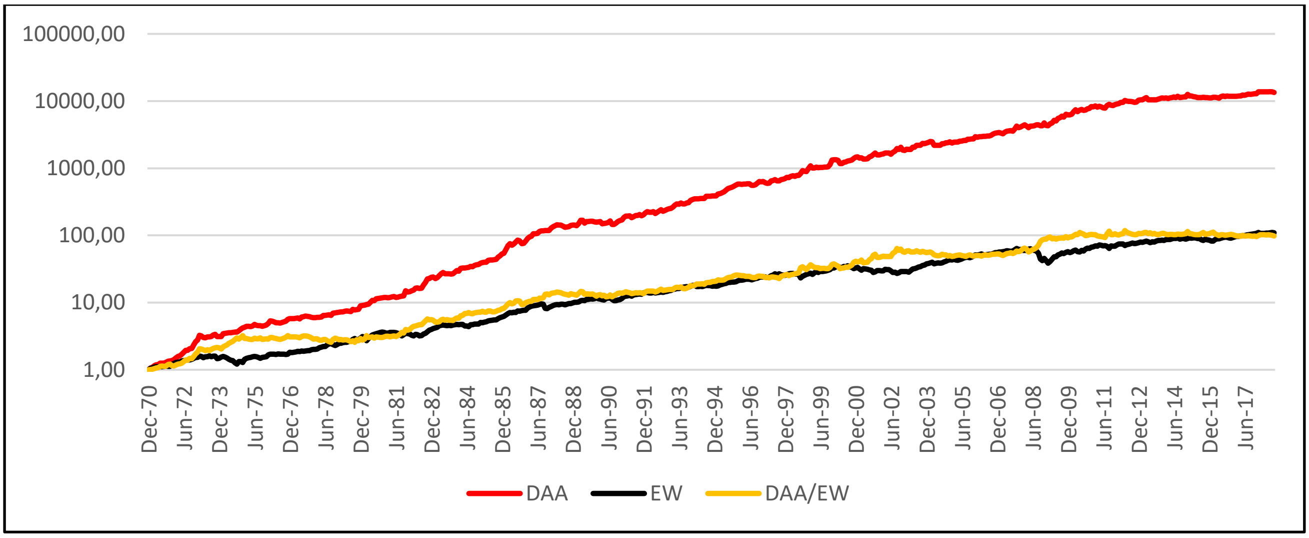

Fig. 22. Log equity DAA1-G12 (B=1, C3=SHV, IEF, UST) with T=2, Dec 1970 – Mar 2018

