A Solution to Goodman's Paradox

Paul FRANCESCHI

University of Corsica
p.franceschi@univ-corse.fr
http://www.univ-corse.fr/~franceschi

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ABSTRACT: In the classical version of Goodman's paradox, the universe where the problem takes place is ambiguous. The conditions of induction being accurately described, I define then a framework of n*-universes, allowing the distinction, among the criteria of a given* n*-universe, between constants and variables. Within this framework, I distinguish between two versions of the problem, respectively taking place: (i) in an* n*-universe the variables of which are colour and time; (ii) in an* n*-universe the variables of which are colour, time and space. Finally, I show that each of these versions admits a specific resolution.*

1. The problem

Goodman's Paradox (thereafter GP) has been described by Nelson Goodman (1946).[1] Goodman exposes his paradox as follows.[2] Consider an urn containing 100 balls. A ball is drawn each day from the urn, during 99 days, until today. At each time, the ball extracted from the urn is red. Intuitively, one expects that the 100th ball drawn from the urn will also be red. This prediction is based on the generalisation according to which all the balls in the urn are red. However, if one considers the property S "drawn before today and red or drawn after today and non-red", one notes that this property is also satisfied by the 99 instances already observed. But the prediction which now ensue, based on the generalisation according to which all the balls are S, is that the 100th ball will be non-red. And this contradicts the preceding conclusion, which however conforms with our intuition.[3]

Goodman expresses GP with the help of an enumerative induction. And one can model GP in terms of the *straight rule* (SR). If one takes (D) for the definition of the "red" predicate, (I) for the enumeration of the instances, (H) for the ensuing generalisation, and (P) for the corresponding prediction, one has then:

(D) R = red
(I) $Rb_1 \cdot Rb_2 \cdot Rb_3 \cdot ... \cdot Rb_{99}$
(H) $Rb_1 \cdot Rb_2 \cdot Rb_3 \cdot ... \cdot Rb_{99} \cdot Rb_{100}$
$\therefore$ (P) Rb_{100}

And also, with the predicate S:

(D*) S = red and drawn before T or non-red and drawn after T
(I*) $Sb_1 \cdot Sb_2 \cdot Sb_3 \cdot ... \cdot Sb_{99}$
(H*) $Sb_1 \cdot Sb_2 \cdot Sb_3 \cdot ... \cdot Sb_{99} \cdot Sb_{100}$ that is equivalent to:
(H'*) $Rb_1 \cdot Rb_2 \cdot Rb_3 \cdot ... \cdot Rb_{99} \cdot \sim Rb_{100}$
$\therefore$ (P*) Sb_{100} i. e. finally:
$\therefore$ (P'*) $\sim Rb_{100}$

The paradox resides here in the fact that the two generalisations (H) and (H*) lead respectively to the predictions (P) and (P'*), which are contradictory. Intuitively, the application of SR to (H*) appears erroneous. Goodman also gives in *Fact, Fiction and Forecast*[4] a slightly different version of the paradox, applied in this case to emeralds.[5] This form is very well known and based on the predicate "grue" = green and observed before T or non-green and observed after T.

The predicate S used in Goodman (1946) presents with "grue", a common structure. P and Q being two predicates, this structure corresponds to the following definition: (P and Q) or (~P and ~Q). In what follows, one will designate by *grue* a predicate having this particular structure, without distinguishing whether the specific form used is that of Goodman (1946) or (1954).

2. The unification/differentiation duality

The instances are in front of me. Must I describe them by stressing their differences? Or must I describe them by emphasising their common properties? I can proceed either way. To stress the differences between the instances, is to operate by *differentiation*. Conversely, to highlight their common properties, is to proceed by *unification*. Let us consider in turn each of these two modes of proceeding.

Consider the 100 balls composing the urn of Goodman (1946). Consider first the case where my intention is to stress the differences between the instances. There, an option is to apprehend the particular and single moment, where each of them is extracted from the urn. The considered predicates are then: *red and drawn on day 1*, *red and drawn on day 2*, ..., *red and drawn on day 99*. There are thus 99 different predicates. But this prohibits applying SR, which requires one single predicate. Thus, what is to distinguish according to the moment when each ball is drawn? It is to stress an essential difference between each ball, based on the criterion of time. Each ball thus is individualised, and many different predicates are resulting from this: drawn at T_1, drawn at T_2, ..., drawn at T_{99}. This indeed prevents then any inductive move by application of SR. In effect, one does not have then a common property to allow induction and to apply SR. Here, the cause of the problem lies in the fact of having carried out an *extreme differentiation*.

Alternatively, I can also proceed by *differentiation* by operating an extremely precise[6] measurement of the wavelength of the light defining the colour of each ball. I will then obtain a unique measure of the wavelength for each ball of the urn. Thus, I have 100 balls in front of me, and I know with precision the wavelength of the light of 99 of them. The balls respectively have a wavelength of 722,3551 nm, 722,3643 nm, 722,3342 nm, 722,3781 nm, etc. I have consequently 99 distinct predicates P3551, P3643, P3342, P3781, etc. But I have no possibility then to apply SR, which requires one single predicate. Here also, the common properties are missing to allow to implement the inductive process. In the same way as previously, it proves here that I have carried out an *extreme differentiation*.

What does it occur now if I proceed exclusively by *unification*? Let us consider the predicate R corresponding to "red or non-red". One draws 99 red balls before time T. They are all R. One predicts then that the 100th ball will be R after T, i.e. red or non-red. But this form of induction does not bring any information here. The resulting conclusion is empty of information. One will call *empty induction* this type of situation. In this case, one observes that the process of unification of the instances by the colour was carried out in a radical way, by annihilating in this respect, any step of differentiation. The cause of the problem lies thus in the implementation of a process of *extreme unification*.

If one considers now the viewpoint of *colour*, it appears that each case previously considered requires a different taxonomy of colours. Thus, it is made use successively:

- of our usual taxonomy of colours based on 9 predicates: purple, indigo, blue, green, yellow, orange, red, white, black
- of a taxonomy based on a comparison of the wavelengths of the colours with the set of the real numbers (*real taxonomy*)
- of a taxonomy based on a single predicate (*single taxon taxonomy*): red or non-red

But it proves that each of these three cases can be replaced in a more general perspective. Indeed, multiple taxonomies of colours are susceptible to be used. And those can be ordered from the coarser (single taxon taxonomy) to the finest (real taxonomy), from the most unified to the most differentiated. We have in particular the following hierarchy of taxonomies:

- TAX_1 = {red or non-red} (single taxon taxonomy)
- TAX_2 = {red, non-red} (binary taxonomy)
- ...
- TAX_9 = {purple, indigo, blue, green, yellow, orange, red, white, black} (taxonomy based on the spectral colours, plus white and black)
- ...
- $TAX_{16777216}$ = {(0, 0, 0), ..., (255, 255, 255)} (taxonomy used in computer science and distinguishing 256 shades of red/green/blue)
- ...
- TAX_R = {370, ..., 750} (real taxonomy based on the wavelength of the light)

Within this hierarchy, it appears that the use of extreme taxonomies such as the one based on a single taxon, or the real taxonomy, leads to specific problems (respectively *extreme unification* and *extreme differentiation*). Thus, the problems mentioned above during the application of an inductive reasoning based on SR occur when the choice in the unification/differentiation duality is carried out too radically. Such problems relate to induction in general. This invites to think that one must rather reason as follows: I should privilege neither *unification*, nor *differentiation*. A predicate such as "red", associated with our usual taxonomy of colours (TAX_9)[7], corresponds precisely to such a criterion. It corresponds to a balanced choice in the unification/differentiation duality. This makes it possible to avoid the preceding problems. This does not prevent however the emergence of new problems, since one tries to implement an inductive reasoning, in certain situations. And one of these problems is naturally GP.

