Department of Economics

Working Paper Series

The Return to Capital and the Business Cycle

08-002

Paul Gomme
Concordia University and CIREQ

B. Ravikumar
University of Iowa

Peter Rupert
University of California, Santa Barbara

Department of Economics, 1455 De Maisonneuve Blvd. West, Montreal, Quebec, Canada H3G 1M8
Tel 514–848–2424 # 3900 · Fax 514–848–4536 · econ@alcor.concordia.ca · alcor.concordia.ca/~econ/repec

The Return to Capital and the Business Cycle*

Paul Gomme
Concordia University and CIREQ
paul.gomme@concordia.ca

B. Ravikumar
University of Iowa
ravikumar@uiowa.edu

Peter Rupert
University of California, Santa Barbara
rupert@econ.ucsb.edu

September 23, 2010

Forthcoming, *Review of Economic Dynamics*

Abstract

A widely cited failing of real business cycle models is their inability to account for the cyclical patterns of financial variables. Perhaps less well known is the fact that the return to capital and equity are identical in the neoclassical growth model. This paper constructs a measure of the return to business capital for the U.S. The S&P 500 return is roughly six times more volatile than the return to business capital. Owing to the equivalence between the returns to capital and equity in the neoclassical growth model, papers in the real business cycle literature that successfully account for the time series variation in the S&P 500 return must fail to account for the time series properties of the return to capital. A fairly basic real business cycle model captures most of the observed variability in the return to capital. What is needed is a theory of the stock market that breaks the equivalence between the returns to equity and capital.

*For helpful comments we thank David Andolfatto, Michele Boldrin, Gian Luca Clementi, Jonas Fisher, Aubhik Khan, Ellen McGrattan, Richard Rogerson, Jian Wang, and seminar participants at University of Wisconsin, University of North Carolina–Greensboro, Seoul National University, University of Guelph, Colgate University, the Federal Reserve Banks of Cleveland, Minneapolis and Philadelphia, and the conference participants at 2006 Midwest Macro, 2006 SED, 2006 CMSG and 2007 Texas Monetary conference. Gomme acknowledges financial support of SSHRC.

1 Introduction

Real business cycle models have been quite successful in explaining the time series behavior of National Income and Products Accounts (NIPA) and associated data. As emphasized by Rouwenhorst (1995), these models have been far less successful in accounting for the properties of financial variables such as the real risk-free rate and the return to equity.

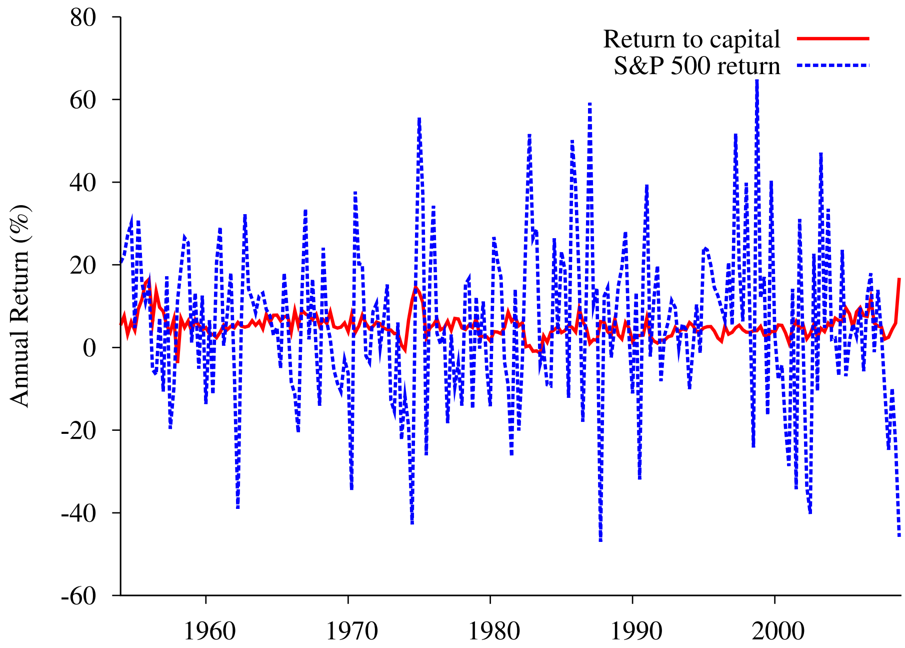
There are two basic points to this paper. First, we establish the equivalence between the return to capital and the return to equity in the neoclassical growth model. On the one hand, the net return to capital can be computed from the marginal product of capital less depreciation and the relative price of investment goods (where consumption plays the role of the numeraire good). On the other hand, the return to equity can be calculated from the dividend and price of equity processes. Section 2.3 makes this equivalence explicit.

Second, armed with this equivalence, one can measure the return to a unit of capital in the model with two different series in the data: the return to equity using the S&P 500 data on prices and dividends, and the return to business capital using NIPA. While the theory says that these two returns should be *identical*, in the data they are not. The emphasis in this paper is on their volatility: The *percent standard deviation* of the S&P 500 quarterly return is considerably more than that of the return to business capital series constructed from NIPA; see Figure 1. Over the 1954Q1–2008Q4 sample period, the volatility of the S&P 500 return is 325.36% while the volatility of the return to capital is only 55.39%.[1]

One traditional strike against the real business cycle model is its inability to account for the S&P 500 return; again, see Rouwenhorst (1995). Jermann (1998) partially accounts for the S&P 500 return by introducing habit persistence and capital adjustment costs to the real business cycle model. Somewhat more recently, Boldrin, Christiano, and Fisher (2001) developed a two sector model with habit persistence and restrictions on factor mobility. By their preferred parameteri-

[1] These figures are in the spirit of "deviations from trend" calculations of other business cycle variables. That is, if $\overline{R}$ is the mean after-tax return in the sample and $\hat{R}_t = \frac{R_t - \overline{R}}{\overline{R}}$ is the deviation at time t from the mean, then the percent standard deviation of the return we report is 100 times the standard deviation of $\hat{R}_t$. The corresponding figures for "raw" standard deviations (i.e., std(R_t)) are 19.27 and 2.86.

Figure 1: After-tax return to the S&P 500 and Capital



zation, their model accounts for 95% of the standard deviation of the S&P 500 return. Recalling the equivalence result described above, Boldrin et al. *could* have compared their model's capital return with the NIPA return to business capital rather than the S&P 500 return. By this measure, the standard deviation of the return to capital in their model is roughly 6 times larger than that in the data. By way of contrast, the real business cycle model constructed in Section 2 — with stochastic taxes on labor and capital income, as well as stochastic labor-embodied technological change and relative price of investment goods — delivers a percentage standard deviation for the return to capital that is almost the same as in the data. Of course, our model in Section 2 is a dismal failure in accounting for the volatility of the S&P 500 return. In other words, neither our model nor the Boldrin et al. model can be considered unqualified successes.

It is important to remember that what Rouwenhorst (1995), Jermann (1998) and Boldrin et al. (2001) did in lining up their models' return to capital with the stock market return was entirely justified from a theoretical point of view. Justified but incomplete since they did not evaluate their models' ability to account for the behavior of the return to business capital. The standard real business cycle model without habit persistence or any frictions does a remarkably good job explaining the volatility of the return to business capital. What is needed is a theory of the stock market that can be added to the real business cycle model in such a way as to break the equivalence between the returns to equity and capital.

The remainder of the paper is organized as follows. The model is presented in Section 2. In the model, there are stochastic processes for: (a) labor income taxes, (b) capital income taxes, (c) labor-embodied technological change, and (d) the relative price of investment goods. As mentioned earlier, Section 2.3 presents the equivalence result between the return to capital and the return to equity. The key result from Section 2 is the expression for the return to capital in the model.

Section 3 describes how to measure the return to business capital, defined as the sum of private nonresidential structures, private nonresidential equipment and software, and private inventories. The calculations for the return to capital, described in Section 3, take into account all taxes paid by the owners of *all* business capital over the period 1954Q1–2008Q4. A number of authors have made conceptually similar calculations using NIPA for specific sectors and for specific types of capital. Poterba (1998) computes an annual return for the *non-financial corporate* sector; Mulligan (2002) calculates the annual return to capital including residential structures; McGrattan and Prescott (2003) compute the annual after-tax return for the *non-corporate* sector. All of these previous studies computed *annual* returns for specific sectors; we compute a *quarterly* return (since that is the frequency typically used in the real business cycle literature) for the entire business sector.

The model is calibrated in Section 4.1. This section also presents SUR estimates of the stochastic processes in the model. The paper's key findings are presented in Section 4.2. The benchmark model accounts for essentially all of the variability of the return to capital (a percentage standard deviation of 59.11% in the model versus 55.39% in the data). Output variability in the benchmark model is considerably higher than in the data, a result that is driven in large part by the stochastic factor income tax rates. When return volatility is expressed relative to the standard deviation of output, the benchmark model accounts for roughly $1/2$ of the observed variability of the return to business capital. A version of the model with labor-embodied technology and relative price of investment shocks only (that is, without stochastic factor income taxes) exhibits a variability of the return to capital that is almost identical to that seen in the data; relative to output volatility, the model delivers 87% of the volatility in the return to capital.

The return to business capital is mildly procyclical, leading the cycle by a quarter. The benchmark model does reasonably well on this score, with a procyclical return to capital, although it lags the cycle by a quarter. The lead-lag pattern of S&P 500 returns is rather different: it is countercyclical and lags the cycle by a quarter. Once more, the behavior of the benchmark model's return is more in accord with the observed return to business capital rather than S&P 500 returns.

