Department of Economics

Working Paper Series

The Return to Capital and the Business Cycle

08-002

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The Return to Capital and the Business Cycle*

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April 7, 2008

Abstract

We measure the return to capital directly from the NIPA and BEA data and examine the return implications of the real business cycle model. Specifically, we construct a quarterly time series of the after-tax return to business capital. The business cycle properties of this return differs considerably from those of the S&P 500 returns. First, its volatility is considerably smaller than that of S&P 500 returns. Second, our measured return is procyclical and leads output by one quarter; S&P 500 returns are *countercyclical* and lead the cycle by four quarters. The standard business cycle model captures almost 50% of the volatility in the return to capital (relative to the volatility of output), and does well in capturing the lead-lag pattern. We consider several departures from the benchmark model; the model with stochastic taxes captures nearly 85% of the relative volatility in the return to capital and the model with high risk aversion captures 80% of the relative volatility. We then include capital gains in our measurement and use a model with investment specific technological change to address the higher volatility in the return to capital. This model accounts for more than 80% of the return volatility, and essentially all of the relative volatility.

*For helpful comments we thank David Andolfatto, Michele Boldrin, Gian Luca Clementi, Aubhik Khan, Ellen McGrattan, Richard Rogerson, Jian Wang, and seminar participants at University of Wisconsin, University of North Carolina–Greensboro, Seoul National University, University of Guelph, Colgate University, the Federal Reserve Banks of Cleveland, Minneapolis and Philadelphia, and the conference participants at 2006 Midwest Macro, 2006 SED, 2006 CMSG and 2007 Texas Monetary conference. Gomme acknowledges financial support of SSHRC.

1 Introduction

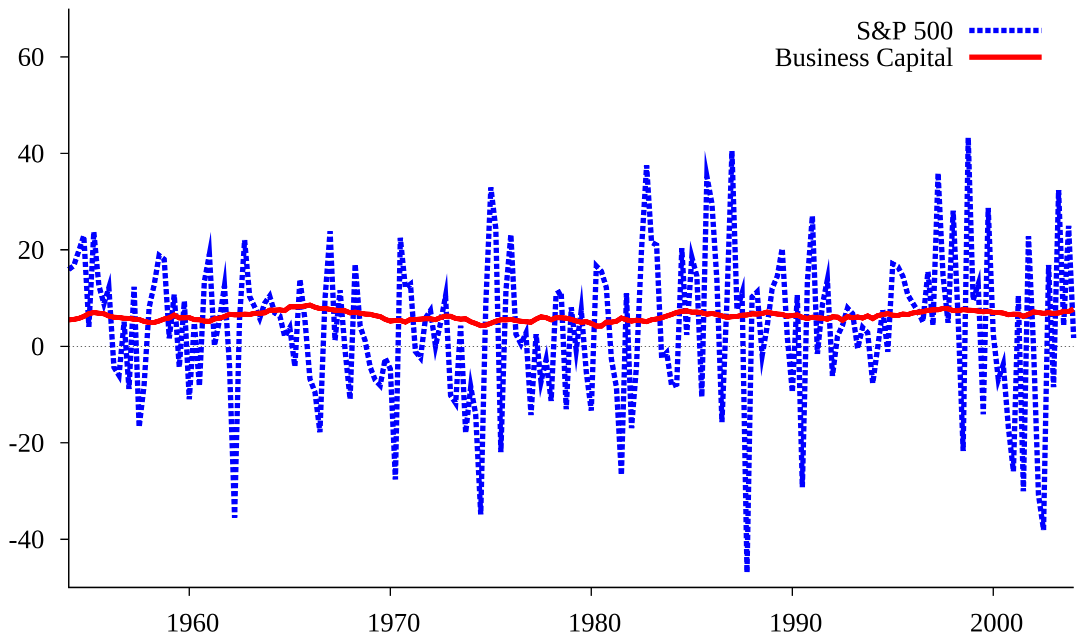
There has been considerable progress in accounting for business cycle fluctuations in aggregate quantities. Using the real business cycle (RBC) framework developed by Kydland and Prescott (1982), many studies have replicated the observed comovements and volatilities of aggregate variables such as output, consumption, investment and hours. In the basic RBC model, changes in total factor productivity alter the real rate of return on a representative unit of capital which, in turn, affects the temporal profiles of consumption, investment, hours, etc. We examine whether the successes in accounting for the aggregate quantities are achieved at the cost of being unable to replicate the time series properties of the return on capital.

In the standard RBC model, one can think of the representative firm as an entity that maximizes the present value of dividends (defined as output minus investment and factor payments to labor). The rate of return to a financial claim to the sequence of the firm's dividends is the same as the rate of return to capital in the RBC model. With this equivalence, Rouwenhorst (1995) used the S&P 500 returns to measure the return on capital. An alternative approach is to follow Poterba (1998), Mulligan (2002) and McGrattan and Prescott (2003) and construct the return by summing all of the relevant income generated by capital and dividing by the stock of capital that generated the income. We follow the latter approach and construct a *quarterly* time series for the after-tax return to business capital. Using our measurement, we reexamine the return implications of the standard RBC model. Specifically, we examine whether the RBC model can account for the business cycle properties of the after-tax return to capital measured by the flow of income accruing to owners of capital.

Our measure of business capital is the sum of private nonresidential structures, private nonresidential equipment and software, and private inventories. Our calculations for the return to capital, described in section 3, take into account all taxes paid by the owners of *all* business capital over the period 1954:1–2003:4. A number of authors have made conceptually similar calculations for specific sectors and for specific types of capital. Poterba (1998) computes annual returns for the *nonfinancial corporate* sector; Mulligan (2002) calculates the annual return to capital including residential structures; McGrattan and Prescott (2003) compute annual after-tax returns for the *non-corporate* sector. All of these previous studies computed *annual* returns; we compute *quarterly* returns since that is the frequency typically used in the RBC literature.

There are three findings of note. First, the return to capital is very smooth relative to the S&P 500 returns; see Figure 1. The *percent standard deviation* of the S&P 500 quarterly returns over the 1954:1–2003:4 sample period is 360.51% while the volatility of our constructed return to capital is only about 13.99%.[1]

Figure 1: After-tax return to the S&P 500 and Capital



The difference in the properties of the returns can be traced back to the difference in the two approaches to measurement noted earlier. The return to the S&P 500 is measured as $\frac{p_{t+1}+d_{t+1}}{p_t} - 1$ where p_s denotes the price of equity and d_s denotes the dividend in period s. It is well known that the volatility in the S&P 500 return is largely due to the volatility in equity prices. Our measurement, however, includes only the flow income per unit of capital, not the variations in

[1]These figures are in the spirit of "deviations from trend" calculations of other business cycle variables. That is, if $\overline{R}$ is the mean after-tax return in the sample and $\widehat{R}_t = \frac{R_t - \overline{R}}{\overline{R}}$ is the deviation at time t from the mean, then the percent standard deviation of the return we report is 100 times the standard deviation of $\widehat{R}_t$. The corresponding figures for "raw" standard deviations (i.e., std(R_t)) are 14.13 and 0.89.

the price of capital. Since the price of capital is assumed to be constant in the basic one-sector RBC model, both theory and our measurement ignore capital gains and take into account only the fluctuations in the flow income generated by capital.

The second finding is that the basic RBC model with logarithmic preferences accounts for about 35% of the volatility in the return to capital. Relative to output volatility, the model accounts for nearly 50% of the volatility in the return to capital. To contrast, Rouwenhorst (1995) showed that the intertemporal marginal rate of substitution (IMRS) or the stochastic discount factor in the basic RBC model is too smooth to account for the volatility in the return to equity. It is not a surprise, given Figure 1, that the basic RBC model fares better under our measure of the return to capital. However, the magnitude – whether the model accounts for little or most of the volatility in our measure of the return to capital – is not obvious.

Third, the basic RBC model does well in capturing the lead-lag pattern of the return to capital, although the model predicts that the return is coincident with the cycle while the data leads by one quarter. The model does decidedly less well in accounting for the lead-lag pattern of S&P 500 returns which are *countercyclical* and lead the cycle by four quarters.

We study a few well known variations of the basic RBC model to examine whether they perform better. A model with indivisible labor generates roughly the same volatility relative to output as the basic RBC model, whereas a model with home production generates only 30% of the relative volatility. Instead of logarithmic preferences, a risk aversion of 5 accounts for 80% of the relative volatility. We also consider an environment with stochastic taxes. This variant, with logarithmic preferences, accounts for almost 85% of the relative volatility. The lead-lag patterns are qualitatively similar to the basic RBC model.

Finally, we use the NIPA data to include capital gains in our measurement of return to capital. To account for the movements in the relative price of capital, we depart from our earlier basic one-sector growth model and use a model with investment specific technological change as in Greenwood, Hercowitz, and Krusell (1997). This model accounts for more than 80% of the volatility in the after-tax return to capital, and all of the relative volatility.

To summarize, the basic RBC model accounts for a large part of volatility in the flow income component of the return to capital, as well as its lead-lag pattern. If variations in the relative price of capital, as documented in the NIPA, are included in the measurement then the model with investment specific technological change accounts for a large part of the volatility in the return to capital. However, under the assumption that *stock* prices reflect the price of the income flow to capital owners, the basic RBC model cannot account for the volatility in the return to equity, nor for its lead-lag pattern.

The rest of the paper is organized as follows. In the next section we set up the economic environment. Our model is essentially the same as the basic RBC model in Prescott (1986). In section 3, we describe our measurement of tax rates and return to capital. In section 4, we study the quantitative implications of the model. Section 5 concludes.

2 Economic Environment

Since the economic environment should be easily recognizable to those familiar with the macroeconomics literature of the past two decades, the model's description is fairly brief. The competitive equilibrium for this model is standard.