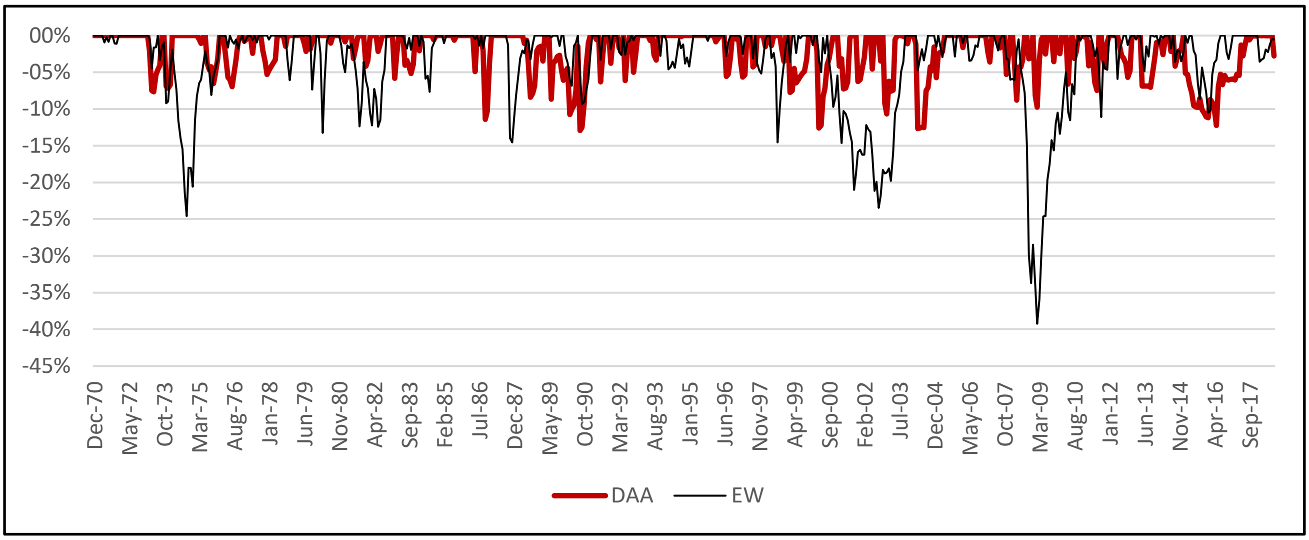

Fig.23. Drawdowns of DAA1-G12 (B=1, C3=SHV, IEF, UST) with T=2, Dec 1970 – Mar 2018

Encouraged by these "aggressive" results, we also tested the problematic DAA-G4 (see section 7), with B=1 and C3=SHV, IEF, UST, with optimized T=4 (=NR, so no risky rotation) on K25/IS, see Fig. 24:

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS	Dec-70	Dec-93	20.9%	12.6%	17.9%	13.8%	4.43	39.4%
OS	Dec-93	Mar-18	14.8%	13.4%	12.5%	9.4%	3.30	50.0%
RS	Mar-08	Mar-18	12.7%	13.4%	10.7%	8.0%	2.57	46.3%
FS	Dec-70	Mar-18	17.8%	13.4%	15.0%	11.3%	3.77	44.9%
Parms:	**T=**	**4**	**B=**	**1**	**NR=**	**4**	**NP=**	**2**

Fig. 24. DAA1-G4 (B=1, C3=SHV, IEF, UST) with T=4, Dec 1970 – Mar 2018 [10]

This DAA1-G4 is much better than the ordinary DAA-G4 (see Fig. 17) with a much better R (17.8 vs. 12.6%) and much better D (13.4 vs. 21.5%) on FS (Dec 1970 -Mar 2018).[18]

Inspired by these appealing results for the "aggressive" DAA1, we will test a minimalistic approach for the risky universe: DAA1-U1 with only one risky US asset R1=SPY (so R1 and therefore T=1 and no rotation), and

[18] For completeness, we also run VAA-G4 with the "aggressive" C3 with T=2 optimized on K25/IS. Compared to the old VAA-G4 (with C3=SHY, IEF, LQD, and T=1, see Fig. 16a), return R on FS (Dec 70 – Mar 2018) is improved (21.4 vs. 18.8%) while drawdown D is only slightly increased (13.3 vs. 13.0%), resulting in a K25/FS=13.6% (12.2% in Fig. 16a). Compared to DAA1-G4 (with T4) in Fig. 24, VAA-G4 with the new C3 and T=2 is also better on R and D. But the average cash fraction CF is highest for VAA-G4 (56%) and lowest for DAA-G4 (25% with B=2, see Fig. 17) with DAA1-G4 in between (45% with B=1).

with B=1 and C3= SHV, IEF, UST (and P2=VWO, BND), see Fig. 25:

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS	Dec-70	Dec-93	17.6%	12.6%	15.1%	11.7%	2.76	39.4%
OS	Dec-93	Mar-18	16.0%	14.4%	13.3%	9.5%	3.34	50.0%
RS	Mar-08	Mar-18	16.2%	14.4%	13.5%	9.6%	3.43	46.3%
FS	Dec-70	Mar-18	16.8%	14.4%	14.0%	10.0%	3.08	44.9%
Parms:	T=	1	B=	1	NR=	1	NP=	2

Fig. 25. DAA1-U1 (B=1, C3=SHV, IEF, UST) with R1=SPY, T=1, Dec 1970 – Mar 2018 [10]

Even in this very simple model with only one risky asset (SPY), we arrive at a return/drawdown combination of better than 16% annual returns and less than 15% drawdowns in all periods, incl. FS (Dec 1970 – Mar 2018).

