

# Internal Wave Beam Reflection Through Simulated Oceanic Pycnocline

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Acknowledgements: **Scott Wunsch** – APL, Johns Hopkins

**R. Joslin** – Office of Naval Research



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College of Engineering

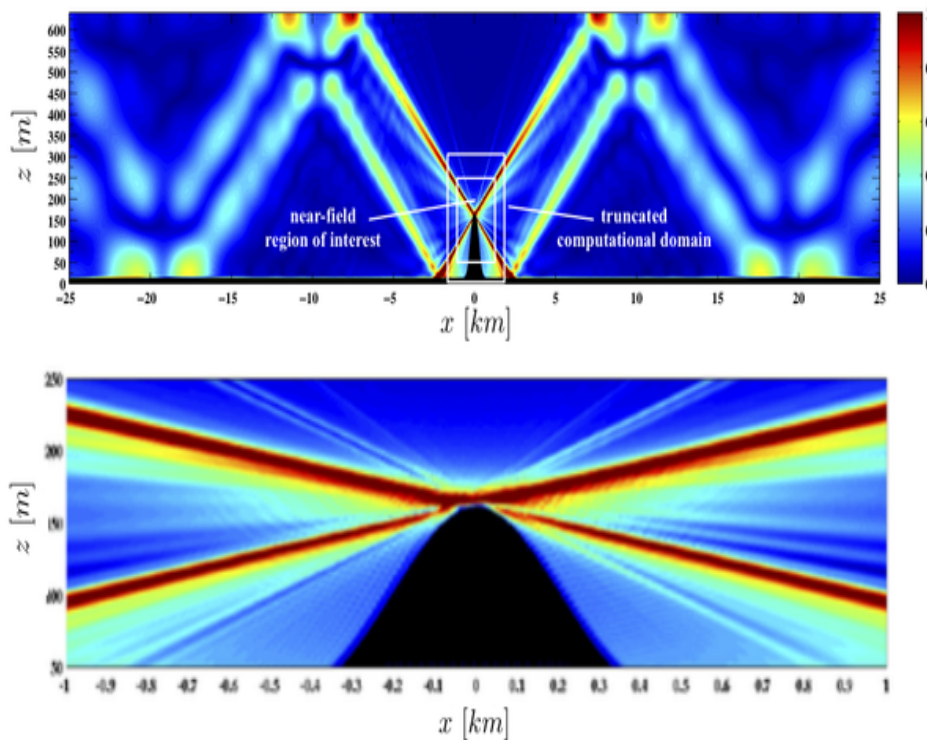


National Science Foundation  
WHERE DISCOVERIES BEGIN

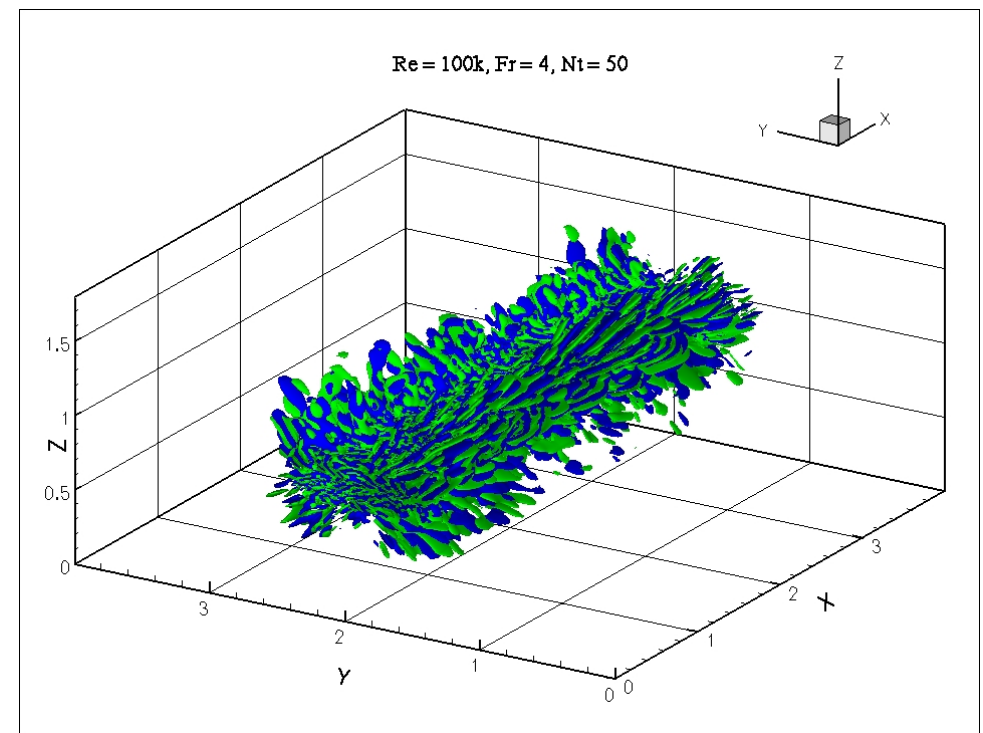


# Motivation

- IW beam generated by ocean topography and turbulent wakes
- Reflection characteristics important to capture energy budgets in large scale oceanic models



Ocean Topography  
(Kraig Winters, UCSD)



Submerged Turbulence

# Objectives

To investigate IGW modulation, reflection, and ducting as it propagates across pycnoclines with a range of characteristics.

## Parameters

$$r = N_{\max}/N_1$$

$$h = \text{Pycnocline width}/\lambda$$

## Measure

Reflection Coef. for  $|\langle p', \mathbf{u} \rangle|$  and  $\Delta N^2$

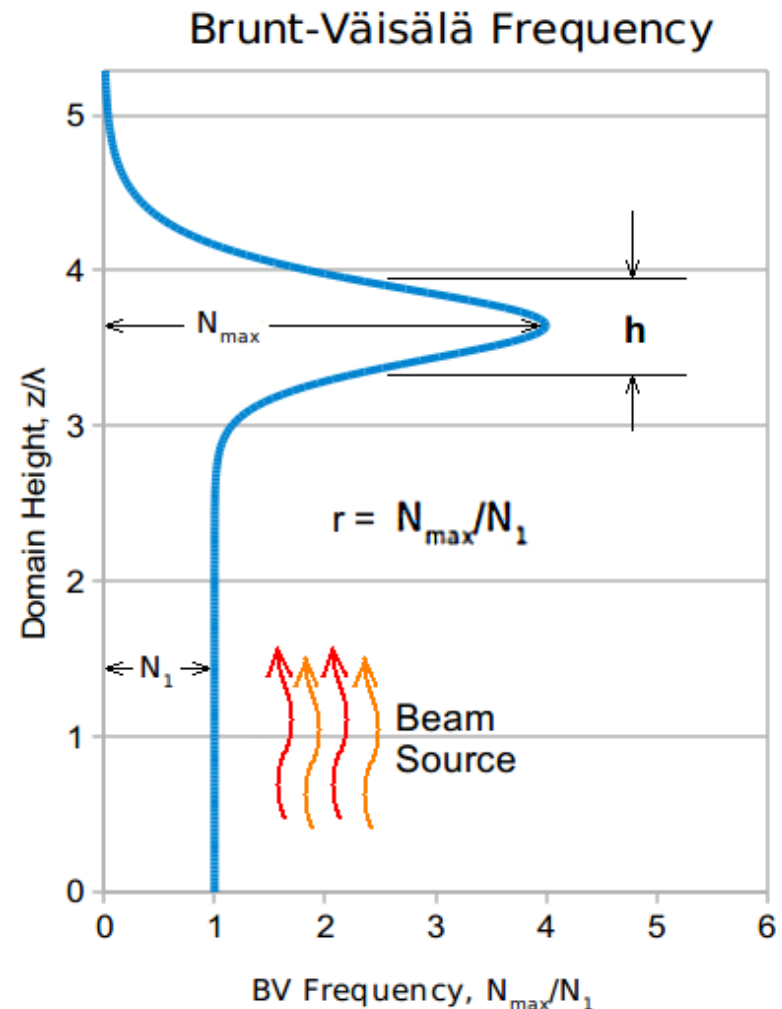
## Compare/Complement

Laboratory experiments

(Wunsch et al., Johns Hopkins APL, 2010)