2.1 Firms

Taking as given the real wage rate, w_t and the rental rate for capital, r_t, the typical firm rents capital, k_t, and hires labor, h_t, to maximize profits,

$$y_t - w_t h_t - r_t k_t.$$

Output is produced according to a constant-returns-to-scale, Cobb-Douglas production function,

$$y_t = z_t k_t^{\alpha} \left(g^t h_t\right)^{1-\alpha}$$

where g is the growth rate of labor-augmenting technological change, and z_t is a random shock to production that follows the stochastic process,

$$\ln z_t = \rho \ln z_{t-1} + \varepsilon_t$$

where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$.

The firm's output can be converted into either consumption, c_t, or investment goods, i_t:

$$c_t + i_t = y_t.$$

2.2 Households

The representative household has preferences over streams of consumption, c_t, and leisure, ℓ_t, summarized by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, \ell_t). \tag{1}$$

The period utility function has the functional form,

$$U(c, \ell) = \begin{cases} \frac{[c\ell^\omega]^{1-\gamma}}{1-\gamma} & \text{if } 0 < \gamma < 1 \text{ or } \gamma > 1, \\ \ln c + \omega \ln \ell & \text{if } \gamma = 1. \end{cases}$$

The household allocates its one unit of time between leisure, ℓ_t, and work, h_t:

$$\ell_t + h_t = 1. \tag{2}$$

The household faces a budget constraint,

$$c_t + i_t = (1 - \tau_\ell) w_t h_t + (1 - \tau_k) r_t k_t + \tau_k \delta k_t + T_t, \tag{3}$$

where τ_ℓ is the tax rate on labor income, τ_k is the tax rate on gross capital income, and T_t is a lump-sum transfer received from the government. $\tau_k \delta k_t$ is a capital depreciation allowance term. The household's capital stock evolves according to

$$k_{t+1} = (1 - \delta) k_t + i_t \tag{4}$$

where δ is the depreciation rate of capital.

The household's problem is to choose contingent sequences for consumption, c_t, leisure, ℓ_t, work, h_t, investment, i_t, and capital, k_{t+1}, so as to maximize lifetime utility, Eq. (1), subject to the constraints, Eqs. (2)–(4), taking as given the wage rate, w_t, rental rate, r_t, taxes, τ_ℓ and τ_k and transfers, T_t.

2.3 Government

The government levies time-invariant taxes on capital income, τ_k, and on labor income, τ_ℓ. It also makes a lump-sum rebate to households, T_t. Government does not directly consume resources; the government sector is included because capital income taxes distort the return to capital, and the focus of this paper is on the after-tax return on capital. The government's budget constraint, then, is

$$T_t = \tau_k r_t k_t - \tau_k \delta k_t + \tau_\ell w_t h_t.$$

2.4 The Return to Capital

Factor market competition and firm profit maximization imply that the rental price of capital satisfies

$$r_t = \alpha z_t k_t^{\alpha-1} \left(g^t h_t\right)^{1-\alpha}$$

The net after-tax *return* to capital, then, is given by

$$R_t = (1-\tau_k)\left[\alpha z_t k_t^{\alpha-1} \left(g^t h_t\right)^{1-\alpha} - \delta\right].$$

In other words, the after-tax return to capital is given by the after-tax marginal product of capital less the depreciation rate.

3 Measurement

In this section we describe the empirical counterparts to our theory in the previous section. As part of this description, we construct a time series for the rate of return to capital. The sample period for the returns data is 1954:1–2003:4.

Construction of the empirical counterparts to the model's variables follows standard procedures in the literature such as those in Cooley and Prescott (1995) and Gomme and Rupert (2007).

The National Income and Product Accounts (NIPA) are the source for much of the derivations. Variables are converted to per capita values using the civilian noninstitutionalized population aged 16 and over. Nominal variables are converted to real ones using a deflator for consumption (non-durables and services), which was constructed from nominal and real consumption so as to conform to our measure of market consumption; on this point, see Greenwood et al. (1997).

In the U.S. economy, the real after-tax rate of return on a representative unit of business capital can be calculated by summing all of the income generated by business capital, subtracting the relevant taxes, and dividing by the stock of capital that generated the income. The income and tax data are found in the NIPA, while the capital stock data is obtained from the Bureau of Economic Analysis (BEA).

There are several issues complicating such a calculation, however. We are interested in obtaining cyclical properties of the return at a quarterly frequency. Unfortunately not all of the necessary data are available quarterly. After presenting the calculations, we will describe the data that is not available at a quarterly frequency, then explain our imputation procedure to construct a quarterly series.

Since we are interested in the return generated from business capital, we must include the income earned from both the corporate and noncorporate sectors. One concern is the income accruing to proprietors. Evidently, this income is partly generated from capital and partly from labor. The generally accepted practice is to allocate proprietors' income to capital and labor in the same proportion as calculated for the economy as a whole; see, for example, Cooley and Prescott (1995) and Gomme and Rupert (2007). That is, if labor's share of national income is $1-\alpha$ and capital's share is α, we attribute the fraction $1-\alpha$ of proprietor's income to labor and the fraction α to capital.

We remove income associated with the housing sector because we are interested in the return to business capital. Our measure of the capital stock will, then, include only those parts that are used in producing market output, and so will exclude residential structures and consumer durables.

While most of the taxes levied against capital income can be obtained fairly directly from the data, those paid by households must be imputed. To obtain the tax rate on general *household income*, we follow the basic methodology of Mendoza, Razin, and Tesar (1994) and Carey and Tchilinguirian (2000). This tax rate, τ_h, is computed as:

$$\tau_h = \frac{\text{Personal Current Taxes}}{\begin{array}{c}\text{Net}\\ \text{Interest}\end{array} + \begin{array}{c}\text{Proprietors'}\\ \text{Income}\end{array} + \begin{array}{c}\text{Rental}\\ \text{Income}\end{array} + \begin{array}{c}\text{Wages and}\\ \text{Salaries}\end{array}}.$$

The tax rate τ_h – distinct from τ_ℓ and τ_k – is an intermediate input into subsequent calculations of the rate of return to capital.

After-tax capital income can be written as:

$$\begin{aligned}
Y_{AT} = {} & \text{Net Operating Surplus} - \text{Housing Net Operating Surplus} \\
& - (1-\alpha)(\text{Proprietor's Income} - \text{Housing Proprietor's Income}) \\
& - \tau_h(\text{Net Interest} - \text{Housing Net Interest}) \\
& - \alpha\tau_h(\text{Proprietor's Income} - \text{Housing Proprietor's Income}) \\
& - \tau_h(\text{Rental Income} - \text{Housing Rental Income}) \\
& - \text{Taxes on Corporate Income} \\
& - \text{Business Property Taxes} \\
& - \text{State and Local Other Taxes}.
\end{aligned}$$

Net operating surplus is defined as value added minus depreciation and payments to labor. As discussed above, the income flows and tax rates have been modified to subtract out the income generated from the housing sector.

Dividing after-tax capital income, Y_{AT}, by the stock of business capital (inventories, market structures and equipment & software) gives the return to capital. After-tax capital income and the stock of inventories are converted to real terms by dividing by the price deflator for personal consumption expenditures while market structures and equipment & software are expressed in real terms (see the quarterly conversion procedure in the next subsection). Thus, the real return can be determined by

$$R_{AT} = \frac{Y_{AT}}{\text{Inventories} + \text{Structures} + \text{Equipment and Software}}.$$

3.1 Annual to Quarterly Conversions

Several series are not available quarterly. Different methods are used to convert the annual series to quarterly. To start, the series State and Local Other Taxes covers such things as licensing fees. It seems reasonable, then, to divide this figure equally across the four quarters. Property taxes (paid by businesses and households) are available quarterly from 1958:1. Prior to this date, the annual observation is repeated for each quarter. Property taxes are not reported separately for businesses and households. It is assumed that the fraction of property taxes paid for by businesses is the same as the fraction of structures owned by businesses.

Quarterly values for all of the housing flows are imputed with the exception of Gross Housing Value Added (GHVA), which is available quarterly. To understand the approach taken here, we will explain the calculation for Net Operating Surplus as an example. Take the observation for GHVA (quarterly), multiply by Net Operating Surplus (annual) divided by GHVA (annual), for the relevant year. That is, apportion the quarterly GHVA to its constituent components using the annual ratios for the appropriate year. This strategy is also used to impute Net Interest, Proprietors' Income and Rental Income for the housing sector.

Quarterly capital stocks are constructed from annual capital stocks and quarterly investment flows (both of which are converted to real by dividing by the consumption deflator for nondurables and services). This procedure requires solving for the depreciation rate that makes the annual capital stocks line up with Q4 of our quarterly capital stock, and be consistent with the quarterly investment flows. For example:

$$
\begin{aligned}
K_{1959Q4} &= K_{1959} \text{ (the annual observation)}\\
K_{1960Q1} &= (1-\delta_{1960})K_{1959Q4} + I_{1960Q1}\\
K_{1960Q2} &= (1-\delta_{1960})K_{1960Q1} + I_{1960Q2}\\
K_{1960Q3} &= (1-\delta_{1960})K_{1960Q2} + I_{1960Q3}\\
K_{1960Q4} &= (1-\delta_{1960})K_{1960Q3} + I_{1960Q4}\\
K_{1960Q4} &= K_{1960} \text{ (the annual observation).}
\end{aligned}
$$

In effect, there are 4 equations (the middle 4) in 4 unknowns: $K_{1960Q1}, K_{1960Q2}, K_{1960Q3}$ and δ_{1960}.

3.2 The Real Return to Capital

The standard deviation of the rate of return to capital is 13.99% over the period 1954:1–2003:4 (see Table 1). As documented in this table (and visually in Figure 1) the rate of return to capital is very smooth relative to the S&P 500 return–the latter is 25 times as volatile.