Thus, it appears that the stake of the choice in the duality unification/differentiation is essential from the viewpoint of induction, because according to whether I choose one way or the other, I will be able or not to use SR and produce valid inductive inferences. Confronted with several instances, one can implement either a process of differentiation, or a process of unification. But the choice that is made largely conditions the later success of the inductive reasoning carried out on those grounds. I must describe both common properties and differences. From there, a valid inductive reasoning can take place. But at this point, it appears that the role of the unification/differentiation duality proves to be crucial for induction. More precisely, it appears at this stage that a correct choice in the unification/differentiation duality constitutes one of the conditions of induction.

3. Several problems concerning induction

The problems which have been just mentioned constitute the illustration of several difficulties inherent to the implementation of the inductive process. However, unlike GP, these problems do not generate a genuine contradiction. From this point of view, they distinguish from GP. Consider now the following situation. I have drawn 99 balls respectively at times T_1, T_2, ..., T_{99}. The 100th ball will be drawn at T_{100}. One observes that the 99 drawn balls are red. They are thus at the same time red and drawn before T_{100}. Let R be the predicate "red" and T the predicate "drawn before T_{100}". One has then:

(I) RTb_1, RTb_2, ..., RTb_{99}
(H) RTb_1, RTb_2, ..., RTb_{99}, RTb_{100}
∴ (P) RTb_{100}

By direct application of SR, the following prediction ensue: "the 100th ball is red and drawn before T_{100}". But this is in contradiction with the data of the experiment in virtue of which the 100th ball is drawn in T_{100}. There too, the inductive reasoning is based on a formalisation which is that of SR. And just as for GP, SR leads here to a contradiction. Call Δ2 this problem, where two predicates are used.

It appears that one can easily build a form of Δ2 based on one single predicate. A way of doing that is to consider the unique predicate S defined as "red and drawn before T_{100}" in replacement of the predicates R and T used previously. The same contradiction then ensues.

Moreover, it appears that one can highlight another version (Δ1) of this problem comprising only one predicate, without using the "red" property which appears useless here. Let indeed T be the predicate drawn before T_{100}. One has then:

(I) Tb_1, Tb_2, ..., Tb_{99}
(H) Tb_1, Tb_2, ..., Tb_{99}, Tb_{100}
∴ (P) Tb_{100}

Here also, the conclusion according to which the 100th ball is drawn before T_{100} contradicts the data of the experiment according to which the 100th ball is drawn at T_{100}. And one has then a contradictory effect, analogous to that of GP, without the structure of "grue" being implemented. Taking into account the fact that only the criterion of time is used to build this problem, it will be denoted in what follows by Δ1-time.

It appears here that the problems such as Δ1-time and Δ2 lead just as GP to a contradiction. Such is not the case for the other problems related to induction previously mentioned[8], which involve either the impossibility of carrying out induction, or a conclusion empty of information. However, it proves that the contradiction encountered in Δ1-time is not of the same nature as that observed in GP. Indeed in GP, one has a contradiction between the two concurrent predictions (P) and (P*). On the other hand, in Δ1-time, the contradiction emerges between on the one hand the conditions of the experiment ($T \geq 100$) and on the other hand the prediction resulting from generalisation ($T < 100$).

Anyway, the problems which have been just encountered suggest that the SR formalism does not capture the whole of our intuitions related to induction. Hence, it is worth attempting to define accurately the conditions of induction, and adapting consequently the relevant formalism. However, before carrying out such an analysis, it is necessary to specify in more detail the various elements of the context of GP.

4. The universe of reference

Let us consider the law (L1) according to which "diamond scratches the other solids". A priori, (L1) strikes us as an undeniable truth. Nevertheless, it proves that at a temperature higher than 3550°C, diamond melts. Therefore in last analysis, the law (L1) is satisfied at a normal temperature and in any case, when the temperature is lower than 3550°C. But such a law does not apply beyond 3550°C. This illustrates how the statement of the conditions under which the law (L1) is verified is important, in particular with regard to the conditions of temperature. Thus, when one states (L1), it proves necessary to specify the conditions of temperature in which (L1) finds to apply. This is tantamount to describing the type of universe in which the law is satisfied.

Let also (P1) be the following proposition: "the volume of the visible universe is higher than 1000 times that of the solar system". Such a proposition strikes us as obvious. But there too, it appears that (P1) is satisfied at modern time, but that it proves to be false at the first moments of the universe. Indeed, when the age of our universe was 10^{-6} second after the big-bang, its volume was approximately equal to that of our solar system.

Here also, it thus appears necessary to specify, at the same time as the proposition (P1) the conditions of the universe in which it applies. A nonambiguous formulation of (P1) thus comprises a more restrictive temporal clause, such as: "at our time, the volume of the visible universe is higher than 1000 times that of the solar system". Thus, generally, one can think that when a generalisation is stated, it is necessary to specify the conditions of the universe in which this generalisation applies. The precise description of the *universe of reference* is fundamental, because according to the conditions of the universe in which one places oneself, the stated law can appear true or false.

One observes in our universe the presence of both constants and variables. There are thus constants, which constitute the fundamental constants of the universe: the speed of light: c = 2,998 x10^8 m/s; Planck's constant: h = 6,626 x 10^{-34} J.s; the electron charge; e = 1,602 x 10^{-19} C; etc. There are on the other hand variables. Among those, one can mention in particular: temperature, pressure, altitude, localisation, time, presence of a laser radiation, presence of atoms of titanium, etc.