To take single-asset universes to the extreme for readers with a higher risk appetite, here we show the result of the same single-asset risky universe (R1=SPY) with T1B1 and now with a single bond cash universe C1=UST (and canary P2= VWO/BND), see Fig. 26:

Period	Start	Stop	R	D	K50	K25	UPI	CF
IS	Dec-70	Dec-93	18.6%	24.2%	12.7%	1.1%	1.85	39.4%
OS	Dec-93	Mar-18	18.3%	14.4%	15.2%	10.9%	3.63	50.0%
RS	Mar-08	Mar-18	19.9%	14.4%	16.6%	11.8%	4.46	46.3%
FS	Dec-70	Mar-18	18.5%	24.2%	12.6%	1.1%	2.59	44.9%
Parms:	T=	1	B=	1	NR=	1	NP=	2

Fig. 26. DAA-U1 with B=1, R1=SPY, C1=UST and T=1, Dec 1970 – Mar 2018 [10]

Even with the (longer maturity) cash of only C1=UST, we arrive at an even higher CAGR return than in case of the C3=SHV, IEF, UST (R/FS=18.5% vs. 16.8%, see Fig. 25 and 26, resp.), but also with more aggressive drawdowns (D=24.2%) in the first period (IS, Dec 1970 - Dec 1993), although for the more recent years the max drawdown is less than 15% with very high returns (19.9% for RS, Mar 2008 – Mar 2018). So, with the present low and rising yields, the alternative C3 cash model (with SHV and UST) might be an interesting alternative (also for VAA, see note 18).

And since there is no rotation between risky assets (NR=1) and between cash bonds (NC=1), total turnover (and therefore transaction costs TTC) is nearly half of that of eg. VAA-G12 (TTC=0.74% vs 1.45% for VAA-G12). However, as said before, with B=1 the average cash fraction CF equals now 45% on FS (Dec 1970 – Mar 2018), which is nearly twice that of DAA with B=2, but still substantially less than that of VAA-G12 (58%).

Finally, to conclude the "aggressive" DAA1 tests, we extended the leveraged cash approach for DAA1-G12 to a leveraged risky approach by also replacing the risky assets SPY, QQQ, IWM, VGK, EWJ, VNQ, GLD, and TLT by their 2x leveraged versions SSO, QLD, UWM, UPV, EZJ, URE, UGL, and UBT, respectively. The results over Dec 1970 – Mar 2018 are R/D= 28.1/15.5% with K25= 15.5% (!), using C3= SHV, IEF, UST, with B=1 and the best (on K25/IS) T=5. More tests of DAA1 with leveraged ETFs and so-called "smart leverage" (with leveraged trades based on unleveraged signals) can be found on TrendXplorer (2018, forthcoming).

Although these "leveraged" results might seem rather spectacular, one could raise some questions since most of these 2x leveraged ETFs (including UST) are rather illiquid. Also, our simulated leveraged proxies from Dec 1969 are somewhat questionable, especially in view of the much higher borrowing rates in the eighties than in the fitted period (mostly from 2009), where most proxy and true (monthly) returns had correlations above 95% between fitted proxies and observed leveraged ETFs. But at least these backtests demonstrates the powerful crash protection of the canary approach (with P2=VWO, BND) combined with asset rotation (as here with the Top5 of the G12 universe), even with very risky (leveraged) assets.

9. Summary and Conclusions

We have introduced a variation of our Vigilant Asset Allocation (VAA, see Keller 2017), called DAA: Defensive Asset Allocation. With DAA crash protection is different from VAA by the introduction of a separate protective or "canary" universe. Crash protection is now determined by breadth momentum (number of bad assets, ie. with non-positive momentum) of this "canary" universe.

With DAA, the usage of this separate canary universe for crash protection lowers cash fractions (CF) and also improves the tracking error with respect to the passive (buy-and-hold) benchmark and limits turnover. For DAA we used the two-asset canary universe of VWO and BND with cash fraction CF= b/B, where b is the number of bad canary assets and B=2 the so-called Breadth parameter.

Otherwise, DAA is equivalent to VAA, with its use of asset rotation and selection based on the well-known relative momentum strategy, so EW-Top T (T<=N, long only), without intrinsic or absolute momentum. Compared to this basic relative strength strategy as eg. in Faber, 2010 and CXO, 2017, we only applied a different (and faster) momentum filter (13612W), besides our novel canary protection.

