

$$1. \quad x_{n+1} = 4x_n - x_n^2, \quad x_0 = 4 \sin^2 \theta$$

$$\Rightarrow x_{n+1} = 4 \sin^2(2^{n+1} \theta)$$

$$x_1 = 16 \sin^2 \theta - 16 \sin^4 \theta$$

$$x_1 = 16 \sin^2 \theta (1 - \sin^2 \theta)$$

$$x_1 = 16 \sin^2 \theta \cos^2 \theta$$

$$* \sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\sin^2(2\theta) = 4 \sin^2 \theta \cos^2 \theta \Rightarrow x_1 = 4(4 \sin^2 \theta \cos^2 \theta)$$

$$\Rightarrow x_1 = 4 \sin^2(2\theta)$$

$$\Rightarrow x_{n+1} = 4 \sin^2(2^{n+1} \theta)$$

$$* \quad x_{n+1} \Rightarrow 4x_n - 4x_n^2 \quad x_0 = \sin^2 \theta$$

$$x_{0+1} = 4x_0 - 4x_0^2$$

$$= 4 \sin^2 \theta - 4 (\sin^2 \theta)^2$$

$$= 4 \sin^2 \theta - 4 \sin^4 \theta$$

$$4 \sin^2 \theta \cos^2 \theta$$

$$\Rightarrow x_1 = \sin^2(2\theta)$$

$$\boxed{x_{n+1} = \sin^2(2^{n+1} \theta)}$$