Table 1: After-tax Returns Data: Selected Moments

	Mean (%)	% Standard Deviation
Business capital	6.29	13.99
S&P 500	4.13	360.51

The quarterly time series for the tax rate on household income, τ_h and the real after-tax return to capital are shown in Table 2. The mean after-tax return to capital, 6.29%, is on the high side of other estimates found in the literature; see, for example, Poterba (1998), Mulligan (2002) and McGrattan and Prescott (2003). Poterba (1998) used data from 1959 to 1996 for the nonfinancial corporate sector and found a mean after-tax return of 3.9%. Mulligan (2002) excludes inventories but includes residential structures and finds the mean after-tax return on capital to be roughly 6%. McGrattan and Prescott (2003) used data from 1880 to 2002 for the noncorporate sector and found a mean after-tax return of 4%. As we report later (in section 4.4), inclusion or exclusion of specific sectors affects the return properties.

4 Quantitative Implications

4.1 Parameters

As has become standard in much of macroeconomics, the calibration procedure involves choosing functional forms for the utility and production functions, and assigning values to the parameters of the model based on either micro-evidence or long run growth facts. Cooley and Prescott (1995)

Table 2: U.S. Return to Capital and Tax Rate on Household Income

	Return to Capital				Tax Rate, τ_h			
Year	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
1954	5.47	5.59	5.78	6.20	11.74	11.69	11.64	11.64
1955	6.79	7.00	6.85	6.75	11.74	11.80	11.95	12.03
1956	6.27	6.02	5.97	5.81	12.26	12.33	12.41	12.47
1957	5.77	5.62	5.55	5.18	12.56	12.59	12.52	12.46
1958	4.95	4.95	5.25	5.68	12.26	12.09	12.24	12.17
1959	5.90	6.48	5.90	5.92	12.42	12.47	12.65	12.80
1960	5.95	5.52	5.49	5.24	13.01	13.09	13.19	13.15
1961	5.17	5.68	5.84	6.16	13.09	13.04	12.95	12.85
1962	6.63	6.52	6.54	6.68	13.00	13.18	13.43	13.62
1963	6.61	6.82	6.86	6.96	13.61	13.50	13.40	13.31
1964	7.49	7.49	7.56	7.41	12.75	11.56	11.78	11.96
1965	8.18	8.19	8.22	8.34	12.57	12.66	12.13	12.07
1966	8.54	8.11	7.82	7.87	12.39	12.98	13.14	13.42
1967	7.65	7.49	7.41	7.35	13.35	13.16	13.43	13.59
1968	6.94	7.08	6.85	6.68	13.71	13.92	15.28	15.63
1969	6.65	6.37	6.16	5.59	16.37	16.46	15.77	15.74
1970	5.21	5.37	5.36	5.04	15.36	15.38	14.44	14.50
1971	5.57	5.55	5.64	5.71	13.71	13.79	13.80	13.97
1972	5.70	5.53	5.98	6.26	15.44	15.61	15.28	14.99
1973	6.22	5.78	5.64	5.69	14.58	14.50	14.68	14.82
1974	5.10	4.74	4.28	4.40	14.95	15.37	15.58	15.59
1975	4.79	5.20	5.50	5.49	15.61	11.85	14.54	14.68
1976	5.60	5.33	5.20	5.08	14.68	15.00	15.29	15.50
1977	5.00	5.65	6.10	5.94	15.68	15.75	15.55	15.70
1978	5.49	5.94	5.91	5.85	15.62	15.81	16.30	16.55
1979	5.57	5.26	4.98	5.10	16.43	16.65	16.98	17.04
1980	4.77	4.21	4.21	5.02	16.54	16.96	17.12	17.10
1981	5.01	5.21	5.88	5.51	17.38	17.66	17.77	17.37
1982	5.24	5.41	5.29	5.11	17.21	17.34	16.76	16.93
1983	5.49	5.67	5.92	6.27	16.45	16.60	15.59	15.52
1984	6.61	7.03	7.26	7.35	15.26	15.16	15.30	15.50
1985	7.08	7.12	6.95	6.68	16.57	14.74	15.85	15.79
1986	6.79	6.62	6.32	6.01	15.43	15.37	15.52	15.91
1987	6.15	6.18	6.45	6.50	15.43	17.26	16.17	16.32
1988	6.71	6.68	6.80	7.10	15.86	15.44	15.39	15.42
1989	6.86	6.71	6.61	6.26	16.06	16.35	16.37	16.40
1990	6.39	6.51	6.01	5.79	16.13	16.20	16.17	16.17
1991	6.02	5.90	5.83	5.62	15.70	15.74	15.74	15.85
1992	6.10	6.04	5.39	6.16	15.30	15.56	15.71	16.06
1993	5.91	6.09	5.86	6.32	15.44	15.89	16.19	16.40
1994	5.74	6.37	6.60	6.78	16.14	16.44	16.18	16.16
1995	6.45	6.39	6.68	6.60	16.35	16.73	16.67	16.82
1996	6.92	7.07	7.16	7.47	17.30	17.72	17.55	17.68
1997	7.55	7.57	7.85	7.78	18.08	18.17	18.35	18.49
1998	7.40	7.40	7.60	7.49	18.51	18.63	18.63	18.75
1999	7.43	7.30	7.16	7.25	18.64	18.76	18.93	19.06
2000	7.02	7.03	6.89	6.55	19.26	19.49	19.40	19.44
2001	6.68	6.65	6.19	6.64	19.74	19.96	16.89	18.75
2002	7.12	7.01	6.83	6.96	16.19	16.00	16.09	16.02
2003	7.02	7.25	7.59	7.61	15.56	15.46	14.02	14.82

provide an overview of the general strategy. A more detailed description of the calibration procedure can be found in Gomme and Rupert (2007).

Table 3: Parameter Values and Steady State

Parameter	Value	Variable	Value
β	0.9907	Hours	0.255
γ	1.0000	Consumption	0.448
ω	1.8643	Output	0.516
α	0.2830	Capital-output ratio	5.951
δ	0.0177	Investment-output ratio	0.131
τ_k	0.5437	Growth rate of output	0.42%
τ_ℓ	0.2263	Average return to capital	5.55%
ρ	0.97019		
σ_ε	0.00765		

In particular, capital's share of income, α, is set to match NIPA data. The parameters governing the stochastic technology shock, ρ and σ_ε, are estimated from regressions using U.S. Solow residuals. The coefficient of relative risk aversion, γ, is set to 1. The growth rate, g, is chosen so that the average growth rate of real per capita output matches that in the U.S. data. The depreciation rate, δ, is set based on BEA data on depreciation and capital stocks. The remaining parameters, ω and β, are chosen so that in steady state, hours of work, h, and the investment-output ratio, i/y, are equal to what is observed in the data. The benchmark parameter values of our model are in Table 3. The tax rates on capital income, τ_k, and on labor income, τ_ℓ, are averages over the years 1954:1–2003:4 and are based on calculations in Gomme and Rupert (2007). For completeness, data on τ_ℓ and τ_k are reported in Table 4.

The steady state of the model for the benchmark parameters are summarized in Table 3. The model is solved by applying a generalized Schur technique to a first-order log approximation of the decision rules around steady state; see Klein (2000).

4.2 Findings

The business cycle moments for the United States covering the period 1954:1–2003:4 are presented in Table 5. With the exception of the returns data, the underlying data has been detrended by tak-