Section 5 takes a close look at Boldrin et al. (2001). Their preferred model has two sectors and habit-persistent preferences. The allocation of factors in the consumption and investment good sectors is determined one period in advance. Concentrating on the volatility of the equity return, they capture 95% of the standard deviation of the S&P 500 return. In computing the return to equity, they implicitly invoke the equivalence between the equity return and the return to capital: They compute the return to equity as the weighted average of the returns to each sector (with the weights given by the relative shares of capital in the two sectors), where sectoral returns are computed from the marginal product of capital with appropriate adjustments for the relative price of investment goods. Almost all of the volatility in the return series in their model is due to movements in the relative price of investment goods. This relative price in their model is almost a factor of six more volatile than the corresponding price series computed from NIPA. In summary, given the equivalence between the stock market return and the return to capital, the fact that they come very close to matching the observed volatility of the stock market return necessarily comes at the price of grossly overstating the volatility of the return to business capital.

Section 6 concludes.

2 The Model

In addition to presenting the model, this section develops an equivalence result between the return to capital on the one hand, and the return to equity on the other. Developing this equivalence requires two different decentralizations – one is the standard decentralization in the neoclassical model and the other is for asset pricing where firms make investment decisions.

2.1 The Neoclassical Decentralization

The representative household seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, \ell_t) \tag{1}$$

subject to

$$\ell_t + n_t = 1 \tag{2}$$

$$c_t + q_t i_t = (1-\tau_{nt}) w_t n_t + (1-\tau_{kt}) r_t k_t + \tau_{kt} \delta q_t k_t + T_t \tag{3}$$

$$k_{t+1} = (1-\delta) k_t + i_t \tag{4}$$

where all of the notation is as in the macroeconomics literature of the past two decades with the possible exception of q_t which is the price of a unit of investment in period t, expressed in units of the consumption good in the same period. Its inverse has the interpretation of investment-specific technological change as in Greenwood, Hercowitz, and Krusell (1997). The term, $\tau_{kt} \delta q_t k_t$, is the capital consumption allowance, reflecting the fact that the U.S. tax code allows firms to write off depreciation of capital against their taxes.

The typical firm faces a sequence of static problems:

$$\max F(k_t, z_t n_t) - r_t k_t - w_t n_t$$

where technological change, z_t, is expressed as labor-embodied to be consistent with balanced growth.

Substituting the time constraint into preferences, consolidating the last two of the household's constraints, and using the equilibrium relationships for factor prices, the relevant Euler equations are:

$$U_1(c_t, 1-n_t) = \lambda_t \tag{5}$$

$$U_2(c_t, 1-n_t) = (1-\tau_{nt}) z_t F_2(k_t, z_t n_t) \lambda_t \tag{6}$$

$$\lambda_t = \beta E_t \left\{ \lambda_{t+1} \left[\frac{(1-\tau_{k,t+1}) F_1(k_{t+1}, z_{t+1} n_{t+1}) + q_{t+1} \tau_{k,t+1} \delta + q_{t+1}(1-\delta)}{q_t} \right] \right\} \tag{7}$$

where λ_t is the Lagrange multiplier associated with the household's consolidated constraint. For future reference, note that the term in square brackets in Eq. (7) is the real, gross return to capital. A unit of capital in period t costs q_t units of consumption good and the payoff in period $t+1$ is: $(1-\tau_{k,t+1}) F_1(k_{t+1}, z_{t+1} n_{t+1})$, the after-tax marginal product of capital; $q_{t+1} \tau_{k,t+1} \delta$, the capital consumption allowance expressed in units of the consumption good; and left-over capital in period $t+1$, worth $(1-\delta) q_{t+1}$ units of consumption good. Thus, the real, gross after-tax rate of return to capital is *not* $(1-\tau_{k,t+1}) F_1(k_{t+1}, z_{t+1} n_{t+1}) + 1 - \delta$ as in the standard one-sector growth model; the relative price of capital goods affects the real return to capital.

The only role of government is to collect and redistribute taxes to satisfy its budget constraint,

$$\tau_{nt} w_t n_t + \tau_{kt} r_t k_t - \tau_{kt} \delta q_t k_t = T_t.$$

The definition of a competitive equilibrium for this economy is entirely standard and is omitted for the sake of brevity.

2.2 The Asset Pricing Decentralization

Here, firms directly acquire capital and so have the following dynamic problem:

$$\max E_0 \sum_{t=0}^{\infty} \Delta_t d_t$$

subject to

$$d_t = (1-\tau_{kt}) \left[F(k_t, z_t n_t) - w_t n_t \right] - q_t i_t + \tau_{kt} \delta q_t k_t \tag{8}$$

and Eq. (4), where d_t is dividends, and Δ_t is a stochastic discount factor. In this case, the Euler equation governing capital accumulation is

$$q_t \Delta_t = E_t \left\{ \Delta_{t+1} \left[(1-\tau_{k,t+1})(F_1(k_{t+1}, z_{t+1} n_{t+1}) - \delta q_{t+1}) + q_{t+1} \right] \right\}. \tag{9}$$

Households have the same preferences as above, Eq. (1), but now face the constraint

$$c_t + p_t s_{t+1} = (1-\tau_{nt}) w_t n_t + (p_t + d_t) s_t$$

where s_t represents the household's equity holdings as of the start of the period, and p_t is the price of equity measured in units of the consumption good in period t. The household's Euler equation governing its purchases of equity is

$$p_t U_1(c_t, 1-n_t) = \beta E_t \left\{ U_1(c_{t+1}, 1-n_{t+1})[p_{t+1} + d_{t+1}] \right\} \tag{10}$$

or,

$$U_1(c_t, 1-n_t) = \beta E_t \left\{ U_1(c_{t+1}, 1-n_{t+1}) \left[\frac{p_{t+1} + d_{t+1}}{p_t} \right] \right\}. \tag{11}$$

Again for future reference, note that the term in square brackets in Eq. (11) is the real, gross return to equity: each equity costs p_t units of consumption good in period t, yields d_{t+1} units of consumption good in period $t+1$ and can be sold for p_{t+1} units of consumption good in period $t+1$.

2.3 An Equivalence Result

Updating the expression for the dividend, Eq. (8), by one period and using the fact that, in equilibrium, $w_t = z_t F_2(k_t, z_t n_t)$,

$$\begin{aligned} d_{t+1} &= (1-\tau_{k,t+1}) \left[F(k_{t+1}, z_{t+1} n_{t+1}) - F_2(k_{t+1}, z_{t+1} n_{t+1}) n_{t+1} \right] \\ &\quad - q_{t+1} \left[k_{t+2} - (1-\delta) k_{t+1} \right] + \tau_{k,t+1} \delta q_{t+1} k_{t+1} \\ &= (1-\tau_{k,t+1}) k_{t+1} F_1(k_{t+1}, z_{t+1} n_{t+1}) - q_{t+1} \left[k_{t+2} - (1-\delta) k_{t+1} \right] + \tau_{k,t+1} \delta q_{t+1} k_{t+1} \end{aligned}$$

where the last line follows from Euler's theorem. This expression can further be rearranged to read

$$\frac{d_{t+1} + q_{t+1} k_{t+2}}{k_{t+1}} = (1-\tau_{k,t+1}) \left[F_1(k_{t+1}, z_{t+1} n_{t+1}) - \delta q_{t+1} \right] + q_{t+1} \tag{12}$$

Substituting this expression into Eq. (9),

$$q_t \Delta_t k_{t+1} = \beta E_t \left\{ \Delta_{t+1} [d_{t+1} + q_{t+1} k_{t+2}] \right\}. \tag{13}$$

If firms act in the best interests of their shareholders (that is, households), then the firm's stochastic discount factor, Δ_t, is the shareholder's (marginal) valuation of a unit of unit of dividends received at t. In other words, $\Delta_t = \beta^t U_1(c_t, \ell_t) = \lambda_t$. Comparing Eq. (10) with Eq. (13) gives $p_t = q_t k_{t+1}$. Dividing both sides of Eq. (12) by p_t and then substituting $p_t = q_t k_{t+1}$ gives

$$\frac{p_{t+1} + d_{t+1}}{p_t} = \frac{(1-\tau_{k,t+1})\left[F_1(k_{t+1}, z_{t+1} n_{t+1}) - \delta q_{t+1}\right] + q_{t+1}}{q_t},$$

where the right-hand side is a rewriting of the term in square brackets in Eq. (7). In other words, the return to capital is identically equal to the return to equity.

2.4 Discussion

(1) There is no reason to take a stand on the source of the variations in the relative price of investment goods, q_t. Whether such fluctuations are endogenous or exogenous are largely irrelevant for the equivalence result.

(2) Fluctuations in the return to capital are *not* driven exclusively by variation in the marginal product of capital. As Eq. (7) makes clear, the fluctuations in the return to capital are also driven by perturbances in the relative price of investment goods, q_t. In other words, "getting the quantities right" in the neoclassical model might not be sufficient to deliver the observed return to capital.

(3) While the return to capital and the return to equity are equivalent in theory, they may not be in the data due to mismeasurement. For instance, (i) the S&P 500 return does not include all of the firms in the economy and (ii) part of the price in S&P 500 could be for the firms' intangible capital. It is hard to imagine how complete coverage of firms or fluctuations in intangible capital could account for the six-fold gap between return to capital and return to equity in Figure 1.