One often tends, when a generalisation is stated, not to take into account the constants and the variables which are those of our universe envisaged in its totality. Such is the case for example when one considers the situation of our universe on 1 January 2000, at 0h. One places then oneself explicitly in what constitutes a section, a slice of our universe. In effect, time is not regarded then a variable, but well as a constant. Consider also the following: "the dinosaurs had hot blood"[9]. Here, one places oneself explicitly in a sub-universe of our where the parameters of time and space have a restricted scope. The temporal variable is reduced to the particular time of the Earth history which knew the appearance of the dinosaurs: the Triassic and the Cretaceous. And similarly, the space parameter is limited to our planet: Earth. Identically, the conditions of temperature are changing within our universe, according to whether one is located at one site or another of it: at the terrestrial equator, the surface of Pluto, the heart of Alpha Centauri, etc. But if one is interested exclusively in the balloon being used for the experimentation within the laboratory of physics, where the temperature is maintained invariably at 12°C, one can then regard valuably the temperature as a constant. For when such generalisations are expressed, one places oneself not in our universe under consideration in his totality, but only in what veritably constitutes a specific part, a restriction of it. One can then assimilate the universe of reference in which one places oneself as a sub-universe of our. It is thus frequent to express generalisations which are only worth for the present time, or for our usual terrestrial conditions. Explicitly or not, the statement of a law comprises a universe of reference. But in the majority of the cases, the variables and the constants of the considered sub-universe are distinct from those allowing to describe our universe in its totality. For the conditions are extremely varied within our universe: the conditions are very different according to whether one places oneself at the 1st second after the big-bang, on Earth at the Precambrian epoch, in our planet in year 2000, inside the particle accelerator of the CERN, in the heart of our Sun, near a white dwarf, or well inside a black hole, etc.

One can also think that it is interesting to be able to model universes the constants of which are different from the fundamental constants of our universe. One can thus wish to study for example a universe where the mass of the electron is equal to 9,325 x10^{-31} kg, or well a universe where the electron charge is equal to 1,598 x 10^{-19} C. And in fact, the toy-universes, which take into account fundamental constants different from those of our familiar universe, are studied by the astrophysicists.

Lastly, when one describes the conditions of a thought experiment, one places oneself, explicitly or not, under the conditions which are related to those of a sub-universe. When one considers for example 100 balls extracted from an urn during 100 consecutive days, one places then oneself in a restriction of our universe where the temporal variable is limited to one period of 100 days and where the spatial location is extremely reduced, corresponding for example to a volume approximately equal to 5 dm^3. On the other hand, the number of titanium or zirconium atoms possibly present in the urn, the possible existence of a laser radiation, the presence or the absence of a sound source of 10 db, etc. can be omitted and ignored. In this context, it is not necessary to take into account the existence of such variables. In this situation, it is enough to mention the variables and the constants *actually* used in the thought experiment. For one can think indeed that the number of variables in our universe is so large that it is impossible to enumerate them all. And consequently, it does not appear possible to characterise our universe in function of all its variables, because one can not provide an infinite enumeration of it. It appears sufficient to describe the considered sub-universe, by mentioning only the constants and the variables which play an effective role in the experiment. Thus, in such situations, one will describe the considered sub-universe by mentioning only the effective criteria necessary to the description of the experiment.

What precedes encourages to think that generally, in order to model the context in which the problems such as GP take place, it is convenient to describe a given universe in terms of variables and constants. This leads thus to define a n-universe ($n \geq 0$) as a universe the criteria of which comprise m constants, and n variables, where the m constants and n variables constitute the *criteria* of the given universe. Within this particular framework, one defines a *temporal 1-universe* (Ω^1T) as a universe comprising only one criterion-variable: time. In the same way, one defines a *coloured 1-universe* (Ω^1C) as a universe comprising only one criterion-variable: colour. One will define also a *coloured and temporal 2-universe* (Ω^2CT) as a universe comprising two criterion-variables: time and colour. Etc. In the same way, a universe where all the objects are red, but are characterised by a different localisation will be modelled by a *localised 1-universe* (Ω^1L) a criterion-constant (red) of which is colour.

It should be noted incidentally that the *n*-universe framework makes it possible in particular to model several interesting situations. Thus, a temporal universe can be regarded as a *n*-universe one of the variables of which is a temporal criterion. Moreover, a universe where one single moment T_0 is considered, deprived of the phenomenon of succession of time, can be regarded as a *n*-universe where time does not constitute one of the variables, but where there is a constant-time. In the same way, an *atemporal* universe corresponds to a *n*-universe no variable of which corresponds to a temporal criterion, and where there is not any time-constant.

In the context which has been just defined, what is it now to be *red*? Here, being "red" corresponds to two different types of situations, according to the type of *n*-universe in which one places oneself. It can be on the one hand a *n*-universe one of the constants of which is colour. In this type of universe, the colour of the objects is not susceptible to change, and all the objects are there invariably red.

The fact of being "red" can correspond, on the second hand, to a *n*-universe one of the criterion-variables of which is constituted by colour. There, an object can be red or non-red. Consider the case of a Ω^1C. In such a universe, an object is red or non-red *in the absolute*. No change of colour is possible there, because no other criterion-variable exists, of which can depend such a variation. And in a Ω^2CT, being red is being red at time T. Within such a universe, being red is being red *relatively* to time T. Similarly, in a coloured, temporal and localised 3-universe (Ω^3CTL), being red is being red at time T and at place L. Etc. In some such universe, being red is being red relatively to other criterion-variables. And the same applies to the *n*-universes which model a universe such as our own.

At this step arises the problem of the *status of the instances* of an object of a given type. What is it thus to be an instance, within this framework? This problem has its importance, because the original versions of GP are based on instances of balls (1946) and emeralds (1954). If one takes into account the case of Goodman (1946), the considered instances are 100 different balls. However, if one considers a unique ball, drawn at times T_1, T_2, ..., T_{100}, one notices that the problem inherent to GP is always present. It suffices indeed to consider a ball whose colour is susceptible to change during the course of time. One has drawn 99 times the ball at times T_1, T_2, ..., T_{99}, and one has noted each time that the ball was red. This leads to the prediction that the ball will be red at T_{100}. However, this last prediction proves to be contradictory with an alternative prediction based on the same observations, and the projection of the predicate S "red and drawn before T_{100} or non-red and drawn at T_{100}"[10].

The present framework must be capable of handling the diversity of these situations. Can one thus speak of an instantiated and temporal 1-universe, or well of an instantiated and coloured 1-universe? Here, one must observe that the fact of being instantiated, for a given universe, corresponds to an additional criterion-variable. For, on the contrary, what makes it possible to distinguish between the instances? If no criterion distinguishes them, it is thus only one and the same thing. And if they are distinct, it is thus that a criterion makes it possible to differentiate them. Thus, an instantiated and temporal 1-universe is in fact a 2-universe, whose 2nd criterion, which makes it possible to distinguish the instances between them, is in fact not mentioned nor explicited. By making explicit this second criterion-variable, it is thus clear that one is placed in a 2-universe. In the same way, an instantiated and coloured 1-universe is actually a 2-universe one of the criteria of which is colour and the second criterion exists but is not specified.

