Options dans le cadre Black-Scholes

TP-2: Pricing Vanna-Volga

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The purpose of this problem set is to explore the Vanna-Volga pricing model. In this problem set, you will use the following functions:

GBSPrice: Price of a vanilla option:

$$P = f(\text{PutCall}, S, K, T, r, b, \sigma)$$

where:

PutCall 'c' for a call, 'p' for a put

b cost of carry: ridk free rate r less dividend yield d

r risk-free rate

```
GBSPrice <- function(PutCall, S, K, T, r, b, sigma) {
  d1 <- (log(S/K) + (b+sigma^2/2)*T)/(sigma*sqrt(T))
  d2 <- d1 - sigma*sqrt(T)

  if(PutCall == 'c')
    px <- S*exp((b-r)*T)*pnorm(d1) - K*exp(-r*T)*pnorm(d2)
  else
    px <- K*exp(-r*T)*pnorm(-d2) - S*exp((b-r)*T)*pnorm(-d1)

px
}
```

GBSVega: Vega ($\frac{\partial P}{\partial \sigma}$) of a Vanilla option:

```
GBSVega <- function(PutCall, S, K, T, r, b, sigma) {
  d1 <- (log(S/K) + (b+sigma^2/2)*T)/(sigma*sqrt(T))
  S*exp((b-r)*T) * dnorm(d1)
}
```

Volatility Interpolation

Given the implied volatility at three strikes, we will use the Vanna-Volga pricing method to interpolate the volatility curve. Assume $r = 0, b = 0, T = 1, \text{Spot} = 100$.

```r
# Benchmark data: (strike, volatility)
VolData <- list(c(80, .32), c(100, .30), c(120, .315))
```

Let's first define an array of pricing functions for the benchmark instruments:

```r
C1 <- function(vol=sigma, spot=Spot) GBSPrice(PutCall='c', S=spot, K=VolData[[1]][1], T=T, r=r, b=b, si

C2 <- function(vol=sigma, spot=Spot) GBSPrice(PutCall='c', S=spot, K=VolData[[2]][1], T=T, r=r, b=b, si

C3 <- function(vol=sigma, spot=Spot) GBSPrice(PutCall='c', S=spot, K=VolData[[3]][1], T=T, r=r, b=b, si

C <- c(C1, C2, C3)
```

1. Write a utility functions to compute the risk indicators, all by finite difference:

```r
Vega <- function(f, vol, spot=Spot) {
  d_vol <- 10e-5
  return( (f(vol+d_vol*vol, Spot)-f(vol-d_vol*vol, Spot))/(2*d_vol) )
}

Vanna <- function(f, vol, spot=Spot) {
  d_vol <- 10e-5
  d_spot <- 10e-5
  return( (f(vol+d_vol*vol, Spot+d_spot*Spot)+
           f(vol-d_vol*vol, Spot-d_spot*Spot)-
           f(vol+d_vol*vol, Spot-d_spot*Spot)-
           f(vol-d_vol*vol, Spot+d_spot*Spot))/(4*d_vol*d_spot) )
}

Volga <- function(f, vol, spot=Spot) {
  d_vol <- 10e-5
  return( (f(vol+d_vol*vol, Spot) - 2*f(vol, Spot) + f(vol-d_vol*vol, Spot))/d_vol**2 )
  }
```

Then, the calculation of vega for the three benchmark options may be performed by:

```r
r<-0
b<-0
T<-1
Spot <- 100
B.vega <- sapply(1:3, function(i) Vega(C[[i]], VolData[[i]][2]))
```

2. Compute vectors of vega, vanna, volga for the three hedge instruments

```r
r<-0
b<-0
T<-1
Spot <- 100
B.vega <- sapply(1:3, function(i) Vega(C[[i]], VolData[[i]][2]))
B.vanna <- sapply(1:3, function(i) Vanna(C[[i]], VolData[[i]][2]))
B.volga <- sapply(1:3, function(i) Volga(C[[i]], VolData[[i]][2]))
B.vega
```

```
## [1]  8.840076 11.834380 11.499491
```

```
B.vanna
```

```
## [1] -14.84369   5.91719  26.87955
```

```
B.volga
```

```
## [1]  4.0722703 -0.2662745  3.5671601
```

3. Choose a new strike for which we want to compute the implied volatility. Let's choose $K = 110$.
4. Compute the risk indicators for a call option struck at that strike.
5. Compute the Vanna-Volga price adjustment and the corresponding implied volatility.
6. Wrap the above logic in a function in order to interpolate/extrapolate the vol curve from $K = 70$ to $K = 130$

Pricing a digital call

Recall that a digital call with strike K pays one euro if $S_T \geq K$, and nothing otherwise.

Using the same logic as in the previous question, price a digital call, maturity $T = 1$, struck at $K = 105$.