3 Measurement of the Return to Capital

To reiterate, the gross, after-tax return to capital, as measured in the model, is the term in square brackets in Eq. (7). The real, net after-tax return to capital, then, is measured by

$$R_t = \frac{(1-\tau_{kt})[F_1(k_t, z_t n_t) - \delta q_t]}{q_{t-1}} + \left(\frac{q_t}{q_{t-1}} - 1\right). \tag{14}$$

The last term (in round brackets) is the net rate of appreciation from holding investment goods from t to $t+1$, expressed in units of consumption goods. For measurement and computations later, we will assume that the model period is one quarter, so the return in Eq. (14) will be annualized as $[(1+R_t)^4 - 1] \times 100\%$.

The task at this stage is to describe empirical counterparts to the theory as laid out above. A key component of this exercise is the construction of a time series for the rate of return to private business capital – the data counterpart to the expression in Eq. (14). An important aspect of the measurement is that housing income components should be removed from a number of income flows factoring into the calculation of the return to private business capital since the model speaks to the market (business) return, not returns to housing or durables. The reader uninterested in these details can skip to Section 3.4 with no loss in continuity.

Much of the data construction follows standard procedures in the literature such as those in Cooley and Prescott (1995) and Gomme and Rupert (2007). The U.S. NIPA are the source for much of the calculations. Nominal variables are converted to real by dividing by a price index for personal consumption of non-durables and services, computed from the corresponding real and nominal series.

Measurement of the return to capital is guided by Eq. (14). The first term on the right-hand side can be computed by dividing total after-tax capital income, net of depreciation, by the previous period's value of capital. The second term is the net change in the relative price of investment goods. To make the analysis conformable with usual practice in the business cycle literature, we have to generate a quarterly time series for returns. Unfortunately, not all of the data is available quarterly and so quarterly series must be imputed; this procedure is described in Section 3.2.

While most of the taxes levied against capital income can be obtained fairly directly from the data, those paid by households must be imputed. To obtain the tax rate on general *household income*, the basic methodology of Mendoza, Razin, and Tesar (1994) and Carey and Tchilinguirian (2000) is followed. This tax rate, τ_h, is computed as:

$$\tau_h = \frac{\textsc{Personal Current Taxes}}{\begin{matrix}\textsc{Net}\\ \textsc{Interest}\end{matrix} + \begin{matrix}\textsc{Proprietors'}\\ \textsc{Income}\end{matrix} + \begin{matrix}\textsc{Rental}\\ \textsc{Income}\end{matrix} + \begin{matrix}\textsc{Wages and}\\ \textsc{Salaries}\end{matrix}}.$$

The tax rate τ_h – distinct from τ_n and τ_k – is an intermediate input into subsequent calculations of the rate of return to capital.

The after-tax return to capital is obtained by dividing total after-tax capital income by the appropriate capital stock. To this end, after-tax capital income can be written as:[2]

$$\begin{aligned}
Y^{AT} = {} & \textsc{Net Operating Surplus} - \textsc{Housing Net Operating Surplus} \\
& - (1-\alpha)(\textsc{Proprietor's Income} - \textsc{Housing Proprietor's Income}) \\
& - \tau_h(\textsc{Net Interest} - \textsc{Housing Net Interest}) \\
& - \alpha\tau_h(\textsc{Proprietor's Income} - \textsc{Housing Proprietor's Income}) \\
& - \tau_h(\textsc{Rental Income} - \textsc{Housing Rental Income}) \\
& - \textsc{Taxes on Corporate Income} \\
& - \textsc{Business Property Taxes} \\
& - \textsc{State and Local Other Taxes}.
\end{aligned}$$

Net operating surplus is defined as value added minus depreciation and payments to labor. The income flows and tax rates have been modified to subtract out the income generated from the housing sector since the model speaks to the return to business capital. Also, as is conventional in the literature, a fraction α of proprietors' income is attributed to capital income, the remaining fraction $1-\alpha$ to labor income. When the aggregate production function is Cobb-Douglas – as assumed below – α corresponds to the exponent on the capital input.

The relative price of investment goods, denoted q^{US}, is computed by dividing price deflator for private nonresidential investment (constructed by dividing nominal private nonresidential invest-

[2] All terms are converted to real by dividing by the aforementioned price deflator for personal consumption of non-durables and services.

ment by its real counterpart) by the price deflator for consumption (that is, of non-durables and services).

Finally, the real, net after-tax return to business capital is given by

$$R_t^{US} = \left[\left(\frac{Y_t^{AT}/4}{K_t^M} + \frac{q_t^{US}}{q_{t-1}^{US}} \right)^4 - 1 \right] \times 100\% \tag{15}$$

where K_t^M denotes the stock of market capital, given by the sum of: inventories; the stock of structures; and the stock of equipment and software. In Eq. (15), both the income flow and capital stock measures are expressed in real terms (that is, each has been divided by the consumption price deflator discussed above). The division by 4 accounts for the fact that quarterly income flows are expressed at an annual rate. The term in brackets, then, is the gross quarterly return. Raising this term to the power 4 annualizes the return. The remaining terms in Eq. (15) converts the gross return to a net return, and expresses the return in percentage points.

There are several important differences between Eq. (14) and Eq. (15).

(1) The model's return, Eq. (14), is computed for a single homogeneous unit of capital while the return for the U.S. is computed as an average for the entire private market capital stock.

(2) The model's return contains terms explicitly accounting for the relative price of new capital goods (q_t and q_{t-1}) while, apart from the capital gain term, Eq. (15) does not. The reason for excluding these relative price terms elsewhere in Eq. (15) is that the Bureau of Economic Analysis's revised methodology for computing capital stocks for the U.S. already accounts for the changes in the quality of capital goods by changes in the relative price of investment goods. In other words, the division by K_t^M in Eq. (15) already embodies the relative price term in Eq. (14).

(3) Since after-tax capital income is measured net of depreciation in Eq. (15), there is no need to explicitly include a term corresponding to δq_t as in Eq. (14).

3.1 Other Tax Rates

Although the tax rates on labor income, τ_n, and capital income, τ_k, do not directly factor into the calculation of the return to capital, these tax rates are used later in simulating the model.

Labor income taxes are given by

$$\begin{aligned}\textsc{Labor Income Taxes} =& \tau_h \left[\textsc{Wages and Salaries} + (1-\alpha)\textsc{Proprietors' Income}\right] \\ & + \textsc{Contributions for Government Social Insurance}\end{aligned}$$

while total labor income is computed via

$$\begin{aligned}\textsc{Labor Income} =& \textsc{Wages and Salaries} + (1-\alpha)\textsc{Proprietors' Income} \\ & + \begin{array}{c}\textsc{Employer Contributions for} \\ \textsc{Government Social Insurance}\end{array}.\end{aligned}$$

The tax rate on labor income can now be computed as

$$\tau_n = \frac{\textsc{Labor Income Taxes}}{\textsc{Labor Income}}.$$

Next, capital income taxes are calculated as

$$\begin{aligned}\textsc{Capital Income Taxes} =& \tau_h \Big[\textsc{Net Interest} + \alpha\textsc{Proprietors' Income} + \textsc{Rental Income} \\ & - \big(\textsc{Housing Net Interest} + \alpha\textsc{Housing Proprietors' Income} \\ & + \textsc{Housing Rental Income}\big)\Big] + \textsc{Corporate Income Taxes} \\ & + \textsc{Real Estate Property Taxes} + \textsc{Other Taxes}.\end{aligned}$$

Capital Income is given by

$$\begin{aligned}\textsc{Capital Income} =& \textsc{Net Operating Surplus} \\ & + \textsc{Consumption of Private Fixed Capital} \\ & - \textsc{Housing Net Operating Surplus} \\ & + (1-\alpha)\big(\textsc{Proprietors' Income} \\ & - \textsc{Housing Proprietors' Income}\big).\end{aligned}$$

So, the tax rate on capital income can now be computed as

$$\tau_k = \frac{\textsc{Capital Income Taxes}}{\textsc{Capital Income}}$$

3.2 Annual to Quarterly Conversions

Several series are not available quarterly. Different methods are used to convert the annual series to quarterly. To start, the series STATE AND LOCAL OTHER TAXES covers such things as licensing fees. It seems reasonable, then, to divide this figure equally across the four quarters. Property taxes (paid by businesses and households) are available quarterly from 1958Q1. Prior to this date, the annual observation is repeated for each quarter. Property taxes are not reported separately for businesses and households. It is assumed that the fraction of property taxes paid for by businesses is the same as the fraction of structures owned by businesses.

Quarterly values for all of the housing flows are imputed with the exception of GROSS HOUSING VALUE ADDED (GHVA), which is available quarterly. To understand the approach taken here, consider the calculation for NET OPERATING SURPLUS. Take the observation for GHVA (quarterly), multiply by NET OPERATING SURPLUS (annual) divided by GHVA (annual), for the relevant year. That is, apportion the quarterly GHVA to its constituent components using the annual ratios for the appropriate year. This strategy is also used to impute NET INTEREST, PROPRIETORS' INCOME and RENTAL INCOME for the housing sector.

Quarterly capital stocks are constructed from annual capital stocks and quarterly investment flows (both of which are converted to real by dividing by the consumption deflator for non-durables and services). This procedure requires solving for the depreciation rate that makes the annual capital stocks line up with Q4 of our quarterly capital stock, and be consistent with the quarterly investment flows. For example:

$$\begin{aligned}
K_{1959Q4} =& K_{1959} \text{ (the annual observation)}\\
K_{1960Q1} =& (1-\delta_{1960})K_{1959Q4} + I_{1960Q1}\\
K_{1960Q2} =& (1-\delta_{1960})K_{1960Q1} + I_{1960Q2}\\
K_{1960Q3} =& (1-\delta_{1960})K_{1960Q2} + I_{1960Q3}\\
K_{1960Q4} =& (1-\delta_{1960})K_{1960Q3} + I_{1960Q4}\\
K_{1960Q4} =& K_{1960} \text{ (the annual observation).}
\end{aligned}$$

In effect, there are 4 equations (the middle 4) in 4 unknowns: $K_{1960Q1}, K_{1960Q2}, K_{1960Q3}$ and δ_{1960}.