Another aspect which deserves mention here, is the question of the reduction of a given *n*-universe to another. Is it not possible indeed, to logically reduce a *n*-universe to a different system of criteria? Consider for example a Ω^3CTL. In order to characterise the corresponding universe, one has 3 criterion-variables: colour, time and localisation. It appears that one can reduce this 3-universe to a 2-universe. That can be carried out by reducing two of the criteria of the 3-universe to one single criterion. In particular, one will reduce both criteria of colour and time to a single criterion of tcolour* (shmolor[11]). And one will only preserve two taxa of tcolour*: G and ~G. Consider then a criterion of color comprising two taxa (red, non-red) and a criterion of time comprising two taxa (before T, after T). If one associates the taxa of colour and time, one obtains four new predicates: red before T, red after T, non-red before T, non-red after T, which one will denote respectively by RT, R~T, ~RT and ~R~T. Several of these predicates are compatible (RT and R~T, RT and ~R~T, ~RT and R~T, ~RT and ~R~T) whereas others are incompatible (RT and ~RT, R~T and ~R~T). At this stage, one has several manners (16)[12] of grouping the compatible predicates, making it possible to obtain two new predicates G and ~G of tcolour*:

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RT ∧ R~T		X				X	X	X				X	X	X		X
RT ∧ ~R~T			X			X			X	X		X	X		X	X
~RT ∧ R~T				X			X		X		X	X		X	X	X
~RT ∧ ~R~T					X			X		X	X		X	X	X	X

In each of these cases, it results indeed a new single criterion of tcolour* (Z), which substitutes itself to the two preceding criteria of colour and time. One will denote by Z_i ($0 \leq i \leq 15$) the taxa of tcolour* thus obtained. If it is clear that Z_{15} leads to the empty induction, it should be observed that several cases corresponding to the situation where the instances are RT lead to the problem inherent to GP. One will note thus that Z_2, i.e. *grue*$_2$ (by assimilating the Z_i to *grue*$_i$ and the Z_{15-i} to *bleen*$_i$) is based on the definition: *grue*$_2$ = red before T *and* non-red after T. It appears here as a conjunctive interpretation of the definition of "grue". In the same way, *grue*$_7$

corresponds to a definition of "grue" based on an *exclusive disjunction*. Lastly, $grue_{12}$ is based on the traditional definition: $grue_{12}$ = red before T or non-red after T, where the disjunction is to be interpreted as an *inclusive disjunction*.

Similarly, it also proves that a Ω^2CT can be reduced to a tcoloured* 1-universe (Ω^1Z). And more generally, a *n*-universe is thus *reducible* to an (*n*-1)-universe (for $n > 1$). Thus, if one considers a given universe, several characterisations in terms of *n*-universe can valuably be used. One can in particular apprehend a same universe like a Ω^3CTL, or like a Ω^2ZL. In the same way, one can represent a Ω^2CT like a Ω^1Z. At this stage, none of these views appears fundamentally better than the other. But each of these two characterisations constitute alternative ways to describe a same reality. This shows finally that a *n*-universe constitutes in fact an abstract characterisation of a real or an imaginary universe. A *n*-universe constitutes thus a *system of criteria*, comprising constants and variables. And in order to characterise a same real or imaginary given universe, one can resort valuably to several *n*-universes. Each of them appears finally as a different characterisation of the given universe, simply based on a different set of primitives.

5. Conditions of induction

The fact that the SR formalism involves the GP effect suggests that the intuition which governs our concept of induction is not entirely captured by SR. It is thus allowed to think that if the formal approach is necessary and useful to be used as support to induction, it does not constitute however a sufficient step. For it appears also essential to capture the intuition which governs our inductive reasoning. Therefore it proves necessary to supplement the formal approach of induction by a semantic approach. Goodman himself provides us with a definition of induction[13]. He defines induction as the projection of characteristics of the past through the future, or more generally, as the projection of characteristics corresponding to a given aspect of an object through another aspect. This last definition corresponds to our intuition of induction. One can think however that it is necessary to supplement it by taking into account the preceding observations[14] concerning the differentiation/unification duality. In that sense, it has been pointed out that induction consists of an inference from instances presenting both common properties and differences. Let the instances-source (instances-S) be the instances to which relate (I) or (I*) and the instance-destination (instance-D) that which is the subject of (P) or (P*). The common properties relate to the instances-S and the differentiated properties are established between the instances-S and the instance-D. The following definition ensues: induction consists precisely in the fact that the instance-D[15] also presents the property that is common to the instances-S, whereas one does vary the criterion (criteria) on which the differences between the instances-S and the instance-D is (are) based. The inductive reasoning is thus based on the constant nature of a property, whereas such other property is variable.

From this definition of induction arise straightforwardly several conditions of induction. I shall examine them in turn. The first two conditions are thus the following ones:

(C1) the instances-S must present some common properties
(C2) the instances-S and the instance-D must present some distinctive properties

This has for consequence that one cannot apply induction in two particular circumstances: firstly (i) when the instances do not present any common property. One will call such a situation a *total differentiation* of the instances. The problems corresponding to this particular circumstance have been mentioned above[16]. And secondly (ii) when the instances do not present any distinctive property. One will call such a situation *total unification*. The problems encountered in this type of situation have also been mentioned previously[17].

It should also be noted that it is not here a question of intrinsic properties of the instances, but rather of the analysis which is carried out by the one who is on the point of reasoning by induction.

Taking into account the definition of induction which has been given, a third condition can be thus stated:

(C3) a criterion-variable is necessary for the common properties of the instances-S and another criterion-variable for the distinctive properties

This refers to the structure of the considered universe of reference. Consequently, two criterion-variables are at least necessary, in the structure of the corresponding universe of reference. One will call that the *minimal condition* of induction. Hence, a 2-universe is at least necessary in order that the conditions of induction can be satisfied. Thus, a Ω^2CT will be appropriate. In the same way, a temporal and localised 2-universe (Ω^2TL) will also satisfy the conditions which have been just defined, etc[18].

It should be noted that another way of stating this condition is as follows: the criterion-variable for the common properties and the criterion-variable for the differentiated properties must be distinct. One should not have confusion between the two. One can call that the *condition of separation* of the common properties and the distinctive properties. Such a principle appears as a consequence of the minimal condition for induction: one must have two criteria to perform induction, and these criteria must be different. If one chooses a same criterion for the common properties and the differentiated properties, one is brought back in fact to one single criterion and the context of a 1-universe, itself insufficient to perform induction.

Lastly, a fourth condition of induction results from the preceding definition:

(C4) one must project the common properties of the instances-S (and not the distinctive properties)

The conditions of induction which have been just stated make it possible from now on to handle the problems involved in the use of SR mentioned above[19]. It follows indeed that the following projections[20] are correct: C°T in a Ω^2CT, C°L in a Ω^2CL, Z°L in a Ω^2ZL, etc. Conversely, the following projections are incorrect: T°T in a Ω^1T, Z°Z in a Ω^1Z. In particular, one will note here that the projection T°T in the Ω^1T is that of Δ1-time. Δ1-time takes indeed place in a Ω^1T, whereas induction requires at the same time common properties and distinctive properties. Thus, a 2-universe is at least necessary. Usually, the criterion of time is used for differentiation. But here, it is used for unification ("drawn before T"). That can be done, but provided that one uses a distinct criterion for the differentiated properties. However, whereas common properties results here from that, the differentiated properties are missing. It thus misses a second criterion - corresponding to the differentiated properties - in the considered universe, to perform induction validly. Thus Δ1-time finds its origin in a violation of the *minimal condition* of induction. One can formulate this solution equivalently, with regard to the *condition of separation*. In effect, in Δ1-time, a same temporal criterion (drawn before T/drawn after T) is used for the common properties and the differentiated properties, whereas two distinct criteria are necessary. It can be thus analysed as a manifest violation of the condition of separation.