Finite-width transition theory

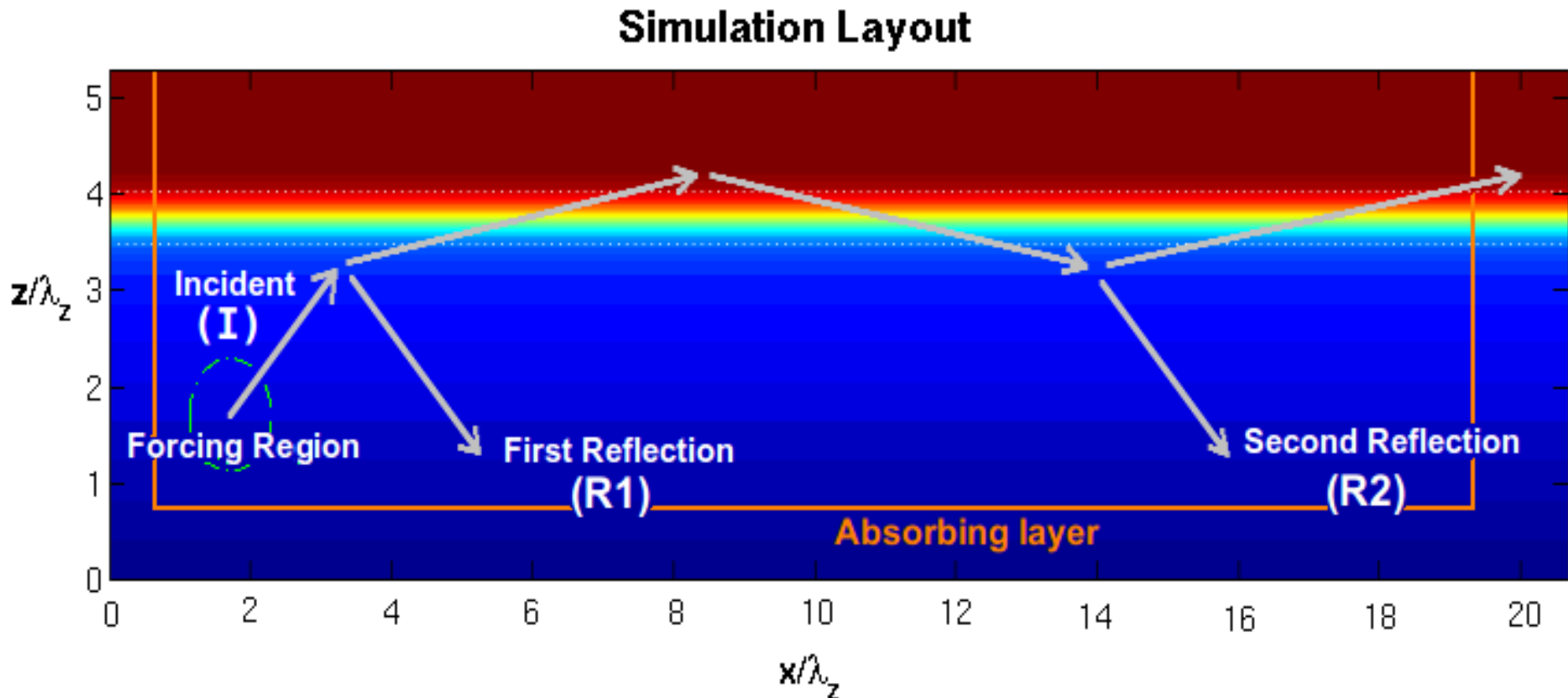
(Mathur, Peacock, MIT, 2009)



# Simulation Layout

Wave steepness = 3%      Incident propagation angle =  $45^\circ$

\*Absorbing layers damp wave beams at domain boundaries

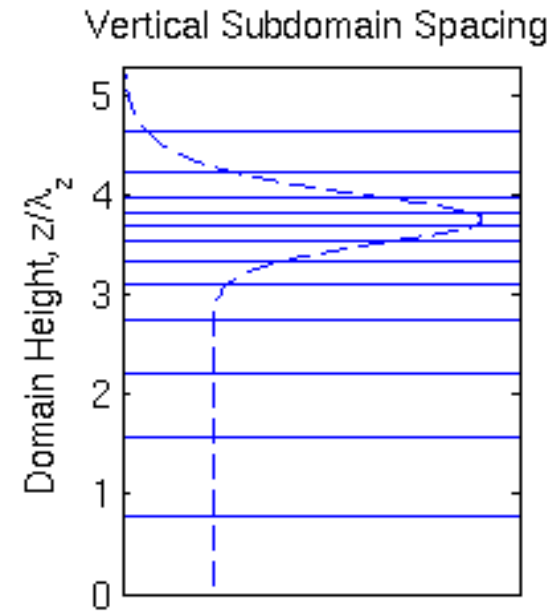
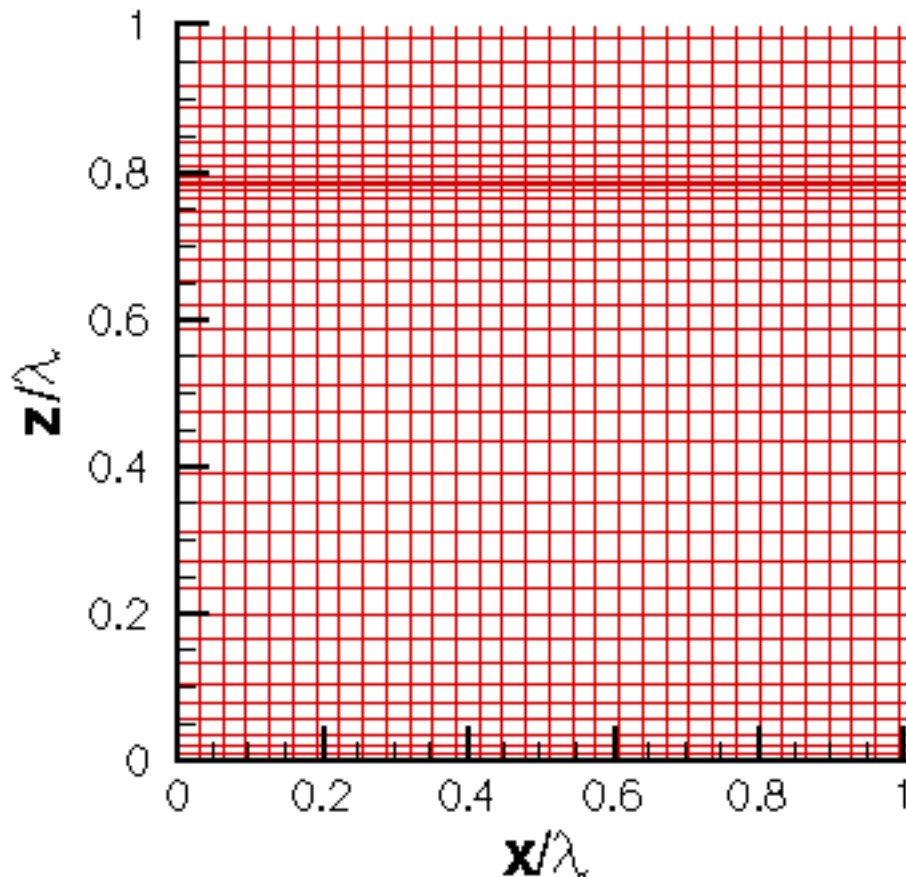


# Computational Approach

## 2D Spectral Multi-domain Penalty Solver

(Diamessis et al. JCP 2005)

**Mesh Grid Plot**



256 – 768 horizontal gridpoints

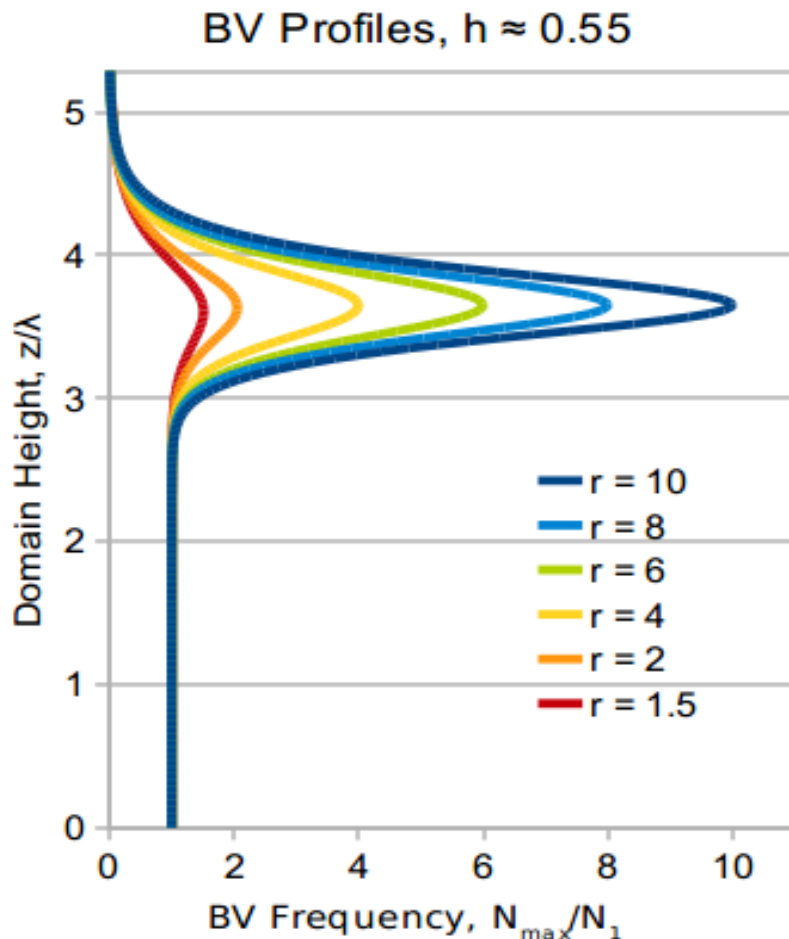
424 – 527 vertical gridpoints

~30 pts./lambda\_x

~40 pts./lambda\_z (everywhere)