3.3 Spliced Data

Two series required splicing. First, hours is measured by private non-farm hours. This series is only available from 1964Q1. For earlier years, the old Citibase series, LHTPRIVA, is used with a level adjustment so that the old and new series coincide in 1964Q1.

Second, Haver Analytics has two series for personal consumption expenditures on housing services. The first series, CSR, ends in 2004Q4; the second series, CSRX, begins in 1959Q1. The series CSR is used up to 1958Q4, with a level adjustment to match CSRX in 1959Q1; CSRX is used starting with 1959Q1.

3.4 Time Series Properties of the Real Return to Capital

The standard deviation of the rate of return to capital is 55.39% over the period 1954Q1–2008Q4 (see Table 1). As documented in this table (and visually in Figure 1) the rate of return to capital is very smooth relative to the S&P 500 return–the latter is roughly 6 times as volatile. Both returns are measured after-tax; for the S&P 500 return, the after-tax calculation is made by multiplying the pre-tax return by $(1-\tau_{kt})$ where τ_{kt} is the tax rate on capital income computed in Section 3.1, and reported in Table 3.[3]

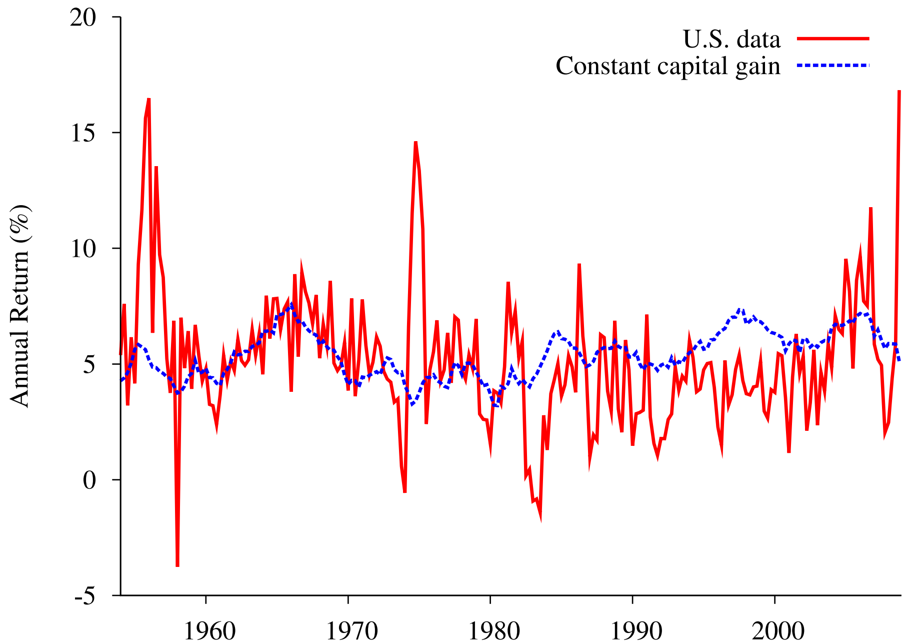
Table 1: After-tax Returns Data: Selected Moments

	Mean (%)	% Standard Deviation
Business capital	5.16	55.39
S&P 500	5.91	325.36

The quarterly time series for the tax rate on household income, τ_h and the real after-tax return to capital are shown in Table 2. The mean after-tax return to capital, 5.16%, is in the middle of other estimates found in the literature. Poterba (1998), using annual data from 1959 to 1996 for the non-financial corporate sector, found a mean after-tax return of 3.9%. Mulligan (2002) excludes

[3] In the S&P 500 return calculation, we have abstracted from taxes on distributions by firms. We thank Ellen McGrattan for pointing out that adjusting for this tax is unlikely to have a significant effect on the S&P 500 return volatility, given the smoothness of the tax rates we report in Table 3; the standard deviation of the capital income tax rate is 0.056.

Figure 2: Return to capital



inventories but includes residential structures and finds the mean after-tax return on capital to be roughly 6%. McGrattan and Prescott (2003) used annual data from 1880 to 2002 for the non-corporate sector and found a mean after-tax return of 4%.

Given the methodology above, it is relatively straightforward to construct other return to capital series. For example, one can construct the return to "all" private capital – that is, business and household capital. Its average return is 3.93% over 1954Q1–2008Q4. For the average return to all private capital to be 3.93 when the average return to business capital is 5.16% necessarily means that the return to housing capital is fairly low. Sure enough, it is at 2.48%.

Figure 2 highlights the importance of the capital gain term in the calculation of the return to business capital. In particular, this figure plots the return to business capital (denoted "U.S. data") against an alternative series that sets the capital gain term in Eq. (15) equal to its sample average. There are two messages to take away from this figure:

(1) The capital gain term adds a substantial amount of volatility to the return to capital: A constant capital gain implies a much smoother return to capital series. The percentage standard deviation of the constant capital gain return series is 18.17% compared to 55.39%, or about $1/3$ of

Table 2: U.S. Return to Capital and Tax Rate on Household Income

	Return to Capital				Tax Rate, τ_h			
Year	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
1954	5.39	7.60	3.21	6.16	11.75	11.69	11.65	11.64
1955	4.16	9.30	11.65	15.62	11.75	11.80	11.96	12.04
1956	16.49	6.34	13.55	9.72	12.27	12.33	12.42	12.48
1957	8.75	5.23	3.75	6.87	12.57	12.60	12.52	12.47
1958	−3.77	7.00	4.82	6.42	12.26	12.09	12.25	12.18
1959	3.93	6.69	5.37	4.27	12.43	12.48	12.67	12.82
1960	4.79	3.26	3.20	2.39	13.03	13.09	13.20	13.16
1961	3.66	5.37	4.31	5.14	13.10	13.04	12.96	12.86
1962	4.72	6.06	5.15	4.95	13.01	13.19	13.44	13.62
1963	5.18	6.48	5.38	6.32	13.62	13.51	13.40	13.31
1964	4.55	7.95	6.10	7.81	12.76	11.56	11.79	11.97
1965	7.83	6.49	7.38	7.68	12.58	12.67	12.13	12.08
1966	3.81	8.89	5.32	8.96	12.39	12.99	13.14	13.43
1967	8.13	7.63	6.74	7.99	13.36	13.17	13.44	13.59
1968	5.25	6.73	5.81	8.59	13.72	13.93	15.29	15.63
1969	5.04	4.71	4.94	5.93	16.37	16.46	15.77	15.74
1970	3.86	7.83	3.61	5.19	15.37	15.39	14.44	14.50
1971	7.79	5.80	4.54	5.08	13.73	13.79	13.80	13.97
1972	6.17	5.77	4.69	4.35	15.44	15.61	15.29	15.00
1973	4.21	3.36	3.49	0.58	14.59	14.50	14.69	14.82
1974	−0.57	6.61	11.51	14.62	14.95	15.36	15.57	15.59
1975	13.36	10.83	2.41	4.75	15.61	11.85	14.54	14.69
1976	5.53	6.89	4.28	4.75	14.68	15.01	15.29	15.50
1977	6.35	4.18	7.04	6.90	15.70	15.75	15.56	15.71
1978	4.75	4.35	5.40	4.82	15.63	15.82	16.31	16.57
1979	6.95	2.84	2.61	2.58	16.46	16.67	17.00	17.08
1980	1.57	3.84	3.74	3.33	16.66	17.00	17.15	17.15
1981	4.87	8.55	6.36	7.28	17.42	17.74	17.87	17.47
1982	5.28	6.00	0.17	0.44	17.28	17.42	16.84	16.97
1983	−0.93	−0.83	−1.41	2.78	16.51	16.67	15.64	15.60
1984	1.28	3.72	4.32	5.02	15.36	15.28	15.44	15.64
1985	3.62	4.11	5.36	4.87	16.76	14.92	16.02	15.95
1986	3.77	9.34	6.21	4.74	15.59	15.52	15.64	16.02
1987	1.00	1.94	1.71	6.27	15.55	17.38	16.28	16.42
1988	6.15	3.83	2.97	6.87	15.96	15.53	15.49	15.51
1989	3.07	2.05	6.04	4.75	16.17	16.47	16.47	16.49
1990	1.47	2.85	2.91	3.00	16.26	16.31	16.29	16.28
1991	7.14	2.71	1.57	1.05	15.79	15.81	15.81	15.91
1992	1.78	1.78	2.60	2.84	15.38	15.65	15.70	16.10
1993	4.88	3.87	4.48	4.26	15.37	15.87	16.18	16.37
1994	6.02	5.29	3.80	3.92	16.02	16.34	16.05	16.00
1995	4.74	5.02	5.06	3.99	16.17	16.54	16.46	16.61
1996	2.28	1.50	5.15	3.22	17.06	17.46	17.27	17.37
1997	3.68	4.77	5.40	4.29	17.78	17.83	17.99	18.11
1998	3.72	3.66	4.01	4.05	18.14	18.25	18.27	18.40
1999	5.10	2.97	2.66	3.89	18.31	18.41	18.57	18.66
2000	3.78	5.45	5.36	3.55	18.88	18.99	18.94	18.99
2001	1.16	4.44	6.30	4.61	19.26	19.39	16.54	18.39
2002	5.33	2.12	3.34	5.61	15.94	15.53	15.59	15.51
2003	2.36	4.64	3.84	5.87	15.10	14.87	13.68	14.48
2004	4.85	7.09	6.49	6.31	14.32	14.34	14.64	14.79
2005	9.55	8.14	4.81	8.62	15.62	15.80	15.88	16.02
2006	9.65	7.72	7.52	11.77	16.49	16.48	16.53	16.63
2007	5.86	5.20	4.95	2.05	17.30	17.34	17.38	17.27
2008	2.47	4.39	5.89	16.84	17.30	14.91	16.08	16.09