We speak of a "bad" or "the best" asset when its (13612) momentum is non-positive or highest, respectively. Let b be the number of bad assets of the canary universe (VWO and BND). The DAA recipe now becomes (with B=2 and assuming T is even[19]):

1. When VWO and BND momentum are both bad (so b=2), invest 100% (=b/B) in the single best bond of the cash universe;
2. When VWO or BND momentum is bad (so b=1), invest 50% (=b/B) in the best T/2 risky assets, equal weight (so EW-T/2), and the other 50% in the best bond of the cash universe;
3. When VWO and BND momentum are both good (so b=0, no bad assets), invest 100% (=1-b/B) in the best (top) T risky assets (out of N>=T risky assets), equal weight (so EW-T).

We found as best "canary" universe the combination of VWO and BND by searching over Dec 1926 – Dec 1970. Applying this canary universe out-of-sample (from Dec 1970), we were able to reduce the average cash fraction CF to less than 30%. This makes DAA less sensitive for cash (or bond) yields, which is particular beneficial in view of recent low rates. It also limits tracking error and turnover.

Therefore, it is instructive to see how DAA fared for a recent sample (RS: Mar 2008 – Mar 2018), see Fig. 27. We also include Total Transaction Costs (TTC)[20] and the parameters N, T, B per strategy in Fig. 27. Bold results indicate best of DAA/VAA. K25 and K50 are Return (R) adjusted for max Drawdown (D) with max drawdown limits of 25% and 50%, respectively (see Section 2), and UPI is the Ulcer Performance Index

[19] When T is uneven, we used the ET formula for a rounded CF, as in VAA, see note 5 and Keller, 2017.
[20] Based on one-sided transaction costs of TC=0.1%

(based on average squared drawdown). In Appendix A, we also show additional results for Mar 2008 – Mar 2018 (RS) for DAA-G12, including the equity (price) lines, drawdown/CF graph and the asset mix.

From Fig. 27, we conclude that both VAA and DAA strategies have much better returns R, drawdowns D, and returns/risk K50, K25 and UPI than the passive benchmark EW-N, which of course wins on CF and TTC.

RS	R	D	K50	K25	UPI	CF	TTC	N	T	B
DAA-G12	**9.9%**	9.0%	**8.9%**	**7.7%**	2.90	**28.5%**	**1.12%**	12	6	2
DAA-U6	**11.6%**	12.6%	**9.9%**	**7.7%**	**3.13**	**25.0%**	**0.69%**	6	5	2
DAA-U15	**11.1%**	12.8%	**9.4%**	**7.3%**	**2.52**	**28.5%**	**1.22%**	15	6	2
DAA-G4	9.4%	20.3%	7.0%	3.0%	1.79	**28.5%**	**0.70%**	4	4	2
VAA-G12	8.6%	**7.1%**	7.9%	7.2%	**2.98**	57.4%	1.56%	12	2	4
VAA-U6	6.8%	**7.1%**	6.3%	5.7%	2.58	73.6%	1.10%	6	6	1
VAA-U15	6.1%	**7.1%**	5.6%	5.1%	2.46	72.9%	1.39%	15	4	3
VAA-G4	**10.5%**	**10.5%**	**9.3%**	**7.7%**	**2.37**	51.2%	1.43%	4	1	1
EW-G12	5.9%	39.2%	2.1%	0.0%	0.56	0.0%	0.0%	12	12	9999
EW-U6	8.0%	39.5%	2.8%	0.0%	0.77	0.0%	0.0%	6	6	9999
EW-U15	8.2%	34.7%	3.8%	0.0%	0.92	0.0%	0.0%	15	15	9999
EW-G4	5.1%	44.9%	0.9%	0.0%	0.37	0.0%	0.0%	4	4	9999

Fig. 27 DAA compared to VAA and EW, Mar 2008 – Mar 2018 [21]

Comparing DAA-G12, -U6 and -U15 with their VAA equivalents, return R and return/risk K50 and K25 are all better for DAA (especially for DAA-U6/U15 with K25= 7.7%/7.3% vs. 5.7%/5.1% for VAA), while drawdown D is worse than VAA but still very limited (<13% over Mar 2008 – Mar 2018, so including the Global Financial Crisis). UPI for DAA-G12 is slightly worse (2.90 vs 2.98 for VAA), while better for U6/U15 (UPI=3.13/2.52 vs. 2.58/2.46 for VAA). Comparing DAA with VAA, VAA-G4 still wins dominantly on performance (as we discussed before), but not with regard to CF and TTC where DAA is substantially better (half of CF and TTC than VAA). For all four strategies, CF for DAA is less than half of VAA while TTC is also lower for DAA.