# Series 1 – Varying Pycnocline Strength

6 Cases:  $r$ -values - 1.5 2 4 6 8 10



- Fill in experimental gap for  $2 < r < 6$

- Compare to finite-width transition linear theory

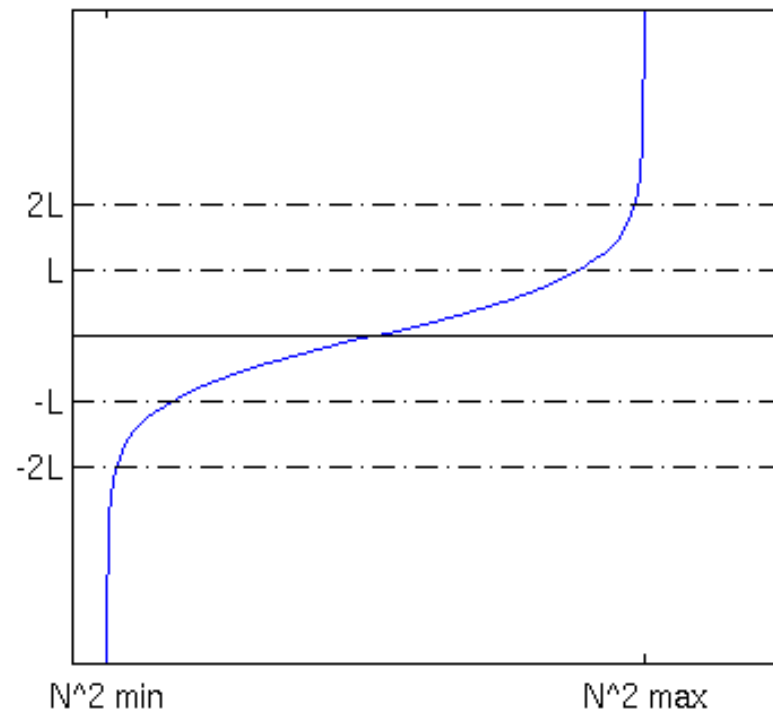
- $R2 = \frac{A_x \text{Ref.}}{A_x \text{Inc.}}$

# Finite-width Transition Linear Theory

Inviscid solutions for transmission coefficients across a  $\tanh(z)$   $N^2$  profile.

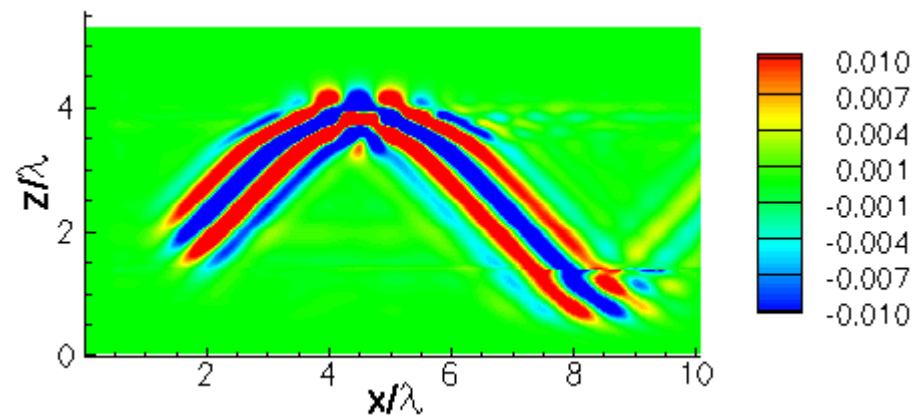
Near perfect transmission occurs when the largest vertical wavelength is much smaller than the transition length,  $L$

$$L^* m \gg 1$$



This is true for all Series 1 cases. We expect perfect transmission!