Table 3: U.S. Tax Rates on Labor and Capital Income

	Tax Rate, τ_n				Tax Rate, τ_k			
Year	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
1954	15.01	14.97	14.92	14.90	48.27	48.06	48.05	47.56
1955	15.21	15.24	15.36	15.41	47.34	46.70	47.26	47.93
1956	15.79	15.83	15.87	15.92	48.83	49.77	48.08	49.29
1957	16.38	16.41	16.30	16.25	49.82	49.35	48.82	48.57
1958	16.06	15.86	16.01	15.90	46.61	46.39	47.24	47.70
1959	16.71	16.72	16.90	17.03	47.59	46.86	47.18	46.48
1960	17.87	17.88	17.97	17.92	47.60	47.58	46.84	47.02
1961	17.95	17.89	17.76	17.65	47.52	46.42	46.66	46.72
1962	18.09	18.26	18.49	18.62	43.77	43.91	44.30	43.75
1963	19.09	18.97	18.84	18.74	43.75	43.99	44.21	44.14
1964	18.13	16.91	17.10	17.24	42.41	42.20	42.26	42.25
1965	17.79	17.86	17.30	17.21	40.60	40.71	40.45	40.78
1966	18.69	19.22	19.45	19.68	40.55	41.29	41.53	40.75
1967	19.81	19.76	19.99	20.17	41.11	41.32	41.46	42.32
1968	20.39	20.59	21.83	22.13	45.43	44.79	45.39	45.97
1969	23.23	23.27	22.57	22.53	46.28	46.43	46.22	48.22
1970	22.19	22.20	21.27	21.28	47.90	46.88	47.04	47.39
1971	20.94	20.96	20.89	21.00	46.64	46.75	45.63	45.07
1972	22.99	23.08	22.71	22.25	45.62	45.79	44.28	44.21
1973	23.16	22.99	23.13	23.16	44.99	46.09	45.48	45.39
1974	23.77	24.20	24.35	24.28	46.61	48.41	51.45	48.04
1975	24.43	20.80	23.27	23.35	43.38	40.87	42.91	42.84
1976	23.72	23.98	24.20	24.34	44.06	44.84	44.84	44.90
1977	24.77	24.79	24.56	24.57	45.81	43.84	42.03	42.32
1978	24.95	25.05	25.46	25.65	42.44	42.43	41.69	42.21
1979	26.09	26.24	26.51	26.53	42.47	42.84	43.40	42.33
1980	26.36	26.60	26.72	26.64	45.10	43.42	44.07	41.30
1981	27.68	28.00	28.07	27.69	40.85	38.45	36.64	36.40
1982	27.84	27.92	27.35	27.40	35.56	35.04	35.22	34.85
1983	27.29	27.41	26.40	26.31	33.56	35.00	34.90	34.28
1984	26.48	26.34	26.45	26.60	34.79	33.35	31.31	31.20
1985	27.95	26.23	27.24	27.18	33.12	32.13	33.01	33.59
1986	27.07	26.99	27.07	27.40	34.43	35.26	36.23	38.40
1987	26.98	28.64	27.56	27.62	37.98	39.07	37.69	36.77
1988	27.65	27.21	27.14	27.18	35.69	36.12	36.92	36.44
1989	27.90	28.21	28.20	28.20	37.92	36.81	36.19	37.36
1990	28.11	28.07	28.08	28.05	36.90	36.51	38.41	39.00
1991	27.91	27.90	27.92	27.97	37.16	36.74	37.02	37.83
1992	27.58	27.77	27.78	28.07	37.73	37.61	37.61	36.80
1993	27.47	27.99	28.29	28.47	38.38	38.57	37.97	39.26
1994	28.21	28.55	28.26	28.18	37.19	37.51	37.69	37.75
1995	28.34	28.66	28.54	28.63	38.76	38.04	36.94	36.52
1996	28.99	29.28	29.08	29.13	35.99	36.30	35.97	35.15
1997	29.50	29.53	29.63	29.71	34.91	34.98	34.76	34.30
1998	29.68	29.75	29.73	29.79	36.31	35.82	35.57	35.64
1999	29.75	29.81	29.91	29.92	35.74	35.74	36.39	36.94
2000	30.13	30.15	30.10	30.15	37.46	37.55	36.70	38.40
2001	30.49	30.63	27.96	29.73	35.67	34.57	32.53	33.44
2002	27.56	27.17	27.22	27.13	31.01	31.09	32.04	32.23
2003	26.85	26.59	25.42	26.17	33.75	32.83	33.12	34.27
2004	26.19	26.14	26.37	26.50	32.87	33.09	33.83	33.83
2005	27.46	27.56	27.59	27.66	36.77	35.74	35.91	36.22
2006	28.23	28.14	28.15	28.14	36.32	36.77	36.83	36.42
2007	28.91	28.88	28.91	28.82	38.43	37.24	37.77	38.55
2008	28.95	26.74	27.86	27.95	33.62	32.63	32.14	31.99

the total volatility of the return to business capital.

(2) While the capital gain term is a non-trivial source of volatility, a model that gets the stochastic properties of the other series "right" (chiefly output and capital) nonetheless makes an important contribution to the volatility of the return to capital.

4 Calibration and Results

4.1 Calibration

Most of the calibration is standard and the discussion is consequently fairly brief. For a more detailed description of the derivations, see Gomme and Rupert (2007). To start, preferences are restricted to be of the constant relative risk aversion variety:

$$U(c,\ell) = \begin{cases} \ln c + \omega \ln \ell & \text{if } \gamma = 1 \\ \frac{[c\ell^{\omega}]^{1-\gamma}}{1-\gamma} & \gamma \in (0,1) \cup (1,\infty). \end{cases}$$

It is well known that these preferences are consistent with balanced growth. For the baseline calibration, the coefficient of relative risk aversion, γ, is set to 1. The remaining preference parameters, ω, the weight on leisure, and β, the discount factor, are chosen to match two averages. First, the representative household works 25.5% of the time, a figure that matches average time spent working in the market as computed from the American Time Use Survey. Second, the average after-tax return on capital is 5.16% (annual), as it is in the data (see Table 1). This second choice was motivated by the fact that the percentage standard deviation of the return to capital is somewhat sensitive to the mean return, and so it is prudent to match the mean return to capital.

The aggregate production function is Cobb-Douglas:

$$y = k^{\alpha}(zn)^{1-\alpha}.$$

Capital's share, α, is assigned the value 0.283 which corresponds to the average private capital income share of output net of housing income. The depreciation rate, δ, has a value of 0.017745. This figure corresponds to the quarterly depreciation rate when the depreciation rate is computed from the Bureau of Economic Analysis's reported depreciation of private nonresidential capital divided by the corresponding capital stock.

The remaining parameters are those describing the stochastic processes. In the baseline calibration, these processes are: labor-embodied technological change, the relative price of investment goods, and the tax rates on labor and capital income. After testing for various lag lengths and retaining only those parameters that we found to be significant, the stochastic processes estimated were:

$$\ln z_t = \text{constant} + \rho_z \ln z_{t-1} + \text{constant} \times t + \varepsilon_{zt} \tag{16}$$

$$\ln \tau_{nt} = \text{constant} + \rho_{n1} \ln \tau_{n,t-1} + \rho_{n2} \ln \tau_{n,t-2} + \varepsilon_{nt} \tag{17}$$

$$\ln \tau_{kt} = \text{constant} + \rho_k \ln \tau_{k,t-1} + \varepsilon_{kt} \tag{18}$$

$$\ln\left(\frac{q_{t+1}}{q_t}\right) = \text{constant} + \rho_{q1} \ln\left(\frac{q_t}{q_{t-1}}\right) + \rho_{q2} \ln\left(\frac{q_{t-1}}{q_{t-2}}\right) + \varepsilon_{qt}. \tag{19}$$

The parameter are estimated via SUR; the results are summarized in Table 4. The correlation matrix of the errors (ordered as: growth in labor-embodied technology, tax rate on labor income, tax rate on capital income, and the relative price of investment) is:

$$\begin{bmatrix} 1.0000 & & & \\ -0.0633 & 1.0000 & & \\ -0.0498 & 0.3094 & 1.0000 & \\ 0.0728 & -0.0386 & -0.0242 & 1.0000 \end{bmatrix}$$

From the data, the mean tax rates were $\overline{\tau}_k = 0.40387$ and $\overline{\tau}_n = 0.24262$, while the technology processes had means $\mu_z = 1.0034$ and $\mu_q = 0.99695$. The latter two values imply a quarterly growth rate of output of 0.4611% and of capital of 0.7684%.