Lastly, the conditions of induction defined above lead to adapt the formalism used to describe GP. It proves indeed necessary to distinguish between the common and the distinctive property(ies). One will thus use the following formalism in replacement of the one used above:

(I) $RT_1 \cdot RT_2 \cdot RT_3 \cdot ... \cdot RT_{99}$
(H) $RT_1 \cdot RT_2 \cdot RT_3 \cdot ... \cdot RT_{99} \cdot RT_{100}$

where R denotes the common property and the T_i a distinctive property. It should be noted here that it can consist of a single object, or alternatively, of instances which are distinguished by a given criterion (which is not concerned by the inductive process) according to *n*-universe in which one places oneself. Thus, one will use in the case of a single instance α, the colour of which is susceptible to change according to time:

(I) $RT_1\alpha \cdot RT_2\alpha \cdot RT_3\alpha \cdot ... \cdot RT_{99}\alpha$

or in the case where several instances $\alpha_1, \alpha_2, ..., \alpha_{99}, \alpha_{100}$ exist[21]:

(I) $RT_1\alpha_1 \cdot RT_2\alpha_2 \cdot RT_3\alpha_3 \cdot ... \cdot RT_{99}\alpha_{99}$

6. Origin of the paradox

Given the conditions of induction and the framework of *n*-universes which have been just defined, one is now in a position to proceed to determine the origin of GP. Preliminarily it is worth describing accurately the conditions of the *universe of reference* in which GP takes place. Indeed, in the original version of GP, the choice of the universe of reference is not defined accurately. However one can think that it is essential, in order to avoid any ambiguity, that this last is described precisely.

The universe of reference in which Goodman (1946) places himself is not defined explicitly, but several elements of the statement make it possible to specify its intrinsic nature. Goodman thus mentions the colours "red" and "non-red". Therefore, colour constitutes one of the criterion-variables of the universe of reference. Moreover, Goodman distinguishes the balls which are drawn at times $T_1, T_2, T_3, ..., T_{100}$. Thus, time is also a criterion-variable of the considered universe. Consequently, one can describe the minimal universe in which Goodman (1946) places himself as a Ω^2CT. Similarly, in Goodman (1954), the criterion-variables of colour (green/non-green) and time (drawn before T/drawn after T) are expressly mentioned. In both cases, one thus places oneself implicitly within the *minimal* framework of a Ω^2CT.

Goodman in addition mentions *instances* of balls or emeralds. Is it necessary at this stage to resort to an additional criterion-variable making it possible to distinguish between the instances? It appears that not. On the one hand indeed, as we have seen previously[22], it proves that one has well a version of GP by simply considering a Ω^2CT and a single object, the colour of which is susceptible to change during the course of time. On the other hand, it appears that if the criterion which is used to distinguish the instances is not used in the inductive process, it is then neither useful as a common criterion, nor as a differentiated criterion. It follows that one can dispense with this 3rd additional criterion. Thus, it proves that the fact of taking into account one single instance or alternatively, several instances, is not essential in the formulation of GP. In what follows, one will be able thus to consider that the statement applies, indifferently, to a single object or several instances that are distinguished by a criterion which is not used in the inductive process.

At this step, we are in a position to replace GP within the framework of *n*-universes. Taking into account the fact that the context of GP is that of a *minimal* Ω^2CT, one will consider successively two situations: that of a Ω^2CT, and then that of a Ω^3CTα (where α denotes a 3rd criterion).

6.1 "Grue" in the coloured and temporal 2-universe

Consider first the hypothesis of a Ω^2CT. In such a universe, being "red" is being red at time T. One has then a criterion of colour for the common properties and a criterion of time for the differentiated properties. Consequently, it appears completely legitimate to project the common property of colour ("red") into the differentiated time. Such a projection proves to be in conformity with the conditions of induction stated above.

Let us turn now to the projection of "grue". One has observed previously[23] that the Ω^2CT was reducible to a Ω^1Z. Here, the fact of using "grue" (and "bleen") as primitives, is characteristic of the fact that the system of criteria used is that of a Ω^1Z. What is then the situation when one projects "grue" in the Ω^1Z? In such a universe of reference, the unique criterion-variable is the tcolour*. An object is there "grue" or "bleen" in the *absolute*. Consequently, if one has well a common criterion (the tcolour*), it appears that the differentiated criterion is missing, in order to perform induction validly. And the situation in which one is placed is that of an extreme differentiation. Thus, such a projection is carried out in violation of the minimal condition of induction. Consequently, it proves that GP cannot take place in the Ω^2CT and is then blocked at the stage of the projection of "grue".

But are these preliminary remarks sufficient to provide, in the context of a Ω^2CT, a satisfactory solution to GP? One can think that not, because the paradox also arises in it in another form, which is that of the projection of tcolour* through time. One can formalise this projection Z°T as follows:

(I*) $GT_1 \cdot GT_2 \cdot GT_3 \cdot \ldots \cdot GT_{99}$
(H*) $GT_1 \cdot GT_2 \cdot GT_3 \cdot \ldots \cdot GT_{99} \cdot GT_{100}$ that is equivalent to:
(H'*) $RT_1 \cdot RT_2 \cdot RT_3 \cdot \ldots \cdot RT_{99} \cdot \sim RT_{100}$
(P*) GT_{100} that is equivalent to:
(P'*) $\sim RT_{100}$

where it is manifest that the elements of GP are still present.

Fundamentally in this version, it appears that the common properties are borrowed from the system of criteria of the Ω^1Z, whereas the differentiated properties come from the Ω^2CT. A first analysis thus reveals that the projection of "grue" under these conditions presents a defect which consists in the choice of a given system of criteria for the common properties (tcolour*) and of a different system of criteria for the differentiated properties (time). For the selection of the tcolour* is characteristic of the choice of a Ω^1Z, whereas the use of time is revealing of the fact that one places oneself in a Ω^2CT. But one must choose one or the other of the reducible systems of criteria to perform induction. On the hypotheses envisaged previously, the choice of the criteria for the common and differentiated properties was carried out within *the same system of criteria*. But here, the choice of the criteria for the common properties and the differentiated properties is carried out within *two different* (and reducible) *systems of criteria*. Thus, the common and differentiated criteria selected for induction are not genuinely distinct. And this appears as a *violation of the condition of separation*. Consequently, one of the conditions of induction is not respected.