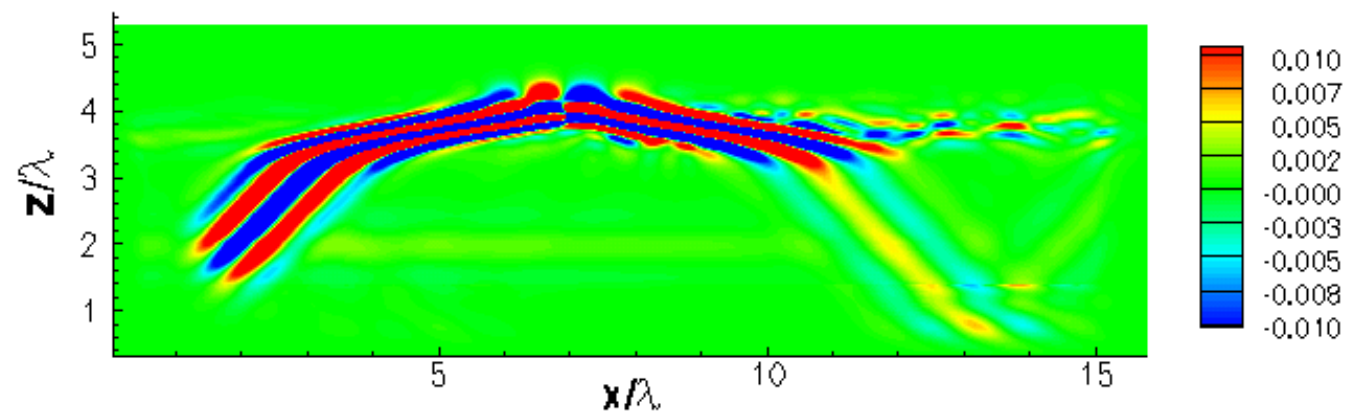
$r = 1.5$   
 $t = 30 \cdot T$



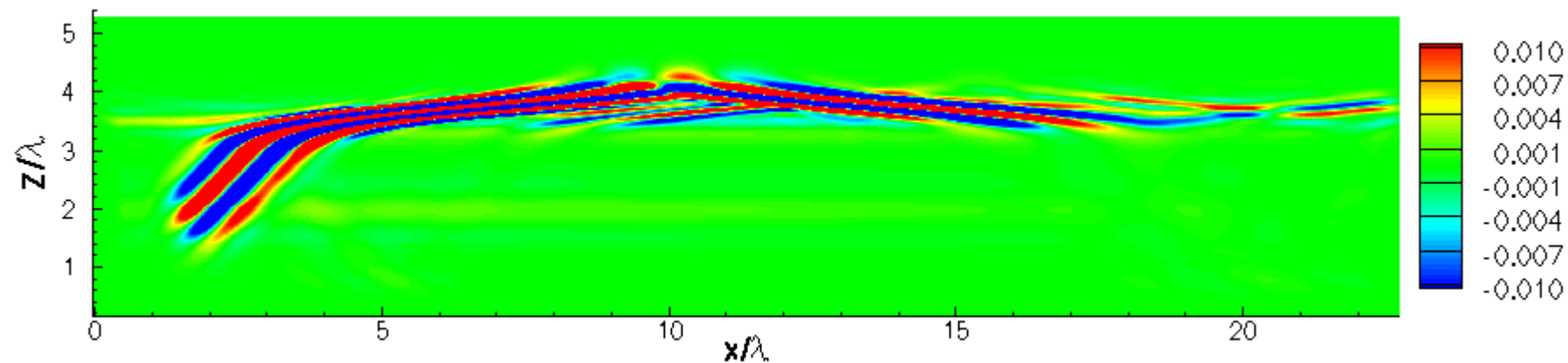
## $\Delta N^2$ Contour Plots

Pycnocline thickness,  $h = 0.55$

$r = 4$   
 $t = 30 \cdot T$



$r = 8$   
 $t = 30 \cdot T$





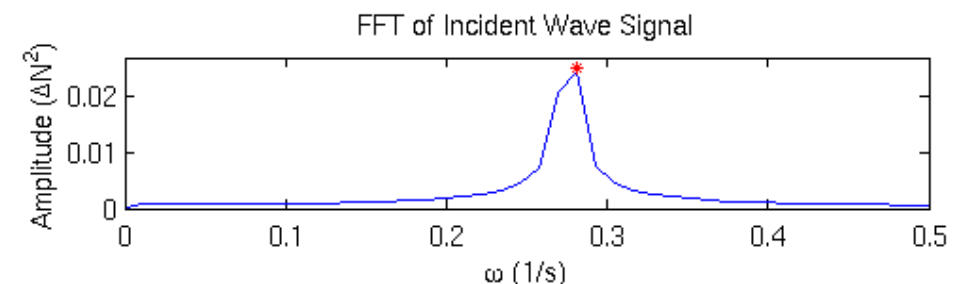
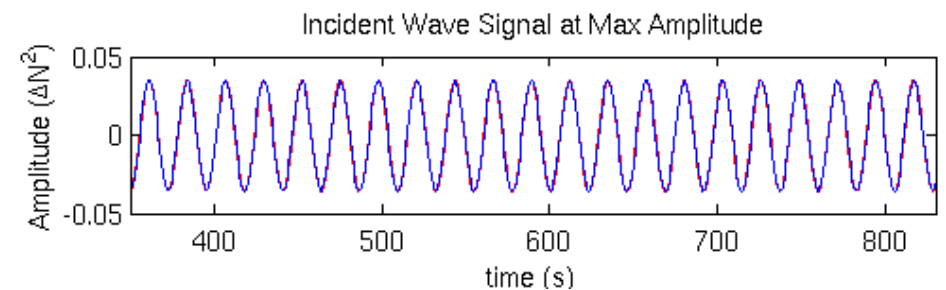
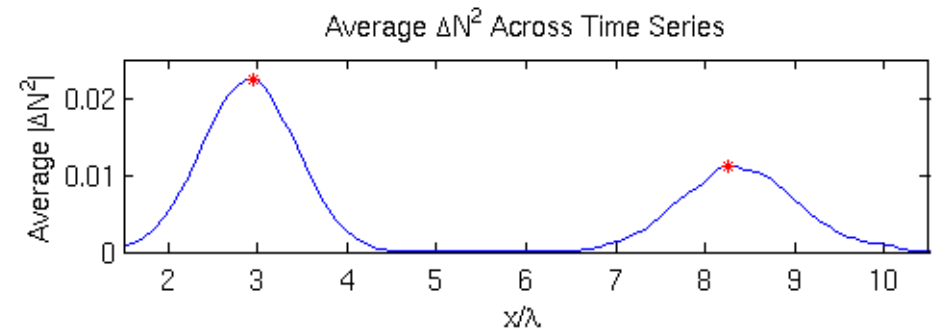
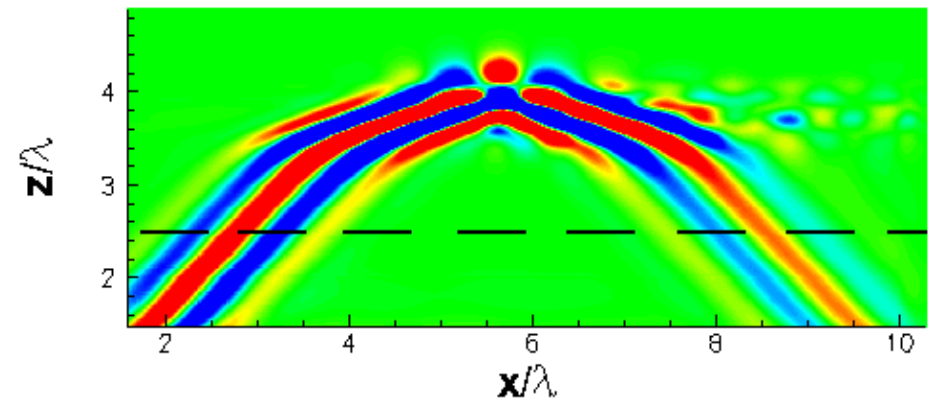
## R2 Measurement Method

1. Take a time-series in the linearly stratified region across  $x$

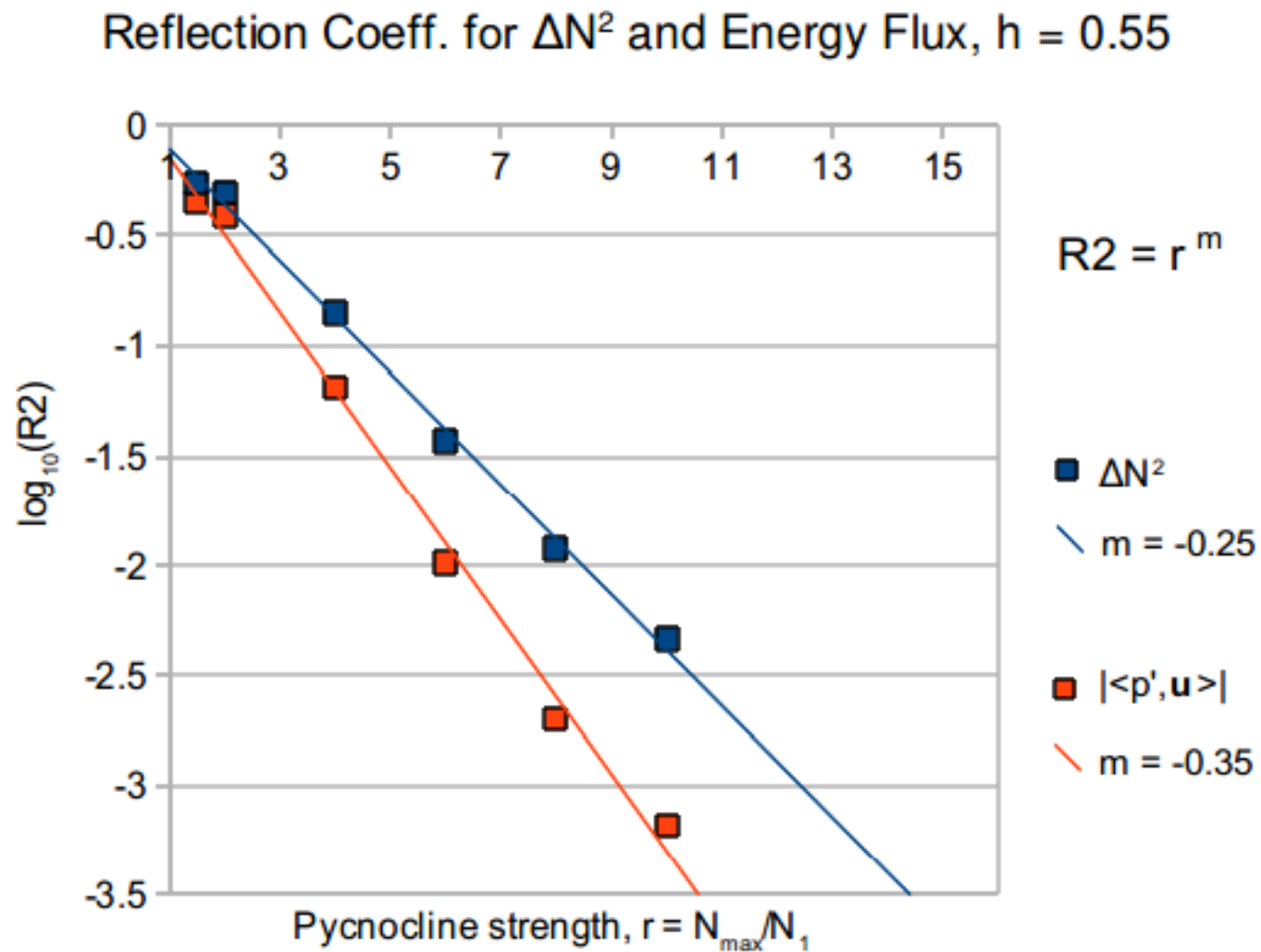
2. Find the  $x$ -location of max average wave amplitude