4.2 Results

The model is solved using a first-order method; see Klein (2000) for details. The balanced growth equations, suitable for solving the model computationally, are summarized in Appendix A. During simulations, the growth is put "back in" where appropriate. For each replication, 288 observations

Table 4: Parameter Estimates and Standard Errors, 1954Q1–2008Q4

Parameter	Estimate	SE
ρ_z	0.9726221	0.0118068
ρ_{n1}	0.7841668	0.0621321
ρ_{n2}	0.204708	0.0617333
ρ_k	0.9724874	0.0147793
ρ_{q1}	0.4593886	0.069097
ρ_{q2}	0.2018417	0.0689491

are generated with the last 188 kept (that is, the same as the number of observations for the U.S. economy from 1954Q1–2008Q4). The model data is logged and Hodrick-Prescott filtered, with the exception of the return to capital which is expressed in percentage deviation terms as $(R_t^k - \overline{R}^k)/\overline{R}^k$, as it is for the U.S. data. The results for 5000 replications are reported in Table 5, along with corresponding moments for the U.S. economy.

The model shares many of the same successes and failures of the standard real business cycle model, and so little time will be spent dwelling on its successes. There are a number of distinct peculiarities:

(1) Overall, there is too much volatility. The percentage standard deviation of output in the model is more than twice that in the data; consumption is roughly 2.4 times too volatile while investment is 4.5 times too volatile.

(2) It is common in the literature to express standard deviations relative to that of output. Even by this metric, investment is too volatile: 5.8 in the model versus 2.7 in the data. The relative volatility of consumption, however, is close to that seen in the data.

It is, perhaps, interesting that these anomalies arise despite the fact that, apart from the price of investment series, the stochastic processes are slightly less volatile than in the data.

Table 6 provides some insights to the dynamics of the benchmark model by considering alternative stochastic processes. More specifically, one or more of the stochastic processes is "turned off," the parameters re-estimated, and the model re-solved and re-simulated. The first shuts down volatility in the relative price of investment, maintaining the stochastic labor-embodied technology

Table 5: Selected Moments

	Standard Deviation	Cross Correlation of Real Output With								
		x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}
U.S., 1954Q1–2008Q4										
Output	1.71	0.12	0.36	0.61	0.84	1.00	0.84	0.61	0.36	0.12
Consumption	0.86	0.19	0.39	0.58	0.74	0.80	0.71	0.56	0.39	0.20
Investment	4.67	−0.18	−0.02	0.21	0.47	0.71	0.81	0.79	0.69	0.51
Hours	1.76	−0.11	0.10	0.34	0.61	0.83	0.88	0.80	0.64	0.44
Productivity	1.01	0.39	0.43	0.44	0.37	0.25	−0.12	−0.37	−0.52	−0.56
Capital	1.27	−0.43	−0.44	−0.41	−0.32	−0.17	0.02	0.21	0.39	0.51
Labor-embodied Tech.	1.63	0.41	0.55	0.66	0.70	0.67	0.33	0.02	−0.24	−0.42
Price of Investment	1.08	−0.07	−0.08	−0.10	−0.10	−0.10	−0.10	−0.08	−0.05	−0.02
Capital tax	3.80	−0.17	−0.12	−0.02	0.04	0.13	0.20	0.24	0.30	0.36
Labor tax	2.89	−0.30	−0.24	−0.10	0.05	0.20	0.30	0.35	0.38	0.35
After-tax return										
Business Capital	55.39	0.13	0.15	0.14	0.17	0.13	0.09	0.08	0.07	0.02
S&P 500	325.36	0.19	0.16	0.09	−0.08	−0.18	−0.22	−0.20	−0.16	−0.08
Benchmark Model										
Output	3.61	0.07	0.20	0.38	0.58	1.00	0.58	0.38	0.20	0.07
Consumption	2.03	−0.09	−0.03	0.06	0.15	0.33	0.39	0.37	0.33	0.27
Investment	21.08	0.11	0.22	0.38	0.53	0.88	0.41	0.22	0.05	−0.07
Hours	4.56	0.11	0.23	0.39	0.56	0.95	0.49	0.29	0.11	−0.02
Productivity	1.56	−0.17	−0.21	−0.26	−0.29	−0.47	−0.09	0.04	0.15	0.21
Capital	2.60	−0.30	−0.24	−0.13	0.03	0.31	0.42	0.48	0.47	0.44
Labor-embodied Tech.	1.37	0.04	0.11	0.20	0.30	0.45	0.31	0.20	0.11	0.04
Capital tax rate	3.71	−0.06	−0.13	−0.22	−0.31	−0.47	−0.32	−0.20	−0.10	−0.02
Labor tax rate	2.72	−0.07	−0.18	−0.32	−0.45	−0.80	−0.47	−0.32	−0.17	−0.06
Price of Investment	1.36	−0.21	−0.22	−0.20	−0.15	−0.06	0.11	0.18	0.21	0.21
Return to capital	59.11	0.06	0.11	0.17	0.26	0.38	0.44	0.19	0.10	0.00

Table 6: Alternative Stochastic Processes

	Standard Deviation	Cross Correlation of Real Output With								
		x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}
Constant capital gain										
Output	3.26	0.08	0.22	0.40	0.58	1.00	0.58	0.40	0.22	0.08
Consumption	1.59	−0.05	0.07	0.23	0.40	0.77	0.61	0.47	0.34	0.23
Investment	15.54	0.14	0.26	0.42	0.57	0.93	0.45	0.28	0.11	−0.02
Hours	4.06	0.12	0.24	0.40	0.55	0.95	0.50	0.33	0.15	0.02
Productivity	1.40	−0.14	−0.18	−0.22	−0.24	−0.43	−0.10	−0.02	0.08	0.13
Capital	1.08	−0.34	−0.25	−0.11	0.07	0.36	0.49	0.57	0.59	0.57
Return to capital	16.67	0.20	0.26	0.33	0.39	0.55	0.32	0.20	0.09	0.00
Constant tax rates										
Output	1.96	0.03	0.17	0.37	0.62	1.00	0.62	0.37	0.17	0.03
Consumption	1.37	−0.09	−0.10	−0.11	−0.12	−0.20	0.11	0.22	0.29	0.30
Investment	15.03	0.07	0.18	0.35	0.55	0.89	0.44	0.19	0.00	−0.11
Hours	1.94	0.07	0.18	0.34	0.53	0.86	0.41	0.16	−0.02	−0.13
Productivity	1.04	−0.08	−0.01	0.07	0.18	0.28	0.40	0.40	0.36	0.30
Capital	2.29	−0.38	−0.33	−0.22	−0.03	0.31	0.44	0.50	0.50	0.46
Return to capital	55.28	0.01	0.07	0.16	0.29	0.45	0.68	0.29	0.16	0.03

shock and tax rates. The second set of results maintains the tax rates at their unconditional means. These results illustrate that stochastic taxes and a stochastic relative price of investment are both responsible for enhancing the volatility of macro aggregates, with stochastic taxes being the prime driver behind the increased volatility of output and hours.[4]

A more traditional business cycle exercise is presented in Table 7 where the only source of fluctuations is the labor-embodied technology shock. In this case, the model does quite well in terms of traditional business cycle moments. The model predicts that output is somewhat less volatile than is observed in the data, a common finding in the business cycle literature. In the model, consumption is roughly half as volatile as output, as it is in the U.S. data. As in much of the business cycle literature, investment volatility relative to that of output is too high relative to the data.

Turning now to the implications for the volatility of returns, the benchmark model performs

[4]In their estimated model, Braun (1994) (using GMM) and McGrattan (1994) (using MLE) find that including stochastic taxes does not increase the volatility of output and hours.

Table 7: Labor-embodied Technology Shocks Only

	Standard Deviation	Cross Correlation of Real Output With								
		x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}
Output	1.39	0.09	0.26	0.46	0.70	1.00	0.70	0.46	0.26	0.09
Consumption	0.65	−0.01	0.16	0.38	0.65	0.98	0.75	0.55	0.38	0.23
Investment	5.62	0.16	0.31	0.50	0.72	0.99	0.66	0.39	0.17	0.01
Hours	0.57	0.18	0.33	0.51	0.73	0.98	0.64	0.36	0.14	−0.02
Productivity	0.84	0.04	0.20	0.41	0.68	0.99	0.74	0.51	0.33	0.17
Capital	0.42	−0.41	−0.31	−0.16	0.06	0.35	0.53	0.63	0.67	0.65
Return to capital	5.81	0.23	0.30	0.38	0.47	0.56	0.37	0.22	0.09	−0.01

well on the volatility of the real, after-tax return to capital. Specifically, the model predicts a percentage standard deviation of 59.11 that is slightly greater than that in the data, 55.39. Relative to output volatility, the model captures about $1/2$ of the variability of the return to capital (16.37 for the model compared to 32.39 in the data).