Table 4: U.S. Tax Rates on Labor and Capital Income

	Tax Rate, τ_ℓ				Tax Rate, τ_k			
Year	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
1951	14.14	14.68	14.99	15.57	67.46	60.20	55.60	56.90
1952	15.96	16.10	16.00	16.02	58.86	58.83	58.34	57.72
1953	15.85	15.68	15.64	15.58	60.25	61.33	61.65	60.78
1954	15.00	14.92	14.91	14.85	58.97	58.55	58.35	57.20
1955	15.20	15.19	15.36	15.41	56.84	55.91	56.46	56.87
1956	15.79	15.83	15.87	15.88	59.22	59.87	58.40	59.37
1957	16.37	16.37	16.29	16.21	60.41	60.02	59.51	60.12
1958	16.02	15.86	15.97	15.86	59.46	59.51	59.55	59.07
1959	16.69	16.70	16.88	16.98	58.31	56.78	58.33	58.22
1960	17.82	17.87	17.96	17.91	59.52	60.73	60.76	61.95
1961	17.91	17.85	17.75	17.61	62.68	60.78	60.65	60.05
1962	18.08	18.22	18.46	18.61	56.82	57.33	57.84	57.21
1963	19.06	18.94	18.84	18.71	57.64	57.39	57.60	57.35
1964	18.09	16.91	17.07	17.21	55.11	55.09	55.18	55.62
1965	17.76	17.83	17.30	17.18	53.10	53.07	52.72	52.78
1966	18.66	19.20	19.43	19.65	52.20	53.37	53.98	53.28
1967	19.78	19.74	19.97	20.14	53.93	54.47	54.74	55.62
1968	20.36	20.54	21.80	22.09	58.79	58.07	58.98	59.80
1969	23.19	23.24	22.53	22.49	60.19	60.85	61.12	64.10
1970	22.15	22.16	21.23	21.25	64.95	63.89	64.08	65.63
1971	20.88	20.89	20.84	20.96	63.15	63.20	62.10	61.52
1972	22.93	23.03	22.65	22.19	62.04	62.78	60.27	59.49
1973	23.09	22.93	23.06	23.09	59.97	61.68	61.25	60.59
1974	23.70	24.14	24.30	24.23	62.63	64.56	67.84	64.55
1975	24.36	20.73	23.22	23.28	59.48	56.26	56.71	56.80
1976	23.65	23.91	24.14	24.28	57.55	59.09	59.37	59.73
1977	24.70	24.72	24.48	24.50	60.77	57.76	55.19	55.67
1978	24.89	24.97	25.38	25.56	56.81	55.53	53.77	54.14
1979	25.99	26.15	26.41	26.41	54.59	55.35	56.08	54.35
1980	26.12	26.48	26.61	26.50	56.81	57.17	57.48	53.07
1981	27.54	27.80	27.87	27.49	52.45	50.11	47.08	47.54
1982	27.65	27.74	27.17	27.26	47.53	46.81	47.12	47.47
1983	27.14	27.24	26.25	26.10	45.52	46.43	46.13	45.11
1984	26.25	26.07	26.16	26.31	45.16	43.46	41.60	41.41
1985	27.60	25.87	26.90	26.86	43.45	42.51	43.78	44.79
1986	26.74	26.69	26.80	27.15	45.03	46.10	47.62	50.18
1987	26.72	28.38	27.31	27.39	49.07	50.48	49.62	49.57
1988	27.42	26.99	26.92	26.97	47.49	47.95	47.67	46.90
1989	27.66	27.96	27.97	27.96	48.79	48.68	48.38	50.22
1990	27.83	27.81	27.82	27.82	49.02	48.95	51.75	52.31
1991	27.69	27.72	27.74	27.80	50.38	51.37	52.27	53.58
1992	27.37	27.57	27.68	27.92	51.62	52.43	54.59	51.53
1993	27.45	27.92	28.21	28.43	52.87	52.62	53.44	52.23
1994	28.29	28.59	28.34	28.29	54.69	52.39	51.63	51.03
1995	28.48	28.81	28.72	28.82	51.99	52.09	50.84	50.83
1996	29.21	29.54	29.36	29.44	49.68	49.31	48.84	47.63
1997	29.82	29.88	29.99	30.09	47.54	47.61	46.96	47.08
1998	30.07	30.13	30.09	30.15	48.62	48.52	47.82	47.73
1999	30.10	30.17	30.28	30.33	47.96	48.45	49.04	48.83
2000	30.50	30.65	30.55	30.58	50.27	49.79	49.76	50.53
2001	30.97	31.22	28.37	30.15	48.81	48.68	49.51	47.01
2002	27.92	27.73	27.81	27.74	44.36	45.17	46.20	46.08
2003	27.43	27.30	25.92	26.63	46.39	45.27	44.24	44.79

Table 5: Selected Business Cycle Moments

	Standard Deviation	Cross Correlation of Real Output With								
		x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}
U.S. Data, 1954:1–2003:4										
Output	1.74	0.11	0.35	0.61	0.84	1.00	0.84	0.61	0.35	0.11
Consumption	0.85	0.19	0.39	0.58	0.73	0.79	0.70	0.55	0.38	0.19
Investment	4.63	−0.20	−0.04	0.20	0.46	0.71	0.81	0.79	0.69	0.50
Hours	1.77	−0.12	0.09	0.34	0.61	0.83	0.89	0.80	0.64	0.43
Productivity	1.01	0.42	0.46	0.46	0.38	0.26	−0.11	−0.36	−0.51	−0.55
Capital	1.21	−0.43	−0.44	−0.40	−0.32	−0.17	0.01	0.19	0.35	0.46
After-tax returns										
Business Capital	13.99	0.20	0.17	0.10	−0.07	−0.19	−0.22	−0.20	−0.15	−0.07
S&P 500	360.51	0.29	0.35	0.39	0.41	0.40	0.28	0.15	0.00	−0.13
Benchmark Model										
Output	1.33	0.10	0.26	0.46	0.71	1.00	0.71	0.46	0.26	0.10
Consumption	0.70	0.02	0.18	0.40	0.66	0.98	0.75	0.54	0.36	0.21
Investment	5.77	0.16	0.31	0.50	0.72	0.98	0.65	0.38	0.17	0.00
Hours	0.48	0.18	0.33	0.51	0.73	0.98	0.64	0.36	0.14	−0.03
Productivity	0.86	0.05	0.21	0.42	0.68	0.99	0.73	0.51	0.32	0.17
Capital	0.44	−0.40	−0.30	−0.15	0.07	0.36	0.54	0.64	0.67	0.65
Return to capital	5.09	0.23	0.29	0.36	0.44	0.53	0.35	0.20	0.08	−0.02

Data sources: With the exception of hours, all variables have been converted from current dollars to real by deflating by the price deflator for consumer nondurables and services. All variables are expressed relative to the civilian noninstitutionalized population aged 16 and over. Output is measured by gross domestic product less gross housing product; consumption by personal consumption expenditures on nondurables and services less gross housing product; investment by private nonresidential fixed investment; hours by private nonfarm payroll hours; productivity is output divided by hours; and capital and the returns series are as described in the text.

ing the logarithm and applying the Hodrick-Prescott filter with a smoothing parameter of 1600. As shown in Figure 1, the returns to the S&P 500 are occasionally negative and so the usual business cycle detrending procedure cannot be applied. Instead, returns are expressed as a percentage deviation from their sample averages, a procedure that is in the same spirit as the Hodrick-Prescott filter.

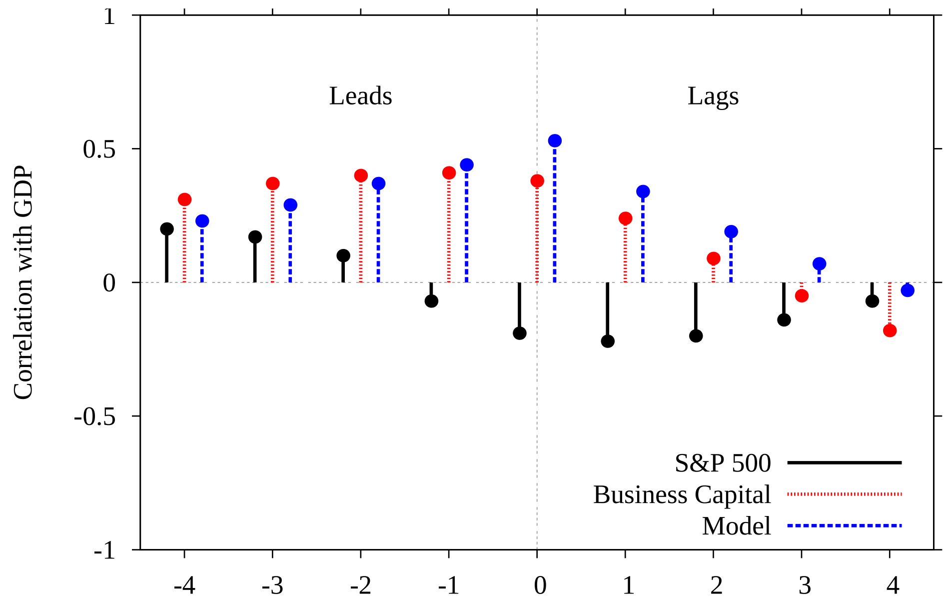
On the real side, the benchmark economy shares many of the successes (and failures) of other RBC models. Models calibrated to the observed Solow residual process typically underpredict the volatility of output; so does our model. In the data, consumption varies less than output while investment varies more; our model delivers this ranking, but underpredicts the volatility of consumption while exaggerating that of investment.

Next, consider the returns data. Recall that in the model, the net after-tax return on capital is given by the after-tax marginal product of capital less the depreciation rate. The model does reasonably well in terms of the average return to capital, predicting a value of 5.55% compared to 6.29% in the data. Keep in mind that the model is not calibrated to the average rate of return.

In the U.S. economy, the return to capital is 8 times more volatile than output and is procyclical. S&P 500 returns are far more volatile – 207 times that of output. These returns are *countercyclical*. To the extent that stock market returns reflect the marginal product of capital, it is odd that its return is countercyclical, albeit weakly (these business cycle facts are not very sensitive to whether the returns are measured after-tax or pre-tax). The model's prediction for the volatility of the return to capital is summarized in Table 5. The rate of return in the benchmark model is about 35% as volatile as in the data. The model predicts that this return is 3.8 times more volatile than output and is strongly procyclical. In the data, the return to capital is 8 times as volatile as output, so the model captures roughly 50% of the relative volatility in the return to capital. If the target was to match the volatility of S&P 500 returns, the model does quite poorly, capturing less than 2% of this relative variability.

The lead-lag pattern of return to capital is quite different from that of the S&P 500 return. While both returns lead the cycle (one quarter for the return to capital; four for S&P 500 returns), the cross-correlation of output with the return to capital is quantitatively different from that with S&P 500 return. As illustrated in Figure 2, the model captures the lead-lag pattern in the return to capital remarkably well. Again, if the target was S&P 500 returns, the model does not perform well.