So, we conclude that DAA in general improves over VAA in terms of return and return/risk, although with slightly higher but still very acceptable drawdowns, while reducing the cash fraction CF from around 60% to less than 30%, resulting also in less reliance on bond yields, less false signals, lower turnover and lower tracking error to market uptrends.

We also looked in Section 9 at an “aggressive” variant of DAA with B=1 and with leveraged assets for risky and cash assets. Because of B=1, cash protection is faster (and “binary”: CF=0/100%) so the average cash fraction CF is higher than for the default DAA (with B=2), but still less than that of VAA. Although our leveraged ETF proxies might raise some questions, we believe it does show the power of our canary approach as very effective crash protection.

[21] Legend: See note 10, with N=NR (# risky assets) and TTC is Total Transaction Costs.

Finally, we did not discuss why (bad momentum of) VWO and BND (aka EEM and AGG) would be a good "canary" in predicting crashes in terms of economic theory. Of course, bad BND performance might reflect higher US bond yields resulting in lower appeal of (US) stocks. See also Fig. 1 and AS, 2017. But explaining the rationale for bad VWO/EEM performance as canary for stocks seems more complicated. It might be because VWO/EEM reflect the sensitivity of Emerging Markets (EM) to crashes in Developed Markets. It might also suggest a currency signal: when the US dollar becomes stronger, EM currencies become relatively weaker pushing down EEM indices (in USD). And we know that the US dollar and US stock markets are often negatively correlated (see eg. Hedgey, 2018). But more research is needed here.

Appendix A Details of DAA-G12

On RS (Mar 2008 – Mar 2018):

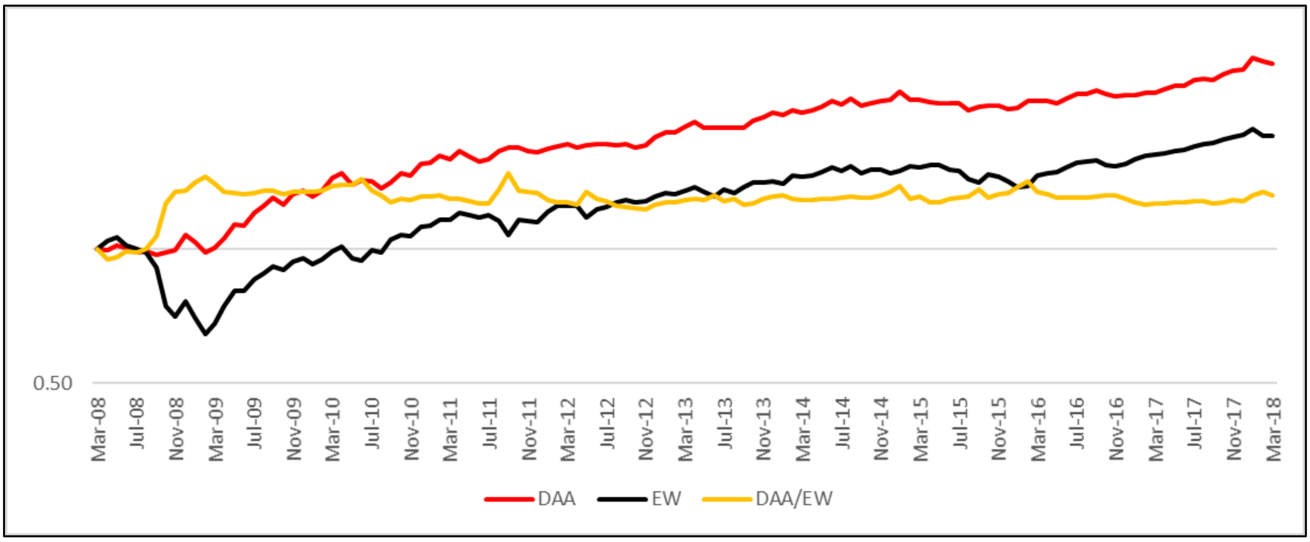

Fig. 28a Log equity DAA-G12 vs. EW-G12 (and relative DAA/EW), Mar 2008 – Mar 2018 (RS)