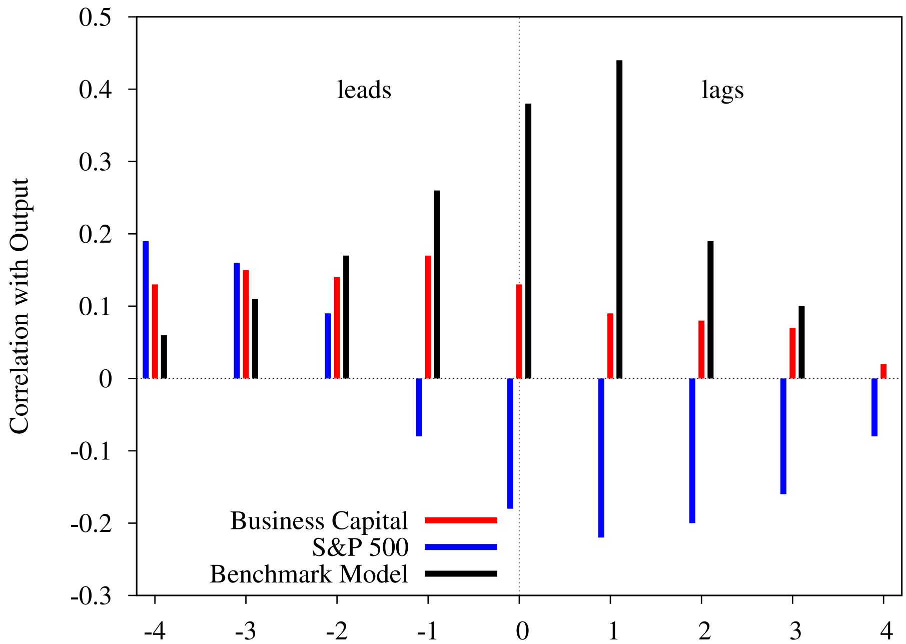
Once more, Tables 6 and 7 provide some insight on the sources of the volatility of the return to capital in the model. When there are only labor-embodied technology shocks (Table 7), the model accounts for 11% of the absolute volatility of the return to capital, or 13% of the volatility relative to that of output. In a sense, this case is loaded against the model since, in the data, volatility in the relative price of investment goods is a large component of the overall volatility in the return to capital. Omitting this term in the model makes it very difficult for the model to replicate the volatility of the return to capital.

Next, in addition to labor-embodied technology shocks, we allow for variation in labor and capital income tax rates (see the top panel of Table 6). Now, the model accounts for 30% of the volatility in the return to capital, although only 16% of the volatility relative to that of output.

Finally, allow for volatility in only the labor-embodied technology and the relative price of investment goods (the bottom panel of Table 6). In this case, the model performs quite well in replicating the volatility of the return to capital: it captures almost 100% of the absolute volatility and 87% of the volatility relative to output.

The model performs quite poorly if the goal were to match the volatility of the equity return

Figure 3: Lead-lag Pattern



as measured by the S&P 500. At best, the model captures 18% of the overall volatility of the real equity return (for the benchmark model), or 15% of the variability measured relative to output (when labor-embodied technology shocks and relative price of investment shocks are in play; bottom panel of Table 6).

The lead-lag pattern of returns tends to receive less attention than the mean return and its volatility. Figure 3 shows that the benchmark model successfully captures the observed procyclical behavior of the return to business capital, although the model return is somewhat too strongly procyclical. The model return lags the cycle by a quarter whereas the return to business capital leads the cycle by a quarter. In contrast, S&P 500 return is countercyclical and lags the cycle by a quarter.

5 A Two-Sector Model with Frictions

Arguably the most successful paper addressing both business cycle facts and asset returns is Boldrin et al. (2001). The purpose of this section is to re-examine the results of their paper in light of the equivalence between the return to capital and the equity return. Presentation of their model is, consequently, brief and the notation follows theirs slavishly.

Their model has habit-persistent preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t - bC_{t-1}) - H_t\right]$$

where b is the degree of habit, and $H_t = H_{c,t} + H_{i,t}$ is total hours worked.

Their most successful model variant is one with two sectors:

$$C_t \leq K_{c,t}^{\alpha}(Z_t H_{c,t})^{1-\alpha} \tag{20}$$

$$K_{c,t+1} + K_{i,t+1} - (1-\delta)(K_{c,t} + K_{i,t}) \leq K_{i,t}^{\alpha}(Z_t H_{i,t})^{1-\alpha}. \tag{21}$$

An important set of frictions is that each of the capital stocks *and* the allocation of labor are predetermined (meaning that period t values are set at time $t-1$). Labor-embodied technology evolves according to

$$Z_t = \exp(x_t) Z_{t-1}, \quad x_t \sim N(\bar{x}, \sigma^2).$$

The labor-embodied technology shock is the only exogenous forcing process in their model.

The two-sector model and the frictions imply that the relative price of capital goods measured in units of consumption goods is not one and is not constant (similar to Section 2.3). In their notation, this price at time t is denoted by $P_{k',t}$ (see page 152, section C). This relative price is endogenously determined within their model.

The real rate of return to a unit of capital allocated to the consumption goods sector can be calculated as follows. Each unit of capital costs $P_{k',t}$ units of consumption good in period t. It yields $\alpha \left[\frac{Z_{t+1} H_{c,t+1}}{K_{c,t+1}}\right]^{1-\alpha}$ units of consumption good in period $t+1$; the left-over capital stock is worth $(1-\delta)P_{k',t+1}$ units of consumption good in period $t+1$. The net rate of return to capital in the consumption goods sector then is

$$r_{c,t+1} = \frac{\alpha \left[\frac{Z_{t+1} H_{c,t+1}}{K_{c,t+1}}\right]^{1-\alpha} + (1-\delta) P_{k',t+1}}{P_{k',t}} - 1. \tag{22}$$

Similarly, the real rate of return to a unit of capital allocated to the investment goods sector can be calculated as follows. Each unit of capital yields $\alpha \left[\frac{Z_{t+1} H_{i,t+1}}{K_{i,t+1}}\right]^{1-\alpha}$ units of *investment good* in period $t+1$, which is worth $P_{k',t+1}\alpha\left[\frac{Z_{t+1}H_{i,t+1}}{K_{i,t+1}}\right]^{1-\alpha}$ units of consumption good in period $t+1$; the left-over capital stock is worth $(1-\delta)P_{k',t+1}$ units of consumption good in period $t+1$. The net rate of return to capital in the investment goods sector then is

$$r_{i,t+1} = \frac{P_{k',t+1}\alpha\left[\frac{Z_{t+1}H_{c,t+1}}{K_{c,t+1}}\right]^{1-\alpha} + (1-\delta)P_{k',t+1}}{P_{k',t}} - 1. \tag{23}$$

The net rate of return to an aggregate unit of capital then is a weighted average of Eq. (22) and Eq. (23). Boldrin et al. use the fraction of capital allocated to each sector as the weight for the sector:

$$r_{t+1} = \frac{K_{c,t+1}}{K_{c,t+1}+K_{i,t+1}} r_{c,t+1} + \frac{K_{i,t+1}}{K_{c,t+1}+K_{i,t+1}} r_{i,t+1}. \tag{24}$$

All returns are computed pre-tax.

Similar to the return to capital in the neoclassical decentralization in Section 2.3, the return to capital in Eq. (24) is derived without any reference to an equity market. Boldrin et al. implicitly invoke the equivalence result by referring to the left hand side of Eq. (24)) as the return to equity and comparing it with the S&P 500 return.

Results for their model are presented in Table 8. While they solve their model using the parameterized expectations approach, here the model is solved by a second-order approximation method; see Gomme and Klein (2009). The second-order method is much easier to implement; the only downside is that it does not do a very good job approximating the mean risk-free return and the mean equity return for their model.[5] Fortunately, here the focus is on return volatility, not the means. The parameter values are as in Boldrin et al. (2001): $\beta = 0.99999$, $\alpha = 0.36$, $\delta = 0.021$, $\bar{x} = 0.0040$, $\sigma = 0.018$ and $b = 0.73$.

A comparison of our Table 8 with Table 2 of their paper shows that the second-order method does quite well in capturing the volatility and correlation pattern of the non-financial variables. The second-order method also does reasonably well in replicating their reported volatility of the return on equity (compare the line labeled “raw” in Table 8 with the corresponding number in their Table 1).

[5] Boldrin et al. (2001) report a mean risk-free rate for their model of 1.20; the second-order method delivers 3.05. Their model delivers an average equity return of 7.84 whereas the second-order method gives 7.30.

Table 8: Boldrin et al. (2001) Model: Selected Moments

	Standard Deviation	Cross Correlation of Real Output With								
		x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}
Output	2.03	0.12	0.30	0.53	0.82	1.00	0.82	0.53	0.30	0.12
Consumption	1.34	0.14	0.32	0.55	0.84	0.95	0.61	0.35	0.16	0.02
Investment	3.47	0.10	0.27	0.48	0.75	0.98	0.91	0.62	0.38	0.18
Hours	1.04	0.10	0.24	0.41	0.62	0.86	0.94	0.59	0.31	0.10
Capital	0.34	−0.49	−0.41	−0.28	−0.08	0.18	0.41	0.55	0.64	0.66
Relative price of investment	11.56	0.19	0.24	0.30	0.36	0.16	−0.39	−0.32	−0.26	−0.21
Return on equity:										
· Percentage Deviation	244.04	0.09	0.10	0.12	0.13	−0.04	−0.43	−0.43	−0.43	−0.26
· Raw	17.70	0.09	0.10	0.12	0.13	−0.04	−0.43	−0.43	−0.43	−0.26

Going now beyond their results, Table 8 also reports the volatility of the return on equity. Relative to the data, their model seems to do quite well, capturing 75% of the overall volatility in the S&P 500 return, or about 63% when volatility is measured relative to that of output.

Recall, though, that the central message of our paper is that the model's return can be lined up with either the stock market return (under one decentralization), or the return to capital (under an alternative decentralization), just as in Boldrin et al. (2001). If the goal is to account for the volatility of the return to capital as measured above, then the volatility of the return in Boldrin et al. (2001) is 4.4 times too large (or 3.7 times too large when measured relative to the standard deviation of output).

This section closes with some further comments on the work of Boldrin et al. (2001).

(1) While they line up the price of investment goods in their model with stock market prices, it seems just as natural to line it up with the NIPA relative price (recall that the return to capital in Eq. (24) is valid even in the absence of a stock market).