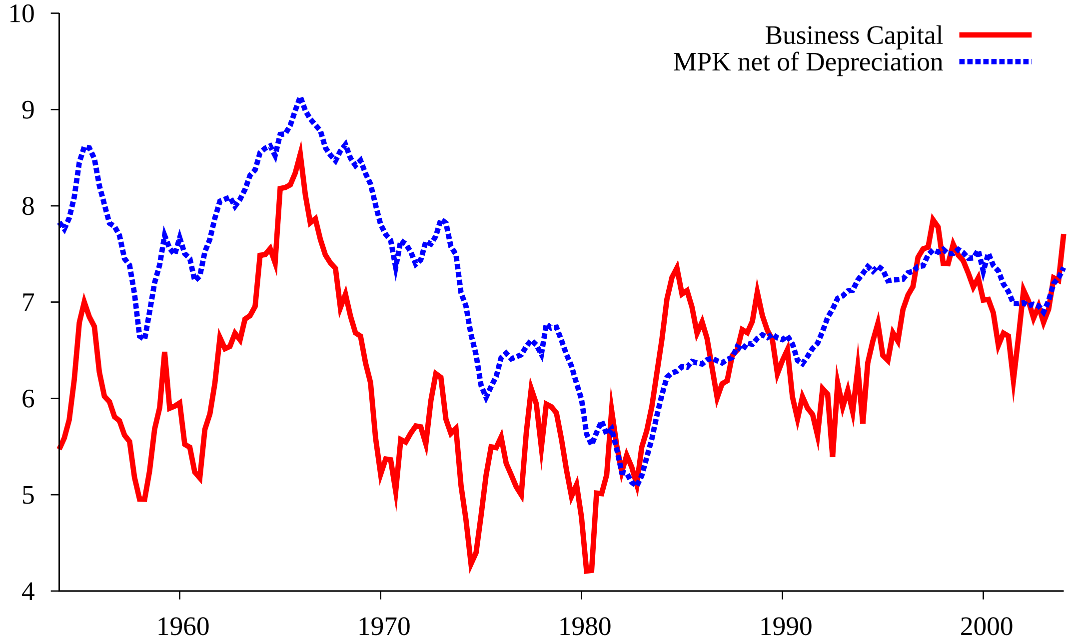
Figure 2: Lead-lag Pattern



4.3 Alternative Models and Parameterizations

A natural question at this stage is whether models in the RBC class could ever deliver the volatility in the rate of return to capital just by successfully delivering the aggregate quantities. One approach to answer this question is to examine the model's after-tax return to capital, $(1-\tau_k)\left[\alpha\left({}^{y_t}/_{k_t}\right)-\delta\right]$, using the time series data on output and capital stock ; i.e., hold fixed τ_k, α and δ as in the model and compute the model's after-tax marginal product of capital. Figure 3 illustrates this time series along with the after-tax rate of return to capital. The volatility of the after-tax marginal product of capital is only 12% less than the volatility of our measure of the rate of return to capital. A model that replicates the time series properties of output and capital stock could potentially generate sufficient volatility in the after-tax marginal product of capital to account for the volatility in the rate of return to capital.

Figure 3: Return to Capital and Marginal Product of Capital



In this subsection, we consider three variants on the benchmark model. The common theme is to explore the model's implications for the volatility of the return to capital. As motivation for these experiments, consider the intertemporal equation governing the accumulation of capital,

$$1 = E_t \left\{ \left(\beta \frac{U_{c,t+1}}{U_{c,t}} \right) \left[1 + (1 - \tau_k) \left(\alpha \left(\frac{y_{t+1}}{k_{t+1}} \right) - \delta \right) \right] \right\}. \tag{5}$$

The first term on the right-hand side is often referred to as the stochastic discount factor or the intertemporal marginal rate of substitution for consumption. The second term is the after-tax gross return to capital. Table 6 summarizes the results for the U.S. data, the benchmark model, and the three variants considered in this subsection. The calibration procedure implies that the average rate of return across model variants are identical.

The first model variant increases the coefficient of relative risk aversion, γ, from 1 to 5. This change has two important implications. First, utility is no longer additively separable between consumption and leisure which implies that the intertemporal marginal rate of substitution now depends not only on consumption but also leisure (hours of work). Second, the representative household will have a stronger utility-smoothing motive as γ increases.[2] Increasing risk aversion

[2]To the extent that introducing habit persistence has effects similar to increasing risk aversion, this experiment is

Table 6: Alternative Models and Parameterizations

	U.S.		Benchmark		Risk Aversion: $\gamma = 5$		Indivisible Labor		Home Production	
	SD	Corr.	SD	Corr.	SD	Corr.	SD	Corr.	SD	Corr.
Output	1.74	1.00	1.33	1.00	1.20	1.00	1.55	1.00	2.02	1.00
Consumption	0.85	0.79	0.70	0.98	0.78	1.00	0.78	0.98	0.91	0.99
Investment	4.63	0.71	5.77	0.98	4.19	0.99	7.09	0.98	61.85	0.29
Hours	1.77	0.83	0.48	0.98	0.32	1.00	0.80	0.98	1.13	0.99
Productivity	1.01	0.26	0.86	0.99	0.88	1.00	0.78	0.98	0.90	0.99
Capital	1.21	−0.17	0.44	0.36	0.33	0.26	0.53	0.36	0.98	0.95
Return to Capital	13.99	−0.19	5.09	0.53	7.76	0.36	5.81	0.54	5.01	0.54

raises the volatility of the return to capital both in absolute terms, and relative to the volatility of output. The model now captures 80% of the relative volatility in the return to capital; the benchmark model around 50%. For the most part, this improvement does not come at the cost of substantially worsening the model's predictions for the real side of the economy.

The second model variant considers Hansen (1985)–Rogerson (1988) indivisible labor. This variant operates more on the return to capital term in Eq. (5). In particular, Hansen showed that indivisible labor could substantially increase the volatility of hours worked. If the variability of capital is not much affected by the introduction of indivisible labor, then we might expect to see more volatility in the marginal product of capital, and so the return to capital; to see this, rewrite Eq. (5) as

$$1 = E_t \left\{ \left(\beta \frac{U_{c,t+1}}{U_{c,t}} \right) \left[1 + (1 - \tau_k) \left(z_{t+1} \alpha \left(\frac{g^{t+1} h_{t+1}}{k_{t+1}} \right)^{1-\alpha} - \delta \right) \right] \right\}. \tag{6}$$

Relative to the benchmark model, introducing indivisible labor increases the volatility of macroaggregates – just as in Hansen. While the variability of the return to capital increases – from 5.09 to 5.81 – its volatility relative to output is essentially unchanged.

The final variant introduces home production; see Benhabib, Rogerson, and Wright (1991) and Greenwood and Hercowitz (1991). Home production is likely to operate primarily through the intertemporal marginal rate of substitution with general equilibrium effects on the marginal product of capital. Allowing agents another margin along which they can smooth utility – namely through

suggestive of the likely effects of introducing habit.

home production – may make them more tolerant of fluctuations in market consumption, the object that appears in Eq. (5). Details of this model are left to the Appendix which also briefly discusses calibration of the home production model. In Table 6, *market* variables are reported for the home production model. The volatility of (market) investment is much higher than that observed in the data. Papers that have successfully addressed the investment volatility issue include Greenwood and Hercowitz (1991), Greenwood, Rogerson, and Wright (1995) and Gomme, Kydland, and Rupert (2001). Most pertinent to the focus of this paper, the home production model implies *lower* volatility (particularly relative to that of output) for the return to capital.

Table 6 provides some insight into factors that are important for accounting for the volatility of the return to capital. Increasing the volatility of output and/or capital increases the variability of the return to capital as seen by comparing the benchmark and indivisible labor models. However, increasing the volatility of these macroaggregates is not sufficient; the home production model has much higher output and market capital stock variability, yet the volatility of of the return to capital is lower than in the benchmark model. In the case of home production, the model also generates a very strong positive correlation between output and market capital, a factor that works against generating high volatility in the return to capital. By way of contrast, the data exhibits a small negative correlation between output and capital. To drive this point home, consider the high risk aversion model. In this case, the volatilities of output and capital are lower than in the benchmark model (factors that would tend to reduce the variability of the return to capital), but the correlation between output and capital is also lower (which tends to raise the volatility of the return to capital); the net result is higher variability in the return to capital.

4.4 A More Traditional Calibration

One of the main points of Gomme and Rupert (2007) is that home production is important for measurement even if the model does not include home production. This approach stands in contrast to much of real business cycle theory that defines economic activity more broadly – at least at the measurement and calibration phase. This subsection investigates the implications of a more traditional calibration strategy that takes a broader view of economic activity. Specifically, we explore the implications of the oft cited Cooley and Prescott (1995) calibration strategy; the interested reader is directed to their paper for more details.

The Cooley and Prescott (1995) calibration proceeds as follows. Given a steady state investment-output ratio of 0.076, an annual capital-output ratio of 3.32, and real growth of 1.56%, the law of motion for capital implies an annual depreciation rate of 6.04% (1.477% quarterly).[3] Cooley and Prescott set the capital share parameter, α, to 0.40 on the basis that since they have defined capital quite broadly, its share of income will correspondingly be higher. They set the risk aversion parameter, γ, to one implying logarithmic utility. Their target for the average fraction of time spent working is 0.31. This target, along with the steady state capital-output ratio, pins down the discount factor, β, and the utility parameter on leisure, ω; for a quarterly frequency, these values are $\beta = 0.9887$ and $\omega = 1.775$. The technology shock process is $\rho = 0.95$ and $\sigma = 0.007$ – fairly close to the values estimated by Gomme and Rupert (2007).

Next, the data used to compare the model differs from that used in the rest of this paper. In particular, housing product and income flows are *not* netted out of any of the series; see the notes to Table 8. The return to (all) capital is measured as

$$\tilde{R}_{AT} = \frac{\tilde{Y}_{AT}}{\textsc{Inventories} + \textsc{Private Fixed Assets}}$$

where PRIVATE FIXED ASSETS is the sum of private nonresidential structures, the stock of private equipment & software, and private residential structures. Notice that government fixed assets as well as consumer durables are omitted from "all capital" since the NIPA do not provide any estimates of the income flows to these assets. After-tax income of all capital is

$$\begin{aligned}\tilde{Y}_{AT} = {} & \textsc{Net Operating Surplus} - (1-\alpha)\textsc{Proprietor's Income} \\ & - \tau_h(\textsc{Net Interest} + \alpha\textsc{Proprietor's Income} + \textsc{Rental Income}) \\ & - \textsc{Taxes on Corporate Income} - \textsc{Business Property Taxes} \\ & - \textsc{Household Property Taxes} - \textsc{State and Local Other Taxes}.\end{aligned}$$

Table 7 summarizes the average rates of return to our measure of business capital as well as

[3] Cooley and Prescott (1995) include population growth in their model; we do not, which implies a larger value for the depreciation rate.