3. FFT signal at location of max amplitude to find RMS amplitude and frequency

Horizontal cross-section for  $x$ - $t$  samples

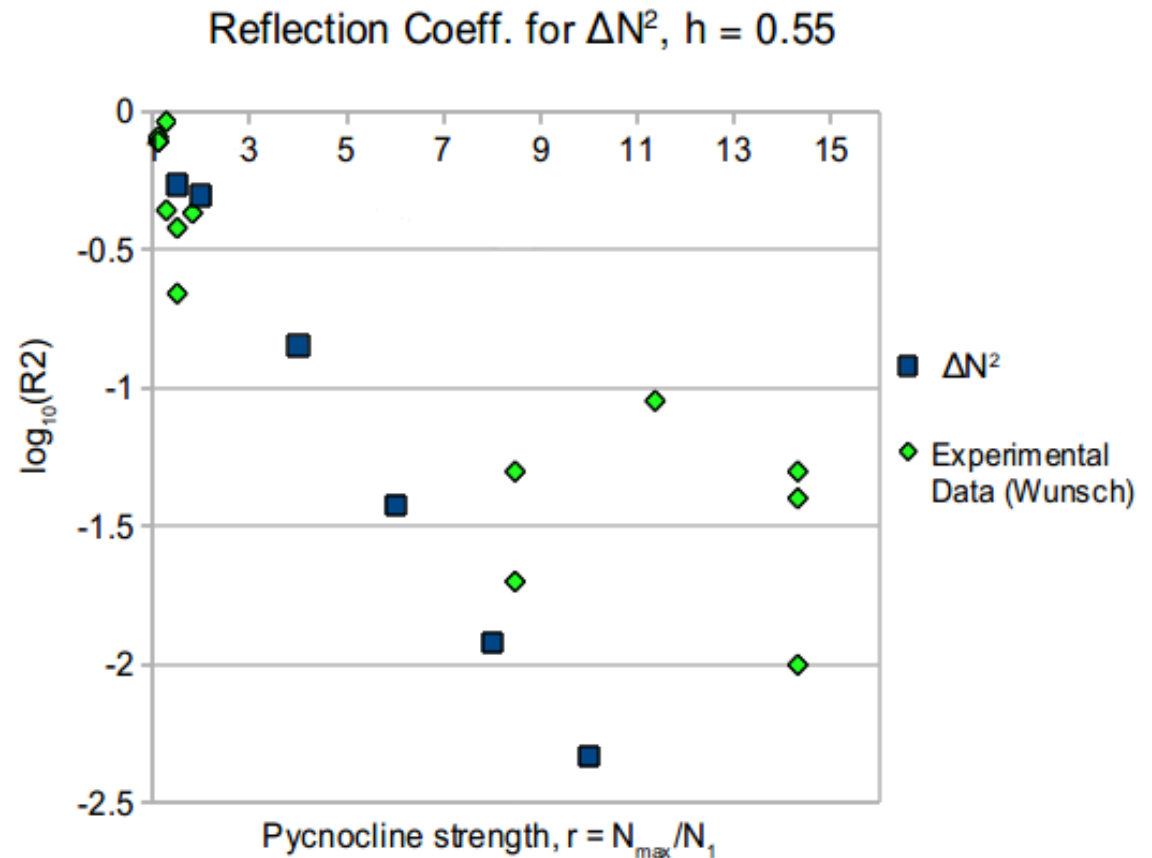


# Measured Reflection Coefficients



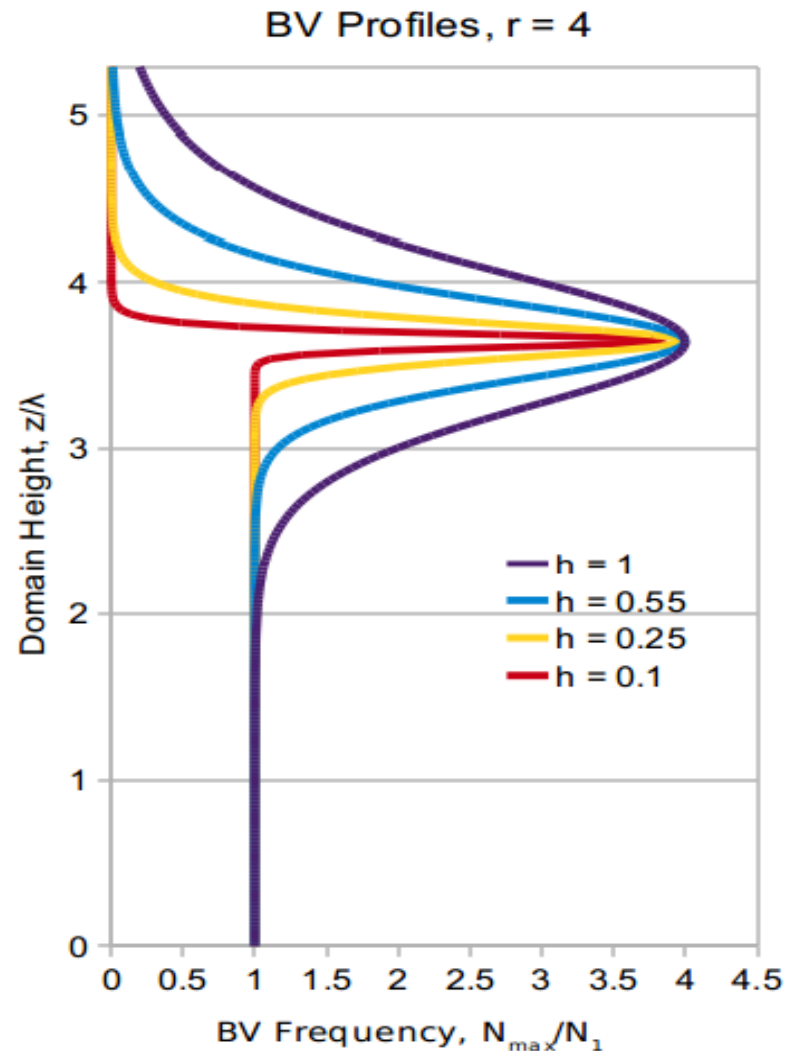
# Measured Reflection Coefficients

- At small  $r$ , results are comparable to laboratory findings
- At higher  $r$ , results differ from lab. Possibly due to experimental noise



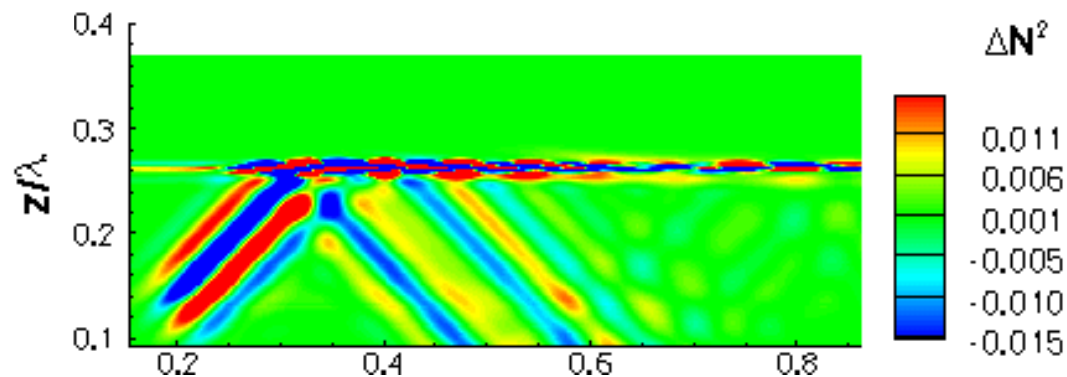
## Series 2 – Varying Pycnocline Thickness

4 Cases:  $h$ -values = 0.1 0.25 0.55 1



- Measure reflection coeff.,  $R_2$ , for **energy flux** and  $\Delta N^2$
- Explore qualitative beam structure inside pycnocline