Table 8 reports that the logged and Hodrick-Prescott filtered relative price in their model has a standard deviation of 11.56; in the data, the relative price has a standard deviation (similarly logged and Hodrick-Prescott filtered) of 1.08. In other words, they require that the relative price of investment goods vary roughly 10 times more than that in the data.

(2) For their model, the annualized return is computed as the sum of the preceding four quarters.[6] More commonly, quarterly returns are annualized by using exponents (that is $R^a = [(1+R^q)^4 - 1] \times 100\%$ where R^q is the quarterly return and R^a the annualized return), or as $R^a = R^q \times 400\%$. Annualizing the return from their model using exponents gives a raw standard deviation of 69.12, and a percentage deviation of 963.39 – nearly three times larger than the observed value of 325.36 for the S&P 500 return.

(3) A final issue relates to the measurement of output. They measure output in their model as the simple sum of consumption and investment: $y = c + i$. Alternatively, one could measure output as $y = c + q\,i$ where $q = P_{k'}$, the relative price that decentralizes their planner's problem. This is the same relative price that appears in the calculations of the sectoral returns, Eqs. (22) and (23).[7] When output is measured with the model-determined relative price, the standard deviation of investment rises from 3.47 to 11.90, and of output from 2.03 to 4.95; in other words, their model's business cycle statistics look much closer to those generated by our benchmark model when the endogenous relative price of investment is used.

6 Conclusions

The key point of this paper is that the return to capital/equity in the neoclassical growth model corresponds to both the stock market return and the return to business capital. In the data, the volatility of the return to business capital is an order of magnitude smaller than that of the stock market return. To date, the literature has focused almost exclusively on the stock market return; prime examples include Rouwenhorst (1995), Jermann (1998) and Boldrin et al. (2001). However, to the degree that these papers successfully account for time series properties of the stock market return, these papers *must* fail miserably on the time series properties of the return to business capital.

[6] Source: Jonas Fisher, private correspondence.

[7] Boldrin et al.'s justification for setting $q = 1$ is that this is how real output was measured prior to chain-weighting. Specifically, NIPA used base period prices which precluded changes in relative prices. Source: Jonas Fisher, private correspondence.

The neoclassical growth model analyzed above – with stochastic labor-embodied technological change, relative price of investment goods, and labor and capital income tax rates – exaggerates slightly the volatility of the return to capital in absolute terms; alternatively, it captures roughly $1/2$ of the volatility measured relative to that of output. The volatility of major macro aggregates like output, consumption, investment and hours worked are considerably larger than that seen in the U.S. data. A version of the model with non-stochastic taxes has a better fit with the usual business cycle moments (apart from that of investment) and captures almost all of the volatility in the return to capital in absolute terms, or 87% of its volatility, measured relative to output.

Just as Boldrin et al. (2001) are largely successful at matching the volatility of the stock market return at the cost of grossly exaggerating the variability of the return to capital, the benchmark neoclassical growth model is largely successful in matching the volatility of the return to capital at the cost of grossly understating the variability of the stock market return. Clearly, what is needed are new models that break the equivalence between the stock market return and the return to capital. In brief, what is needed is a (new) theory of the stock market to add to the neoclassical growth model.

A Balanced Growth Transformations

Recall that the relevant Euler equations and constraints are:

$$(1-\tau_{nt})z_tF_2(k_t,z_tn_t)U_1(c_t,1-n_t)=U_2(c_t,1-n_t) \tag{25}$$

$$\begin{aligned} q_tU_1(c_t,1-n_t)=\beta E_t\Big\{&U_1(c_{t+1},1-n_{t+1}) \\ &\times\left[(1-\tau_{k,t+1})\left(F_1(k_{t+1},z_{t+1}n_{t+1})-\delta q_{t+1}\right)+q_{t+1}\right]\Big\} \end{aligned} \tag{26}$$

$$c_t+q_ti_t=y_t=F(k_t,z_tn_t) \tag{27}$$

$$k_{t+1}=(1-\delta)k_t+i_t \tag{28}$$

Assumptions:

1. Relative price of investment goods: the growth rate of the relative price of investment goods,

q_t/q_{t-1} is stationary. Along the balanced growth path, $q_t/q_{t-1} = \mu_q$.

2. Market production is Cobb-Douglas: $F(k_t, z_t n_t) = k_t^{\alpha}(z_t n_t)^{1-\alpha}$. Assume that $z_t = \mu_z^t \tilde{z}_t$ where $\tilde{z}_t$ is stationary and μ_z is the growth rate of labor-embodied technological change.

3. Utility function: $U(c,\ell)$ is homogeneous of degree $(1-\gamma)$ in c. Consequently, $U_1(c,\ell)$ is homogeneous of degree $-\gamma$ in c while $U_2(c,\ell)$ is homogeneous of degree $(1-\gamma)$ in c.

For computational purposes, it is necessary to deal with stationary variables. Let $\tilde{}$ denote variables rendered stationary by dividing by $\mu_z q_t^{\frac{\alpha}{\alpha-1}}$ and $\hat{}$ variables made stationary by dividing by $\mu_z q_t^{\frac{1}{\alpha-1}}$. At this juncture, it is helpful to note that $\tilde{y}_t$ and $\tilde{c}_t$ are stationary, as are $\hat{k}_t$ and $\hat{\imath}_t$.

Eqs. (25)–(28) can, then, be rewritten as:

$$(1-\tau_{nt})(1-\alpha)\tilde{z}_t\left(\frac{\hat{k}_t}{\tilde{z}_t n_t}\right)^{\alpha} U_1(\tilde{c}_t, 1-n_t) = U_2(\tilde{c}_t, 1-n_t) \tag{29}$$

$$\begin{aligned} U_1(\tilde{c}_t, 1-n_t) = \beta E_t \Bigg\{ & \frac{q_{t+1}}{q_t}\left[\mu_z\left(\frac{q_{t+1}}{q_t}\right)^{\frac{\alpha}{1-\alpha}}\right]^{-\gamma} U_1(\tilde{c}_{t+1}, 1-n_{t+1}) \\ & \times\left[(1-\tau_{k,t+1})\alpha\left(\left(\frac{\tilde{z}_t n_{t+1}}{\hat{k}_{t+1}}\right)^{1-\alpha} - \delta\right) + 1\right]\Bigg\} \end{aligned} \tag{30}$$

$$\tilde{c}_t + \hat{\imath}_t = \tilde{y}_t = \hat{k}_t^{\alpha}(\tilde{z}_t n_t)^{1-\alpha} \tag{31}$$

$$\mu_z\left(\frac{q_{t+1}}{q_t}\right)^{\frac{1}{\alpha-1}}\hat{k}_{t+1} = (1-\delta)\hat{k}_t + \hat{\imath}_t \tag{32}$$

References

Boldrin, Michele, Christiano, Lawrence J., and Fisher, Jonas D.M. "Habit Persistence, Asset Returns, and the Business Cycle." *American Economic Review* 91 (March 2001): 149–166.

Braun, R. Anton. "Tax Disturbances and Real Economic Activity in the Postwar United States." *Journal of Monetary Economics* 33 (1994): 441–462.

Carey, David, and Tchilinguirian, Harry. "Average Effective Tax Rates on Capital, Labour and Consumption." Working Papers Series 258, OECD (2000).

Cooley, Thomas, and Prescott, Edward C. "Economic Growth and Business Cycles." In Thomas Cooley, editor, "Frontiers of Business Cycle Research," Princeton, N.J.: Princeton University Press (1995): 1–38.

Gomme, Paul, and Klein, Paul. "Second-order Approximation of Dynamic Models without the use of Tensors." Working Paper 09-004, Concordia University (2009).

Gomme, Paul, and Rupert, Peter. "Theory, Measurement, and Calibration of Macroeconomic Models." *Journal of Monetary Economics* 54 (March 2007): 460–497.

Greenwood, Jeremy, Hercowitz, Zvi, and Krusell, Per. "Macroeconomic Implications of Capital-embodied Technological Change." *American Economic Review* 87 (June 1997): 342–362.

Jermann, Urban J. "Asset Pricing in Production Economies." *Journal of Monetary Economics* 41 (April 1998): 257–275.

Klein, Paul. "Using the Generalized Schur Form to Solve a Multivariate Linear Rational Expectations Model." *Journal of Economic Dynamics and Control* 24 (September 2000): 1405–1423.

McGrattan, Ellen R. "The Macroeconomic Effects of Distortionary Taxation." *Journal of Monetary Economics* 33 (1994): 573–601.

McGrattan, Ellen R., and Prescott, Edward C. "Average Debt and Equity Returns: Puzzling." *American Economic Review* 93 (May 2003): 392–397.

Mendoza, Enrique G., Razin, Assaf, and Tesar, Linda L. "Effective Tax Rates in Macroeconomics: Cross-Country Estimates of Tax Rates on Factor Incomes and Consumption." *Journal of Monetary Economics* 34 (December 1994): 297–323.

Mulligan, Casey B. "Capital, Interest, and Aggregate Intertemporal Substitution." Working Paper 9373, National Bureau of Economic Research (2002).

Poterba, James M. "Rate of Return to Corporate Capital and Factor Shares: New Estimates Using Revised National Income Accounts and Capital Stock Data." *Carnegie-Rochester Conference Series on Public Policy* 48 (June 1998): 211–246.

Rouwenhorst, K. Geert. "Asset Pricing Implications of Equilibrium Business Cycle Models." In Thomas Cooley, editor, "Frontiers of Business Cycle Research," Princeton, N.J.: Princeton University Press (1995): 294–330.