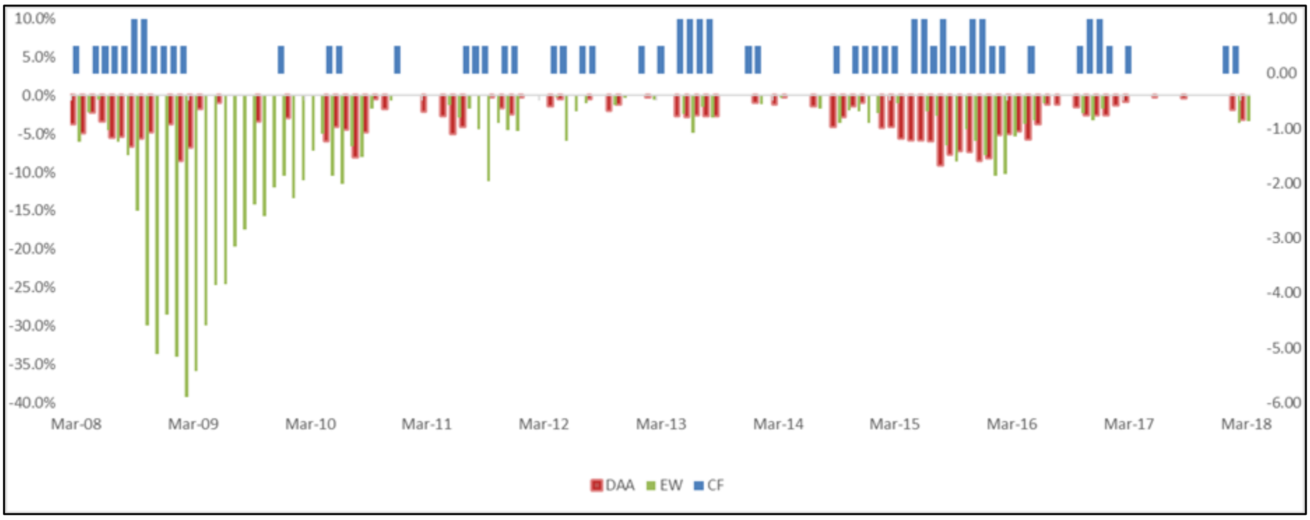

Fig. 28b Drawdowns and CF of DAA-G12 and EW-G12, Mar 2008 – Mar 2018 (RS)