However, the projection Z°T has a certain intuitive support, because it is based on the fact that the notions of "grue before T" and "grue after T" have a certain intuitive meaning. Let us then disregard the violation of the conditions of the induction which has been just mentioned, and consider thus this situation in more detail. In this context, GP is always present, since one observes a contradiction between (P) and (P'*). It is with this contradiction that it is worth from now on being interested. Consider the particular step of the equivalence between (H*) and (H'*). One conceives that "grue before T" is assimilated here to RT, because the fact that the instances-S are red before T results clearly from the conditions of the experiment. On the other hand, it is worth being interested by the step according to which (P*) entails (P'*). According to the classical definition[24]: "grue" = $\{RT \wedge R{\sim}T, RT \wedge {\sim}R{\sim}T, {\sim}RT \wedge {\sim}R{\sim}T\}$. What is it then to be "grue after T"? There, it appears that a "grue" object can be $R{\sim}T$ (this corresponds to the case $RT \wedge R{\sim}T$) or ${\sim}R{\sim}T$ (this correspond to the cases $RT \wedge {\sim}R{\sim}T$ and ${\sim}RT \wedge {\sim}R{\sim}T$). In conclusion, the object can be either $R{\sim}T$ or ${\sim}R{\sim}T$. Thus, the fact of knowing that an object is "grue after T" does not make it possible to conclude that this object is ${\sim}R{\sim}T$, because this last can also be $R{\sim}T$. Consequently, the step according to which (P*) involves (P'*) appears finally *false*. From where it ensues that the contradiction between (P) and (P'*) does not have any more a raison d'etre.

One can convince oneself that this analysis does not depend on the choice of the classical definition of "grue" ($grue_{12}$) which is carried out, by considering other definitions. Consider for example the definition based on $grue_9$: "grue" = $\{RT \wedge {\sim}R{\sim}T, {\sim}RT \wedge {\sim}R{\sim}T\}$ and "bleen" = $\{RT \wedge R{\sim}T, {\sim}RT \wedge R{\sim}T\}$. But in this version, one notes that one does not have the emergence of GP, because the instances-S, which are RT, can be at the same time "grue" and " bleen". And the same applies if one considers a conjunctive definition ($grue_2$) such as "grue" =

$\{RT \wedge \sim R\sim T\}$. In such a case indeed, the instances-S are "grue" only if they are RT but also ~R~T. However this does not correspond to the initial conditions of GP in the Ω^2CT where one ignores if the instances-S are ~R~T.

One could also think that the problem is related to the use of a taxonomy of tcolour* based on two taxa (G and ~G). Consider then a taxonomy of tcolour* based on 4 taxa: $Z_0 = RT \wedge R\sim T$, $Z_1 = RT \wedge \sim R\sim T$, $Z_2 = \sim RT \wedge R\sim T$, $Z_3 = \sim RT \wedge \sim R\sim T$. But on this hypothesis, it appears clearly that since the instances-S are for example Z_1, one finds himself replaced in the preceding situation.

The fact of considering "grue after T", "grue before T", "bleen before T", "bleen after T" can be assimilated with an attempt of expressing "grue" and " bleen" with the help of our own criteria, and in particular that of time. It can be considered here as a form of anthropocentrism, underlain by the idea to express the Ω^1Z with the help of the taxa of the Ω^2CT. Since one knows the code defining the relations between two reducible *n*-universes - the Ω^1Z and the Ω^2CT - and that one has partial data, one can be tempted to elucidate completely the predicates of the foreign *n*-universe. Knowing that the instances are GT, G~T, ~GT, ~G~T, I can deduce that they are respectively {RT, ~RT}, {R~T, ~R~T}, {~RT}, {R~T}. But as we have seen, due to the fact that the instances are GT and RT, I cannot deduce that they will be ~R~T.

The reasoning in this version of GP is based on the apparently inductive idea that what is "grue before T" is also "grue after T". But in the context which is that of the Ω^1Z, when an object is "grue", it is "grue" in the absolute. For no additional criterion exists which can make its tcolour* vary. Thus, when an object is GT, it is *necessarily* G~T. And from the information according to which an object is GT, one can thus conclude, by *deduction*, that it is also G~T.

From what precedes, it ensues that the version of GP related to the Z°T presents the apparent characters of induction, but it does not constitute an authentic form of this type of reasoning. Z°T thus constitutes a disguised form of induction for two principal reasons: first, it is a projection through the differentiated criterion of time, which constitutes the standard mode of our inductive practice. Second, it is based on the intuitive principle according to which everything that is GT is also G~T. But as we have seen, it consists here in reality of a deductive form of reasoning, whose true nature is masked by an apparent inductive move. And this leads to conclude that the form of GP related to Z°T analyses itself in fact veritably as a *pseudo-induction*.

6.2 "Grue" in the coloured, temporal and localised 3-universe

Consider now the case of a Ω^3CTα. This type of universe of reference also corresponds to the definition of a *minimal* Ω^2CT, but it also comprises one 3rd criterion-variable[25]. Let us choose for this last a criterion such as localisation[26]. Consider then a Ω^3CTL. Consider first (H) in such a 3-universe. To be "red" in the Ω^3CTL, is to be red at time T and at location L. According to the conditions of GP, colour corresponds to the *common* properties, and time to the *differentiated* properties. One has then the following projection C°TL:

(I) $RT_1L_1 \cdot RT_2L_2 \cdot RT_3L_3 \cdot \ldots \cdot RT_{99}L_{99}$
(H) $RT_1L_1 \cdot RT_2L_2 \cdot RT_3L_3 \cdot \ldots \cdot RT_{99}L_{99} \cdot RT_{100}L_{100}$
$\therefore$ (P) $RT_{100}L_{100}$

where taking into account the conditions of induction, it proves to be legitimate to project the common property ("red") of the instances-S, into differentiated time and location, and to predict that the 100th ball will be red. Such a projection appears completely correct, and proves in all points in conformity with the conditions of induction mentioned above.

What happens now with (H*) in the Ω^3CTL? It has been observed that the Ω^3CTL could be reduced to a Ω^2ZL. In this last *n*-universe, the criterion-variables are tcolour* and localisation. The fact of being "grue" is there relative to location: to be "grue", is to be "grue" at location L. What is then projected is the tcolour*, i.e. the fact of being "grue" or "bleen". There is thus a common criterion of tcolour* and a differentiated criterion of localisation. Consequently, if it is considered that the instances-S are "grue", one can equally well project the property common "grue" into a differentiated criterion of localisation. Consider then the projection Z°L in the Ω^2ZL:

(I*) $GL_1 \cdot GL_2 \cdot GL_3 \cdot \ldots \cdot GL_{99}$
(H*) $GL_1 \cdot GL_2 \cdot GL_3 \cdot \ldots \cdot GL_{99} \cdot GL_{100}$
$\therefore$ (P*) GL_{100}

Such a projection is in conformity with the conditions mentioned above, and constitutes consequently a valid form of induction.