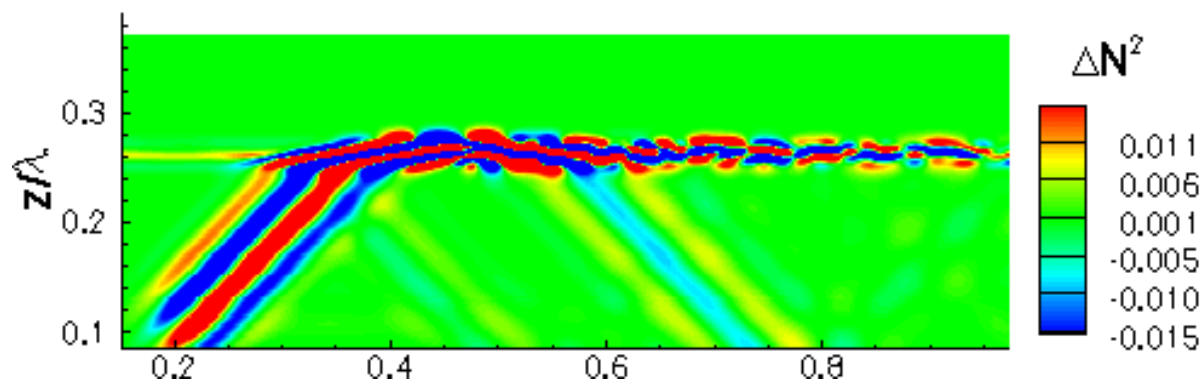
$h = 0.1$   
 $t = 30 \cdot T$



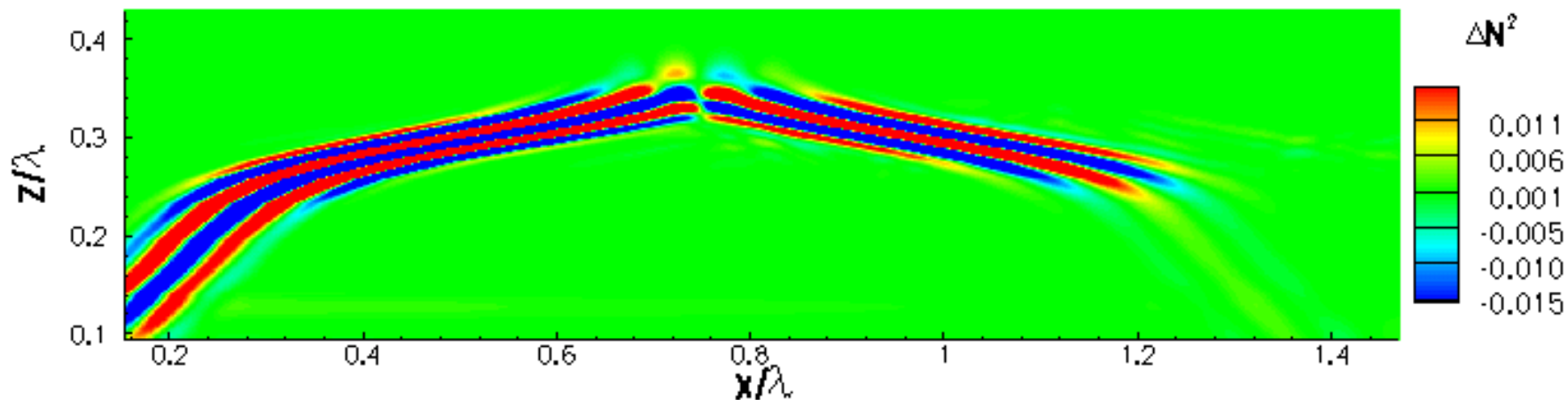
## $\Delta N^2$ Contour Plots

Pycnocline strength,  $r = 4$

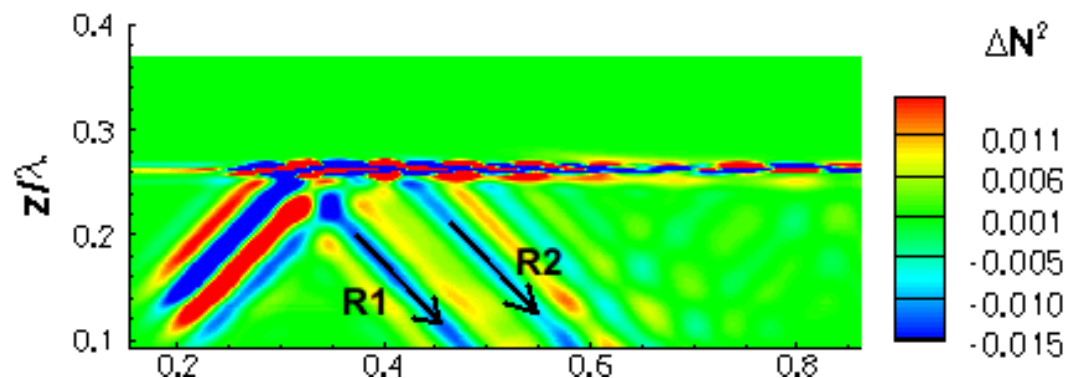
$h = 0.25$   
 $t = 30 \cdot T$



$h = 1$   
 $t = 30 \cdot T$



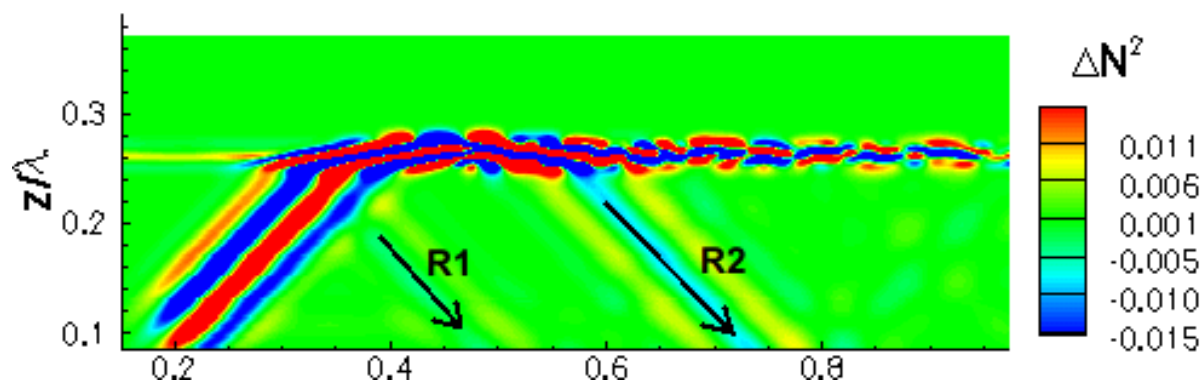
$h = 0.1$   
 $t = 30 \cdot T$



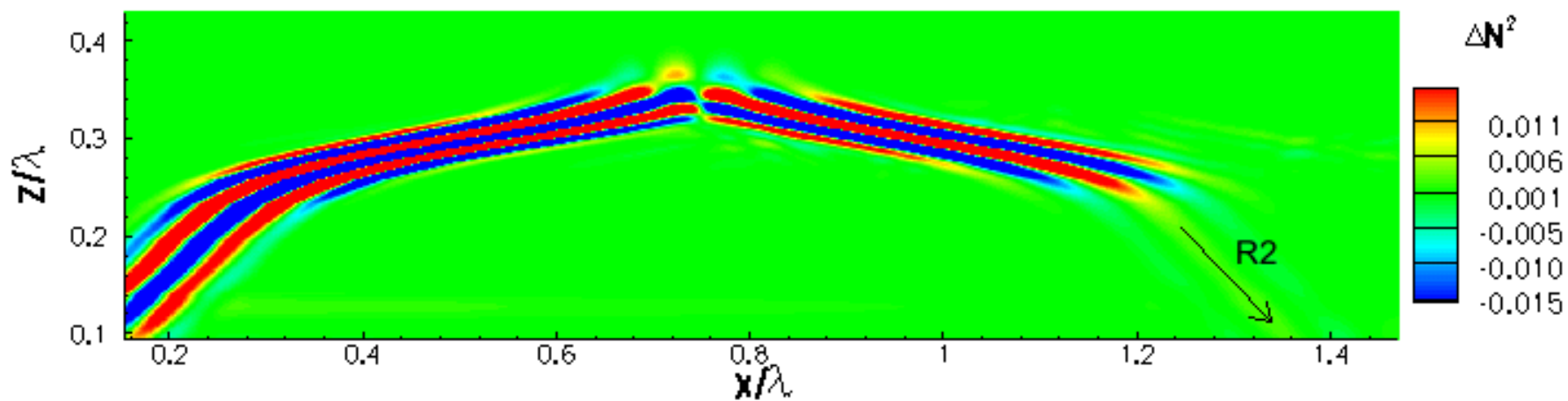
## $\Delta N^2$ Contour Plots

Pycnocline strength,  $r = 4$

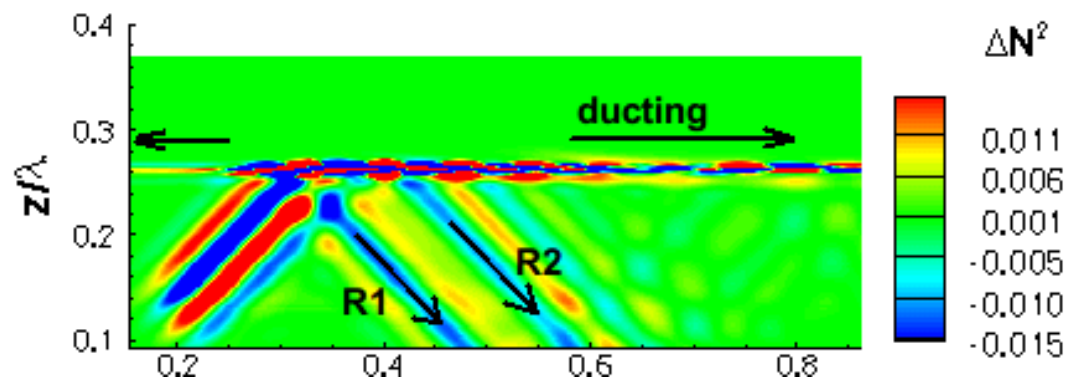
$h = 0.25$   
 $t = 30 \cdot T$



$h = 1$   
 $t = 30 \cdot T$



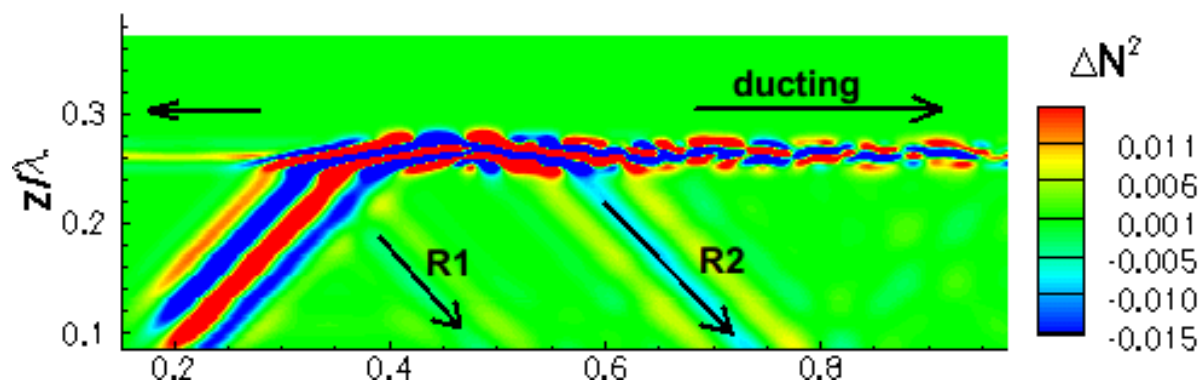
$h = 0.1$   
 $t = 30 \cdot T$



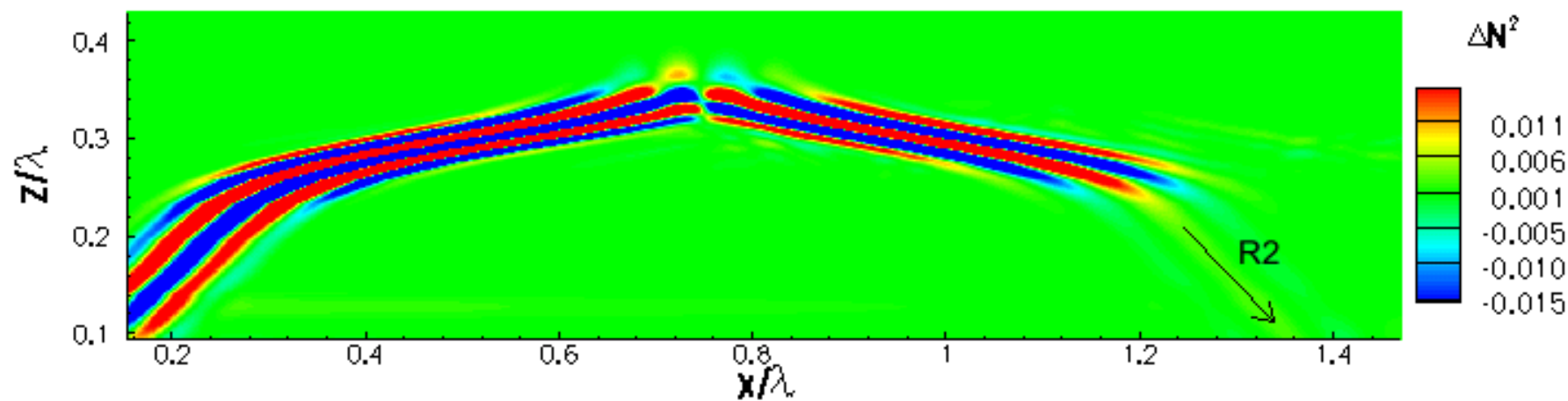
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Pycnocline strength,  $r = 4$