Table 7: Rates of Return for Different Measures of Capital, 1954:1–2003:4

	Pre-tax	After-tax	Implied τ_k
Business Capital	10.62%	6.29%	41.2%
All Capital	7.75	5.03	35.12
Housing Capital	4.24	3.54	18.

Figure 4: After-tax Returns on Capital

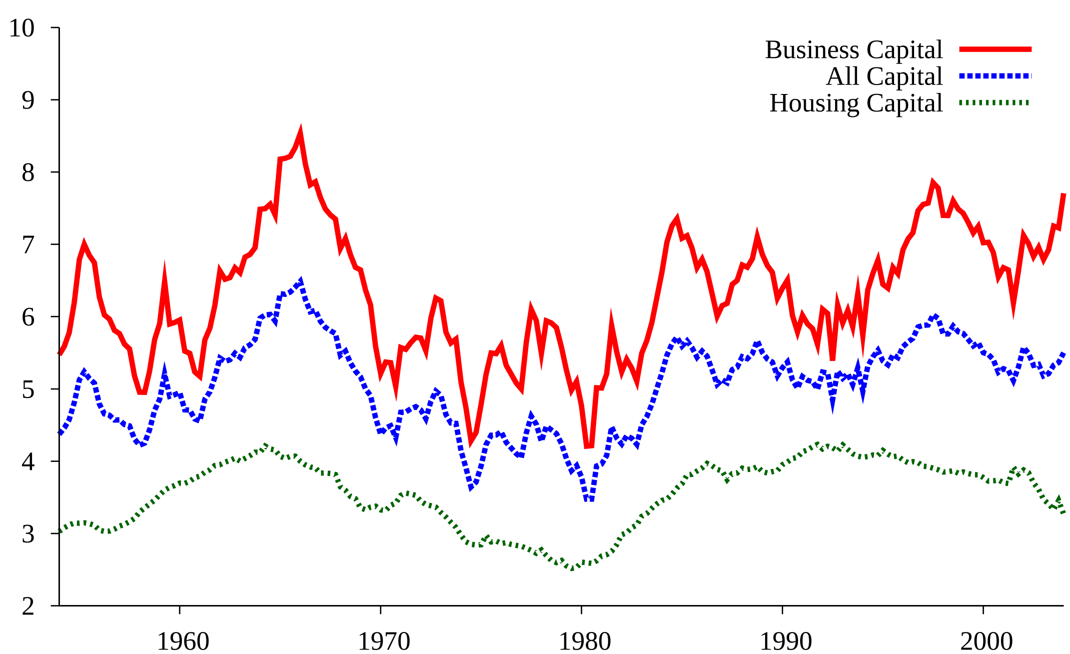


Table 8: Alternative Calibration Strategy

	Standard Deviation	Cross Correlation of Real Output With								
		x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}
U.S. Data, 1954:1–2003:4										
Output	1.66	0.11	0.36	0.62	0.85	1.00	0.85	0.62	0.36	0.11
Consumption	0.79	0.22	0.43	0.63	0.78	0.83	0.74	0.58	0.40	0.19
Investment	4.66	0.19	0.38	0.61	0.81	0.92	0.83	0.64	0.39	0.13
Hours	1.78	−0.15	0.07	0.34	0.61	0.83	0.88	0.79	0.63	0.41
Productivity	1.00	0.44	0.46	0.43	0.32	0.18	−0.17	−0.40	−0.53	−0.55
Capital	1.17	−0.42	−0.32	−0.18	−0.02	0.16	0.34	0.49	0.60	0.66
After-tax Return										
All capital	12.80	0.26	0.32	0.36	0.38	0.36	0.24	0.11	−0.01	−0.13
Model										
Output	1.38	0.09	0.25	0.46	0.70	1.00	0.70	0.46	0.25	0.09
Consumption	0.44	−0.09	0.08	0.30	0.58	0.93	0.76	0.60	0.45	0.32
Investment	4.44	0.15	0.30	0.49	0.72	0.99	0.67	0.40	0.19	0.02
Hours	0.68	0.17	0.32	0.51	0.73	0.99	0.65	0.37	0.16	−0.01
Productivity	0.72	0.02	0.19	0.40	0.67	0.99	0.74	0.52	0.34	0.19
Capital	0.29	−0.43	−0.33	−0.19	0.03	0.31	0.50	0.61	0.65	0.64
Return to Capital	6.03	0.22	0.30	0.39	0.48	0.59	0.42	0.27	0.15	0.05

Data sources: With the exception of hours, all variables have been converted from current dollars to real by deflating by the price deflator for consumer nondurables and services. All variables are expressed relative to the civilian noninstitutionalized population aged 16 and over. Output is measured by gross domestic product; consumption by personal consumption expenditures on nondurables and services; investment by private fixed investment plus purchases of consumer durables; hours by private nonfarm payroll hours; productivity is output divided by hours; capital by the sum of private fixed assets, the stock of consumer durables, and the stock of inventories (with conversions to quarterly as in section 3.1); and the returns series are as described in the text.

all capital and the implied tax rate on capital income. Figure 4 displays the after-tax returns on business capital, all capital, and housing capital. The return to all capital is a weighted average of the returns to business and housing capital where the weights are given by the relative sizes of the capital stocks. We should note that the "all capital" rate of return calculations embody capital stocks with very different rates of returns. In particular, the pre-tax return to business capital is almost 2.5 times the return to housing capital. (The return to housing capital can be obtained by subtracting business capital income from all capital income, then dividing by the stock of residential structures.) The after-tax returns, however, differ by a factor of only 1.8. In general, these rates of return and the implied capital income tax rate are related by

$$R^{\text{after-tax}} = (1 - \tau_k) R^{\text{pre-tax}}. \tag{7}$$

As shown in Table 7, the implied tax rate on housing capital is roughly 40% of that associated with business capital. Thus, aggregating these capital stocks into "all capital" may be problematic.

Business cycle moments for both the U.S. economy (new measurement) and the model (Cooley and Prescott calibration) are summarized in Table 8. Apart from the rate of return on capital, the U.S. business cycle properties are quite similar to those reported in Table 5. The percentage standard deviation of the return to all capital is roughly 90% that of business capital. The smaller variability of the return to all capital can be largely attributed to the fact that the return to housing capital is considerably smoother than that earned on business capital. The model's prediction for the variability of the return to capital is higher than that of the benchmark model (6.03% versus 5.09%), and the model can account for a higher fraction – around 50% – of the volatility in the return to (all) capital. In terms of volatility relative to output variability, the model accounts for roughly 55% of the variability in the return to all capital.

4.5 Stochastic Taxes

The "fit" between the return to capital and the after-tax marginal product of capital is rather poor in Figure 3. In the data, capital's share of income, depreciation and tax rates all vary, and allowing all of these parameters to vary markedly improves the visual fit of the marginal product of capital.

Figure 5: Return to Capital and Marginal Product of Capital

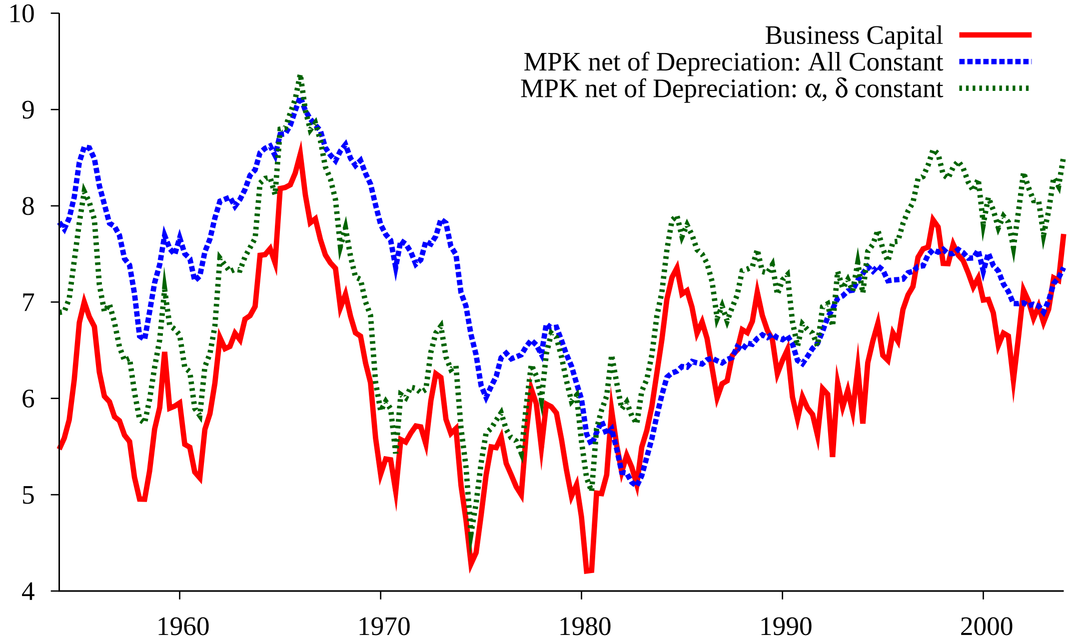


Table 9: SUR Estimation Results

x_t	x_{t-1}	x_{t-2}	time	constant	SD(ε)
$\ln z_t$	0.9781957		0.0000294	0.0545838	0.0076589
	(0.0112304)		(0.0000251)	(0.0257348)	
$\tau_{\ell t}$	0.6738393	0.3096548	0.00000202	0.0043901	0.0057163
	(0.0659515)	(0.0691569)	(0.000025)	(0.0041647)	
τ_{kt}	0.94086		−0.0000463	0.0370209	0.0135034
	(0.0219333)		(0.0000229)	(0.0140522)	

Most of the action, though, is due to changes in tax rates over time. Figure 5 illustrates the time series for $(1-\tau_{kt})\left[\alpha\left(y_t/k_t\right)-\delta\right]$ where α and δ are fixed but the output, capital and tax rate are allowed to vary as in the data.