In this context, one can project valuably a predicate having a structure identical to that of "grue", in the case of emeralds. Consider the definition "grue" = green before T or non-green after T, where T = 10 billion years. It is known that at that time, our Sun will be extinct, and will become gradually a dwarf white. The conditions of our atmosphere will be radically different from what they currently are. And the temperature will rise in particular in considerable proportions, to reach 8000°. Under these conditions, the structure of many minerals will change radically. It should normally thus be the case for our current emeralds, which should see their colour modified, due to the enormous rise in temperature which will follow. Thus, I currently observe an emerald: it is "grue" (for

T = 10 billion years). If I project this property through a criterion of *location*, I legitimately conclude from it that the emerald found in the heart of the Amazonian forest will also be "grue", in the same way as the emerald which has been just extracted from a mine from South Africa.

At this stage, one could wonder whether the projectibility of "grue" is not intrinsically related to the choice of a definition of "grue" based on inclusive disjunction ($grue_{12}$)? Nevertheless, one easily checks by using an alternative definition of "grue" that its projection remains valid[27].

It should be noticed that one has here the expression of the fact that the taxonomy based on the tcolour* is coarser than that based on time and colour. In effect, the former only comprises 2 taxa (grue/bleen), whereas the latter presents 4 of them. By reducing the criteria of colour and time to a single criterion of tcolor*, one has replaced 4 taxa (RT ∧ R~T, RT ∧ ~R~T, ~RT ∧ R~T, ~RT ∧ ~R~T) by 2. Thus, "grue" constitutes from this point of view a predicate coarser than "red". The universe which is described did not change, but the *n*-universes which are systems of criteria describing these universes are different. With the tcolour* thus defined, one has less predicates at its disposal to describe a same reality. The predicates "grue" and "bleen" are for us not very informative, and are less informative in any case that our predicates "red", "non-red", "before T", etc. But that does not prevent however "grue" and "bleen" to be projectibles.

Whereas the projection of "grue" appears valid in the Ω^2ZL, it should be noticed however that one does not observe in this case the contradiction between (P) and (P'*). For here (I*) is indeed equivalent to:

(I'*) $RT_1L_1 \cdot RT_2L_2 \cdot RT_3L_3 \cdot \ldots \cdot RT_{99}\ L_{99}$

since, knowing according to the initial data of GP that the instances-S are RT, one valuably replaces the GL_i by the RT_iL_i (i < 100). But it appears that on this hypothesis, (P*) does not involve:

(P'*) $\sim RT_{100}L_{100}$

because one does not have an indication relating to the temporality of the 100th instance, due to the fact that only the localisation constitutes here the differentiated criterion. Consequently, one has well in the case of the Ω^3CTL a version built with the elements of GP where the projection of "grue" is carried out valuably, but which does not present a paradoxical nature.

7. Conclusion

In the solution to GP proposed by Goodman, a predicate is projectible or nonprojectible in the *absolute*. And one has in addition a correspondence between the entrenched[28]/non-entrenched and the projectible/nonprojectible predicates. Goodman in addition does not provide a justification to this assimilation. In the present approach, there is no such dichotomy, because a given predicate P reveals itself projectible in a given *n*-universe, and nonprojectible in another *n*-universe. Thus, P is projectible *relatively* to such universe of reference. There is thus the *projectible/nonprojectible relative to such n-universe* distinction. And this distinction is justified by the conditions of induction, and the fundamental mechanism of induction related to the unification/differentiation duality. There are thus *n*-universes where "green" is projectible and others where it is not. In the same way, "grue" appears here projectible relative to certain *n*-universes. Neither *green* nor *grue* are projectible in the absolute, but only relative to such given universe. Just as of some other predicates, "grue" is projectible in certain universes of reference, but nonprojectible in others[29].

Thus, it proves that one of the causes of GP resides in the fact that in GP, one classically proceeds to operate a dichotomy between the projectible and the nonprojectible predicates. The solutions classically suggested to GP are respectively based on the distinction temporal/nontemporal, local/non-local, qualitative/nonqualitative, entrenched/non-entrenched, etc. and a one-to-one correspondence with the projectible/nonprojectible distinction. One wonders whether a given predicate P* having the structure of "grue" is projectible, in the *absolute*. This comes from the fact that in GP, one has a contradiction between the two concurrent predictions (P) and (P*). One classically deduces from it that one of the two predictions must be rejected, at the same time as one of the two generalisations (H) or (H*) on which these predictions are respectively based. Conversely, in the present analysis, whether one places himself in the case of the *authentic* projection Z°L or in the case of the *pseudo-projection* Z°T, one does not have a contradiction between (P) and (P'*). Consequently, one is not constrained any more to reject either (H) or (H*). And the distinction between projectible/nonprojectible predicates does not appear indispensable any more[30].

How is the choice of our usual *n*-universe carried out in this context? *N*-universes such as the Ω^2CT, the Ω^3CTL, the Ω^2ZL etc. are appropriate to perform induction. But we naturally tend to privilege those which are based on criteria structured rather finely to allow a maximum of *combinations* of projections. If one operates from the criteria Z and L in the Ω^2ZL, one restricts oneself to a limited number of combinations: Z°L and L°Z. Conversely, if one retains the criteria C, T and L, one places oneself in the Ω^3CTL and one has the possibility of projections C°TL, T°CL, L°CT, CT°L[31], CL°T, TL°C. One has thus a maximum of combinations. This seems to encourage to prefer the Ω^3CTL to the Ω^2ZL. Of course, pragmatism seems to have to play a role in the choice of the best alternative of our criteria. But it seems that it is only one of the multiple factors which interact to allow the optimisation of our criteria to carry out the primitive operations of grouping and differentiation, in order to then be able to generalise, classify, order, make assumptions or forecast[32]. Among these factors, one can in particular mention: pragmatism, simplicity, flexibility of implementation, polyvalence[33], economy in means, power[34], but also the nature of our real universe, the structure of our organs of perception, the state of our scientific knowledge, etc[35]. Our usual *n*-universes are optimised with regard to these various factors. But this valuably leaves room for the choice of other systems of criteria, according to the variations of one or the other of these parameters[36].