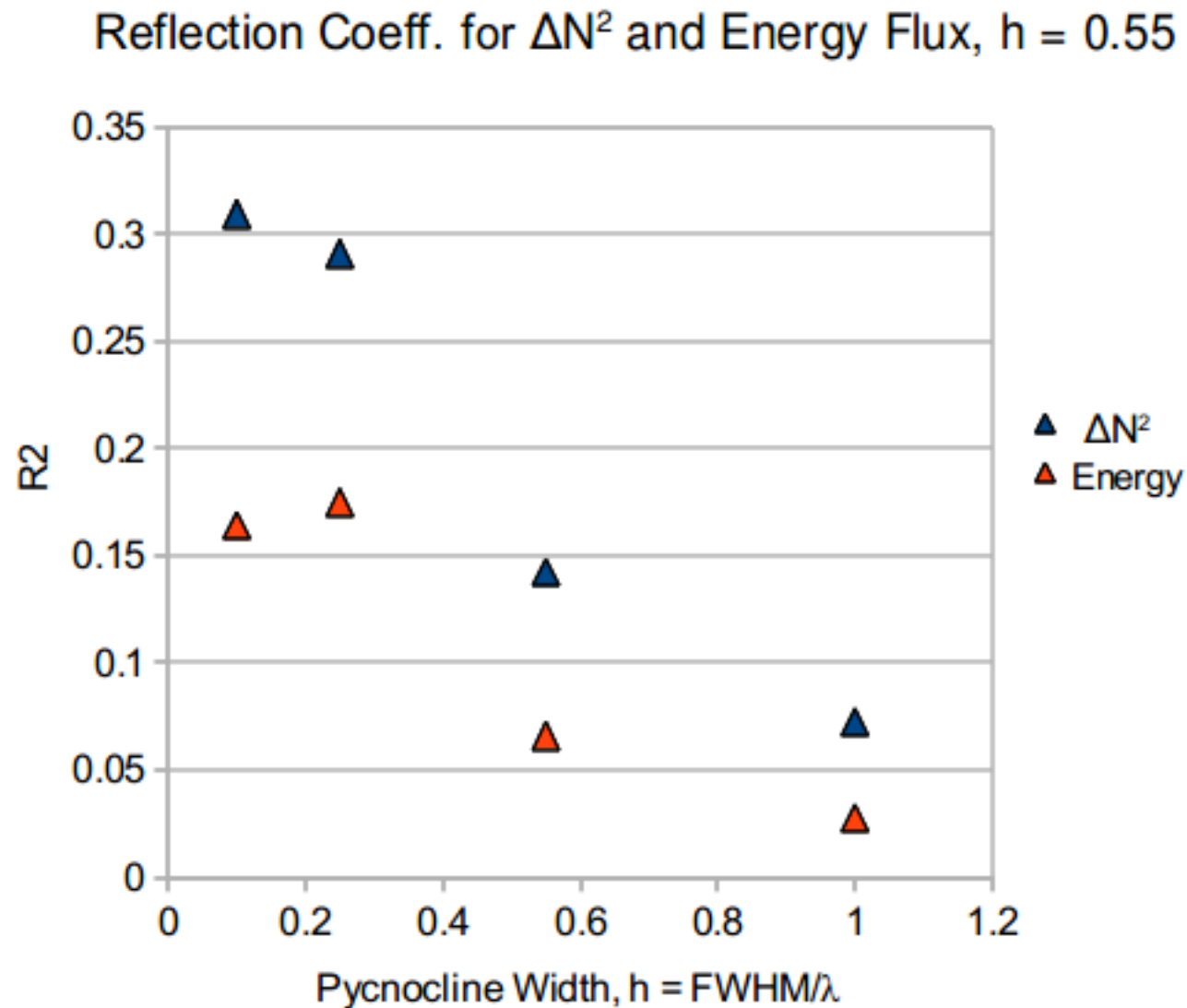
$h = 0.25$   
 $t = 30 \cdot T$



$h = 1$   
 $t = 30 \cdot T$

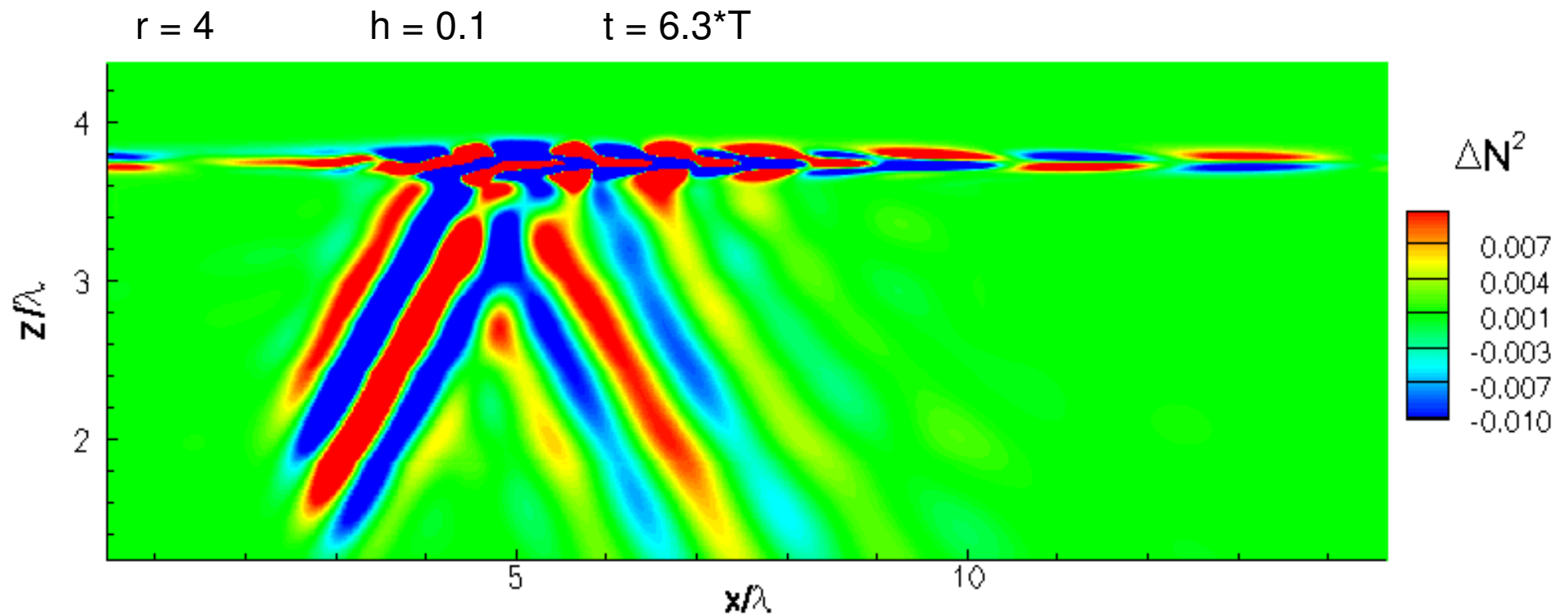


# Measured Reflection Coefficients



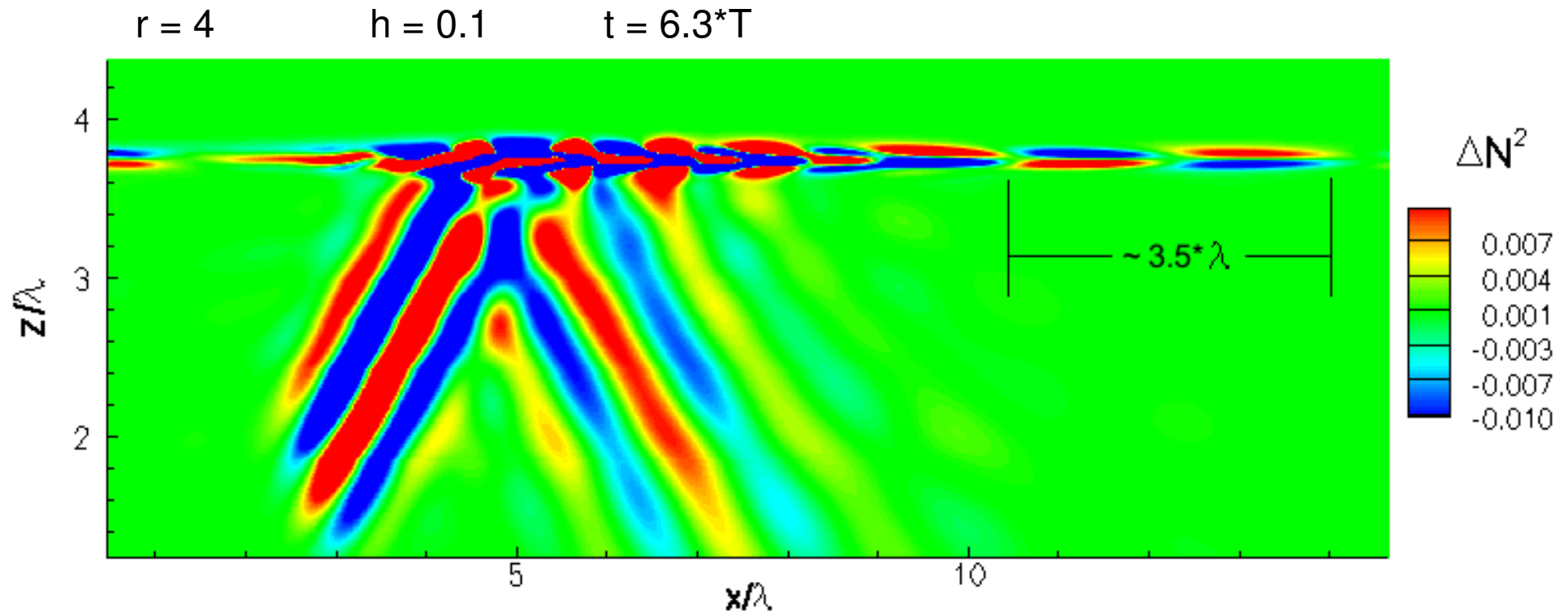


## Curious Phenomena within Thin Pycnocline ( $h = 0.1$ )



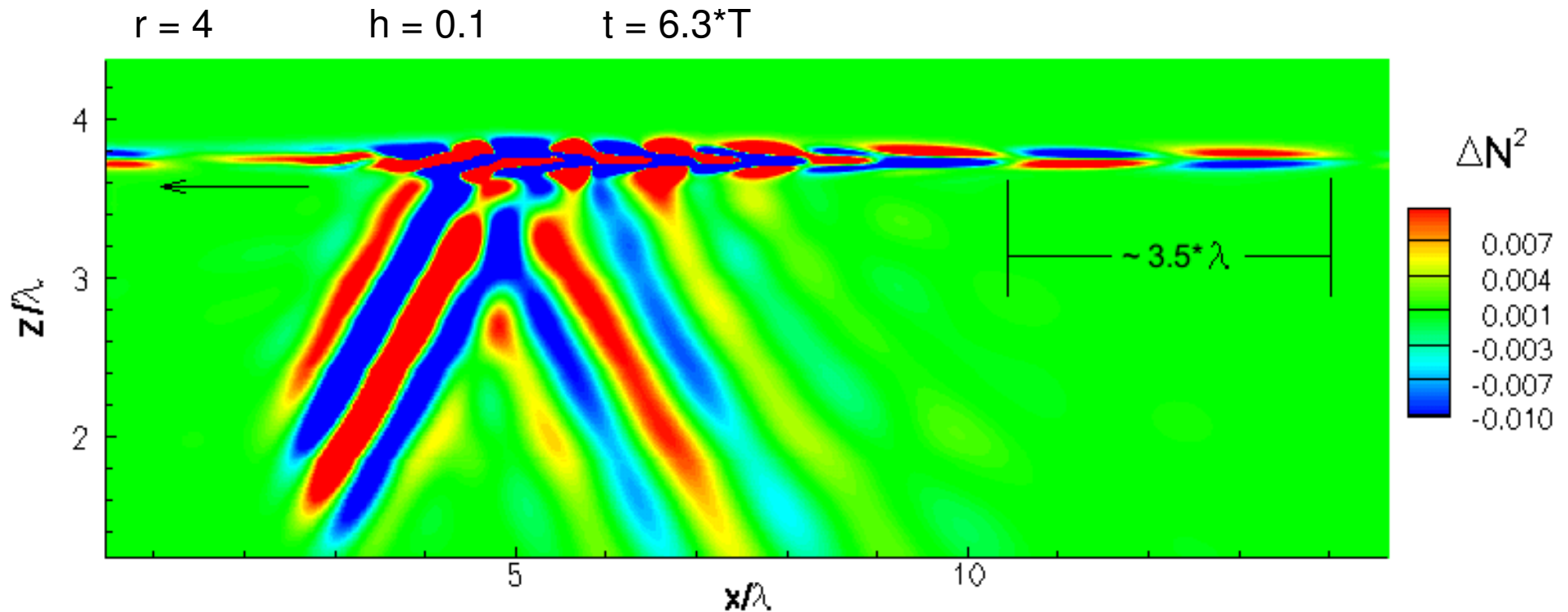
## Curious Phenomena within Thin Pycnocline ( $h = 0.1$ )

- Fast moving packets, large length scale



## Curious Phenomena within Thin Pycnocline ( $h = 0.1$ )

- Fast moving packets, large length scale
- Rearward modal structure



# Conclusions

- At a fixed pycnocline thickness,  $h = 0.55$ , reflection varies from strong reflection to full absorption from the range of  $2 < r < 6$
- $R_2$  decays to zero according to a power law
- Results differ from laboratory observations and from theory
- Variable pycnocline thickness reveals bidirectionally propagating vertical modal structure, and greater ducted energy

# Next Steps

- Investigate wave decay inside pycnocline
  - Viscous dissipation?
- Investigate modal structure and quick beams in pycnocline
- Adjust transition length independently from pycnocline width
  - How does beam change as transition approaches a sharp gradient?
- Explore: High amplitude incident beams, shallow angle of incidence, high Reynolds number
- Model sub-critical shear flow inside pycnocline