This subsection investigates how stochastic taxes contribute to the model's prediction for the volatility in the return to capital. A preliminary step is to estimate the joint process of the technology shock and the tax rates on labor and capital income. The tax rates are as reported in Table 4. Estimation results are summarized in Table 9 over the sample period, 1954:1–2003:4. A time trend is included in each regression equation to absorb any secular trends in the variables. The Solow residual and tax rate on capital income only require one lag; the tax rate on labor income requires two. All three shock processes exhibit considerable persistence. The correlation matrix of the residuals (ordered as ε_{zt}, $\varepsilon_{\ell t}$ and ε_{kt}) is :

$$\begin{bmatrix} 1.0000 & & \\ -0.0888 & 1.0000 & \\ -0.3257 & 0.2241 & 1.0000 \end{bmatrix} \tag{8}$$

The stochastic processes are given by the autocorrelation coefficients and standard deviations of the innovations (see Table 9) and the correlations of the innovations (see Eq. (8)).

As in Mendoza et al. (1994), we are measuring tax payments instead of statutory tax rates. First, our measurement accounts for subsidies and tax shelters. Second, even if the statutory tax rates change only at an annual frequency, stabilization policies over the cycle have the unintended effect of changing the effective tax rates at a much higher frequency. Third, as noted in Table 9, the autocorrelation in the capital income tax rate is high indicating that there is not much variation in the rate from one quarter to the next.

The upper panel of Table 10 repeats the U.S. observations, and the lower panel presents the results when taxes are stochastic and preferences are logarithmic. Here, variation in tax rates makes the model's predictions for macroaggregates more volatile – particularly investment and the return to capital. Volatility in the return to capital is now just over 78% of that seen in the data (compared with about 35% for the benchmark model); relative to output, the model now accounts for just about 85% of the volatility in the return to capital (compared with nearly 50% for the benchmark model).

4.6 Investment Specific Technological Change and Capital Gains

In this section we extend the model to allow for capital gains coming through changes in the price of capital. In particular, the model is closely related to that of Greenwood et al. (1997) which incorporates capital-embodied technological change.

The household's problem changes only slightly compared to the baseline model. The main difference between this problem and the baseline is that the state of investment-specific technol-

Table 10: Results for Stochastic Taxes

	Standard Deviation	Cross Correlation of Real Output With								
		x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}
U.S. Data, 1954:1–2003:4										
Output	1.74	0.11	0.35	0.61	0.84	1.00	0.84	0.61	0.35	0.11
Consumption	0.85	0.19	0.39	0.58	0.73	0.79	0.70	0.55	0.38	0.19
Investment	4.63	−0.20	−0.04	0.20	0.46	0.71	0.81	0.79	0.69	0.50
Hours	1.77	−0.13	0.09	0.34	0.61	0.83	0.89	0.80	0.64	0.43
Productivity	1.01	0.42	0.46	0.46	0.38	0.26	−0.11	−0.36	−0.51	−0.56
Capital	1.21	−0.43	−0.44	−0.41	−0.32	−0.18	0.01	0.19	0.35	0.46
After-tax returns										
Business capital	13.99	0.31	0.37	0.40	0.41	0.38	0.24	0.09	−0.05	−0.18
Stochastic Tax Model										
Output	1.61	0.10	0.25	0.45	0.66	1.00	0.66	0.45	0.25	0.10
Consumption	0.80	−0.04	0.09	0.28	0.49	0.80	0.65	0.51	0.38	0.26
Investment	9.14	0.16	0.29	0.46	0.63	0.92	0.53	0.32	0.12	−0.02
Hours	1.18	0.14	0.26	0.41	0.54	0.84	0.46	0.28	0.10	−0.02
Productivity	0.88	−0.01	0.11	0.27	0.48	0.69	0.59	0.43	0.31	0.20
Capital	0.65	−0.34	−0.25	−0.10	0.10	0.38	0.53	0.61	0.63	0.61
Return to Capital	10.97	0.21	0.28	0.35	0.42	0.53	0.34	0.21	0.09	−0.00

ogy changes over time. Let q_t represent the state of investment-specific technological change; its inverse, $1/q_t$, is the price of investment goods (relative to consumption goods). Note that in the baseline structure the price of investment goods relative to consumption is fixed and equal to one. Therefore the law of motion for capital and the budget equation become:

$$k_{t+1} = (1-\delta)k_t + q_t i_t \tag{9}$$

$$c_t + i_t = (1-\tau_\ell)w_t h_t + (1-\tau_k)r_t k_t + \tau_k \delta \frac{k_t}{q_t} + T_t \tag{10}$$

The second term from the end in Eq. (10) includes the price of investment goods which is necessary for the balanced growth transformations. A possible justification is that it reflects the original cost of capital goods.

The return to capital with investment specific technological change is given by:

$$R_t^{IS} = (1-\tau_k)(\alpha z_t k_t^{\alpha-1}\left(g^t h_t\right)^{1-\alpha})q_t + (\tau_k\delta + 1 - \delta)\frac{q_t}{q_{t+1}} \tag{11}$$

Details of solving for the model's balanced growth path are omitted here, but can be found in Gomme and Rupert (2007).

Data

It is necessary to specify the data corresponding to the price of capital, $1/q$, in the model. We determine $1/q$ by dividing nominal private non-residential fixed capital by real private non-residential fixed capital obtained from the National Income and Product Accounts (NIPA).

Since the NIPA report data at an annual rate, we need to transform the annual numbers to quarterly. Since the BEA simply multiplies quarterly figures by 4 to make them annual, we divide by 4 to "undo" what the BEA has done, giving

$$\hat{R}_t^K = \left[1 + \frac{Y_t^K/4}{K_t} + \left(\frac{q_t}{q_{t-1}} - 1\right)\right]^4 - 1 \tag{12}$$

In Figure 6, we plot our earlier measure of the after-tax return to business capital and our current measure that includes fluctuations in the relative price of capital. The return including capital gains is roughly four times as volatile as the return without capital gains. (Recall from Figure 1 that the S&P 500 return is about 25 times as volatile as our return without capital gains.)

Figure 6: Return to Capital

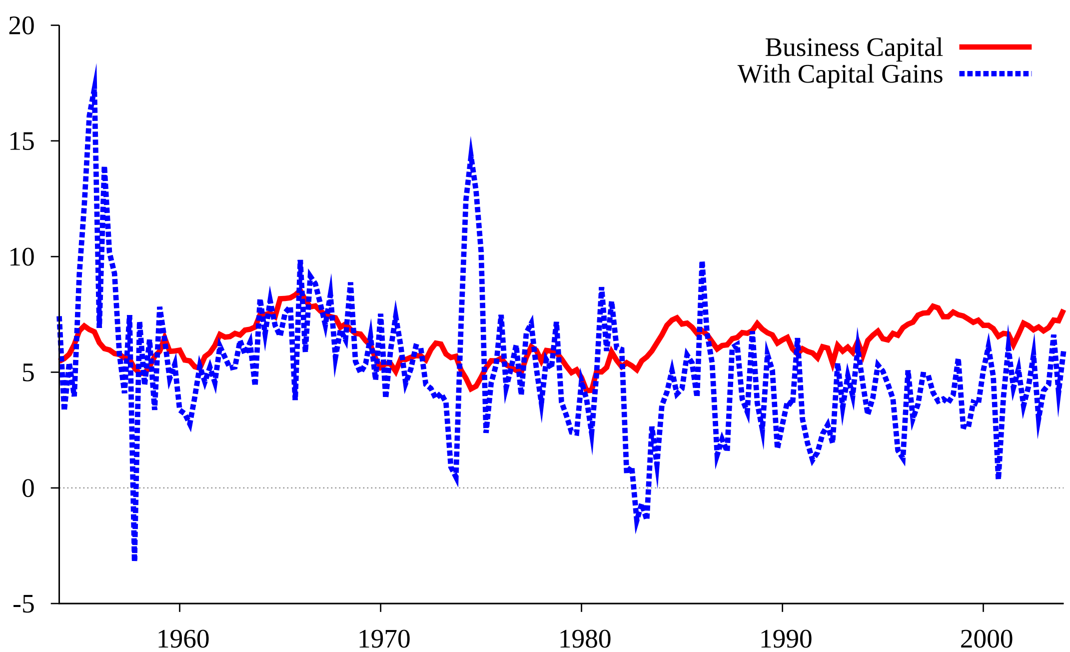


Table 11: SUR Estimation Results

x_t	$\ln z_t$	$\ln q_t$
x_{t-1}	0.9699564	1.396578
	(0.0118151)	(0.0692421)
x_{t-2}		−0.2413925
		(0.1185867)
x_{t-3}		−0.1606142
		(0.0693956)
time	0.0000465	−0.0000407
	(0.0000262)	(0.0000149)
constant	0.0734401	0.0060821
	(0.0270714)	(0.0035141)
SD(ε)	0.0076501	0.0052525

Table 12: Results for Investment Specific Technological Change

	Standard Deviation	Cross Correlation of Real Output With								
		x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}
U.S. Data, 1954:1–2003:4										
After-tax return, business capital	54.55	0.09	0.10	0.15	0.18	0.18	0.15	0.12	0.06	0.03
Investment Specific Technological Change Model										
Output	1.39	0.10	0.26	0.46	0.71	1.00	0.71	0.46	0.26	0.10
Consumption	0.86	−0.02	0.10	0.25	0.43	0.65	0.55	0.45	0.35	0.25
Investment	8.45	0.14	0.27	0.42	0.61	0.84	0.53	0.27	0.09	−0.05
Hours	0.78	0.15	0.26	0.40	0.58	0.78	0.48	0.23	0.05	−0.07
Productivity	0.91	0.02	0.17	0.35	0.58	0.85	0.66	0.50	0.35	0.21
Capital	0.80	−0.27	−0.17	−0.03	0.15	0.39	0.53	0.59	0.60	0.56
Return to Capital	44.57	−0.00	0.05	0.12	0.23	0.38	0.22	0.15	0.09	0.05

Simulation Results

To generate model output, it is necessary to obtain the joint shock process for the Solow residuals and price of capital. Our estimates suggest that the relative price of investment series needs three lags; the final results are summarized in Table 11. The correlation matrix of the residuals (ordered as ε_{zt} and ε_{qt}) is :

$$\begin{bmatrix} 1.0000 & \\ 0.0221 & 1.0000 \end{bmatrix} \tag{13}$$

The business cycle moments are given in the bottom panel of Table 12.

