Parameter Estimation

Ejercicio 1 (theoretical)

(1) Sea $A(x_1, x_2, \ldots, x_n)$ donde $A \sim N(\mu, \sigma)$ con parámetros μ y σ. Muestre que los estimadores máximos verosímiles son:

(a) $\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i$ (b) $\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

Desarrollo...

Fn de verosimilitud

$$L(\mu) = \prod_{i=1}^{n} f(x_i \mid \mu) = \prod_{i=1}^{n}\left[\frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{1}{2\sigma^2}\cdot(x_i-\mu)^2}\right] \rightarrow$$

$$\rightarrow \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n \underbrace{\left(e^{-\frac{1}{2\sigma^2}\cdot\sum_{i=1}^{n}(x_i-\mu)^2}\right)}_{*} = \left((2\pi\sigma^2)^{-n/2}\right)(*)$$

$e^a \cdot e^b = e^{a+b}$

$$\rightarrow \ln(2\pi\sigma^2)^{-n/2} + \ln\left(e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2}\right)$$

$$\rightarrow \underbrace{-\frac{n}{2}\ln(2\pi\sigma^2)}_{cte} - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2 \cdot \underbrace{\ln(e)}_{1}$$

$$\frac{dL(\mu)}{d\mu} = \cancel{0} - \left(\frac{1}{2\sigma^2}\right)(2)\left(\sum_{i=1}^{n}(x_i-\mu)^1\right)(-1)$$

$$\rightarrow 0 = \frac{\sum_{i=1}^{n}(x_i-\mu)}{\sigma^2} \rightarrow \sum_{i=1}^{n}(x_i-\mu) = 0 \rightarrow \sum_{i=1}^{n} x_i - \sum_{i=1}^{n}\mu \rightarrow$$

$\sigma^2 \neq 0$

$$n\mu = \sum_{i=1}^{n} x_i \rightarrow \hat{\mu} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)$$

b(a)

Para σ^2:

ln de la función de verosimilitud

$$L_n(L(\mu,\sigma^2)) = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\frac{dL(\mu,\sigma^2)}{d\sigma^2} = -\frac{n}{2\sigma^2} + \frac{1}{2(\sigma^2)^2}\sum_{i=1}^{n}(x_i-\mu)^2 = 0$$

$$\rightarrow -n + \frac{1}{\sigma^2}\sum(x_i-\mu)^2 \rightarrow n\sigma^2 = \sum_{i=1}^{n}(x_i-\mu)^2 \rightarrow \sigma^2 = \frac{1}{n}\left(\sum(x_i-\bar{x})\right)$$