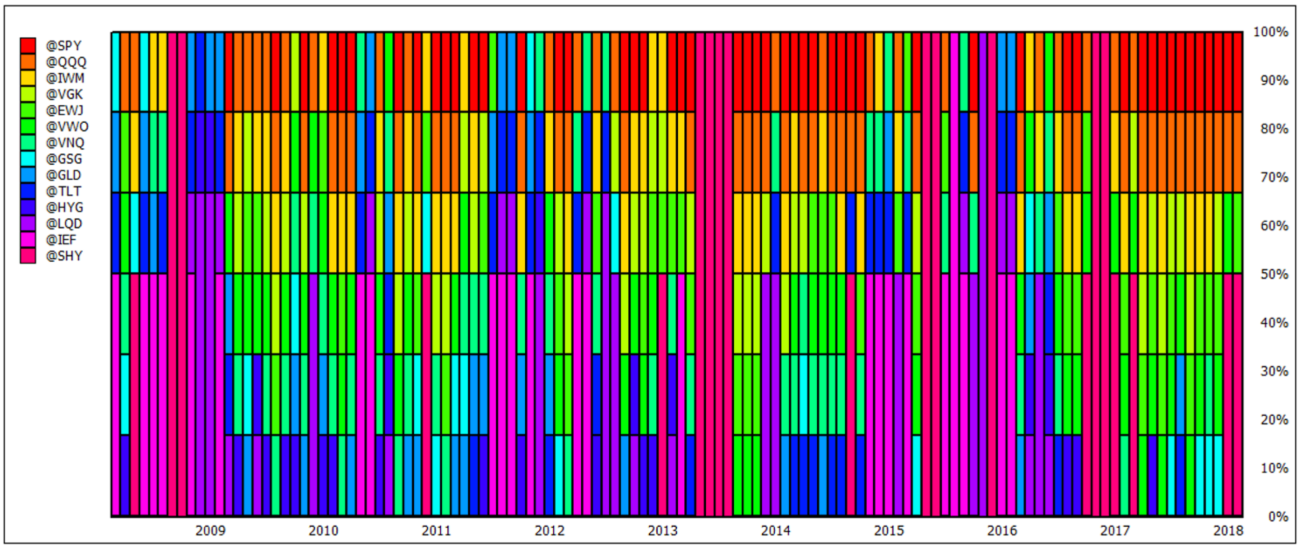

Fig. 28c Allocation mix for DAA-G12, Mar 2008 – Mar 2018 (RS)

On FS (Dec 1970 – Mar 2018):

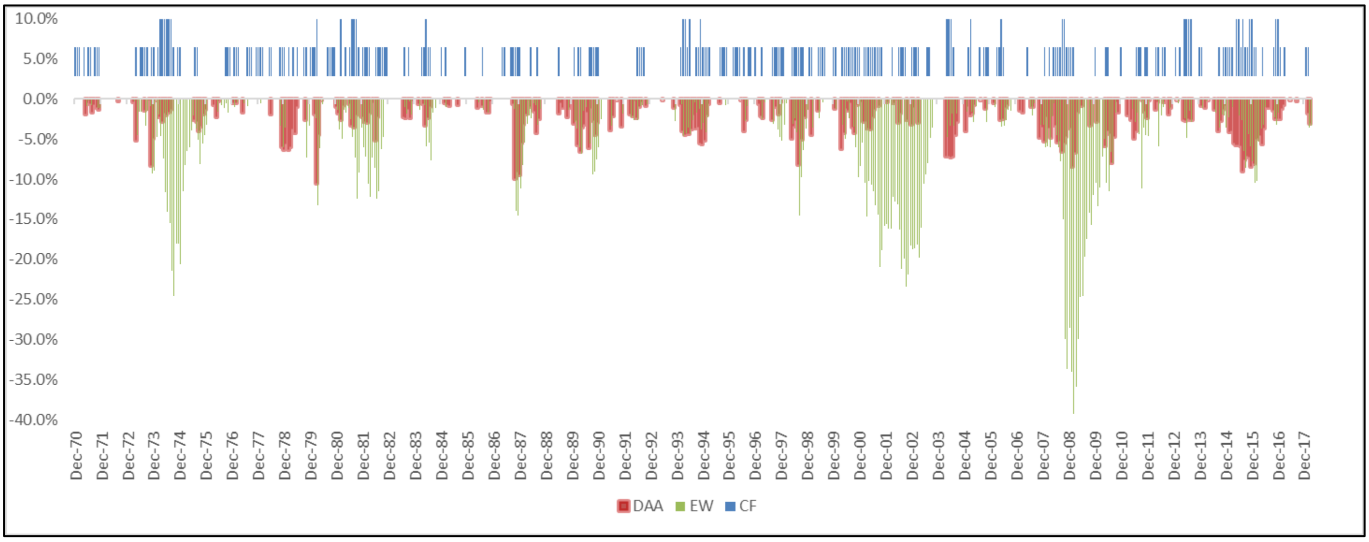

Fig. 28d Drawdowns and CF of DAA-G12 and EW-G12, Dec 1970 – Mar 2018 (FS)

Appendix B. Literature

Allocate Smartly (AS), 2017, Vigilant Asset Allocation, https://allocatesmartly.com/vigilant-asset-allocation-dr-wouter-keller-jw-keuning/

Antonacci, G., 2013a, Absolute Momentum: A Simple Rule-Based Strategy and Universal Trend Following Overlay, SSRN 2244633