The results show that the percentage standard deviation of the return to business capital is much higher for the investment specific technological change model. The percentage standard deviation of the return to capital in the standard model is 5.09; while for the model with investment specific shocks it is 44.57. To be consistent with the theory, the corresponding target for the standard model was 13.99, whereas the target now is 54.55. As is evident, the model with investment specific technological change accounts for more than 80% of the observed volatility in the return to capital, and essentially all of the volatility relative to output.

5 Conclusions

We constructed a time series for the after-tax return to capital and showed that its behavior is substantially different from the S&P 500 returns. Our measure of the return to capital is considerably smoother (by a factor of 25) and has a higher mean. The standard real business cycle model accounts for close to 50% of the volatility in the return to capital relative to that of output. We considered three variants of the standard model – high risk aversion, indivisible labor and home production. The high risk aversion model delivers 80% of the relative volatility in the return to capital, the indivisible labor model delivers over 45%, while the home production model just over 30%.

Allowing taxes to vary over time also produces very promising results. The joint stochastic process for total factor productivity, the capital income tax, and the labor income tax were estimated from the data. The addition of stochastic taxes allows the model to capture almost 85% of the volatility of the return to capital (relative to the variability of output), up from the roughly 50% for the baseline model.

Allowing for fluctuations in the relative price of capital implies alterations in both theory and measurement. When the theory is extended to include investment specific technological change and the measurement is extended to include the capital gains documented in NIPA, the model accounts for more than 80% of the volatility in the return to capital, and all of the volatility relative to output.

As noted in the Introduction, the return to capital and the return to equity are the same in the basic RBC model. The high risk aversion variant of the RBC model delivers 80% of the relative volatility in the return to capital, but delivers just 3% of the relative volatility in the return to the S&P 500.[4] To date, work in the literature has focused on "fixing" the standard RBC model so that it can account for the business cycle properties of the S&P 500 returns. From a theoretical point of view, there is no reason to prefer S&P 500 returns over the return to capital as computed in this paper. Our return to capital series presents a serious problem for papers in the literature since success in mimicing S&P 500 returns has come at the cost of failing to account for returns computed from NIPA and capital stock data. As is evident from Figures 1 and 2, in order to simultaneously account for the volatilities and the lead-lag patterns of the return to capital and the S&P 500 return, we have to break the equivalence in the model between the return to capital and the return to equity. To the extent that the S&P 500 return does not reflect the return on a representative unit of capital in aggregate models (see Mulligan (2002), for instance), it might be useful to construct a general equilibrium theory of publicly traded firms and of "non-traded" firms.[5] Another approach to break the equivalence might be to introduce limited participation in equity markets. Such theories might deliver a smooth return to capital and a volatile return to equity via volatile stochastic discount factors that price equity. Whether such theories deliver the observed lead-lag patterns in the return to capital and return to equity is a question for future research.

[4] Jermann (1998) examines an RBC model with habit persistence and adjustment costs to capital while Boldrin, Christiano, and Fisher (2001) examine a two-sector growth model with habit persistence and restrictions on factor mobility across sectors. Both papers account for almost the entire observed volatility in S&P 500 returns.

[5] The S&P 500 capital is less than 40% of the total private fixed capital in the U.S. economy.

Appendix: Home Production

The market sector, denoted by the subscript M, produces output according to the technology

$$y_{Mt} = z_{Mt} k_{Mt}^{\alpha} \left(g^t h_{Mt}\right)^{1-\alpha}, \tag{A.1}$$

where y_M is the amount of output, k_M denotes the beginning of period capital stock, h_M denotes hours worked, g is the growth rate of labor-augmenting technical change, and z_M denotes the state of disembodied technical progress. Output in the market sector can be allocated between consumption goods and investment goods such that

$$c_{Mt} + i_{Mt} + i_{Ht} = y_{Mt},$$

where c_M denotes market consumption, i_M market investment, and i_H home investment.

The representative firm's problem is to choose k_{Mt} and h_{Mt} in order to

$$\max z_{Mt} k_{Mt}^{\alpha} \left(g^t h_{Mt}\right)^{1-\alpha} - w_t h_{Mt} - r_t k_{Mt}$$

where w_t is the real wage rate, and r_t is the real rental rate on market capital.

Consumption goods in the home sector (denoted by H subscripts) use labor and home capital according to the technology

$$c_{Ht} = k_{Ht}^{\theta} \left(g^t h_{Ht}\right)^{1-\theta}. \tag{A.2}$$

Market and home capital evolve according to

$$k_{Mt+1} = (1-\delta_M) k_{Mt} + i_{Mt} \tag{A.3}$$

$$k_{Ht+1} = (1-\delta_H) k_{Ht} + i_{Ht}. \tag{A.4}$$

The representative household has preferences over market consumption, c_{Mt}, home consumption, c_{Ht}, market hours, h_{Mt}, and home hours, h_{Ht}, summarized by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_{Mt}, c_{Ht}, h_{Mt}, h_{Ht}), \quad 0 < \beta < 1, \tag{A.5}$$

where

$$U(c_M, c_H, h_M, h_H) = \begin{cases} \frac{[C(c_M, c_H)(1-h_M-h_H)^{\omega}]^{1-\gamma}}{1-\gamma} & \text{if } 0 < \gamma < 1 \text{ or } \gamma > 1, \\ \ln C(c_M, c_H) + \omega \ln(1-h_M-h_H) & \text{if } \gamma = 1, \end{cases} \tag{A.6}$$

Table 13: Long Run Averages for the Home Production Model

Observation	Value
Capital's share of market income	0.283
Depreciation of market capital (annual)	0.069113
Depreciation of home capital (annual)	0.059981
Market investment as a share of market output	0.1306
Home investment as a share of market output	0.1571
Market hours	0.255
Home hours	0.24

where C is the "aggregate" of market and home consumption, described by:

$$C(c_m, c_h) = \begin{cases} \left[\psi c_m^{\xi} + (1-\psi) c_h^{\xi}\right]^{1/\xi} & \text{if } \xi \in (-\infty, 0) \cup (0, 1) \\ c_m^{\psi} c_h^{1-\psi} & \text{if } \xi = 0. \end{cases} \tag{A.7}$$

Implicit in Eq. (A.6) is an assumption that the individual's time endowment is equal to one.

Given the initial conditions k_{M0} and k_{H0}, the representative agent's problem is to choose $\{c_{Mt}$, c_{Ht}, h_{Mt}, h_{Ht}, k_{Mt+1}, $k_{Ht+1}\}_{t=0}^{\infty}$ in order to maximize Eq. (A.5) subject to Eqs. (A.2)–(A.4), and

$$c_{Mt} + i_{Mt} + i_{Ht} = (1-\tau_\ell) w_t h_{Mt} + (1-\tau_k) r_t k_{Mt} + \tau_k \delta_M k_{Mt} + T_t.$$

where T_t is the transfer from the government in period t.

The government satisfies its budget constraint,

$$\tau_\ell w_t h_{Mt} + \tau_k r_t k_{Mt} - \tau_k \delta_M k_{Mt} = T_t$$

In steady state, the model must be consistent with long run averages observed in the U.S. data; for data details, see Gomme and Rupert (2007). These long run averages are summarized in Table 13. The first three of these long run averages directly determine the parameters α, δ_M and δ_H. The coefficient of relative risk aversion, γ, is set to one which implies logarithmic utility. The curvature parameter in the consumption aggregator, ξ, is set to 0.4 based on estimates by McGrattan, Rogerson, and Wright (1997) and Rupert, Rogerson, and Wright (1995). The remaining parameters, β, ω, ψ and θ, are set to match the remaining four long run averages in Table 13. The parameter values are summarized in Table 14. Finally, the properties of the stochastic technology process are as for the benchmark model.

Table 14: Home Production Model Parameter Values

Parameter	Description	Value
α	Capital's share of market income	0.283
δ_M	Depreciation of market capital (quarterly)	0.0177
δ_H	Depreciation of home capital (quarterly)	0.0153
γ	Coefficient of relative risk aversion	1
ξ	Curvature parameter in consumption aggregator	0.4
τ_k	Capital income tax rate	0.3014
τ_ℓ	Labor income tax rate	0.2263
β	Discount factor	0.9907
ω	Utility weight on leisure	0.7489
ψ	Parameter on market consumption in consumption aggregator	0.5307
θ	Capital's share in home production	0.3083
ρ	Autoregressive parameter of technology shock	0.96405
σ	Standard deviation of innovation to technology shock	0.00818

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