[1] Nelson Goodman, "A Query On Confirmation", *Journal of Philosophy*, vol. 43 (1946), p. 383-385; in *Problems and Projects*, Indianapolis, Bobbs-Merrill, 1972, p. 363-366.
[2] With some minor adaptations.
[3] See Goodman "A Query On Confirmation", p. 383: "Suppose we had drawn a marble from a certain bowl on each of the ninety-nine days up to and including VE day and each marble drawn was red. We would expect that the marble drawn on the following day would also be red. So far all is well. Our evidence may be expressed by the conjunction "$Ra_1 \cdot Ra_2 \cdot ... \cdot Ra_{99}$" which well confirms the prediction Ra_{100}." But increase of credibility, projection, "confirmation" in any intuitive sense, does not occur in the case of every predicate under similar circumstances. Let "S" be the predicate "is drawn by VE day and is red, or is drawn later and is non-red." The evidence of the same drawings above assumed may be expressed by the conjunction "$Sa_1 \cdot Sa_2 \cdot ... \cdot Sa_{99}$". By the theories of confirmation in question this well confirms the prediction "Sa_{100}"; but actually we do not expect that the hundredth marble will be non-red. "Sa_{100}" gains no whit of credibility from the evidence offered."
[4] Nelson Goodman, *Fact, Fiction and Forecast*, Cambridge, MA, Harvard University Press, 1954.
[5] *Ibid*., p. 73-4: "Suppose that all emeralds examined before a certain time *t* are green. At time *t*, then, our observations support the hypothesis that all emeralds are green; and this is in accord with our definition of confirmation. [...] Now let me introduce another predicate less familiar than "green". It is the predicate "grue" and it applies to all things examined before *t* just in case they are green but to other things just in case they are blue. Then at time *t* we have, for each evidence statement asserting that a given emerald is green, a parallel evidence statement asserting that that emerald is grue."
[6] For example with an accuracy of 10^{-4} nm.
[7] Or any taxonomy which is similar to it.
[8] See §2 above.
[9] This assertion is controversial.
[10] Such a remark also applies to the statement of Goodman, *Fact, Fiction and Forecast*.
[11] As J.S. Ullian mentions it, "More one 'Grue' and Grue", *Philosophical Review*, vol. 70 (1961), p. 386-389, in p. 387.
[12] I. e. C(0, 4)+C(1, 4)+C(2, 4)+C(3, 4)+C(4, 4) = 2^4, where C(*p*, *q*) denotes the number of combinations of *q* elements taken *p* times.
[13] See Goodman, "A Query On Confirmation", p. 383: "Induction might roughly be described as the projection of characteristics of the past into the future, or more generally of characteristics of one realm of objects into another."
[14] See §2 above.
[15] One can of course alternatively take into account several instances-D.
[16] See §2 above.
[17] *Ibid*.
[18] For the application of this condition, one must take into account the remarks mentioned above concerning the problem of the status of the instances. Thus, one must actually compare an instantiated and temporal 1-universe to a 2-universe one of the criteria of which is temporal, and the second criterion is not explicitly mentioned. Similarly, an instantiated and coloured 1-universe is assimilated in fact to a 2-universe one of the criteria of which is temporal, and the second criterion is not specified.
[19] See §3 above.
[20] With the notations C (colour), T (time), L (localisation) and Z (tcolour*).
[21] However, since the fact that there exists one or more instances is not essential in the formulation of the given problem, one will obviously be able to abstain from making mention of it.
[22] See §4.
[23] *Ibid*.
[24] It is the one based on the inclusive disjunction ($grue_{12}$).
[25] A same solution applies, of course, if one considers a number of criterion-variables higher than 3.
[26] All other criterion distinct from colour or time, would also be appropriate.
[27] In particular, it appears that the projection of a conjunctive definition ($grue_2$) is in fact familiar for us. In effect, we do not proceed otherwise when we project the predicate "being green before maturity and red after maturity" applicable to tomatoes, through a differentiated criterion of location: this is true of the 99 instance-S observed in Corsica and Provence, and is projected validly to a 100th instance located in Sardinia. One can observe that such a type of projection is in particular regarded as nonproblematic by Jackson (Franck Jackson, "'Grue'", *Journal of Philosophy*, vol. 72 (1975), p. 113-131): "There seems no case for regarding 'grue' as nonprojectible if it is defined this way. An emerald is grue $_1$ just if it is green up to *T* and blue thereafter, and if we discovered that all emeralds so far examined had this property, then, other things being equal, we would probably accept that all emeralds, both examined and unexamined, have this property (...)." If one were to replace such a predicate in the present analysis, one should then consider that the projection is carried out for example through a differentiated criterion of localisation (p. 115).
[28] Goodman, *Fact, Fiction and Forecast*.
[29] The account presented in J Holland, K Holyoak, R. Nisbett and P. Thagard (*Induction*, Cambridge, MA; London, MIT Press, 1986) appears to me to constitute a variation of Goodman's solution, directed towards the computer-based processing of data and based on the distinction integrated/non-integrated in the default hierarchy. But Holland's solution presents the same disadvantages as that of Goodman: what justification if not anthropocentric, does one have for this distinction? See p. 235: "Concepts such as "grue", which are of no significance to the goals of the learner, will never be

generated and hence will not form part of the default hierarchy. (...) Generalization, like other sorts of inference in a processing system, must proceed from the knowledge that the system already has".

The present analysis also distinguishes from the one presented by Susan Haack (*Evidence and Inquiry*, Oxford; Cambridge, MA, Blackwell, 1993) because the existence of natural kinds does not constitute here a condition for induction. See p. 134: "There *is* a connection between induction and natural kinds. [...] the reality of kinds and laws is a necessary condition of successful inductions". In the present context, the fact that the conditions of induction (a common criterion, a distinct differentiated criterion, etc.) are satisfied is appropriate to perform induction.

[30] A similar remark is made by Franck Jackson in conclusion of his article ("'Grue'", p. 131): "[...] the SR can be specified without invoking a partition of predicates, properties or hypotheses into the projectible and the nonprojectible". For Jackson, all noncontradictory predicates are projectible: "[...] *all* (consistent) predicates are projectible." (p. 114). Such a conclusion appears however stronger than the one that results from the current analysis. Because for Jackson, all predicates are thus projectible in the *absolute*. However in the present context, there are no projectible or nonprojectible predicates in the absolute. It is only *relative* to a given *n*-universe, that a predicate P reveals projectible or nonprojectible.

More generally, the present analysis distinguishes fundamentally from that of Jackson in the sense that the solution suggested to GP does not rest on the counterfactual condition. This last appears indeed too related to the use of certain predicates (*examined*, *sampled*, etc.). On the other hand, in the present context, the problem is considered from a general viewpoint, independently of the particular nature of the predicates constituting the definition of *grue*.

[31] Such a projection corresponds for example to the generalisation according to which "the anthropomorphic statue-menhirs are of the colour of granite and date from the Age of Bronze".

[32] As Ian Hacking underlines it, *Le plus pur nominalisme*, Combas, L'éclat, 1993, p. 9: "Utiliser un nom pour une espèce, c'est (entre autres choses) vouloir réaliser des généralisations et former des anticipations concernant des individus de cette espèce. La classification ne se limite pas au tri : elle sert à prédire. C'est une des leçons de la curieuse "énigme" que Nelson Goodman publia il y a quarante ans." My translation: "To use a name for a species, it is (among other things) to want to carry out generalisations and to form anticipations concerning the individuals of this species. Classification is not limited to sorting: it is used to predict. It is one of the lessons of the strange "riddle" which Nelson Goodman published forty years ago."

[33] The fact that a same criterion can be used at the same time as a common and a differentiated criterion (while eventually resorting to different taxa).

[34] I.e. the number of combinations made possible.

[35] This enumeration does not pretend to be exhaustive. A thorough study of this question would be of course necessary.

[36] I thank the editor of Dialogue and two anonymous referees for very helpful comments on an earlier draft of this paper.