Antonacci, G., 2013b, Risk Premia Harvesting Through Dual Momentum SSRN 2042750

Antonacci, G., 2014, Dual Momentum Investing, McGraw Hill (book)

Asness, C.S., A. Frazzini, R. Israel, and T.J. Moskowitz, 2014, Fact, Fiction and Momentum Investing. Journal of Portfolio Management, Fall 2014, SSRN 2435323

Beekhuizen, P. and W.G. Hallerbach, 2015, Uncovering Trend Rules, SSRN 2604942

CXO, 2017, Momentum Investing/SACEMS with Different Alternatives for Cash, https://www.cxoadvisory.com/30180/momentum-investing/sacems-with-different-alternatives-for-cash/

Faber, M. T., 2007, A Quantitative Approach to Tactical Asset Allocation, Journal of Wealth Management, Spring 2007. Updated in Faber (2013).

Faber, M. T., 2010, Relative Strength Strategies for Investing, SSRN: 1585517

Faber, M. T., 2013, A Quantitative Approach to Tactical Asset Allocation, SSRN 962461. Update of Faber (2007).

Faber, M.T., 2018, QTAA Backtester 2018, The Idea Farm (subscription), https://theideafarm.com/about/

Faber, Nathan, 2015, The Search for Crisis Alpha: Weathering the Storm Using Relative Momentum, ThinkNewfound.com (paper)

French, K.R., 2018, Data Library, https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Goldberg, L.R., and O. Mahmoud, 2016, Drawdown from Practice to Theory, arxiv.org/pdf/1404.7493.pdf

Harding, D., G. Nakou and A. Nejjar, 2003, The Pros and Cons of 'Drawdown' as a Statistical Measure of Risk for Investments 2003, AIMA Journal, april 2003, p.16

Hedgey, 2018, The Investing Implications of a Strong US Dollar, https://app.hedgeye.com/insights/68178-the-investing-implications-of-a-strong-u-s-dollar

Hurst, B., Y.H. Ooi, and L.H. Pedersen, 2012, A Century of Evidence on Trend-Following Investing, working paper, AQR Capital Management.

Jegadeesh, N., and S. Titman, 1993, Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency, Journal of Finance XLVIII, 65/91.

Keller, W.J. and A. Butler, 2014, A Century of Generalized Momentum; From Flexible Asset Allocations (FAA) to Elastic Asset Allocation (EAA), SSRN 2543979

Keller, W.J., A. Butler, and I. Kipnis, 2015, Momentum and Markowitz: A Golden Combination, SSRN 2606884

Keller, W.J., and J.W. Keuning, 2016, Protective Asset Allocation (PAA), SSRN 2759734

Keller, W.J., and J.W. Keuning, 2017, Vigilant Asset Allocation (VAA), SSRN 3002624

Levine, A. and L. H. Pedersen, 2015, Which Trend is Your Friend?, SSRN 2603731

Magdon-Ismail,M., A. Atiya, 2004, An Analysis of the Maximum Drawdown Risk Measure, Risk 2004/10

Martin, 1987, The Ulcer Index, http://www.tangotools.com/ui/ui.htm

Moskowitz, T., Y.H. Ooi, and L.H. Pedersen, 2011, Time Series Momentum, Working Paper nr. 79, The Initiative on Global Markets, University of Chicago.

Newfound, 2015, Two Centuries of Momentum, Thinknewfound.com (paper)

Smithsonian,2016, The Story of the Real canary in the Coal Mine, www.smithsonianmag.com/smart-news/story-real-canary-coal-mine-180961570/

TrendXplorer, 2016, Prospecting Dual Momentum With GEM, https://indexswingtrader.blogspot.com/2016/10/prospecting-dual-momentum-with-gem.html

TrendXplorer, 2017, Index Mapping for ETF Proxies, https://indexswingtrader.blogspot.com/2017/03/index-mapping-for-etf-proxies.html

TrendXplorer, 2018, Presenting Keller Ratio, https://indexswingtrader.blogspot.com/2018/04/presenting-keller-ratio.html

TrendXplorer, 2018, DAA on Steroids, forthcoming, https://indexswingtrader.blogspot.com

Zakamulin, V., 2015, Market Timing with Moving Averages: Anatomy and Performance of Trading Rules, SSRN 2585056